Astronavigation

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A Method for Determining Exact Position by the Stars

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Preface

As a mathematician, I conceived the idea to write about the scientific part of navigation long before GPS became available to the public. From the outset, it was meant to be invariant with regard to the fast-changing technology available to navigators. It was also meant to be a manual that would make the navigator less dependent on the availability of other ephemerides. However, I did not want to turn the clock back by ignoring the state of developments in the fields of calculator and PC technology. Even the most casual observer must admit that we have achieved a level in our general education that renders anyone who does not know how to operate a personal computer illiterate. Today, the average student aged twelve or above should be able to handle the algebraic symbolic language as employed in all advances calculators without even understanding the underlying mathematics of the formulae used in navigation. I also wanted to draw a clear dividing line between the art and science of navigation (based on clear definitions, the important concept of stating the underlying assumptions, and the rigorous applications thereof) and the laws of physics as they apply to navigation.

I also wanted to show that the approximate methods used in celestial navigation, which are based on the original methods of Capt. Sumner and St. Hilair, are merely special cases of a general mathematical method that consists in approximating the two transcendental equations—the fundamental equations of navigation—by two linear equations.

A similar statement can be made with regard to the Lunar/Distance method for finding the approximation to the time at sea. This method is also an approximation to the problem of solving the Fundamental Equation with respect to the parameter of time. Therefore, it is also no longer necessary to look for the elusive error analysis of the Line of Position (LOP) method. Not knowing the error or the realistic upper bound for it (as a function of distance and azimuth from the true position) has always been a shortcoming for this method.

From a mathematical point of view, the distinction between Celestial Navigation and Astro Navigation is very clear. Celestial Navigation is an approximate method requiring an estimated position. Astro Navigation is an exact method not requiring an estimated position. Furthermore, in Celestial Navigation it is necessary to select the Celestial Objects very carefully to, one, avoid improper spacing and, two, choosing them too close to the zenith of the observer. In addition, the initial guess or Dead Reckoning Position (DRP or DR) needs to be sufficiently close to the true position. Therefore, in Celestial Navigation, all of the azimuths of the assumed triangles have to be sufficiently close to the true azimuths.

On the other hand, this does not apply to Astro Navigation (AN). There are very few cases where the methods of AN fail to work, as, for instance, in cases where the observed bodies are too close together or in such pathological cases where the given parameters are erroneous. (Those cases are referred to in future chapters as "Ill-Conditioned".)

Basically, this book is about anything pertaining to navigation that can be quantitatively expressed and ultimately computed. This book also does not rely on the use of any special calculator or PC. Nor does it depend on any other algorithms than the generally accepted mathematical ones. In theory, this amounts to saying that in the event where no calculator is available, a navigator can solve his positioning problem by using the provided formulae and a copy of the old, standby Logarithmic Tables.

I wanted to provide the reader with several independent methods for determining a position and other relevant data. I have also provided more than one method to give the navigator several alternatives for doing so. This also applies to choosing a suitable formula for calculating refraction and dip, which are considered by some to be the most limiting factors to accuracy in Celestial Navigation.

In addition to the above objectives, I have addressed several problems unique to navigation such as the problem of navigating without a sextant or even a clock. These problems have been addressed analytically and not just for emergency purposes but also to examine their underlying principles. All of the final formulae presented here are governed by a self-imposed rule for simplicity and comprehensibility and can be evaluated on any scientific calculator without the user even understanding their mathematical derivations. I would also like to stress that this book is not an exercise in spherical trigonometry (although it employs some basic spherical trigonometry equations sparingly).

In the first part of this book the Earth is assumed to be a sphere. Only in the section that deals with parallaxes is the Earth assumed to be a Spheroid of Revolution. The first chapter deals with the concept of Terrestrial Navigation and provides the rigorous formulae needed for Rhumb Line and Great Circle navigation. The chapter also covers the basics of the underlying mathematical projections on which navigational charts are based. In particular, it provides error analysis for approximating Mercator Plotting Sheets by the Non-Mercator Plotting Sheets that are sold commercially or are self-made. Furthermore, it also provides an analytic estimate for approximating a small area of the surface of the Earth by a plane Euclidian surface.

For centuries navigators have been using the Line of Position (LOP) method to avoid numerous complex mathematics and trigonometric calculations. In this book, I use the "Orientation Schematic" that enables a navigator to apply the exact formulae for determining his or her position at sea or in the air. In this context, the number of required trigonometric calculations in the Exact Method does not exceed the number of the same calculations required by the LOP method.

In addition to the derivations of the exact formulae, I have included other independent methods and, at least, one iterative method for determining position. In addition to the exact methods, I have covered approximate methods and the least square method of error analysis as applied to the non-exact formulae of navigation. However, it should be noted that this method becomes meaningless in cases where the data used is sufficiently erroneous.

