How to Average Celestial Sights for Optimum Accuracy

The beauty of celestial navigation is its transparency, meaning when we get a position fix using standard good procedures we can be completely confident that it is right, and if we make an error on the way to this fix, it is usually obvious even before we get the fix completed. With celestial navigation, we not only find our position independent of the external world, we have a way to evaluate its accuracy. This is not the case, for example, with GPS. We have to believe the GPS output based on our faith in the system, a procedure rather more appropriate to religion than to navigation.

Part of the "standard good procedure" mentioned above, however, is the taking of multiple sights of each body used in the fix. If we take just one or two sights, we run the risk of starting out with less than the best possible data. These are human measurements, and we can all make an error, even in the best of conditions — an imperceptible little bump-twist of the dial, just as we release it, or some chance coincidence of the vessel's heel angle and our sextant rocking angle can lead to an imprecise sight, or just a blunder, we read the dial (of sextant or watch) as 23 and then record 32. Each of these is possible, but it is very unlikely that they would happen twice in the same series of sights.

The motivation of taking multiple sights is to average out these individual errors. Even if there were no blunders or mistakes, conventional celestial navigation is right on the edge of the ultimate capability of both man and sextant. About the best accuracy we can hope for is some 0.5 miles in final position, and even this level of accuracy depends mostly on how we correct the process for vessel motion during the sights. And we are striving for this level of using a hand-held instrument that measures angles only to within about 0.2 miles and astronomical data that is only accurate to within about 0.1 miles. In short, to get the optimum accuracy in the final product, we must do every thing as careful as possible.

A key question then is simply how accurate are the sights themselves? Any navigator taking multiple sights of the same body will naturally observe some spread in the data. The sights will not be precise, but will scatter about the proper value. The extent of the scatter is some fraction of an arc minute — maybe optimistically about 0.2' in ideal conditions with good instrument in experienced hands, but more likely it will be about twice that (0.4') in good conditions. In poor conditions, it may be more like 1' or even 2'. The question at hand here is, how do you know what that scatter is and how do we average it out.

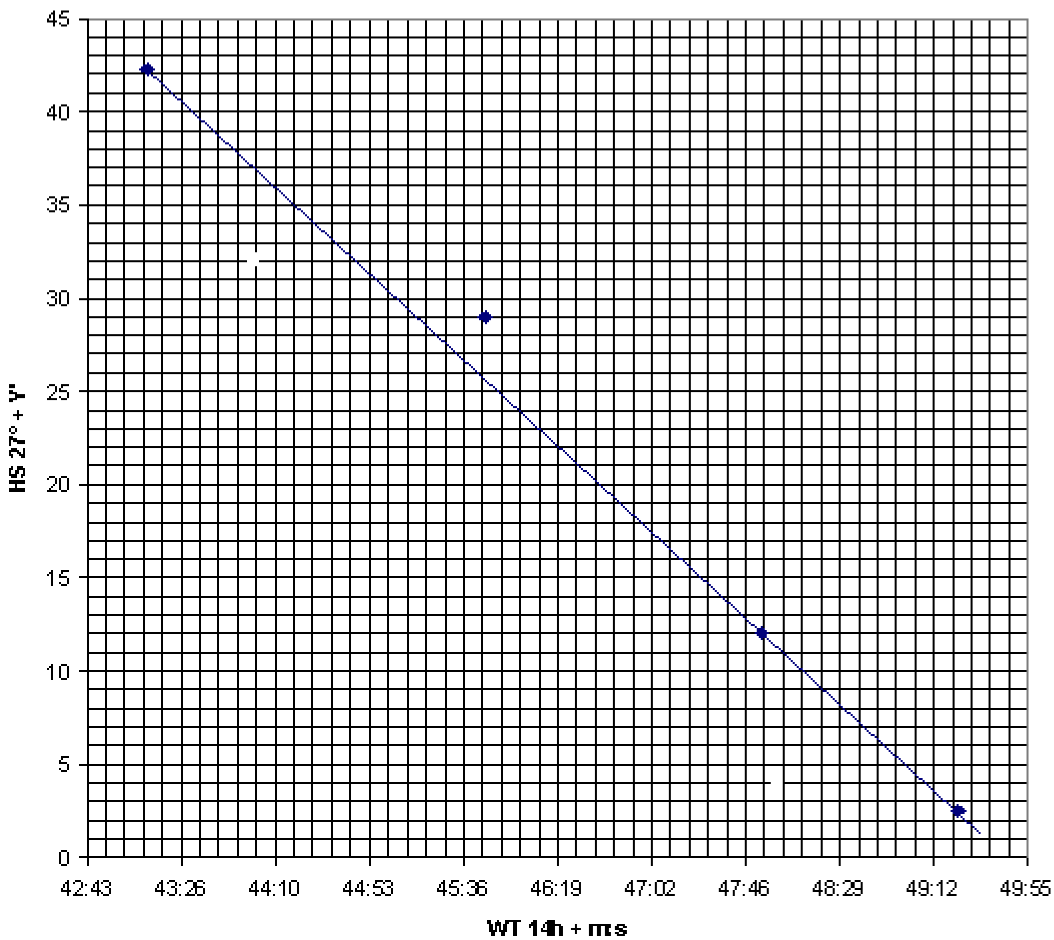
We should note here, that even beginners, after good instruction, can do a series of sights well within 1' in good conditions. The use of a sextant is not a magic art, and all who care to learn can, with only modest practice and good instruction.

