

Explanatory Supplement to the Astronomical Almanac

THIRD EDITION

Edited by

Sean E. Urban
US Naval Observatory

P. Kenneth Seidelmann
University of Virginia

Foreword by Roger Sinnott



UNIVERSITY SCIENCE BOOKS
Mill Valley, California

Foreword	xxvii
Preface	xxxii
Contributing Authors	xxxiii
1 Introduction to Positional Astronomy	1
1.1 Introduction	1
1.1.1 Purpose	1
1.1.2 Introduction to The Astronomical Almanac	1
1.1.3 The Astronomical Almanac and The Explanatory Supplement	3
1.1.4 Conceptual Changes since the Last Edition	3
1.1.5 Implementation of the Changes	7
1.1.6 Deletions from This Edition and The Astronomical Almanac	9
1.2 Timescales and Calendars	9
1.2.1 Atomic Timescales	9
1.2.2 Dynamical Time	10
1.2.3 Timescales Based on Earth's Rotation	11
1.2.4 Coordinated Universal Time (UTC)	13
1.2.5 Enumeration of Dates	14
1.3 Celestial and Terrestrial Coordinates	15
1.3.1 Coordinate Systems and Frames	15
1.3.2 Celestial Coordinate Systems	18
1.3.3 Terrestrial Coordinate Systems	20
1.3.4 The Rotation of the Earth	24
1.3.5 Connections between Terrestrial and Celestial Coordinates	25
1.3.6 Effects of the Position and Motion of the Object and Observer	27
1.4 Orbital Motion	29
1.4.1 Motion in Two-Body Systems	29
1.4.2 Ephemerides	33
1.4.3 Perturbations by and on Extended Bodies	35
1.5 Solar System	36
1.5.1 Sun	36
1.5.2 Planets	37
1.5.3 Satellites	37
1.5.4 Other Bodies	38
1.5.5 Planetary Coordinates	38

1.6	Astronomical Phenomena	38
1.6.1	<i>Rising, Setting, and Twilight</i>	39
1.6.2	<i>Meridian Transit</i>	39
1.6.3	<i>Conjunction, Opposition, and Elongation</i>	40
1.6.4	<i>Eclipses, Occultations, and Transits</i>	40
1.6.5	<i>Satellite Phenomena</i>	41
1.6.6	<i>Physical Observations of the Sun, Moon, and Planets</i>	42
	References	43
2	Relativity for Astrometry, Celestial Mechanics and Metrology	45
2.1	Introduction	45
2.2	Newtonian Formalism	46
2.3	Special Relativity and the Metric Tensor	48
2.4	Einstein's Theory of Gravity	50
2.5	The Problem of Observables	52
2.5.1	<i>The Ranging Observable</i>	52
2.5.2	<i>The Spectroscopic Observable</i>	52
2.5.3	<i>Astrometric Observables</i>	54
2.6	The Post-Newtonian Framework	55
2.6.1	<i>Coordinate Timescales and Proper Time</i>	56
2.6.2	<i>The Barycentric Celestial Reference System</i>	56
2.6.3	<i>BCRS versus ICRS</i>	59
2.6.4	<i>The Geocentric Celestial Reference System</i>	59
2.6.5	<i>Coordinate Transformations</i>	62
2.6.6	<i>Relativistic Potential Coefficients</i>	62
2.7	Applications	63
2.7.1	<i>Other Versions of the Geocentric Celestial Reference System</i>	63
2.7.2	<i>Rotating Reference Systems</i>	64
2.7.3	<i>The Einstein-Infeld-Hoffmann Equations of Motion</i>	64
2.7.4	<i>Equations of Motion in the GCRS</i>	66
2.7.5	<i>Astrometry</i>	66
2.7.6	<i>VLBI Observations</i>	68
2.8	Relativistic Timescales, their Definitions and Units	68
2.8.1	<i>Practical Transformations of Timescales</i>	68
2.8.2	<i>Scaled Coordinate Timescales</i>	69
2.8.3	<i>Units of Timescales</i>	70
	References	71
3	Time	75
3.1	Introduction	75
3.1.1	<i>Absolute (or Newtonian) Time</i>	75
3.1.2	<i>Proper Time</i>	75
3.1.3	<i>Coordinate Time</i>	76
3.2	Time and the Earth's Rotation	76
3.2.1	<i>Sidereal Time</i>	77
3.2.2	<i>UT1</i>	80

