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A NAVIGATION COMPUTER

O. E. BROWN AND J. J. NASSAU, Case Institute of Technology

1. **Introduction.** This paper presents a mathematical description of a computer of the circular slide rule type prepared for the graphical solution of the spherical triangle, particularly as applied to the astronomical triangle. The computer is rectangular and on each side a transparent rotor is attached with the pivot point at the lower left hand corner. On each rotor appear two one-parameter families of curves, one family consisting of constant altitude curves and the other of curves of constant azimuth angle. Through each transparent rotor appears the *grid* made up of many dots. These dots define two sets of curves, one for constant meridian angle (*i.e.*, local hour angle with a range from 0° to 180°) and the other set for constant declination. The meridian angles represented are integral degrees while the declinations used are those of 44 navigation stars and a few integers under 30°.

Figure 1 shows the grid, Figure 2 the rotor, both with some intermediate curves and dots removed to permit the necessary reduction in size.

The finished computer has the following properties:

1. At one setting of a rotor both the altitude and azimuth angle are obtained.

2. The errors in altitude are rarely over four minutes of arc and their average without regard to sign is under two minutes.

3. Although, from the map projection point of view, each grid and rotor is of a size to represent $\frac{1}{8}$ of the celestial sphere, the one grid and two rotors are sufficient to solve any spherical triangle.

4. The declinations of each of 44 navigation stars are plotted permanently on the grid.

5. The entire device is about 11 by 12 inches and less than $\frac{1}{4}$ of an inch thick.

2. **A description of the mathematical principles of the computer.**

The Astronomical Triangle. The fundamental problem of finding an observer's position by celestial observation is that of solving the PZS spherical triangle for h and z, given d, L and t. The vertices of this triangle are Z, the observer's zenith, P, the celestial pole above the horizon, and S, the heavenly body observed and hereafter referred to as the *star*. The arcs SP, PZ, and ZS are respectively the complements of the star's declination, the observer's assumed latitude, and the star's altitude. Given the quantities d, L, and t, the