The other very important point to note here is the actual extent of the scatter is not crucial to the final accuracy of the fix, assuming that it is truly random, and assuming we know how to evaluate it properly. In other words, we can get just as good a fix out of sights that scatter by an arc minute or more among individual sights as we can from a set of sights that are consistent to within a few tenths of a minute of each other. And this is precisely the subject of this discussion: How to do that evaluation.

The problem is not a trivial one because the sextant height itself is changing with time, so we naturally expect each one to be different from the last one. Looking to the east, the sight angle will be getting bigger with time as the boy rises, and looking east they will get smaller as the boy sets. Measuring a sextant altitude for a fix is not like measuring your pulse rate, which you can just do many times in a row and then perform a simple average to get an accurate value. We must evaluate a sequence of sights and use their relative values to determine if individual ones might be in doubt. A set of sample data is shown below.

DR 47° 38' N, 122° 20'W (Gasworks Park on Lake Union in Seattle), ZD=+8h, WE = 7s fast, HE = 7 feet. IC = 3.5' off the scale. Sights of the sun's lower limb. Note we must use dip short for the ultimate evaluation of these sights, but that is not the issue at hand.

WT	**Hs**	
14h 43m 11s	27° 42.3'	
14h 45m 46s	27° 29.0'	
14h 47m 53s	27° 12.0'	
14h 49m 23s	27° 02.5'	This data is plotted below to illustrate the trend.



There are two ways to average these sights for the best LOP (line of position). If you have a calculator programmed to do celestial navigation or a software product that does it on a computer, then the simplest approach is to simply to the sight reduction of each line from the DR position and make a list of the LOPs as follows:

WT	**LOP** (Assumed position = DR position, HE = 0 ft)
14h 43m 11s	a = 17.8' T 221
14h 45m 46s	a = 21.6' T 221
14h 47m 53s	a = 18.8' T 222
14h 49m 23s	a = 19.4' T 222

At this point since we have removed the time dependence of the sight altitude, we can simply average the four a-values to get a = 19.4' T 222. This would be a reasonably good way to have combined the 4 independent sights into one "best value." In other words, with software at hand, this is one way to "average sights."

Looking at the list of LOPs and also at the plot of the Hs values, it appears that the second sight (14:45:46) is too high. It might seem reasonable to drop this one out of the list and average the rest. This would give: a = 18.7' T 222. This is a big effect, 19.4 to 18.7 is a shift of 0.7 miles in this average LOP. We could take the attitude that our "careful" detection of one sight that was out of line has improved our accuracy by more than half a mile. In other words, computing all the a-values, noting which ones if any are well outside the other values, throwing them out and averaging the rest could be considered a second level of "sight averaging."

All of this presumes we have a calculator or software so we can sight reduce from the same assumed position a whole series of sights in a short time. Note that you can indeed use the same position for multiple sights with tables using Mike Pepeprday's S-Table method, but we are not covering that here. Even if we could use tables for this analysis this is way too long and tedious to do so for a series of sights. When we are using tables, we want to "average" the Hs values directly and then just carry out a single sight reduction on the best value sight that represents all the set.

