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Estimation of Altitude Accuracy of Punctual Celestial Bodies Measured with Help of Digital Still Camera

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ABSTRACT: Measurement of altitude traditionally made with sextant may be done with help of digital still camera. Factors influencing accuracy of this measurement done with help of such a device are described in the paper. Values of errors introduced by each of these factors are estimated basing on example technical data of typical digital camera. This analysis shows, which factors are the most important and if accuracy of altitude is sufficient for purposes of celestial navigation.

1 ALTITUDE MEASUREMENT WITH DIGITAL STILL CAMERA

1.1 *Demand accuracy*

Demand accuracy of single altitude measurement may vary in dependence from the assumed number of measurements to be done. The target is the suitable accuracy of position, which may be obtained by increasing the number of measurements, when single measurement has small accuracy. As a reference approximately 1′ (one angular minute) may be assumed as demand accuracy of altitude. Digital still camera may be used for measurement of angular distance, but it is necessary to investigate, if with its technical parameters obtainment of required accuracy is possible.

1.2 *Typical digital still camera*

Features of popular digital still cameras:

1 mostly equipped with charge-coupled device (CCD) image sensor, residual with complementary metal-oxide-semiconductor (CMOS);
2 color image sensor with Bayer pattern mosaic, 24 bit color depth;
3 image sensors array covered with micro-lens;
4 equipped with mechanical shutter;
5 effective diameter of objective aperture about 15 mm, comparable with diagonal of array;
6 number of active pixels over 3 million with array format 3:4, dimensions of array for these numbers in pixels minimum 1500×2000;
7 square pixel, pixel size 5 μm, regarding 6° array dimensions in mm 10×7.5 and diagonal 12.5;
8 angle of field of view along longer side about 50°, along shorter 40°; with the usage of zoom 3× it is adequately 20° and 15°;
9 variable focal length of objective and variable focusing distance allowing registration of image from infinity to few centimeters, regarding 6°, 7° and 8° focal length in pixels is equal (2000/2)/tan(50°/2)=2145, it is 10.7 mm.

Values of technical parameters of particular models may differ from listed above. Particularly big differences are within diameter of objective, number of pixels and pixel size (features 5°, 6°, 7°). Increasing number of pixels within the same dimension of array may be noticed in models introduced on market. In this situation pixels are smaller. Maintenance the same level of luminous energy on single pixel required increasing of light flow and this may be done by widening objective area. Regarding 8°, measurement of altitude with help of digital still camera is limited to 50°.

1.3 *Calculation of angular distance*

Assuming:

- focusing distance is set on infinity – focal plane, which coincides with image sensor array is in focus of objective,
- image is formed accordingly to rules of central projections on plane – plane of array is perpendicular to optical axis,
- pixels create orthogonal Cartesians coordinate system,

unit vector of direction p of punctual object P recorded on array, in orthogonal coordinate system $Oxyz$ set by main plane of objective and its optical axis (Figure 1) may be calculated from

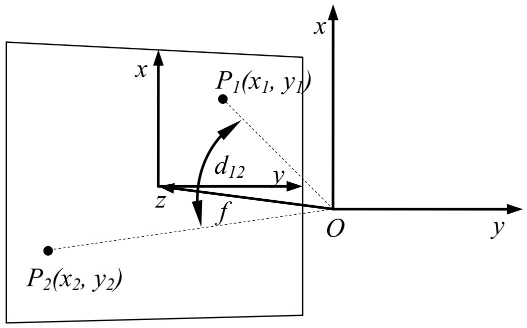


Fig. 1. Association of image sensor array with coordinate system Oxyz set by main plane of objective Oxy and its optical axis Oz

$$p = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \dfrac{x}{\sqrt{x^2+y^2+f^2}} \\ \dfrac{y}{\sqrt{x^2+y^2+f^2}} \\ \dfrac{f}{\sqrt{x^2+y^2+f^2}} \end{bmatrix} \tag{1}$$

where x and y are coordinates of P on array plane in relation to point indicated by optical axis, f focal length, directions of axis Ox and Oy are equal to appropriate directions on matrix and direction Oz agree with optical axis.