In a separate and independent section of the first part of this book, I have devoted space to the error analysis of navigational data, specifically to the analysis of random errors, thereby providing navigators with a method for assessing their own proficiency with respect to measuring sextant altitudes simultaneously with the corresponding watch-time.

I have also attempted to dispel the notion that the Lunar Distance method is the only practical method for a sailor to determine time and therefore longitude at sea. I have provided the reader with an approximate iterative method for calculating time that is based solely on altitude and azimuth observations.

If the reader uses the first part of this book alone, he will have to depend fully on the availability of a current Nautical Almanac (NA). However, the second part of this book provides the reader with an option that replaces the NA.

Very little knowledge of astronomy is required to understand and use the first part of this book as a navigation manual and to use the provided formulae, the reader merely has to be familiar the trigonometric functions. Spherical trigonometric formulae have been kept to a bare minimum.

However, the second part of this book employs the basics of Positional Astronomy. It covers all the relevant topics as related to navigation and develops the formulae for precession, nutation, equation of the center, the equation of the equinox, the equation of time, the equation of the vertical, parallaxes, aberration and proper motion.

As I said, the main objective of this book is to provide the navigator with a comprehensive set of formulae that solely involve polynomials and trigonometric series that can be evaluated on any scientific calculator. With the help of those formulae, some additional data on star positions, and data on the perturbation of the Kepler motion of the Moon and planets, the navigator will no longer have to depend on the current NA. In cases where additional data is not available, he or she can still determine their position by relying on the data for the Sun and selective stars (provided herein). However, in the later cases, they will not be able to use the Moon and the planets for navigation.

Since the development of suitable formulae for an ephemeris was based on the motion of the Earth, Moon, and several planets about the Sun, or, more precisely, about their common center of masses, it was necessary to treat the Earth as a giant gyro that moves under the influence of the gravitational forces of the aforementioned bodies and describes a nearly elliptical orbit about the Sun. Therefore, in terms of Astro-Dynamics, we are dealing with a multi-body problem of a rigid body that can only be solved by means of employing approximations.

In terms of the underlying mathematics, we require the application of orthonormal transformations to the rigid coordinate system of the gyro Earth that, in turn, rotates about an inert system of coordinates that move along the orbit Sun–Earth. The problem of actually solving the equations that govern the motion of gyro Earth under the influence of said gravitational forces can be done only by employing approximations. Therefore, all formulae and data pertaining to the motion of the moon and the planets are approximations.

Of course, astronomers have been able to determine and predict the position of heavenly bodies without the use of explicit astro-mechanics a long time before Newton developed the concept of modern mechanics. However, the development of viable ephemerides that depend primarily on observations constitutes a tremendous task that requires years of effort on the part of many astronomers.

Sparrows Point, USA K.A. Zischka

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About the Author

K.A. Zischka was born in 1933 in Germany. After graduating from a Teacher College in Leipzig, he went into teaching and studied Chemistry by correspondence at the Technische Hochschule Dresden. He then returned to studying full time Mathematics and Physics at the Friedrich Schiller University in Jena where, after four years, he advanced to the level of Vordiplom-pre Master's degree. Subsequently, he was admitted to the Johann Wolfgang Goethe University in Frankfurt where he graduated with a Diploma in Mathematics and Theoretical Physics—Master's Degree. He then worked at the Research Center of the AEG in Frankfurt and continued studying towards a Dr. of Science Degree at the Technische Hochschule in Darmstad.

After graduating with distinction, he became Special Lecturer and then Assistant Professor at the University of Saskatchewan in Saskatoon, Canada. Later, because of his interest in sailing, he accepted an appointment at the University of Windsor, Ontario where he was elevated to the rank of Full Professor.

During his stay at the U. of W., he taught Applied Mathematics and Mathematical Physics. While on sabbaticals, he held a Visiting Appointment as Professor at the University in Karlsruhe, Germany and also had an appointment at the D.I. of F.R. in Bombay (Mumbai), India. Later, he worked for 2 years at a university in West Africa.

While living in Canada, he became a sailor, a bush-pilot, canoeist and something of an adventurer. (He kayaked solo down the Abititi River into the Hudson Bay.) He is also an avid fisherman, hunter, and survivalist and has owned several sail-boats and float planes.

For the past 20 years, he has also been traveling and living in various parts of Central America and studying Anthropology, Theoretical Medicine, and Architecture. He recently acquired a 41-foot Sea Wolf Ketch in which he plans to retire to Mexico.

Introduction

The diversity of the phenomena of nature is so great, and the treasures hidden in the heavens so rich, precisely in order that the human mind shall never be lacking in fresh nourishment.