3.3	Dynamical Time	81
3.3.1	<i>Ephemeris Time</i>	82
3.3.2	<i>TDT/TDB</i>	83
3.4	Atomic Time	83
3.4.1	<i>International Atomic Time, TAI</i>	84
3.4.2	<i>Terrestrial Time, TT</i>	84
3.5	Civil Time	85
3.5.1	<i>Coordinated Universal Time, UTC</i>	85
3.5.2	<i>Time Zones</i>	86
3.6	Barycentric and Geocentric Coordinate Time	88
3.6.1	<i>Barycentric Coordinate Time, TCB and Barycentric Ephemeris Time, T_{eph}</i>	88
3.6.2	<i>Geocentric Coordinate Time, TCG</i>	89
3.7	Julian Date	91
3.8	Summary of Useful Relationships	92
3.9	Timekeeping	93
3.9.1	<i>Devices</i>	93
3.9.2	<i>International Agencies</i>	95
3.10	Time Dissemination	96
3.10.1	<i>High-Frequency Radio Time Signals</i>	97
3.10.2	<i>Low-Frequency Radio Time Signals</i>	97
3.10.3	<i>LORAN-C</i>	97
3.10.4	<i>Global Positioning System (GPS)</i>	98
3.10.5	<i>GLONASS</i>	98
3.10.6	<i>GALILEO</i>	98
3.10.7	<i>Two-Way Satellite Time Transfer (TWSTT)</i>	99
3.10.8	<i>Time Transfer by Internet</i>	99
3.11	Relativistic Effects	99
3.11.1	<i>Clock Transport from a Rotating Reference Frame</i>	99
3.11.2	<i>Non-rotating Local Inertial Reference Frame</i>	100
3.11.3	<i>Electromagnetic Signal Transfer from a Rotating Reference Frame</i>	101
3.11.4	<i>Electromagnetic Signal Transfer from a Non-rotating, Local Inertial Frame</i>	101
	References	101
4	The Fundamental Celestial Reference System	105
4.1	Summary	105
4.2	The ICRS, the ICRF, and the HCRF	105
4.3	Background: Reference Systems and Reference Frames	108
4.4	The Effect of Catalog Errors on Reference Frames	109
4.5	Late 20th Century Developments	111
4.6	ICRS Implementation	112
4.6.1	<i>The Defining Extragalactic Frame</i>	112
4.6.2	<i>The Frame at Optical Wavelengths</i>	113
4.6.3	<i>Standard Algorithms</i>	114