Direction of this unit vector is the same as on real object (opposite turn). Angular distance d_{12} between real objects, recorded on array as points P_1 and P_2 may be obtained from scalar product of their unit vectors p_1 and p_2

$$\cos d_{12} = p_1 \circ p_2$$

$$\cos d_{12} = x'_1\, x'_2 + y'_1\, y'_2 + z'_1\, z'_2 \tag{2}$$

$$\cos d_{12} = \frac{x_1 x_2 + y_1 y_2 + f^2}{\sqrt{x_1^2 + y_1^2 + f^2}\sqrt{x_2^2 + y_2^2 + f^2}}$$

2 FACTORS INFLUENCING ACCURACY

2.1 *Inaccuracy of lens area, inhomogeneity of glass and optical aberrations of objective*

Distortion of direction of light ray result from these elements should not exceed the angle of theoretical resolving power of objective, arising from diffraction of rays on diaphragm. This may be treated as a rule during production of optical elements. For objective with circular aperture this angle is given by formula

$$u = 1.22\frac{\lambda}{D} \tag{3}$$

where λ length of wave, D diameter of objective aperture (diaphragm), u in radians (Wagnerowski 1959). For yellow-green light λ=0.000556 mm and objective aperture D=15 mm this angle u=0.15′.

2.2 *Inaccuracy of placing of pixels optical centers on nodes of virtual net with equal mesh*

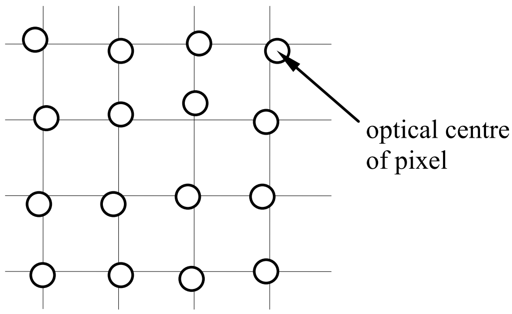


Fig. 2. Inaccuracy of placing of pixels optical centers on nodes of virtual net with equal mesh

There is no direct information about this in technical specifications of digital image sensors. In case of CMOS, basing on characteristic of this technology one may conclude that this error is two orders smaller then pixel size. But regarding feature 3° performance of micro-lens layer has the main implication here. On microscopic photographs of micro-lens layer error arising from this is imperceptible in comparison to pixel size.

2.3 *Non-perpendicular pixels arrangement*

This information may be found in technical specifications of some manufacturers for example Dalsa company (Dalsa 2002).

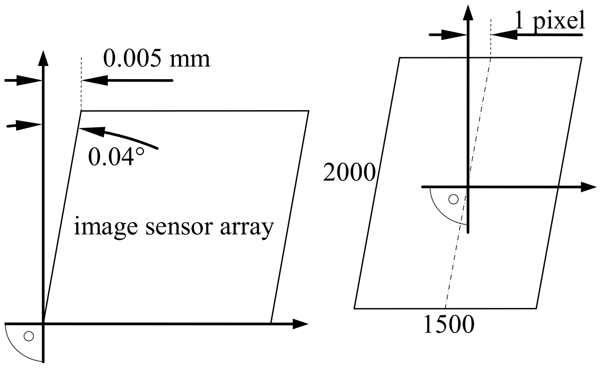


Fig. 3. Non-perpendicular pixels arrangement on image sensor array

In the specification quoted in references (CCD image sensor, 1024×1024 pixels, pixel size 7.4 μm) non-perpendicular pixels arrangement allowance is 0.005 mm measuring displacement of parallel edges perpendicularly to them (Figure 3). For this sensor the angle between directions of main axis of array may differ from right angle about 0.0378°. At distant of 1000 pixels (feature 6°) from the middle of array it is no more then 1 pixel for this angle. This error may be neglected if altitude is measured parallel to one of directional axis (parallel to one of the edges of array), because it shifts mutually celestial body and visible horizon parallel to the last one, not influencing the altitude.

2.4 *Inaccuracy of arrangement of pixels at one plane*

According to specification mentioned above allowance of arrangement of pixels at one plane is <7μm – about pixel size (Figure 4).

