This shows that the Sumner line lies so nearly in a due east-and-west direction that a difference of longitude of 30′ makes a difference of latitude of only 1′.

From an azimuth table or diagram, it is found that the azimuth of the sun corresponding to Lat. 50° N. Dec. 22° N. and H. A. $6^m\ 10^s$ E., is N. 177° E. Therefore, using the values given by the earlier solution, the line is defined as follows:

$$A \begin{cases} 50° & 00' & 51'' & N. \\ 40 & 00 & 00 & N. \end{cases} \qquad \text{Line runs N. 87° E.}$$

The direction of the line thus given and the one found from the double coordinates may be shown to agree as in examples before given.

## FINDING THE INTERSECTION OF SUMNER LINES.

**381.** The intersection of Sumner lines may be found either graphically or by computation.

**382.** Graphic Methods.—Each line may be plotted upon the chart of the locality in which the ship is being navigated and the intersection thus found. The details of the plotting will be obvious, whether the line is defined by two of its points, or by one point and its direction. This plan will commend itself especially when the vessel is near shore, as the chart in use will then probably be one of conveniently large scale, and it will be an advantage to see where the position falls with reference to soundings and landmarks.

**383.** When clear of the land it is often inconvenient to follow this plan; a large scale chart may not be at hand, it may not be desired to deface the chart with numerous lines, or the necessary space for chart work may not be available. In such a case, the following method[a] is recommended, as it obviates the disadvantages of the other.

To understand the principle of this method, suppose that the lines are defined by the latitude and longitude of two points of each, and consider that they are plotted on a chart which is constructed upon a sheet of elastic rubber. It is evident that if, while holding it fast in the direction of the meridians, we stretch this rubber along the lines of the parallels in a uniform manner until the length of each minute of longitude is made to equal a minute of latitude, the chart, while losing its accuracy as portraying actual conditions on the earth's surface, still correctly represents the positions of the various points in terms of the new coordinates which have been created, namely, those in which a minute of latitude is equal to a minute of longitude. Thus, if on the true chart a point is $m$ minutes north and $n$ minutes east of another, on the stretched one it will still be $m$ minutes north and $n$ minutes east, the only difference being that the minutes of longitude will now be of a different length; and if on the original chart the two Summer lines intersect at a point $m$ minutes north and $n$ minutes east (on the original scale) of some definite point of one of the lines, the intersection on the stretched chart will lie $m$ minutes north and $n$ minutes (of the new scale) to the east of the same point.

A stricter mathematical conception of the stretched chart and its properties may perhaps be obtained by considering the chart of the locality to be projected (with the eye at the zenith) upon a plane which passes through one of the meridians and makes an angle with the plane of the horizon which is equal to the latitude; each minute of longitude will then be increased by multiplying it by the secant of the latitude, and thus becomes equal to a minute of latitude.

From a consideration of the properties of this hypothetical chart it may be seen that the following rule may be laid down: If two or more Sumner lines be plotted by their latitude and longitude upon any sheet of paper, using a scale whereon latitude and longitude are equal regardless of the latitude of the locality, the intersection of those lines, measured by coordinates on the scale employed, correctly represents the intersection of the lines as it would be measured upon a true chart.

It follows from this that we may plot Sumner lines upon any piece of paper, measuring the coordinates with an ordinary scale ruler, and assigning any convenient length for the mile; the larger the scale the more accurate will be the determination. Or, what is even more convenient, we may employ "profile paper," whereon lines are ruled at right angles to each other and at equal distances apart, in which case no scale ruler is needed.

One caution must be observed in using this method; all longitudes employed on the paper for any purpose must be those of the scale, namely, one minute of longitude equals one minute of latitude. For instance, if the two Sumner lines be taken at different times, in bringing the first up to the position of the second by the intermediate run, that run must be laid down to scale; that is, the easting or westing must appear as so many minutes of longitude, not so many miles. To do this enter the traverse table with course and distance run, and pick out latitude and departure; then, by means of the middle latitude, convert departure into minutes of longitude and bring the first line to the second by laying off so many minutes of latitude north or south, and so many of longitude east or west.