4.6.4	<i>Relationship to Other Systems</i>	114
4.6.5	<i>Data in the ICRS</i>	115
4.7	Formulae	117
	References	118
5	Terrestrial Coordinates and the Rotation of the Earth	123
5.1	Introduction	123
5.2	Terrestrial Reference Systems	124
5.2.1	<i>Reference Systems and Reference Frame Concepts</i>	124
5.2.2	<i>The Figure of the Earth and Geodetic Reference Surfaces</i>	127
5.2.3	<i>Geodetic Datums</i>	128
5.2.4	<i>Geocentric, Geodetic, and Astronomical Coordinates</i>	131
5.2.5	<i>Local Coordinate Systems</i>	134
5.3	Gravity, Tides, and Motions of the Crust	148
5.3.1	<i>Modeling the Earth's Gravity Field</i>	149
5.3.2	<i>Representation of the Earth's Gravity Field</i>	150
5.3.3	<i>Solid Earth Tides</i>	153
5.3.4	<i>Ocean Tide Model</i>	158
5.3.5	<i>Site Displacement Due to Ocean and Atmospheric Loading</i>	162
5.3.6	<i>Plate Motions</i>	169
5.3.7	<i>Tidal Effects in UT1</i>	170
5.3.8	<i>Tidal Effects in Polar Motion</i>	171
5.4	Monitoring the Orientation of the Earth	175
5.4.1	<i>Very Long Baseline Interferometry</i>	176
5.4.2	<i>Global Navigation Satellite System</i>	177
5.4.3	<i>Laser Ranging</i>	179
5.4.4	<i>Doppler Orbitography and Radiopositioning Integrated by Satellite (DORIS)</i>	181
5.4.5	<i>Historical Methods</i>	181
5.4.6	<i>Alternative Techniques</i>	183
5.4.7	<i>International Services</i>	184
	References	187
6	Precession, Nutation, Polar Motion, and Earth Rotation	199
6.1	Introduction	199
6.2	The Celestial Reference System (CRS) and the Terrestrial Reference System (TRS)	200
6.2.1	<i>Reference Systems in The Astronomical Almanac</i>	202
6.2.2	<i>The Conventional International Origin and the International Terrestrial Reference Frame (ITRF)</i>	203
6.2.3	<i>The Celestial Intermediate Reference System and the Celestial Intermediate Pole (CIP)</i>	204
6.2.4	<i>The Equinoxes</i>	206
6.3	The Equation of the Equinoxes, $E_e(T)$	207

6.4	The Equation of the Origins, $E_o(T)$	208
6.5	The J2000.0 to ICRS Frame Bias Matrix	208
6.6	Precession and Nutation	209
6.6.1	<i>The IAU 2000 Precession-Nutation Theory and IAU 2006/2000A</i>	209
6.6.2	<i>Precession</i>	211
6.6.3	<i>Nutation</i>	223
6.7	The Celestial Intermediate Origin (CIO)	227
6.8	Time and Earth Orientation	229
6.8.1	<i>Apparent Solar Time, Mean Solar Time, and the Equation of Time</i>	230
6.8.2	<i>Greenwich Mean Time</i>	231
6.8.3	<i>Universal Time (UT)</i>	232
6.8.4	<i>Earth Rotation Angle (ERA or θ)</i>	233
6.8.5	<i>Sidereal Time</i>	234
6.8.6	<i>Conversions between Universal Time and Sidereal Time</i>	235
6.8.7	<i>Conversion of Local Apparent Sidereal Time to UT1</i>	237
6.8.8	<i>Local Time and Hour Angle</i>	238
6.9	The Terrestrial Intermediate Origin (TIO)	239
6.9.1	<i>The Ephemeris Meridian</i>	239
6.9.2	<i>Polar Motion</i>	240
	References	244
7	Positions	249
	Introduction	249
7.1	Summary of Systems, Places, and Coordinates	250
7.1.1	<i>Reference Systems</i>	251
7.1.2	<i>Places</i>	252
7.1.3	<i>Coordinates</i>	253
7.1.4	<i>Summary of Notations</i>	257
7.2	Components Used in the Reduction of Positions	258
7.2.1	<i>Space Motion</i>	258
7.2.2	<i>Parallax</i>	261
7.2.3	<i>Aberration</i>	263
7.2.4	<i>Gravitational Light-Deflection</i>	270
7.2.5	<i>Celestial "Of Date" Positions</i>	272
7.2.6	<i>Terrestrial "Of Date" Positions</i>	275
7.2.7	<i>Refraction</i>	277
7.3	Reduction of Positions from the BCRS to the GCRS	281
7.3.1	<i>Proper Place</i>	281
7.3.2	<i>Local Place</i>	281
7.3.3	<i>Astrometric Place</i>	281
7.3.4	<i>Differential Astrometry</i>	283
7.4	Algorithms for the Reductions of Positions	283
7.4.1	<i>Apparent and Intermediate Place Algorithm for Planets</i>	283
7.4.2	<i>Apparent and Intermediate Place Algorithm for Stars</i>	290

