FORUM

Longitude without Time

Francis Chichester

In 1964 during the second single handed transatlantic race Colonel H. G. Hasler in *Jester* lost the correct time and, having no radio, could not get a time check. Thus he could not obtain the longitude but, being a fine seaman and practical navigator, made a landfall on Nantucket light-vessel by a method used by old sailing ship skippers. This was to position himself on the correct parallel of latitude and sail due east or west to the destination.

I wondered what I would do in the same predicament. The problem of longitude had plagued all navigators until the invention of the marine chronometer. Before that the only way of finding longitude was by making a lunar observation which consisted of measuring the angle between the Moon and planets or the Sun and comparing it, after many corrections, with the angle tabulated in the Nautical Almanac. These lunar distances are not now given in the Almanac so that nowadays it would be necessary to calculate the distance between the Moon and Sun or planets before starting the observation. The only text I could lay my hands on at short notice which showed how to work a lunar was the 1840 edition of Raper's *Navigation*.

The amount of computation to work out a lunar distance was astounding and the idea of undertaking this calculation at sea in a small yacht, especially single handed, horrified me.

Pondering the problem I thought of a simple solution.

Latitude is first accurately determined by meridian altitude and then, at a convenient time, a normal Sun and Moon observation is made. The Sun-Moon fix will vary in position according to the G.M.T. used when computing it, but the only correct value of G.M.T. is that which will place this fix on the already known latitude. The correct fix derived from the correct G.M.T. must be where the line joining two Sun-Moon fixes calculated for guessed-at G.M.T.s cuts the known latitude. The solution depends on the rapid change of the Moon's position which causes a large difference of altitude for a comparatively small difference of time between observations.

An example, using Oporto (on the same latitude as Nantucket) as the desired landfall, follows.

The altitudes of the Sun and the Moon are observed at an estimated G.M.T. of, say, 16.00. Three altitudes of the Sun, six of the Moon and finally three more of the Sun are taken. If the times by a stopwatch for the Sun and Moon observations are not then exactly equal, the Sun's altitude is then interpolated to make them coincide. (This is easy to do by noting the Sun's rate of change of altitude in the sight reduction tables.)

Using the standard air navigation tables (H.O. 249 = A.P. 3270) these sights are then reduced using an assumed G.M.T. half an hour earlier than it is guessed to be (i.e. $15^h\,30^m$ G.M.T.); the resultant position lines are plotted and a fix obtained. On the small-scale chart I used, this position was 41°45′ N., 7°40′ W.

The same observation is then reduced using a G.M.T. half an hour later than estimated (i.e. $16^h 30^m$ G.M.T.) and the Sun and Moon position lines plotted for this new time. This gave a position 40° 15′ N., 22°17′ W.

At the nearest convenient time before or after the Sun-Moon observation the Sun or a star is then observed for latitude. Accurate time is not needed for this observation. The compass will indicate when the Sun is true north or south and altitude shots taken over a period of a few minutes will enable the meridian altitude to be deduced from the highest altitude of the series. Note the latitude thus obtained; in this case it was 41° N.

The two fixes are joined and where the line cuts the observed latitude (41°) gives the correct position at the time of the Moon observation (change in latitude since the meridian altitude must be allowed for). In this example the longitude at the time of the Moon observation was pricked off as 15°06′ W., which showed an error of 6′; accordingly the G.M.T. at the time of the Moon observation was $16^h 00^m 24^s$ giving an error of 24^s.

For greater accuracy the Sun-Moon fixes should be recomputed for times within a few minutes either side of $16^h 00^m 24^s$ G.M.T., say at $16^h 10^m$ and $15^h 50^m$. Where the line joining these two fixes cuts the latitude 41° would give a more accurate longitude.

Mr. D. H. Sadler comments:

So far as I can discover, without a prolonged search, Mr. Chichester's actual method of determining position at sea without reference to G.M.T. is new; and it is certainly elegant. However, the fundamental principle on which it is based is old, and is the same as that underlying the conventional observation of 'lunar distance', namely that of determining G.M.T. from the position of the Moon in its orbit; the difference is that the lunar distance is measured from the horizon (i.e. an observed altitude) instead of from the Sun or a star.

This variation of the lunar distance method was, of course, well understood and investigated two hundred years ago. It suffers from several disadvantages compared with the conventional method, and it is for this reason that (apparently) it was never greatly used in practice; the disadvantages are:

(a) *observational*: the altitude can only be derived when the horizon is clearly defined, which implies freedom from low cloud or haze, and an illuminated horizon;

(b) *precision*: apart from other sources of error, the precision of an observed altitude is limited by that of the dip of the horizon, and is consequently much less than that of a lunar distance to the Sun or star;

(c) *applicability*: the method can only be used to full precision when the Moon's orbit is nearly perpendicular to the horizon, whereas the Sun, planets and the zodiacal stars are always sufficiently close to the orbit; it is of no value to observe the Moon's altitude near the meridian.

The advantages are a considerable simplification in the reduction, since only the standard processes of calculating the altitude and applying altitude corrections to the observed altitude are involved. The actual method proposed by Mr. Chichester combines the determination of G.M.T. with that of a conventional fix. It provides a vivid, and easily understood, demonstration of the effect of varying the assumed value of G.M.T., and makes possible a realistic assessment of the uncertainty in the deduced longitude; and it possesses the great merit of simplicity and elegance.

