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Abstract

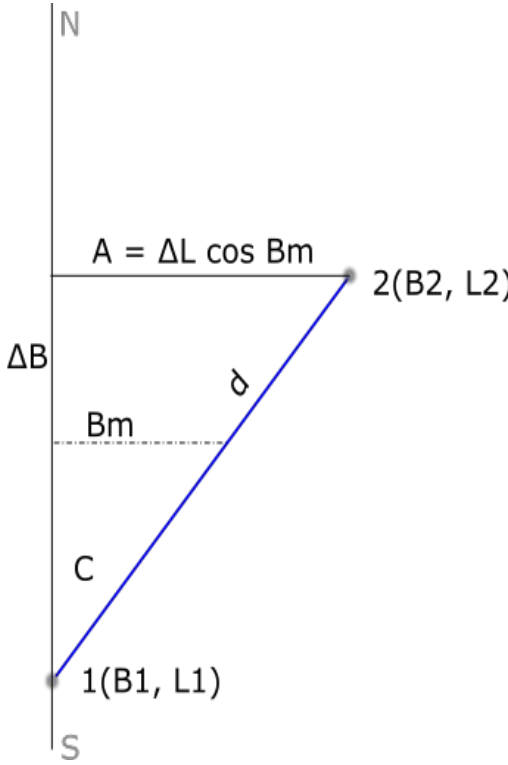
Dead reckoning involves the determination of one’s present or future position by projecting the ship’s course and distance run from a known position. A closely related problem is that of finding the course and distance from one known point to another. For short distances, these problems are easily solved directly on charts, but for trans-oceanic distances, a purely mathematical solution is often a better method. Collectively, these methods are called The Sailings (*Bowditch*).

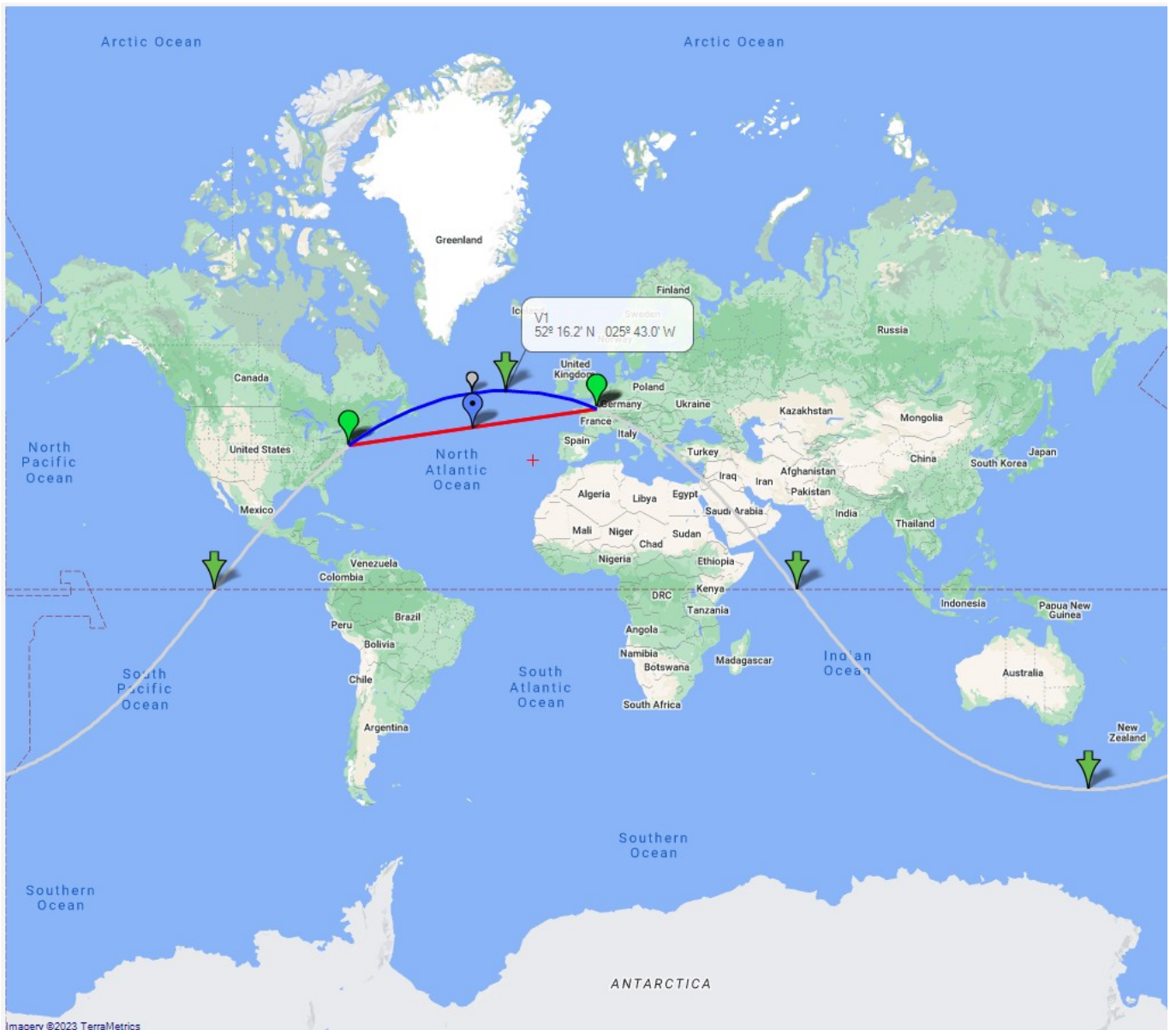
This article does not intend to give an explanation of what the sailings are, but to be a compendium of the various existing mathematical methods, emphasizing the consistency of the models and their accuracy, and the precision of the calculations.