7.4.3	<i>Topocentric Place Algorithm</i>	293
7.4.4	<i>Summary of the Algorithms</i>	297
7.4.5	<i>Approximate Algorithms</i>	298
References		301
Further Reading		303
8	Orbital Ephemerides of the Sun, Moon, and Planets	305
8.1	Fundamental Ephemerides	305
8.2	Previous Ephemerides Used in <i>The Astronomical Almanac</i>	306
8.2.1	<i>Ephemerides Prior to 1984</i>	306
8.2.2	<i>Ephemerides from 1984 through 2002</i>	307
8.2.3	<i>Ephemerides Starting in 2003</i>	307
8.3	Equations of Motion	307
8.3.1	<i>Point-Mass Interactions</i>	308
8.3.2	<i>Solar System Barycenter</i>	309
8.3.3	<i>Figure Effects</i>	309
8.3.4	<i>Lunar Gravity Coefficients</i>	310
8.3.5	<i>Lunar Physical Libration: Coordinates</i>	311
8.3.6	<i>Physical Libration Differential Equations</i>	311
8.3.7	<i>The Moment of Inertia Tensor</i>	312
8.3.8	<i>Time Derivative of the Inertia Tensor</i>	312
8.3.9	<i>Principal Moments</i>	313
8.3.10	<i>Figure–Point Mass Torques upon the Moon</i>	313
8.3.11	<i>Figure–Figure Torque upon the Moon</i>	313
8.3.12	<i>Acceleration of the Moon from Earth Tides</i>	314
8.4	The Numerical Integration of DE405/LE405	315
8.4.1	<i>Estimated Integration Error</i>	315
8.4.2	<i>Adopted Constants</i>	315
8.5	Observational Data Fit by the Planetary and Lunar Ephemerides	316
8.5.1	<i>Optical Data</i>	316
8.5.2	<i>Meridian Transits</i>	316
8.5.3	<i>Photographic and CCD Astrometry</i>	317
8.5.4	<i>Occultation Timings</i>	317
8.5.5	<i>Astrolabe</i>	317
8.5.6	<i>Radiometric Emission Measurements</i>	317
8.5.7	<i>Ranging Data</i>	317
8.5.8	<i>Orbiter Range Points</i>	320
8.5.9	<i>Lander Range Data</i>	320
8.5.10	<i>VLBI Data</i>	320
8.5.11	<i>Lunar Laser Range Data</i>	320
8.6	The Orientation of DE405/LE405	321
8.6.1	<i>Adjustment of DE405 onto the ICRF Reference Frame</i>	321
8.7	Various Formulae Used in the Reduction of the Observational Data	321
8.7.1	<i>Phase Corrections</i>	322
8.7.2	<i>Corrections to Precession and Equinox Drift</i>	322
8.7.3	<i>Computation of Ranges</i>	322

8.7.4	<i>Antenna Location</i>	323
8.7.5	<i>Time Delay for Relativity</i>	323
8.7.6	<i>Time Delay for the Solar Corona</i>	323
8.7.7	<i>Time Delay for the Troposphere</i>	324
8.7.8	<i>Modeling the Surface of Mercury</i>	324
8.7.9	<i>Modeling the Surface of Venus</i>	324
8.7.10	<i>Modeling the Surface of Mars—Closure Points</i>	325
8.7.11	<i>Viking Lander Computations</i>	325
8.8	Initial Conditions and Constants of DE405/LE405	325
8.8.1	<i>Asteroids</i>	322
8.9	Positional Errors of the Planetary and Lunar Ephemerides	332
8.9.1	<i>Inner Four Planets: Ephemerides from Ranging and VLBI</i>	332
8.9.2	<i>Uncertainties from Asteroid Perturbations</i>	333
8.9.3	<i>Outer Planets: Reliance on the Classical Optical Observations</i>	333
8.9.4	<i>Planetary Positional Errors in the Almanacs, 1984–2002</i>	333
8.9.5	<i>Planetary Position Uncertainties in 2003</i>	333
8.10	Keplerian Elements for Approximate Positions of the Major Planets	338
8.10.1	<i>Formulae for Using the Keplerian Elements</i>	339
8.10.2	<i>Solution of Kepler's Equation, $M = E - e \sin E$</i>	340
8.10.3	<i>Approximate Accuracies of the Keplerian Formulae</i>	341
8.11	The Availability of Ephemerides	341
	References	342
9	Planetary Satellites and Rings	347
9.1	Introduction	347
9.2	Orbital Elements and Perturbations	347
9.2.1	<i>Orbital Elements</i>	347
9.2.2	<i>Secular Perturbations of the Orbit</i>	349
9.2.3	<i>Perturbations due to Commensurabilities</i>	351
9.2.4	<i>Long-Period Perturbations by Other Satellites</i>	352
9.3	Planetocentric Rectangular Coordinates	353
9.4	Traditional Method	356
9.4.1	<i>The Apparent Orbit</i>	356
9.4.2	<i>Calculating Tabulated Values</i>	360
9.5	Modern Method	361
9.5.1	<i>The Apparent Orbit</i>	361
9.5.2	<i>Calculating Tabulated Values</i>	363
9.6	Satellite Data in <i>The Astronomical Almanac</i> and Notation	365
9.6.1	<i>Presented Data</i>	365
9.6.2	<i>Notation</i>	365
9.7	The Satellites of Mars	367
9.8	The Satellites of Jupiter	369
9.8.1	<i>The Galilean Satellites</i>	369
9.8.2	<i>Jupiter's Fifth Satellite, Amalthea</i>	373
9.8.3	<i>Jupiter's Sixth through Thirteenth Satellites</i>	375