In the case where the Sumner is defined by one position and its line of direction, it is not correct to lay down the angle to the meridian on the hypothetical chart, for all angles are distorted thereon. The best way is to find another position on the line by assuming a second latitude ten or twenty miles removed from that of the point given, entering the traverse table with the angle that the line makes with the meridian as a course, and abreast the latitude taking out the departure; convert departure into difference of longitude, and plot the second point by its coordinates from the first.

Example: Let it be required to find the intersection, by each of the methods, of the following lines:

$$A_1 \begin{cases} 40° & 00' & N. \\ 63 & 15 & W. \end{cases} \qquad A_2 \begin{cases} 40° & 20' & N. \\ 63 & 07 & W. \end{cases}$$

$$B_1 \begin{cases} 40 & 05 & N. \\ 63 & 03 & W. \end{cases} \qquad B_2 \begin{cases} 40 & 15 & N. \\ 63 & 12 & W. \end{cases}$$

[a] Suggested by Lieut. G. W. Logan, U. S. Navy.

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Figure 45 shows the intersection, (1) by Mercator chart, (2) by scale, and (3) on profile paper, as follows:

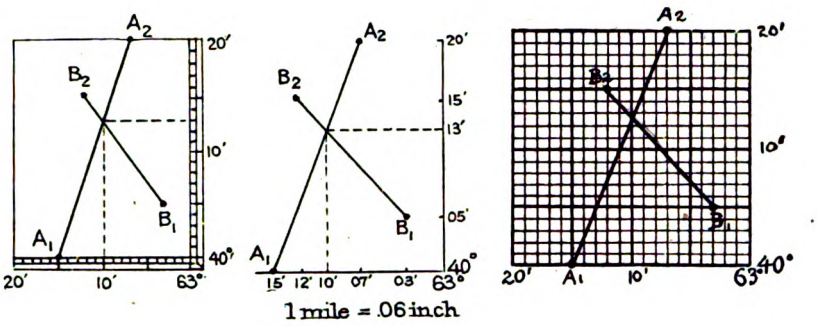



FIG. 45.

Intersection: $\begin{cases} 40° & 12'.8 \text{ N.} \\ 63 & 09 .9 \text{ W.} \end{cases}$

Suppose, in the example just given, the first line had been defined as follows:

$$A \begin{cases} 40° & 00' \text{ N.} \\ 63 & 15 \text{ W.} \end{cases} \qquad \text{Line runs N. } 17° \text{ E.}$$

To find a second coordinate by which to plot it, proceed as follows:

In Table 2, for 17°: Lat. 20′ N., Dep. 6.1 m. E. For Mid. Lat.: 40°, Dep. 6.1 m., diff. long. 8′.0 E. Hence, as previously given:

$$A_1 \begin{cases} 40° & 00' \text{ N.} \\ 63 & 15 \text{ W.} \end{cases} \qquad A_2 \begin{cases} 40° & 20' \text{ N.} \\ 63 & 07 \text{ W.} \end{cases}$$

**384.** METHODS BY COMPUTATION.[a]—The finding of the intersection of two Sumner lines by computation may be divided into two cases:

*Case I.* When one line lies in a NE.–SW. direction, and the other in a NW.–SE. direction.

*Case II.* When both lie in a NE.–SW., or both in a NW.–SE. direction.

Suppose, first, that the lines are defined by the latitude and longitude of two points of each, and for the simplification of the problem consider the lines projected on a plane passing through one of the meridians and making an angle with the plane of the horizon equal to the latitude, the properties of which were explained under the graphic method, (art. 383); this saves the necessity of converting minutes of longitude into miles of departure before the solution and converting them back again afterwards; as all points are thus projected in corresponding relative positions, the results are as exact as if the longer method be followed of dealing with minutes of latitude and longitude of unequal length.