Chart approach

Middle-Latitude Sailing

A method that combines plane sailing and parallel sailing. Plane sailing is used to find difference of latitude and departure when course and distance are known, or vice versa. Parallel sailing is used to inter-convert departure and difference of longitude. The mean latitude is normally used for want of a practicable means of determining the middle latitude, the latitude at which the arc length of the parallel separating the meridians passing through two specific points is exactly equal to the departure in proceeding from one point to the other (*Bowditch*).

Difference in latitude and departure	Equations
 <p>A: Departure Bm: Middle latitude</p>	$\Delta L = \frac{A}{\cos B_m}$ $B_m = \frac{B_1 + B_2}{2}$ <p>From the triangle it follows that:</p> $\Delta B = d \cos C$ $A = d \sin C$ <p>hence $\tan C = \frac{A}{\Delta B}$</p> $d = \sqrt{\Delta B^2 + A^2}$ <p>Constraints:</p> <ul style="list-style-type: none"> • $B_m < 60^\circ$ • $d < 200 \text{ nm}$ ($d < 375 \text{ nm Valtaggio}$) • $\Delta B < 5^\circ$



GC: nodes and vertices (green arrows).

Great Ellipse Sailing

Composite sailing

Composite sailing is a modification of great-circle sailing to limit the maximum latitude, generally to avoid ice or severe weather near the poles.

If B_v is the latitude of the limiting parallel, the longitude of the GC at that B_v is:

$$\cos L_{vx} = \frac{\tan(B_x)}{\tan(B_v)} \text{ for } x = 1, 2$$

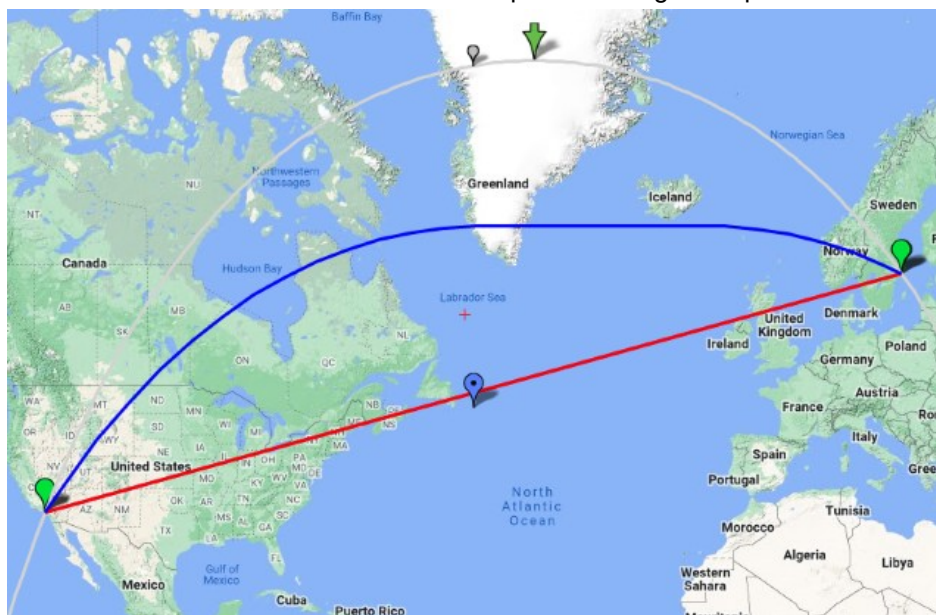
The point of departure and the destination are used successively as point X. Solve the two great circles at each end of the limiting parallel, and use parallel sailing along the limiting parallel.

The screenshot shows a software interface for composite sailing calculations. The main map displays a route from New York to London. A blue line represents the composite sailing route, which follows a great-circle path (grey) but is constrained to a limiting latitude of 47°N. A red line represents the rhumb line. The software interface includes a 'Sailing Calculator' with input fields for departure and destination coordinates, a 'Waypoints' table, and a 'Solution' table.

Lat	Long
36.9616666	-75.7033333
37.3011390	-75
39.5126063	-70
41.3899915	-65
42.9597663	-60
44.2457114	-55
45.2882011	-50
46.0438887	-45
46.5956177	-40
46.9204427	-35
47	-30.268725
47	-18.547506
46.9320990	-15
46.6499331	-10
46.1438422	-5

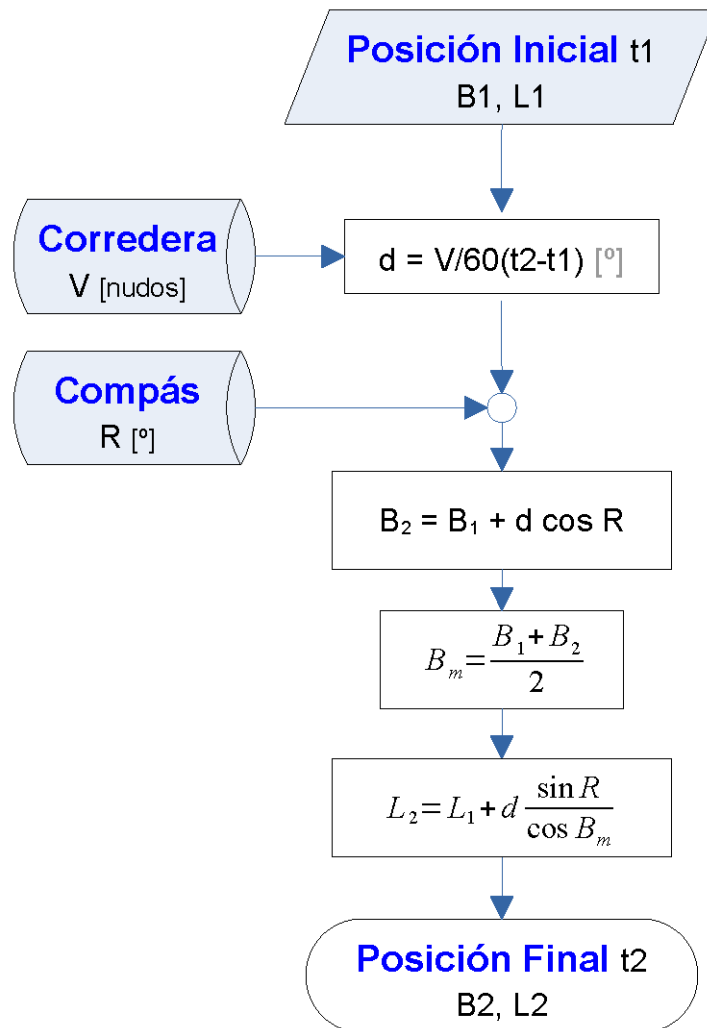
RL ML	38° 57.7' N	L1	079° 42.2' W	C	d	B2	49° 39.1' N	L2	001° 29.8' W
Warning	d(81) > 5°	Warning	d > 200						
RL SE	38° 57.7' N		079° 42.2' W	81.1	3376.9	49° 39.1' N		001° 29.8' W	
RL NS	38° 57.7' N		079° 42.2' W	81.2	3389.4	49° 38.1' N		001° 29.8' W	
RL EE	38° 57.7' N		079° 42.2' W	81.2	3387.6	49° 39.1' N		001° 29.8' W	
RL EN	38° 57.7' N		079° 42.2' W	81.2	3387.6	49° 38.1' N		001° 29.8' W	
GC SE	38° 57.7' N		079° 42.2' W	55.8	3284.5	49° 39.1' N		001° 29.8' W	