9.9	The Rings and Satellites of Saturn	376
9.9.1	<i>The Rings of Saturn</i>	376
9.9.2	<i>The Satellites of Saturn</i>	378
9.10	The Rings and Satellites of Uranus	388
9.11	The Satellites of Neptune	393
9.11.1	<i>Triton</i>	393
9.11.2	<i>Nereid</i>	394
9.12	The Satellites of Pluto	396
	References	397
10	Physical Ephemerides of Solar System Bodies	399
10.1	Introduction	399
10.2	Rotational Elements and Cartographic Coordinates	399
10.2.1	<i>Planetocentric and Planetographic Coordinate Systems</i>	401
10.2.2	<i>Coordinate Systems of Small Bodies</i>	406
10.3	Phases and Magnitudes	410
10.3.1	<i>Absolute and Apparent Magnitudes</i>	410
10.3.2	<i>Surface Brightness</i>	412
10.4	The Apparent Disk of a Solar System Body	415
10.4.1	<i>Sub-Earth Points and Sub-Solar Points</i>	417
10.4.2	<i>The Greatest Defect of Illumination</i>	422
10.4.3	<i>The Apparent Magnitudes of the Minor Planets</i>	422
10.5	Physical Ephemeris of the Sun	426
10.6	Physical Ephemeris of the Moon	427
10.6.1	<i>Calculating Librations</i>	429
10.7	Physical Ephemerides of the Planets	434
10.7.1	<i>Mercury</i>	435
10.7.2	<i>Venus</i>	435
10.7.3	<i>Mars</i>	436
10.7.4	<i>Jupiter</i>	436
10.7.5	<i>Saturn</i>	436
10.7.6	<i>Uranus</i>	437
10.7.7	<i>Neptune</i>	437
10.8	Physical Ephemerides of the Dwarf Planets	437
10.9	Physical Ephemerides of the Satellites	438
10.9.1	<i>Satellites of Mars</i>	438
10.9.2	<i>Satellites of Jupiter</i>	439
10.9.3	<i>Satellites of Saturn</i>	441
10.9.4	<i>Satellites of Uranus</i>	441
10.9.5	<i>Satellites of Neptune</i>	443
10.9.6	<i>Satellites of Pluto</i>	447
	References	447