**385.** *Case I. One line NE.–SW., and the other NW.–SE.*—Suppose the two lines, projected as described, are as shown in figure 46, $A_1 A_2$ and $B_1 B_2$; for the present assume that the two points, $A_1$ and $B_1$, have a common latitude. Drop the perpendicular PO from the intersection; then the latitude of the intersection will be a distance OP above the common latitude of $A_1$ and $B_1$, and its longitude will be a distance $A_1O$ to the right of $A_1$ and $B_1O$ to the left of $B_1$.

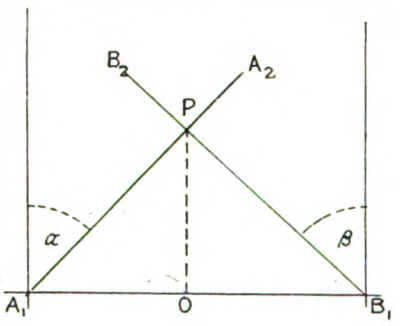



FIG. 46.

Find the angles $\alpha$ and $\beta$ from the traverse table (Table 2), they being taken out with the difference of latitude between the two points of the same line in the column Lat. and the difference of longitude in the column Dep. (Do not overlook the fact that we are dealing now with the plane of projection and that $\alpha$ and $\beta$ are not the angles made by the Sumner line with meridians on the earth's surface.) The solution may now be accomplished by either of two methods:

($a$) Observe that the case is the same as if a ship were steaming along the line $A_1 B_1$ and took the first bearing of the point P when at $A_1$, at an angle from the course equal to $90° - \alpha$, and the second bearing when at $B_1$, at an angle from the course equal to $90° + \beta$, with an intervening run equal to the difference of longitude $A_1 B_1$; or, she may be considered as steaming from $B_1$ to $A_1$, in which case the first angle is $90° - \beta$ and the second $90° + \alpha$. Picking out of Table 5 B, corresponding to the angles given, the quantity in the *second* column, we shall have the ratio of the distance of *passing abeam*, OP, to the distance $A_1 B_1$; multiply the difference of longitude by this ratio, and we shall have the actual length of OP. Then entering the traverse table with this as a latitude and $\alpha$ as a course, we find in the departure column the distance $A_1O$ by which the longitude of OP is defined; it is recommended also to pick out $B_1O$, using the angle $\beta$, which affords a proof of the correctness of all work done after the finding of $\alpha$ and $\beta$.

[a] Suggested by Lieut. G. W. Logan, U. S. Navy.

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(*b*) The second method is to find by trial and error some latitude such that its departure corresponding to $\alpha$, *plus* its departure corresponding to $\beta$, equals the difference of longitude $A_1 B_1$; then the point will be defined by the latitude, and by its longitude from $A_1$ and $B_1$, the agreement of the longitude as established from the different points furnishing a check upon the operation.

EXAMPLE: Find the intersection of the following Sumner lines:

| | | | | | |
|---|---|---|---|---|---|
| $A_1$ | 49° 40′ N. 6 55.3 W. | $A_2$ | 50° 00′ N. 7 20.0 W. | +20′ lat. +24.7 long. | Line runs NW.–SE. $\alpha = 51°$. |
| $B_1$ | 49 40 N. 6 32.5 W. | $B_2$ | 50 00 N. 6 11.3 W. | +20 lat. −21.2 long. | Line runs NE.–SW. $\beta = 47°$. |

Longitude $A_1 B_1 = 22'.8$.

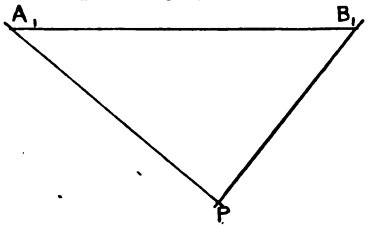



FIG. 47.

First draw a rough sketch (fig. 47) to illustrate the direction of coordinates.

Notice that $A_1$ is west of $B_1$. The line through $A_1$ runs NW.–SE. That through $B_1$, NE.–SW. The intersection is therefore south of both, east of $A_1$, and west of $B_1$.