Bowditch 2019 - 1213. Composite Sailing Example 1:

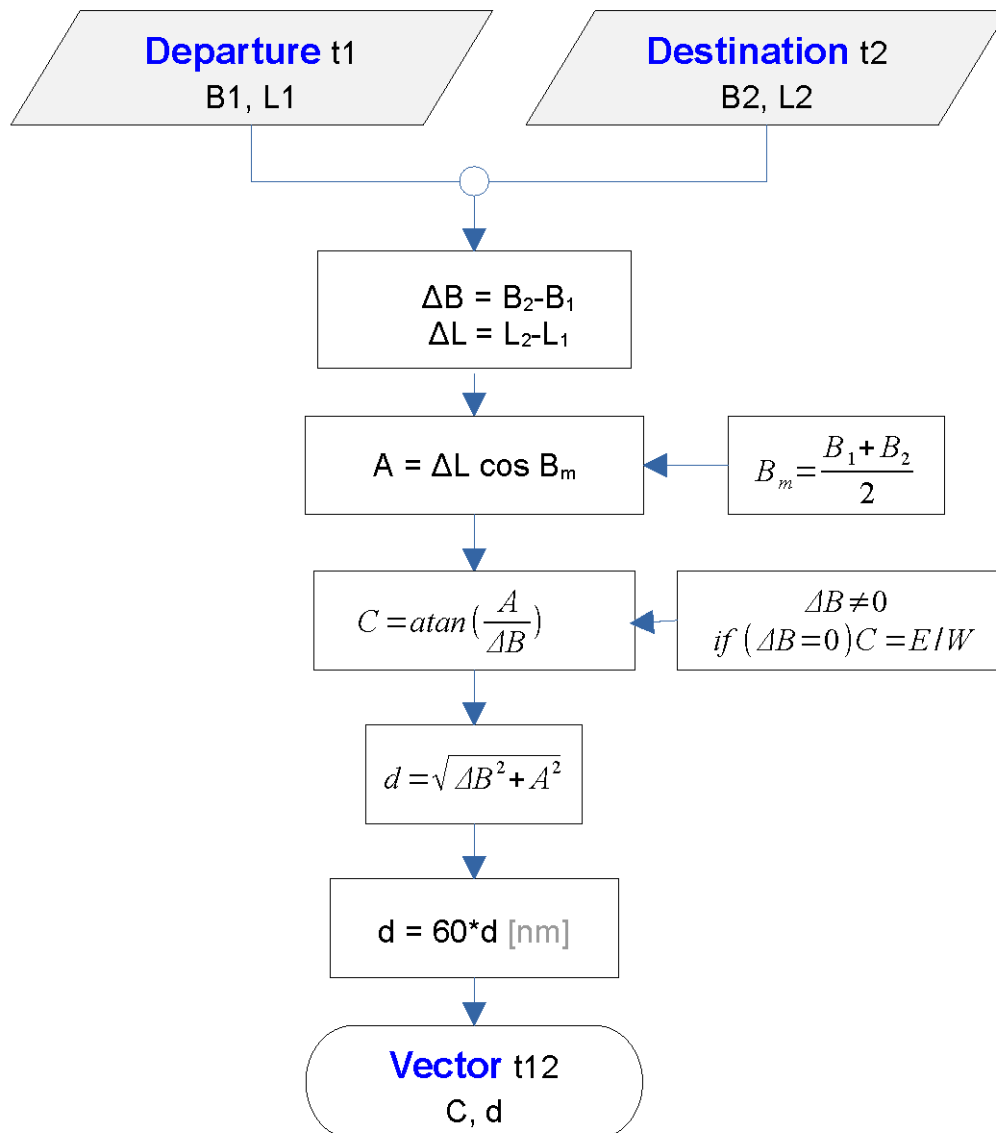


Rhumb line (red), GC (grey) and composite sailing (blue).

Middle-Latitude Sailing



Middle-Latitude Sailing



A3. Software

Chart Work / Sailings

Available at the author's web site.

Chart Work - Traditional paper chart work on an ECS

Open Sea Map

Chart Work

- Sailings
- Calculator
- Coastal Navigation
- Celestial Navigation
- Radiationavigation
- About...

Time

Now Date: 18/01/2023

UTC: 08:05:22

Dead Reckoning

Latitude: 43° 19.0' N

Longitude: 002° 00.0' W

Sailing Calculator Parameters Route

Calculation: C, d

Type of route: GC route

Limiting B: 50 0 0 N

Departure: Latitude 28 18 16.00 N, Longitude 18 48 31.00 W

Route: Course 254.072083°, Distance 2478.06532 nm

Destination: Latitude 16 58 13.00 N, Longitude 61 45 57.00 W

Waypoints

Lat	Lng
23.304444	-18.808611
28.1785477	-20