11 Eclipses of the Sun and Moon	453
11.1 Introduction	453
<i>11.1.1 Eclipse Data Available from the Nautical Almanac Offices</i>	<i>454</i>
<i>11.1.2 Corrections to the Ephemerides</i>	<i>456</i>
11.2 The Occurrence of Lunar and Solar Eclipses	457
<i>11.2.1 Overview</i>	<i>457</i>
<i>11.2.2 Geocentric Least Angular Separation</i>	<i>457</i>
<i>11.2.3 Occurrence of Lunar Eclipses</i>	<i>459</i>
<i>11.2.4 Occurrence of Solar Eclipses</i>	<i>461</i>
11.3 Solar Eclipses	464
<i>11.3.1 Fundamental Equations: Introduction</i>	<i>464</i>
<i>11.3.2 Besselian Elements</i>	<i>464</i>
<i>11.3.3 Coordinates of the Observer</i>	<i>470</i>
<i>11.3.4 Conditional and Variational Equations</i>	<i>474</i>
<i>11.3.5 Calculation of General Solar Eclipse Phenomena</i>	<i>477</i>
<i>11.3.6 Local Circumstances</i>	<i>487</i>
11.4 Lunar Eclipses	491
<i>11.4.1 Introduction</i>	<i>491</i>
<i>11.4.2 Computations</i>	<i>492</i>
11.5 Eclipses Online	495
<i>11.5.1 Solar Eclipses</i>	<i>495</i>
<i>11.5.2 Lunar Eclipses</i>	<i>497</i>
11.6 Transits	499
References	502
Further Reading	502
12 Astronomical Phenomena	505
12.1 General Aspects of the Night Sky	505
12.2 Configurations of the Sun, Moon, and Planets	506
<i>12.2.1 Interesting Phenomena of the Sun, Earth, and Moon</i>	<i>506</i>
<i>12.2.2 Geocentric Phenomena</i>	<i>508</i>
<i>12.2.3 Heliocentric Phenomena</i>	<i>511</i>
12.3 Risings, Settings, and Twilight	511
<i>12.3.1 Sunrise, Sunset, and Twilight</i>	<i>512</i>
<i>12.3.2 Moonrise and Moonset</i>	<i>514</i>
<i>12.3.3 Formulae Associated with Rising and Setting</i>	<i>515</i>
<i>12.3.4 Illumination</i>	<i>518</i>
12.4 Occultations	521
<i>12.4.1 Occultations of Stars</i>	<i>521</i>
<i>12.4.2 Occultations of Planets</i>	<i>524</i>
12.5 Pole Star Tables	525
<i>12.5.1 Derivation of the Pole Star Coefficients</i>	<i>527</i>
References	528

13 Stars and Stellar Systems	529
13.1 Introduction	529
13.1.1 <i>Data Presentation</i>	529
13.1.2 <i>Sources of Data</i>	530
13.1.3 <i>Sources of Position</i>	530
13.2 Stellar Data	531
13.2.1 <i>Bright Stars</i>	531
13.2.2 <i>Double Stars</i>	532
13.2.3 <i>Photometric Standards</i>	534
13.2.4 <i>Radial Velocity Standards</i>	536
13.2.5 <i>Variable Stars</i>	537
13.2.6 <i>Exoplanets and Host Stars</i>	538
13.2.7 <i>Pulsars</i>	539
13.3 Clusters and Galaxies	539
13.3.1 <i>Open Star Clusters</i>	539
13.3.2 <i>Globular Star Clusters</i>	541
13.3.3 <i>Bright Galaxies</i>	542
13.3.4 <i>Quasi-Stellar Objects</i>	544
13.4 Wavelength Specific Sources	545
13.4.1 <i>Radio-Source Positional Calibrators</i>	545
13.4.2 <i>Radio-Flux Calibrators</i>	546
13.4.3 <i>X-Ray and Gamma Ray Sources</i>	547
References	548
14 Computational Techniques	555
14.1 Introduction to Computing Techniques	555
14.2 Interpolation and Subtabulation	559
14.2.1 <i>Introduction and Notation</i>	559
14.2.2 <i>Interpolation Formulae</i>	560
14.2.3 <i>Inverse Interpolation</i>	561
14.2.4 <i>Polynomial Representations</i>	561
14.2.5 <i>Chebyshev Polynomials</i>	562
14.3 Plane and Spherical Trigonometry	563
14.4 Matrix and Vector Techniques	565
14.4.1 <i>Matrix Multiplication</i>	565
14.4.2 <i>Rotation of Axes Using Matrices</i>	565
14.4.3 <i>Spherical Coordinates Using Vectors</i>	567
14.4.4 <i>Specific Coordinate Transformations</i>	569
14.5 Numerical Calculus	572
14.5.1 <i>Numerical Differentiation</i>	572
14.5.2 <i>Numerical Integration</i>	574
14.6 Statistics	578
14.6.1 <i>Accumulation of Error</i>	578
14.6.2 <i>The Method of Least-Squares</i>	579
References	583