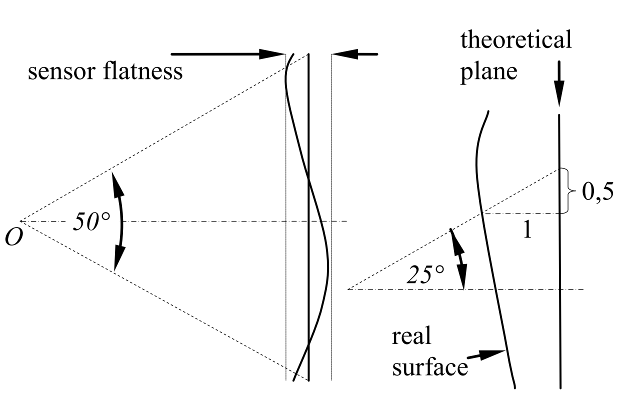


Fig. 4. Inaccuracy of arrangement of pixels at one plane

Maximal distance between real surface of the array and theoretical plane is equal half of the allowance. Error of object location arising from this increases with angular distance of object from point on array indicated by optical axis (called later as focus). For object recorded near boundary of field of view for data of feature 8° error is maximally a half of the maximal distance. For example, if value of allowance is equal two pixel sizes, maximal error of location may be half of the pixel.

2.5 *Non-perpendicular direction of optical axis to array plane*

Array is placed on chip, chip in socket, socket on electronic board and this is connected with objective by camera casing. If this construction is not calibrated to make array and main plane of objective parallel, then each of the connections carries error in their parallelism. According to specification mentioned above parallelism allowance of array and plane of chip casing is equal 1.4/100 (0.8°).

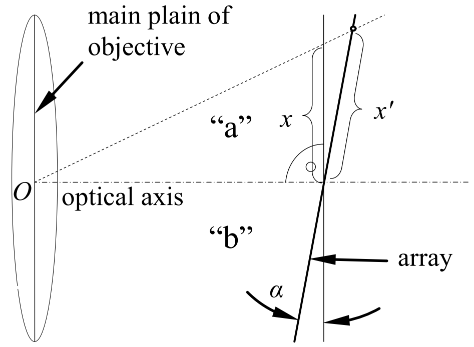


Fig. 5. Non-perpendicular direction of optical axis to array plane.

Difference Δx between measured distance x' and true distance x (for array perpendicular to optical axis) at part of array open from main plane of objective (area "a" on Figure 5) is positive and given by formula

$$\Delta x_a = x' - x = x' - \frac{\cos\alpha}{\frac{\sin\alpha}{f} + \frac{1}{x'}} \tag{4}$$

and at part open to opposite side (area "b") negative

$$\Delta x_b = x' - x = x' - \frac{\cos\alpha}{\frac{1}{x'} - \frac{\sin\alpha}{f}} \tag{5}$$

where α is angle of array deviation from perpendicular to optical axis and f focal length. For focal lengths short in comparison to measured distance x' values Δx are considerable and regarding 8° they reach 8 pixels for α=1° (Figure 6).

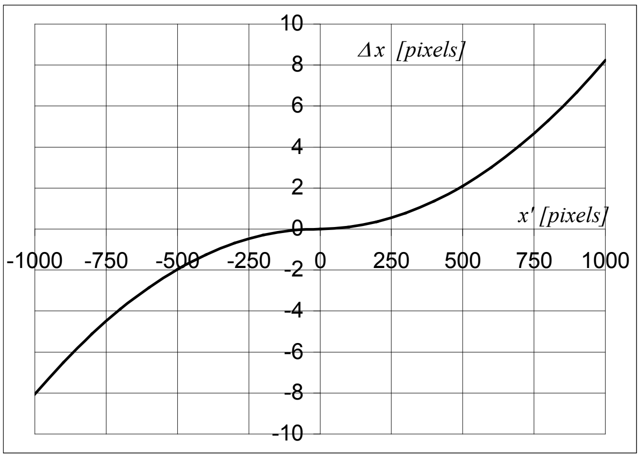


Fig. 6. Difference Δx between distance of object from focus x′ measured on array non-perpendicular to optical axis and true distance x on perpendicular array in function of x′.