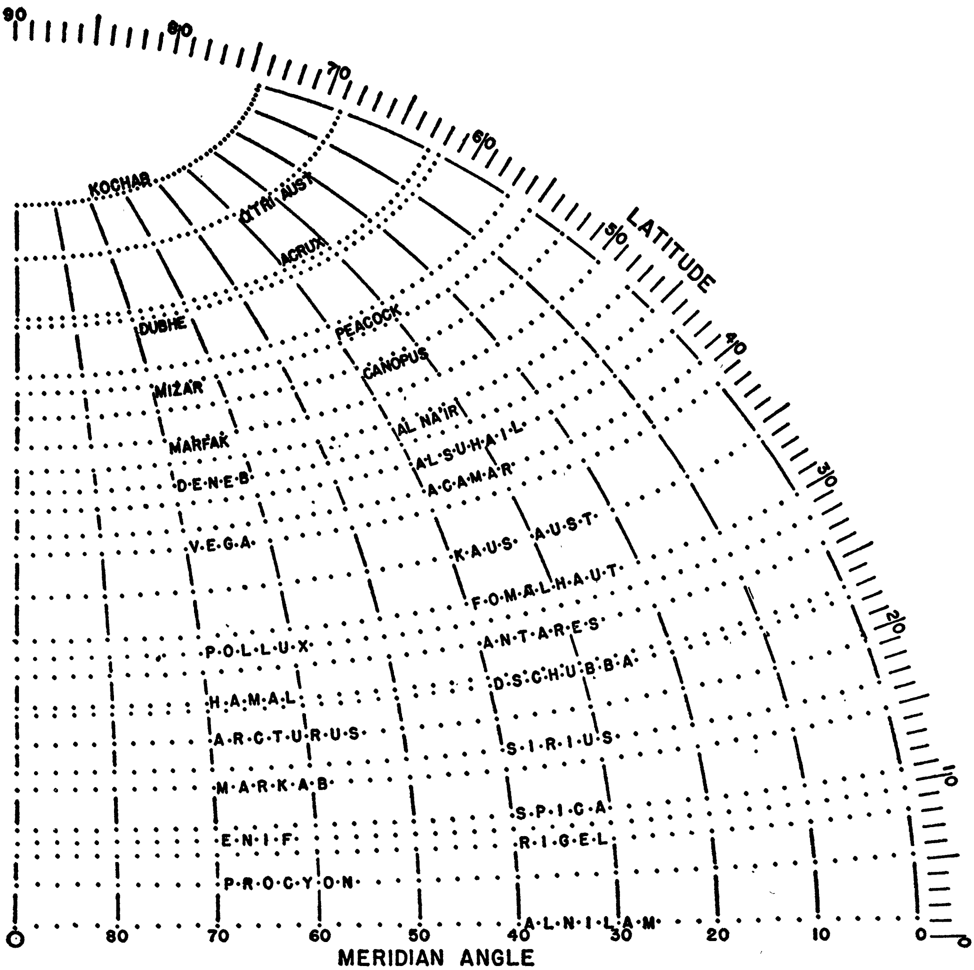

Fig. 1

quantities h and z are determined. They may be computed in any numerical case by using the formulas of spherical trigonometry or they may be found for given integral values in such tables as H.O. 214,* and H.O. 218.

* U.S. Hydrographic Office.

Graphical Solution. The required quantity, h, is expressed in terms of the known quantities d, L, and t by the law of cosines as