Furthermore. we will show that the above simplified averaging of calculator solutions is not guaranteed to yield the correct result. It could well be that shifting the 19.4 to the 18.7 was a mistake.

The motivation for throwing out the second sight was that it was well off the line that the other 3 fit onto. We assumed that nature was kind in this regard... i.e. three in a row must be right. And clearly we need some method to evaluate which sights are far enough off to not consider...i.e. to conclude that they may be blunders or isolated errors. If a sight is say 5 miles off, it is almost certainly wrong, and if we keep it, we will be pulling the average off the proper value. So how do we proceed?

The trick is simple. We fine tune the line. In the last example, we simply took the best line that fit the most data points and drew it. It could have been any line, meaning in this case a line with any slope to it. But the trick is *we know the slope*. This does not have to be a free ranging option in selecting the best line. We can calculate the proper slope and then just slide that line up and down the page for the "best fit" to the data. Once we have that line in place, we have firmer ground to stand on as we pitch out some of our sights.

To calculate the slope, we do a sight reduction at some time near the beginning of the sight session to get Hc and Zn (we do not need the latter) and then do the same thing at some time near the end of the sight session. In the above example, we chose to do the computation at 14h 44m 00s to get Hc = 26° 3.3', and then again at 14h 48m 00s to get Hc = 25° 35.7'. This tells us that the sun at this time and place was setting at a rate of 63.3-35.7 = 27.6' per 4 minutes of time.

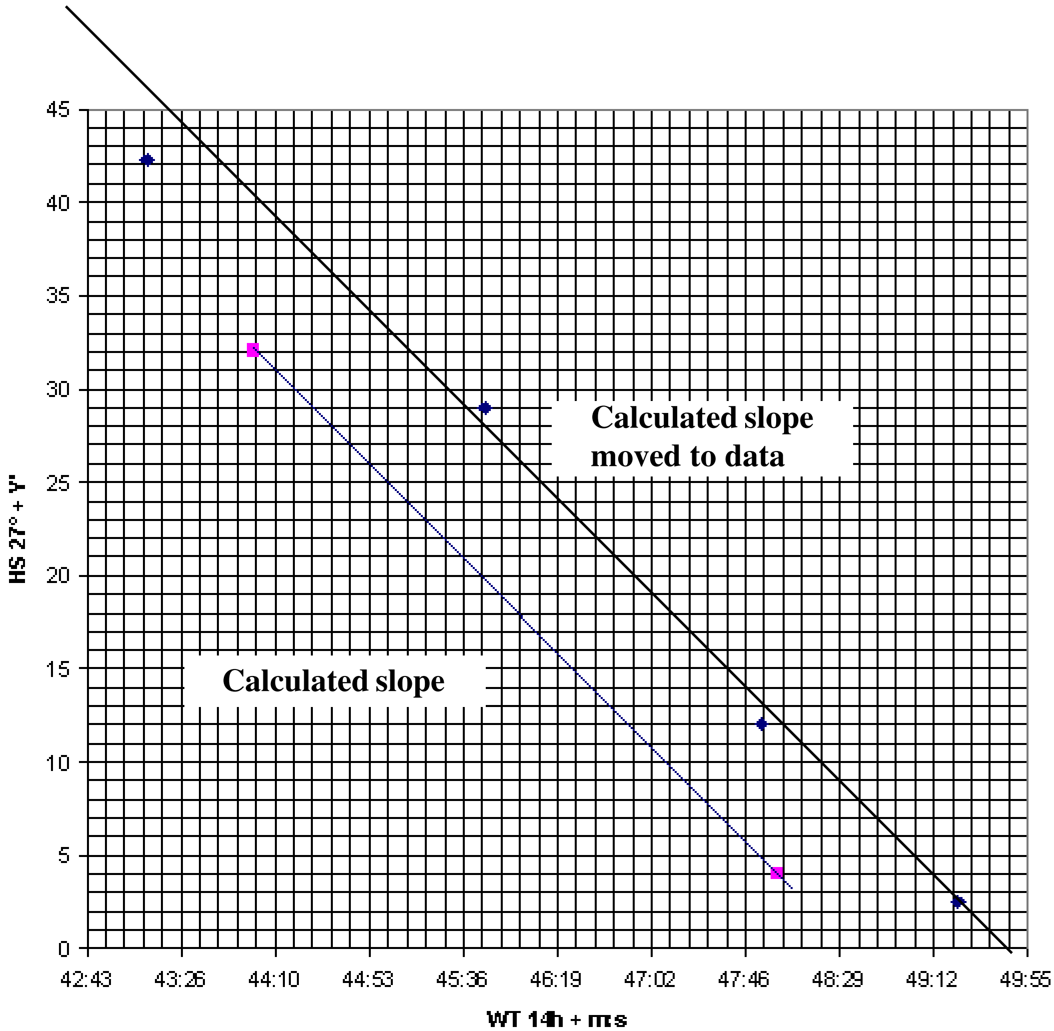
We can now draw in that line of the same plot with the data and then use parallel rules or plotter to move it up to the data. Just slide this line up and down to get the best fit position. There is no rotating it now for the best fix. We know that all good sights will be rising or falling at this rate. To draw the line, just choose any convenient place on the 1444 line to mark the first Hc then at 1448 mark another Hc that is 28' lower. Note that actual values of Hc used here do not matter so long as they are plotted on the right times and are 28’ apart.