15 Calendars	585
15.1 Introduction	585
15.1.1 Overview	585
15.1.2 Uses of Calendars	586
15.1.3 Astronomical Bases of Calendars	586
15.1.4 Astronomical, Observational and Arithmetic Calendars	587
15.1.5 Visibility of the Crescent Moon	588
15.1.6 Non-astronomical Cycles and the Week	588
15.1.7 Historical Eras and Chronology	589
15.1.8 The Christian Era	589
15.1.9 Dates	591
15.1.10 Julian Day Numbers and Julian Dates	591
15.1.11 Luni-solar Calendars	592
15.1.12 Accuracy of Calendars	593
15.2 The Ancient Egyptian Calendar	593
15.2.1 History of the Egyptian Calendars	593
15.2.2 Rules of the Egyptian Calendar	594
15.3 The Roman and Julian Calendars	594
15.3.1 Introduction	594
15.3.2 Divisions of the Roman Months	595
15.3.3 Caesar's Reform	595
15.3.4 The Julian Calendar in Medieval Europe	597
15.3.5 Rules for the Julian Calendar	597
15.4 The Gregorian Calendar	598
15.4.1 History of the Gregorian Reform	598
15.4.2 Rules for the Civil Use of the Gregorian Calendar	599
15.4.3 Rules for the Ecclesiastical Calendar	599
15.4.4 Calculation of the Date of Easter Sunday	600
15.5 The Jewish Calendar	601
15.5.1 History of the Jewish Calendar	601
15.5.2 Rules for the Modern Jewish Calendar	602
15.5.3 Rules for Postponement (Dehiyyot)	603
15.5.4 Determining Tishri 1	604
15.5.5 Determining the Length of the Year	605
15.5.6 Terminology of the Jewish Calendar	606
15.6 The Islamic Calendar	606
15.6.1 Introduction	606
15.6.2 History of the Islamic Calendar	607
15.6.3 Rules for the Arithmetic Islamic Calendar	607
15.7 The Indian Calendars	608
15.7.1 History of Indian Calendars	608
15.7.2 The Traditional Indian Solar Calendar	609
15.7.3 The Traditional Indian Luni-solar Calendar	609
15.7.4 Rules of the Reformed Saka Calendar	610
15.8 The Chinese Calendar	610
15.8.1 History of the Chinese Calendar	610
15.8.2 Sexagenary Cycle	612

15.8.3	<i>Major and Minor Terms</i>	612
15.8.4	<i>Rules for the Modern Chinese Calendar</i>	613
15.8.5	<i>Finding the First Day of a Chinese Year</i>	614
15.8.6	<i>Finding the First Day of Each Month in a Chinese Year</i>	615
15.9	The French Republican Calendar	616
15.10	The Bahá'í Calendar	616
15.11	Calendar Conversion Algorithms	617
15.11.1	<i>Introduction</i>	617
15.11.2	<i>Calculating the Day of the Week</i>	618
15.11.3	<i>Interconverting Dates and Julian Day Numbers</i>	618
15.11.4	<i>Converting Dates in the Jewish Calendar</i>	619
15.11.5	<i>Calculating the Date of Easter</i>	621
15.12	Calendar Conversion Programs	621
	References	622
A	Bases and Constants	625
A.1	Bases	625
A.2	Astronomical Constants	625
A.2.1	<i>CODATA</i>	626
A.2.2	<i>Future of the Astronomical Unit</i>	626
	References	626
B	Acronyms	627
C	Glossary	633
	Index	661