(1) $$\sin h = \sin d \sin L + \cos d \cos t \cos L.$$

By the substitution

(2) $$\sin d = y, \qquad \cos d \cos t = x,$$

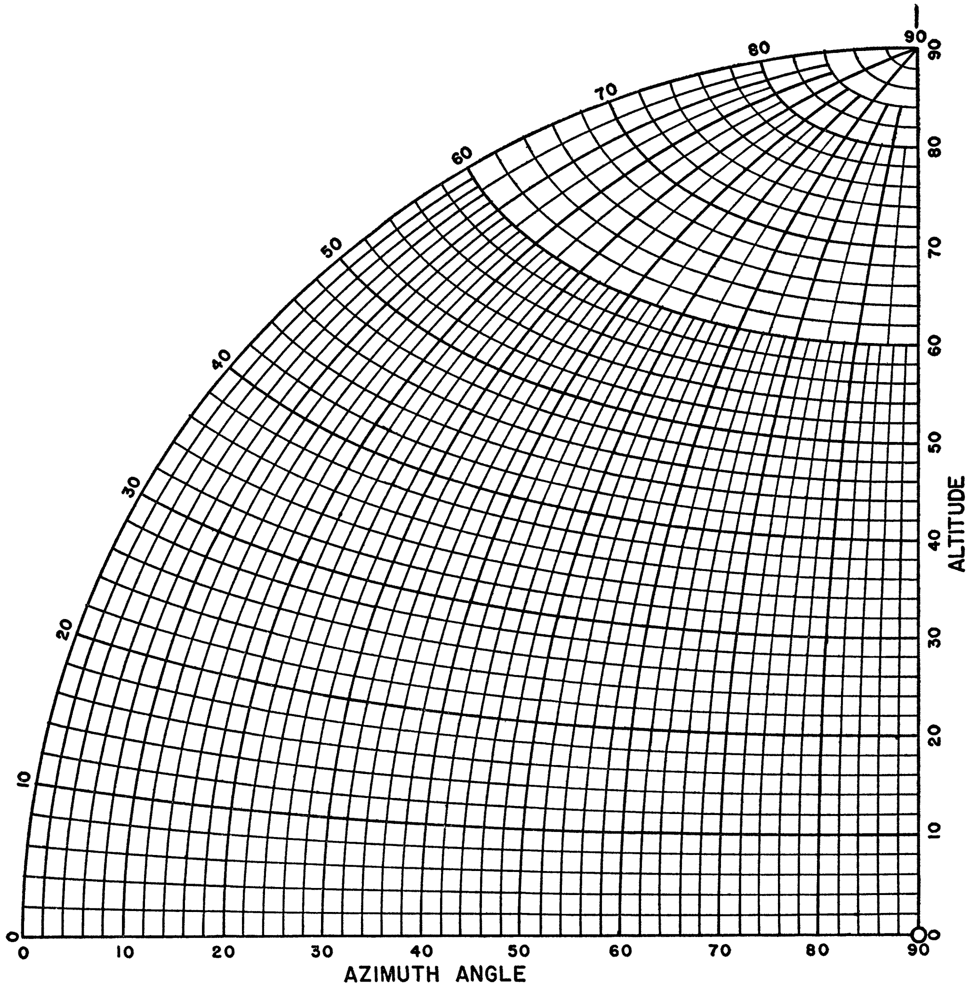

Fig. 2

Equation (1) becomes

(3) $$x \cos L + y \sin L - \sin h = 0,$$

which is the equation of a straight line with L as normal angle and sin h as normal intercept. Equations (2) are used to define a coördinate system. It is apparent that the "d=constant curves" are straight lines parallel to the x axis while the "t=constant curves" are ellipses with common major axis. The figure, thus constructed, ultimately becomes the grid. These two families of curves are transformed to improve the scale and are replaced by dots at their points of intersection. Integer declinations are also replaced by the actual declinations of the 44 stars selected.

Applying the law of cosines again to the same triangle, we obtain:

$$\sin d = \sin h \sin L + \cos h \cos z \cos L, \tag{4}$$

which, after the substitution,

$$y' = \sin h, \qquad x' = \cos h \cos z \tag{5}$$

becomes

$$x' \cos L + y' \sin L - y = 0. \tag{6}$$

With L and y constant this equation defines another straight line with normal intercept y and the same normal angle L as that of the previous line. Obviously the coördinate system defined by Equations (5) is exactly similar to that defined by Equations (2), except that now the straight lines represent constant values of h while the ellipses correspond to constant values of z. As in the previous case, the figure thus obtained is transformed to improve the scale and becomes the rotor of the computer.

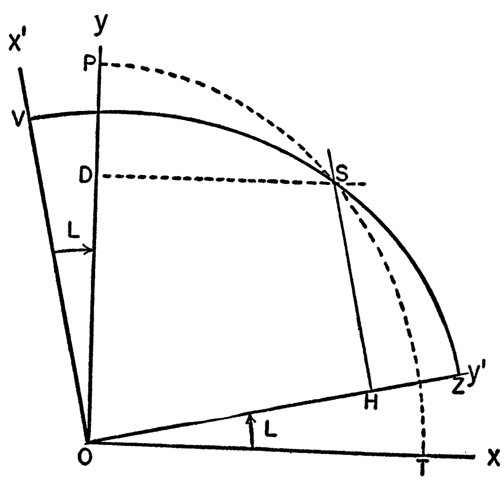

Fig. 3

Since equations (3) and (6) have the same normal angle, and the normal intercept of each is the ordinate of the other, the two may be superimposed as shown in Figure 3. The dotted ellipse PST represents one of the hour angle curves of the grid and corresponds to an assumed value of t. The dotted line SD is one of the grid lines of constant declination to be marked on the finished computer by the name of the corresponding star. The straight line SH perpendicular to OY', and the ellipse ZSV represent, respectively, constant altitude and constant azimuth curves on the rotor. The angle L of the figure is the angle through which the rotor has been turned and corresponds to the assumed latitude of the problem. This is essentially the underlying principle of the computer. The star S is located on the grid from its given coördinates (t, d) on the XOY-system, the rotor is turned through the assumed latitude L and the coördinates (z, h) of the star are read on the $X'OY'$-system.