Absolute values of differences in both areas for the same values of x' are almost equal. Therefore if focus is lying exactly at half a way on straight line between both objects, then this error compensate itself. In practice such a measurement is difficult to be done, but one may assume certain allowance in distance, with which observer is able to divide section between to points on two even parts. Assuming, that after such a division one part may be shorter then another maximally about 1/3, in extreme case error of distance is 3 pixels.

2.6 *Pixel size*

Location of punctual object on image sensor array may be obtained with accuracy of half of the pixel size.

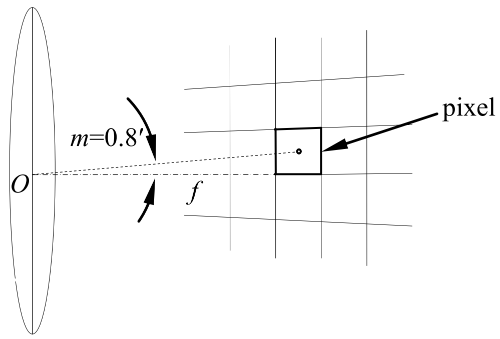


Fig. 7. Accuracy of location of punctual object arising from pixel size

Regarding 9° angular resolving power m near focus is equal arctan(0.5⊙1/2145) =0.8′.

2.7 *Variable angular resolving power*

Scale of the image increases with the distance from focus. With constant spatial resolving power of array, angular resolving power m in radians is given by equation

$$m = \arctan\left(0{,}5 \cdot \frac{1}{f} \cos^2 \beta\right) \tag{6}$$

where β is angular distance from focus and f focal length in pixels.

Accuracy of location in spherical coordinate system calculated on location on array is minimal for objects imaged in focus and increases with distance from this point. Regarding 8° and 9° angular resolving power near boundary of field of view is equal 0.66′.

Assuming, that interior spot and the first ring of diffractive image of star projects on array, diameter of star image is equal 4×u=0.6′. It may be contained within one pixel and may occupy no more then 4 pixels. For single-bit depth of colors accuracy of location increases with the square root of number of pixels affected – for 4 pixels – twice. Image of star recorded on array has in fact diameter about 3-4 pixels. It is the result of algorithm of obtaining colors (feature 2°) and for this reason the image is enlarged 1 pixel to each direction. It not improves accuracy anyway.

2.8 *Differentiation in location of middle of array and focus*

Coordinates of focus are necessary for calculation of altitude. With regard on lack of these data coordinate of middle of array are accepted in turn. If accepted coordinates differ from focus (Figure 8)

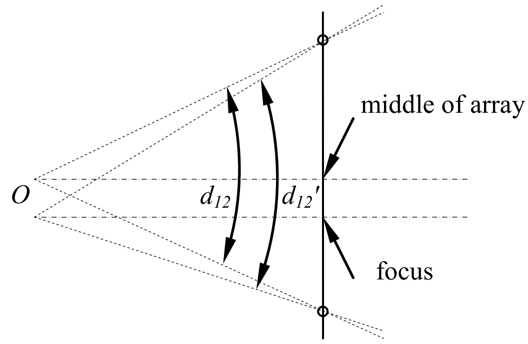


Fig. 8. Focus is not in the middle of array

about shifts Δx and Δy along appropriate axis, then true distance d_{12}' is given by formula

$$\cos d_{12}' = \frac{(x_1+\Delta x)(x_2+\Delta x)+(y_1+\Delta y)(y_2+\Delta y)+f^2}{\sqrt{(x_1+\Delta x)^2+(y_1+\Delta y)^2+f^2}\sqrt{(x_2+\Delta x)^2+(y_2+\Delta y)^2+f^2}} \quad (7)$$

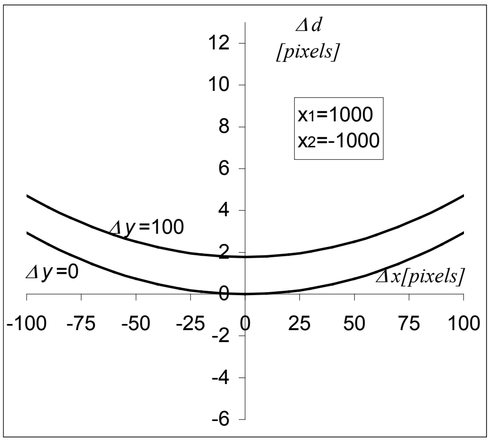
where x and y with indexes 1, 2 are coordinates of points 1, 2 in relation to the middle of array.