27.5339951	-25
26.7006532	-30
25.6770626	-35
24.4621268	-40
23.0555539	-45
21.4583877	-50
19.6736161	-55
16.9702777	-61.765833

Calculate Log file

Solution

RL ML	B1	L1	C	d	B2	L2
RL ML	28° 18.3' N	019° 48.5' W	254.0	2474.2	16° 58.2' N	061° 46.0' W
RL SE0	28° 18.3' N	019° 48.5' W	254.0	2469.1	16° 58.2' N	061° 46.0' W
RL SE	28° 18.3' N	019° 48.5' W	254.0	2469.1	16° 58.2' N	061° 46.0' W
RL LX	28° 18.3' N	019° 48.5' W	254.1	2478.1	16° 58.2' N	061° 46.0' W
RL SE	28° 18.3' N	019° 48.5' W	254.1	2473.1	16° 58.2' N	061° 46.0' W
RL BN	28° 18.3' N	019° 48.5' W	254.1	2473.1	16° 58.2' N	061° 46.0' W
GC SE	28° 18.3' N	019° 48.5' W	253.4	2460.1	16° 58.2' N	061° 46.0' W
GD VCT	28° 18.3' N	019° 48.5' W	253.5	2464.6	16° 58.2' N	061° 46.0' W

GPX OUT Open GPX directory Open CPN Test

Route

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GPX OUT Open GPX directory Open CPN Test

Destination