ON THE GEOMETRICAL SOLUTION OF THE NAVIGATIONAL TRIANGLE

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The *United States Naval Institute Proceedings* for March, 1952, contain a paper by Joseph B. Breed III entitled "The Navigational Triangle: How to Solve It by Drawing It." In this paper Mr. Breed capably expresses the conviction that although the supplanting of the trigonometric solution of the navigational triangle by tabulated solutions represents a great forward step in celestial navigation, instruction in the newer methods often leaves the student in ignorance of the basic relationships in the navigational triangle. Mr. Breed does not propose that spherical trigonometry be reinstated in the navigational curriculum. Instead, he suggests, as an instructional tool, a geometrical solution of the navigational triangle that requires only a protractor, a straight edge, and a pair of compasses. The paper describes with admirable clarity how to obtain the computed altitude of a celestial object by constructing the triangular pyramid whose vertex is the center of the earth; whose three sides are the extended radii of the earth through the nearer geographic pole, the geographical position of the celestial object, and the navigator's position; and whose base is the intersection of the pyramid with a plane tangent to the earth at the nearer pole.

Mr. Breed suggests that his readers "Work out several navigation problems with this graphic method (azimuth, great circle course and distance, and star identification as well as line of position problems)." The triangular pyramid that Mr. Breed discusses, although sufficient for the determination of computed altitude or great circle distance, will not yield the azimuth or the great circle course. The following construction extends Mr. Breed's solution to include the azimuth, without which, the solution of a line of position problem is not complete.

In Figure 1, E is the center of the earth; Z is the navigator's position; P is nearer geographic pole; S is the geographic position of the celestial object observed. ZP′ and ZS′ are tangent to the earth at Z and intersect the extended radii EP and ES respectively. The desired azimuth angle is the angle at Z in the plane triangle P′ZS′. This triangle is the base of a triangular pyramid whose vertex is at E. The three angles between the three radii to P, Z, and S are the complements of the latitude of Z, of the altitude of the celestial object viewed from Z, and of the declination of the celestial object.

Figure 2 shows how a flattened version of the pyramid of Figure 1 can be constructed and the azimuth angle obtained. First, draw EZ to any convenient scale. Next, draw EP_1 so as to make an angle of $90°\text{-}L_0$ with EZ, and ES so as to make an angle of $90°\text{-}H_c$ with EZ. (H_c will already have been obtained from Mr. Breed's construction.) Extend EP_1 and ES until they meet the perpendicular to EZ drawn

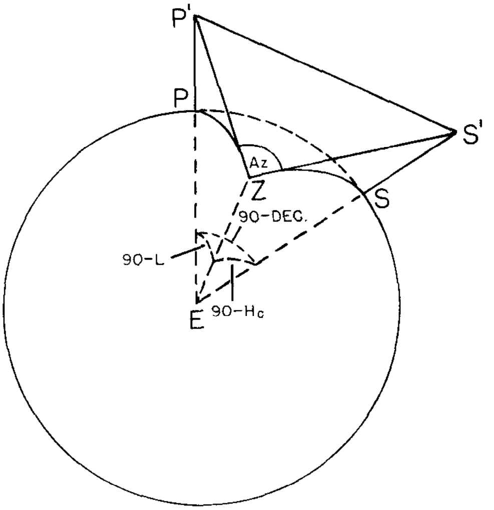

Figure 1

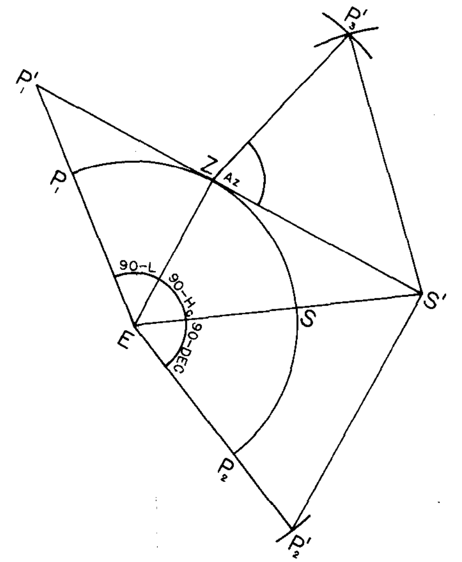

Figure 2

through Z at P_1' and S′. Next draw EP_2', equal in length to EP_1', so as to make an angle 90°-Dec. with ES′. Then strike an arc from Z with radius ZP_1' and an arc from S′ with radius $S'P_2'$. Angle $P_3'ZS'$ is the desired azimuth angle.

Geometrical solids of the sort used in this and Mr. Breed's constructions are often employed in deriving the fundamental formulae of spherical trigonometry, but they have not been as widely exploited in the manner suggested by Mr. Breed as their instructiveness merits. The author has found Mr. Breed's approach, as extended in this paper, a very useful manner of presenting the derivation of the tabulated data in HO 214 and HO 249 without introducing the concepts of trigonometry.