(*a*) To solve by Table 5 B: First bearing $(90° - \alpha) = 39°$; second bearing $(90° - \beta) = 137°$. Corresponding ratio, 0.43, multiplied by $22'.8 = 9'.8$ lat. (The angles $90° - \beta$ and $90° + \alpha$ would have given the same ratio, 0.43.) Then (Table 2) with $\alpha = 51°$, lat. $= 9'.8$, dep. $= 12'.1$; and with $\beta = 47°$, lat. $= 9'.8$, dep. $= 10'.5$.

Hence, intersection:

9′.8 S. of lat. 49° 40′ N. = 49° 30′.2 N.
12 .1 E. of long. 6 55.3 W. = 6° 43 .2 W.
10 .5 W. of long. 6 32.5 W. = 6° 43 .0 W. } check.

(*b*) To solve by Table 2:

| Assuming lat ..... | 5′ | 8′ | 10′ | 9′.9 |
|---|---|---|---|---|
| Dep. for 51° ..... | 6.2 | 9.7 | 12.3 | 12.2 |
| Dep. for 47° ..... | 5.3 | 8.5 | 10.7 | 10.6 |
| Sum ..... | 11.5 | 18.2 | 23.0 | 22.8 |

Hence, intersection:

9′.9 S. of 49° 40′ = 49° 30′.1.
12 .2 E. of 6 55.3 = 6 43 .1
10 .6 W. of 6 32.5 = 6 43 .1 } check.

It may be seen that the results by the two methods substantially agree.

**386.** *Case II. Both lines NE.-SW., or both NW.-SE.*—Consider the lines as drawn in figure 48, and continue the assumption that $A_1$ and $B_1$ have a common latitude. The differences from the first case by both methods simply involve a change of signs.

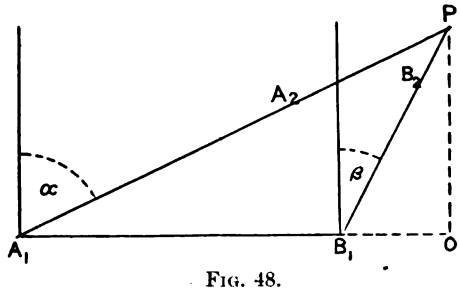



FIG. 48.

(*a*) If the ship is steaming from $A_1$ toward $B_1$, the first angle from the keel line is $90° - \alpha$, and the second, $90° - \beta$; if steaming from $B_1$ toward $A_1$, the first angle is $90° + \beta$, and the second $90° + \alpha$; in other words, either add both angles to 90° or subtract both from 90° and enter with the smaller angle as the first bearing.

(*b*) It may be seen that $OA_1 - OB_1 = A_1 B_1$; in other words, to solve by the second method, the values must be so found that the *difference* of the corresponding departures equals the difference of longitude, instead of their sums, as before.

EXAMPLE: Find the intersection of the Sumner lines defined below:

| | | | | | |
|---|---|---|---|---|---|
| $A_1$ | 49° 30′ N. 5 24.8 W. | $A_2$ | 49° 50′ 5 21.5 | +20′ lat. −3.3 long. | Line runs NE.-SW. $\alpha = 9°$. |
| $B_1$ | 49 30 N. 5 25.8 W. | $B_2$ | 49 50 4 52.1 | +20 lat. −33.7 long. | Line runs NE.-SW. $\beta = 59°$. |

$A_1 B_1 = 1'.0$.

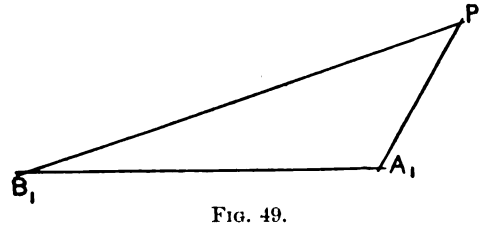



FIG. 49.

In this case (fig. 49) $B_1$ is west of $A_1$, the lines both run NE.–SW., and $\beta$ is the greater angle; therefore intersection lies to the north and east of both points.