Johannes Kepler (German Astronomer, 1571–1630) [33]

This book is based on the premise that the Earth, as viewed by a navigator, can be approximated by a sphere. A more accurate description would result if one adopts the concept of the Earth as a Geoid. However, for astronomical and navigation purposes it suffices to consider the planet to be a Spheroid of Revolution with its minor axis coincident with the North/South axis of the Celestial Sphere.

For the navigator, treating the Earth as a sphere has two great advantages: the first is that it conforms to the concept of the Celestial Sphere; and the second is that the corresponding geometry is fairly simple. (Spherical trigonometry is easier than Elliptical Geometry.)

The first part of this book treats the Earth as a sphere with the single exception of the derivation of formula for the parallax of the Moon (Sect. 3.3), where we have to take the oblateness of the Earth into consideration to distinguish between the astronomical and geocentric altitudes.

The role which accurate charts play in navigation cannot be overemphasized. Without accurate charts, the celestial coordinates become meaningless. Therefore, it is imperative that the navigator has a basic knowledge of Cartography (and, if possible, Geodesy). The making of accurate charts is based on mathematical projections. Because of that, the most relevant projections used in making nautical and aeronautical charts, as well as star finders, are discussed in various sections of this book.

In addition to having accurate charts at hand, the navigator should also be familiar with the accuracy of the plotting sheets. Plotting sheets that employ a longitude scale that is constantly proportional to the latitude scale for a wide range of latitudes (Non-Mercator plotting sheets) introduce an additional error in the plotting techniques that, by themselves, are based on the false premise that the azimuth of the assumed or Dead Reckoning position (DR) is always sufficiently close to the true azimuth. An analysis of Non-Mercator plotting sheets is provided in Sect. 1.3.

Because of the ramifications that the use of Non-Mercator plotting sheets have on the accuracy of the plotting techniques, the first part of this book also addresses the problems of flattening the Earth locally. For the purpose of navigation, elliptic or spherical geometry can be approximated by Euclidean geometry in the small. Practically speaking, this shows the size of an area on the surface of the Earth that can be approximated by an equal area on the tangent plane and depicted on a sheet of paper if drawn according to a suitable scale. In this case, great circles become straight lines and the sum of the angles of a corresponding spherical triangle equate to a hundred and eighty degrees.

The book also includes an elementary exercise for the interested reader to show clearly that despite knowing the geometry in the small, one cannot tell for sure what the geometry in the large—the true shape—might be.

Historically, it has been a long time since Bernard Riemann (1826–1866) conceived of spaces different from the Euclidean ones that result in geometries that, in the small, can be approximated by what we actually perceive. Another distinguished mathematician, Nickolai Lobachevsky (1792–1856) actually proved that in the infinitesimally small neighborhood of a point on his hyperbolic plane the trigonometric formulae of Euclidean geometry were also valid. Much later, the great physicist Albert Einstein (1879–1955) tried to convince us with his General Theory of Relativity that our real space must have a curvature, i.e., that it cannot be a Euclidean space.

In the 1830s, Lobachevsky considered the possibility of replacing Euclidean geometry with his own astronomical space but failed to convince the experts because of the absence of supporting experimental data since the vast distances required in his space were of the order of $10^7 \times$ a (where 'a' denotes the length of the semi-major axis of our solar system). The failure of providing experimental proof, of course, does not disprove his theory and is very similar in nature to the failure of proving that the Earth is a sphere by measuring the angles of a triangle on the surface of the planet and showing that the sum of the three angles is different from 180°. This experiment had been carried out rigorously by the distinguished mathematician K.F. Gauss (1777–1855).

As you begin to read this book, you might ask yourself: Why astronavigation and not Celestial Navigation? Are not they the same thing? The reasons for the distinction between the two are not based on etymological criteria but on astronomical and mathematical ones.

Astronavigation is an independent navigation system based on the observation of stars, the Sun (which is also a star), the planets and the Moon. It distinguishes itself from the concept of Celestial Navigation, as still practiced by the majority of navigators, by its independence on other navigational systems such as Dead Reckoning and intelligent guessing. Celestial Navigation as practiced by navigators around the world is merely a method that improves on an initial approximation to the true position of the ship or aircraft. Hence it is always assumed that the navigator more or less knows his position.

Celestial Navigation has been used all around the world for centuries. This book does not question its usefulness as an aid to navigation provided the user is familiar with its limitations. So far, no one has come up with an adequate error analysis. However, if the navigator already has a sufficiently close approximation to his true position, Celestial Navigation will provide an improved position provided the navigator follow certain well-established procedures for the selection of the heavenly bodies employed.