Value of difference Δd in pixels calculated from

$$\Delta d = f(d_{12} - d_{12}') \quad (8)$$

between angular distance d_{12} calculated assuming focus is in the middle of array and true distance d_{12}' in function of Δx and Δy is presented on Figure 9.

Calculation is made for maximal possible measurement of distance, parallel to the edge and through the middle of array, regarding features 6° and 9° ($y_1=y_2=0$, $x_1=1000$, $x_2=-1000$, $f=2145$) – upper graph – and with distance section asymmetrical in relation to middle of array – lower graph. Relevant error may reach significant values (over ten and so pixels) and may be predominant among listed above.



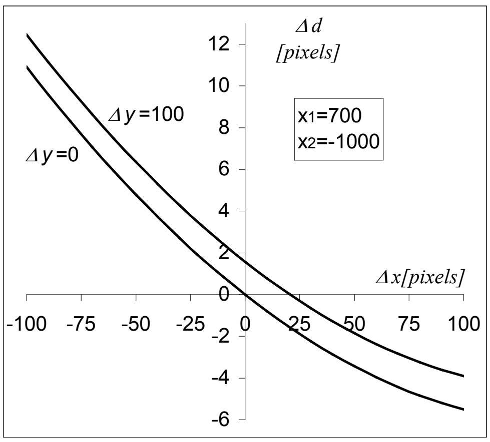


Fig. 9. Error Δd of angular distance resulting from shifting of focus about values Δx and Δy in relation to middle of array; for symmetrical (upper) and asymmetrical (lower) measurements.

2.9 *Summary of factors 1-8*

Only error introduced by pixel size has random characteristic. Factors mentioned in items 1, 2, 3, 4, 5, 8 introduce error with pattern characteristic. It depends on location of object image on array and is constant for given device unit during exploitation. Theoretically it may be determined and then taken into account during image processing. Assuming that this is not accomplish and due to discretion of object location on array it has to be treated as random error. Factors mentioned in items 1, 2 introduce errors with specific values spreading on relatively small area (within 1 pixel) – local range. Factors described in 3, 5, 8 introduce errors with trends spreading on whole array and factor described in 4 may has such nature too.

2.10 *Inaccuracy of focal length*

Camera is to be calibrated before measurement due to variable focal length of objective (feature 9°). Aim of calibration is to determine focal length f from formula (2) basing on measurement of known angular distance, for instance between two stars. Proportion of error of distance to distance during calibration should be as small as possible, because it determine relative accuracy of focal length.

Condition for calibration:

- stars in vicinity of the same altitude or at very high altitude (reduced influence of refraction),
- distance between stars as long as possible,
- middle point of array in vicinity of midway point between stars,
- section connecting stars parallel to one of the edges of array.

Because the same formula is used for calculation of altitude, then if celestial body and point below on visible horizon are situated near points of stars during calibration, then altitude measurement has comparative characteristic. In the situation errors with trends spreading on whole array have only residual influence on measured distance, resulting from inaccurate composition of points during measurement and calibration.

3 CONCLUSIONS

Exact measurement of distance with help of digital still camera require first of all:

- coordinates of point on array indicated by optical axis (focus of objective),
- angle of array deviation from perpendicular to optical axis and its direction on array,

– focal length.

Except focal length (published anyway with insufficient accuracy), these data do not appear in technical information of user manuals. Therefore error of measurement may exceed the acceptable level even several times. So calibration of camera aiming calculation of required data is essential.

At comparative measurement error compose from local range errors and predominant error arising form pixel size. In this case error of measurement is about 1′ but fitting of measurement of altitude in place, where stars were during calibration is difficult. At high stability of focal length during exploitation of camera (between switching on/off) it is worth to do many calibrations, with keeping only first condition from mentioned in 2.10. Then more of measurements of altitude may be treated as comparative.

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