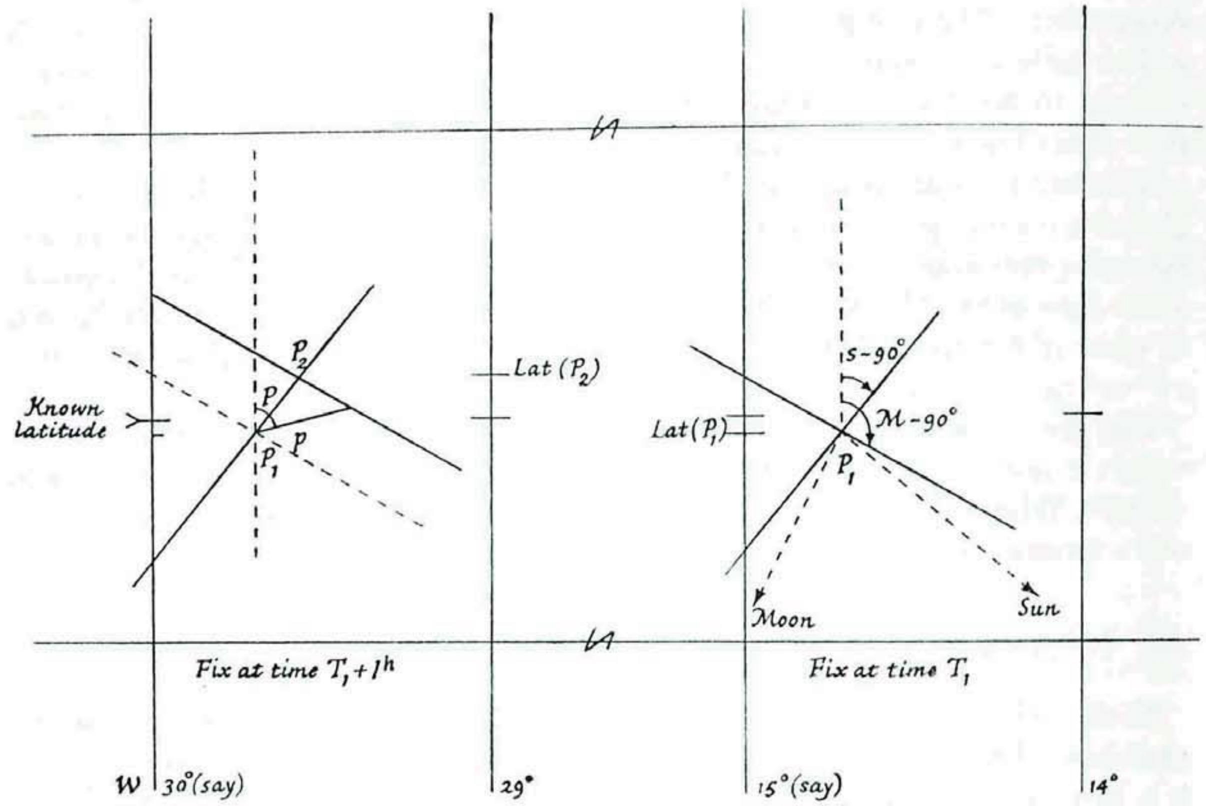

FIG. 1. Typical plot for two fixes 1 hour apart

It is of interest to enquire how the precision varies with the different positions of the Sun (or, of course, a star) and the Moon; for example, a little consideration will show that the Sun must not be near the meridian, as otherwise the change in latitude will be too small. The diagram illustrates a typical plot for two fixes P_1 and P_2 corresponding to G.M.T.s one hour apart. The second fix is obtained from the first by moving it 15° westwards, and then translating the Moon position line through a small distance p in position angle P, where

$$p \sin P = 41'.0 - v \qquad p \cos P = d$$

in which v and d are the hourly variations of the Moon's G.H.A. and Dec. in *The Nautical Almanac*; P is always between 60° and 120°. If the azimuths of the Sun and Moon are S and M respectively, then (approximately) the difference in latitude between the two fixes P_1 and P_2 is

$$\frac{p \cos (M - P) \sin S}{\sin (M - S)}$$

and the error in the latitude, due to small errors e_S and e_M in the observed altitudes of the Sun and Moon is, approximately,

$$\frac{e_S \sin M + e_M \sin S}{\sin (M - S)}$$

In order to obtain the greatest precision in G.M.T. (and thus in longitude), the ratio between the error and the difference of latitude must be as small as possible, that is, the factor

$$\frac{e_S \sin M + e_M \sin S}{p \cos (M - P) \sin S}$$

must be small. Clearly sin S should not be small (the Sun cannot be near the meridian) and neither should cos $(M - P)$ (the Moon also cannot be near to the meridian); the optimum conditions are when the Sun and Moon are both near the prime vertical, preferably one east and one west of the meridian. The expression is independent of the angle of cut of the two position lines, but obviously too small an angle must be avoided.

In Mr. Chichester's example, P is 72°.5, M is about 280° and S about 258°; so that the conditions are good, although the angle of cut is small and the Moon, only 60 hours before New Moon, must have been difficult to observe. Actually, the errors in longitude corresponding to errors of 1′ in the observed altitudes (or, of course, also in the calculated altitudes and in the altitude corrections) are about 30′ for both the Sun and the Moon—almost the maximum precision attainable; the error of 6′ is thus fortuitously small. For such calculations the working unit in altitude should certainly be 0′.1; the use of sight reduction tables for air navigation, with a tabular precision of 1′, introduces unnecessary sources of error.

It is curious that this method has not been used before; or, if it has, is not well known. Perhaps it is a consequence of the fact that the present position-line fix only dates from the time when 'lunars' had already become obsolete.