The Transformation. As the reader will observe, the lines and ellipses described previously in connection with the XOY-system make up an orthographic projection of the hour circles and parallels of the celestial sphere. The corresponding curves of the rotor represent the "parallels" of constant altitude and the vertical circles through Z. Computers of the same fundamental nature as the one described here have been built using the orthographic projection as well as some using a stereographic projection. These two projections have some advantage in preparation because of the lines, ellipses, and circles which make up their coördinate systems. The orthographic projection exhibits great radial crowding around the outside while the stereographic projection has its greatest crowding at the center. A suitable projection in our case is obtained by transforming the orthographic projection by means of the polar coördinate equations

$$\theta' = \theta, \qquad r' = \arcsin r.$$

The resulting figure is known as an *azimuthal equidistant* projection about the point O. All distances from O are true while the circumferential scale varies from the radial scale to $\pi/2$ times the radial scale.

Reduction to One Quadrant. Any quadrant of the grid figure is like any other quadrant except possibly for a reflection, and the same is true of the rotor figure. Thus one quadrant of grid figure and two quadrants of rotor figure are sufficient to represent any configuration. One quadrant of the grid figure is printed in duplicate on both sides of an opaque sheet and one transparent rotor quadrant is mounted over each.

Reference Lines of the Grid and Rotor. The grid carries rows of dots corresponding to the declination of the 44 navigation stars and a few additional rows for integer declinations under 30°. The dots on each row are spaced one degree apart, thus forming the family of hour circles. Each dot, therefore, represents the position of a star of known d and t, taken to the nearest integer. The rotor carries the set of equal altitude curves one degree apart and azimuth lines two degrees apart. The latitude scale is marked on the grid at the limiting circumference of the rotor (Figure 1).

3. Uses. The computer is primarily a navigation instrument for securing the necessary quantities (h, z) to establish the line of position. The value of t is obtained in the usual way with the assumed longitude adjusted so that t is equal to the nearest integer. Knowing the value of t and the coördinates of the star observed, the corresponding dot of the grid is located and circled with a pencil. The rotor is turned to the assumed latitude and the value of h is read to the nearest minute by eye interpolation between the constant altitude curves. The value of z is obtained likewise and read to the nearest degree. In observing a body in the solar system the required dot is penciled in at its proper position on the grid and h and z obtained as before. The surface of the grid is matte-finished to permit marking and erasing.

The above procedure conforms to the standard method for determining the line of position. The computer may also be used for identifying a star from its approximate altitude and azimuth.

4. **Accuracy.** The device is amazingly accurate. The navigator wants the azimuth only to the nearest degree but desires the altitude to the nearest few minutes. H.O. 218 gives the altitude to the nearest minute of arc while H.O. 214 gives the altitude to the nearest tenth of a minute. Bearing in mind that one minute of arc amounts to one nautical mile and the additional fact that the sextant reading, with which the computed altitude is to be compared, is probably in error by a few minutes it appears that this computer is sufficiently accurate for position fixing in the air and in surface craft if an error of a very few miles can be tolerated. The computer was used in long over-ocean flying with results essentially as stated above.

THE JEEP PROBLEM: A MORE GENERAL SOLUTION

C. G. PHIPPS, University of Florida

1. **Introduction.** In this MONTHLY, January, 1947, N. J. Fine discusses the problem of advancing a single jeep a given distance beyond a base of supply. He finds where the dumps must be established in order that the amount of gasoline used is a minimum. This problem is one of a more general class whose solution I wish to discuss in detail here.

Now a solution which yields the minimum amount of gasoline for a given distance is also a solution which yields the maximum distance for a given initial amount, and conversely. It so happens that a solution for the latter conditions appears much easier to obtain. This is therefore the type of problem to be treated first.

Before formulating the various problems in detail, it is necessary to define the special terms that will be used. A group of jeeps traveling together will be referred to as a *caravan*. A *station* is defined as a point where it is necessary to establish a dump, or where the number of jeeps in the caravan is changed. The *home station* is the fixed base from which all jeeps originally set out. A *stage* is defined as the distance between two successive stations. In general, the dif-