(*a*) By Table 5 B: First course $(90° + \alpha) = 99°$; second course $(90° + \beta) = 149°$; ratio $0.67 \times 1'.0 = 0'.7$; or, first course $(90° - \beta) = 31°$; second course $(90° - \alpha) = 81°$; ratio $= 0.67$, as before.

$\alpha = 9°$, lat. $= 0'.7$, dep. $= 0'.1$; and $\beta = 59°$, lat. $= 0'.7$, dep. $= 1'.2$.

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**Hence**, intersection:

0′.7 N. of 49° 30′ N. = 49° 30′.7 N.
0 .1 E. of 5 24 .8 W. = 5 24 .7 W.
1 .2 E. of 5 25 .8 W. = 5 24 .6 W. } check.

(*b*) By Table 2:

| Assuming lat ..... | 2′.0 | 0′.5 | 0′.6 |
|---|---|---|---|
| Dep. for 9° ..... | 0 .3 | 0 .1 | 0 .1 |
| Dep. for 59° ..... | 3 .3 | 0 .9 | 1 .1 |
| Difference ..... | 3 .0 | 0 .8 | 1 .0 |

**Hence**, intersection:

0′.6 N. of 49° 30′ = 49° 30′.6
0 .1 E. of 5 24 .8 = 5 24 .7
1 .1 E. of 5 25 .8 = 5 24 .7 } check.

**387.** In discussing these cases, we have assumed that there was a point of one line which had a common latitude with a point of the other line; this would be the case if two lines were worked from time sights taken at the same time. It may occur, however, either that they have not a common latitude, but do have a common longitude, as in the case of two lines worked from $\varphi'$ $\varphi''$ (latitude) sights taken at the same time; or that they have neither a common latitude nor a common longitude, as with one time sight and one latitude sight, or with two sights taken at different times.

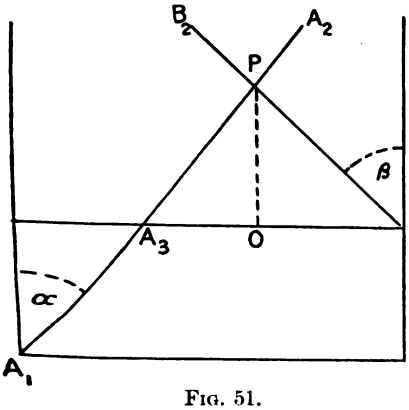
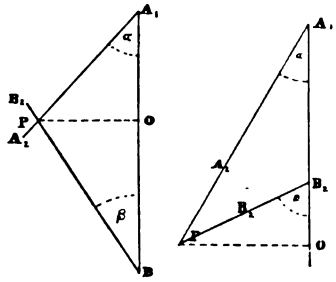


Fig. 51.


Fig. 50.

In case there is a common longitude (fig. 50), which will be rather a rare one, the problem is worked with OP as a *longitude* coordinate; the modification of the other method will suggest itself, the principal change rendered necessary being due to the fact that the angles from the course in Table 5 B will be complementary to what they were before, as we are now dealing with angles to the meridian instead of angles to the parallel.

When there is no common coordinate of either latitude or longitude, the simplest way of solving is first to find some point on one line which corresponds in latitude with one of the points on the other line, then solve as before.

Thus, in figure 51, given $A_1$ $A_2$ and $B_1$ $B_2$, find $\alpha$ and $\beta$, and thence the longitude of a point $A_3$ corresponding to the difference of latitude between $A_1$ and $B_1$ on the course $\alpha$; then find intersection of $A_3$ $A_2$ and $B_1$ $B_2$ in the usual way.

EXAMPLE: Let it be required to find the intersection of Sumner lines as follows:

$A_1$ 25° 30′ S., 115 22 E. $A_2$ 25° 50′ S., 115 40 E. +20′ lat. +18 long. Line runs SE.–NW. $\alpha$=42°.