Currently, the availability of sophisticated programmable calculators has given Celestial Navigation a new image. For with such calculators, it is immaterial how often the user has to apply certain subroutines in their program. It is feasible to repeat the process of obtaining a "fix" using the LOP method by merely substituting the previously used DR or assumed position with the so obtained resulting fix and by pushing one more button, thereby obtaining a successive fix. Once the difference between the two successive results becomes negligible, the CN navigator has found his true position—perhaps so, but not necessarily true. In the end, we are back to experimental mathematics and nothing said above about Celestial Navigation has really changed. In Sect. 2.3, this book presents an iterative method that covers the method mentioned above. The criteria used by the proponents of said method merely constitute a necessary condition for the convergence, but not a sufficient one.

In astronavigation the determinations for a "fix" is obtained independently of the DR or assumed position. It requires only one more additional parameter in order to eliminate the ambiguity inherent with the exact solution. This parameter can be readily obtained by one more astronomical observation as, for instance, by noting the approximate azimuth of two celestial objects or by using an approximate latitude.

The observer obtains his position by using exact spherical trigonometric formulae. Any resulting errors due to errors in measuring altitudes, time, and parameters like refraction, SD and HP, can be evaluated analytically since the underlying functions are analytical ones. Whenever numerical methods are being employed their error bounds can be determined.

From a mathematical point of view, this book is based on the question of the solvability of a system of two transcendental equations, also known as Fundamental Equations of Navigation (FEN), since each of these two equations represents a circle of equal altitude on the celestial sphere with the celestial object in its center—solvability not only with regards to latitude and longitude, but also with regards to the variable of "time" which enters those equations through the "time dependency" of all the other parameters. This book also analyzes the problems of the existence, non-uniqueness, and ill-conditioning of solutions of FEN and also provides approximate methods for solving it.

A famous physicist once said, "Look at the equation. It tells you everything you want to know." This book shows that methods of celestial navigation turn out to be special cases of the mathematical methods of linearization and iteration. In this context, astronavigation can provide navigators with their approximate position anywhere on the globe without prior knowledge of other approximations. It can also provide the navigator with the approximate time of observation by using the Equation of Computed Time.

My motives for writing the second part of this book derive from a long search for a comprehensible, concise and compact manual on Positional Astronomy which included the necessary ephemerides for the Sun and stars which were normally at a navigator's disposal. By compact, I mean, something that a navigator could carry in the large cargo pocket of his or her pants. By comprehensive, I mean, something that would give the navigator the relevant equations at a glance and also enable him to review the underlying theory should he or she care to do so.

When this idea of designing a manual first came to me, GPS was not available. Navigators had to rely on bulky tables and almanacs. Today, a wide variety of electronic devices have freed navigators from the need for such bulky books and tables. However, as anyone who has spent any time at sea knows, salt water and electronics are not compatible. At one point or another, some or all of a person's electronic devises can and do fail at sea. Part II of this book covers such an emergency and should make the reader fairly independent of the software as used in celestial navigation.

But aside from having something to cover an emergency, those of us who are seriously interested in all aspects of navigation like to have something handy to review, if not even to learn some of the basics of positional astronomy as part of the knowledge necessary to understand ephemerides. Accordingly, the objective of the second part of this book is twofold: namely to provide the reader with a concise and comprehensible manual on positional astronomy as it applies to astronavigation; and to furnish him with the concise algorithms for finding the position of the sun and the 57 navigational stars at any given instant.

All the algorithms used in the second part of this book can be executed on any simple, inexpensive scientific calculator, thus freeing a navigator from being tied to a programmable calculator or a PC.

The formulae for the algorithms are either exact ones or approximations to the exact ones. Therefore, the resulting algorithms vary with respect to their degree of accuracy leading to the development of three types of ephemerides.

Type 1: Low precision ephemerides for the sun and navigational stars should yield numerical results $\pm 1'$.

Type 2: Intermediate precision ephemerides should provide results with an accuracy of $\pm 20''$.

Type 3: Compressed low precision ephemerides should come fairly close to the low precision ephemerides.

All the algorithms provided in this book can be executed without knowing the underlying theory and therefore, this manual can be compressed to a few pages that will easily fit into a shirt pocket.

Appendix A includes all the data needed for the ephemerides for the 57 navigational stars. It includes RA, δ, μ_{α}, μ_{δ}, and Π. Therefore, the manual holder is not dependent on the availability of other publications.

For the reader who is also interested in Astro-Dynamics, a brief section on the earth as a gyro has been included showing where some of the limitations of a strictly theoretical approach enter the quantitative analysis of the theory of orbits of rigid and non-rigid bodies.

I should like to disclaim this as a "complete" treatise on the discipline of astronavigation. It is, however, more complete than anything I am aware of or have found on the subject of Celestial Navigation to date.