If we are using software, it is trivial to do these two sight reductions, it takes just a minute or two. But if we are using tables, we must do one more trick to make it work. We can’t just choose two arbitrary times, since we won’t be able to get a whole-degrees value of LHA for use in the tables. So we simply do it once properly from a time early in the sights or just before it (in this case 1444), and then get Hc from that, and then look up the Hc for the same declination and latitude but for an LHA which is 1° higher. This will give the Hc for 4 minutes later (1448). You can then use these two values to figure the slope. If the d-value in the sight reduction tables is the same at both LHAs, then you do not even need to figure the d-correction since that will not change the slope. Using Pub 249 for the above calculations, you get Hc at 1444 (Lat = 48° N, Dec = S 6°, LHA = 38°) = 26° 32' and at 1448 (Lat = 48° N, Dec = S 6°, LHA = 39°) = 26° 04' which gives the same 28' per 4 minutes slope that we got by computer.

The next figure shows this result applied to the sight data at hand. Note the interesting result that poor ole Sight No.2 that we were so ready to throw out was probably quite OK. The problem sight was No. 1, which looks off by more than 4' or so!

It’s way too late to say “in short” but at least “in summary” this is a good trick to know about for evaluating sextant sights. It is especially valuable on those days when the electronics have failed completely and the sky has been overcast for some days and you are anxious for at least a good LOP and all you get is 3 hurried sights in broken cloud cover and choppy seas and you have to make the best of these.

Once the data is analyzed in this manner, we then just select one sight from the good data line and sight reduce it. In this example, we could use the last sight, right on the line, or we could use sight No 2 and take 1' off the value (it was not way out of line at all!), or sight No. 3 and add 1' or use the purely fictitious sight at 47:02 with an Hs of 27° 19.0'. We did not actually measure a height at this last time, but if we had, that is most likely what we would have gotten. Any one of these sights is the proper “average” of the full set of four, and it took the full set of four and some reasoning to pick it out.



References:

Optimizing sextant sights is discussed in Bowditch section 1609 of recent editions. Unfortunately this is a shortened presentation compared to earlier versions and sadly omits the slope fitting. If available, check section 1507 of 1977 or earlier editions which discuss this most important technique. (They discontinued the discussion most likely because they had used Pub 214 to compute the slope and that pub went out of print. But you can, obviously, compute the slope with any sight reduction method.)

See also: "Precision Celestial Navigation Experiments (Jan 1958 to May 1961)" carried out by Capt. Henry H. Schufeldt, USNR... there is a report on this in the *Navigation Journal* a publication of the Royal Institute of Navigation, Vol 15, No3.

For ongoing studies of celestial navigation, you cannot beat the Newsletter of the Foundation for the Promotion of the Art of Navigation. See links at www.starpath.com. A most wonderful organization for those interested in navigation.