$B_1$ 25 15 S., 115 37 E. $B_2$ 25 35 S., 115 30 E. +20 lat. − 7 long. Line runs NE.–SW. $\beta$=19°

Find where $B_1$ $B_2$ crosses parallel 25° 30′ S.
$\beta$=19°, lat.=+15′, dep.=−5′.1. Hence, the line $B_3$ $B_2$ becomes:

$B_3$ 25° 30′ S., 115 31.9 E. $A_1$ $B_3$=9′.9 Line runs NE.–SW. $\beta$=19°.

The directions of the lines (fig. 52) require us to follow Case I. $A_1$ is west of $B_3$. The line through $A_1$ runs SE.–NW., and that through $B_3$, SW.–NE. Therefore, the intersection is south of $A_1$ and $B_3$, east of $A_1$, and west of $B_3$.

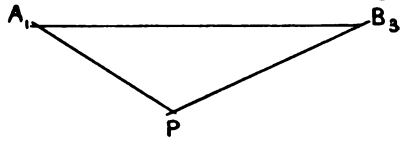


Fig. 52.

(*a*) By Table 5 B. (90°−$\alpha$)=48°, (90°+$\beta$)=109°. Ratio 0.81 × 9′.9=8′.0 lat.; $\alpha$=42°, lat.=8′.0, dep.=7′.2. $\beta$=19°; lat. =8′.0, dep.=2′.7.

**Hence**, intersection:

8′ S. of 25° 30′ S. = 25° 38′ S.
7.2 E. of 115 22 E. = 115 29.2 E.
2.7 W. of 115 31.9 E. = 115 29.2 E. } check.

(*b*) By Table 2:

| Assuming lat ..... | 6′ | 8′ |
|---|---|---|
| Dep. for 42° ..... | 5.5 | 7.2 |
| Dep. for 19° ..... | 2.1 | 2.7 |
| Sum ..... | 7.6 | 9.9 |

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Intersection:

8′ S. of 25° 30′ = 25° 38′
7.2 E. of 115 22 =115 29.2 } check.
2.7 W. of 115 31.9 =115 29.2 }

**388.** The following is a summary of the method when lines are given by coordinates of two points of each:

(*a*) *By Table 5 B.*

1. Write down lines; find $\alpha$ and $\beta$.
2. If there are no points which have a common latitude, reduce one point of one line to latitude of some given point of the other.
3. Write down difference of longitude.
4. Draw rough sketch to illustrate direction of point of intersection.
5. Enter Table 5 B:

*Case I*, angles $(90° - \alpha)$ and $(90° + \beta)$ *or* $(90° - \beta)$ and $(90° + \alpha)$.

*Case II*, angles $(90° + \alpha)$ and $(90° + \beta)$ *or* $(90° - \beta)$ and $(90° - \alpha)$.

Take out ratio from *second* column, and multiply by difference of longitude; this gives difference of latitude of intersection from the common latitude.

6. Find departure corresponding respectively to $\alpha$ and $\beta$ with latitude; this gives differences of longitude to the point of intersection from the respective points of common latitude.

(*b*) *By Table 2.*

1. Write down lines; find $\alpha$ and $\beta$.
2. If there are no points which have a common latitude, reduce one point of one line to latitude of some given point of the other.
3. Write down difference of longitude.
4. Draw rough sketch to illustrate direction of point of intersection.
5. Enter Table 2, at pages $\alpha$ and $\beta$; find by trial some latitude at which—

*Case I*, the sum of the corresponding departures equals the total difference of longitude;

*Case II*, the difference of the corresponding departures equals the total difference of longitude.

These give differences of latitude and longitude to the point of intersection from the respective points of common latitude.

**389.** If the lines, instead of being defined by coordinates of two points, are defined by the coordinates of one point of each with its direction as deduced from the azimuth of the body, it will be better not to consider the projection on the fictitious plane through the meridian, as there will then be no advantage in so doing. In this case, consider the angles of the lines with the meridian, as given, $\alpha$ and $\beta$; reduce the difference of longitude $A_1 B_1$ to departure, and use this in miles instead of the $A_1 B_1$ in minutes; and when $A_1O$ and $B_1O$ are found, being in miles of departure, they must be reduced to minutes of longitude before being applied to the longitude of $A_1$ and $B_1$.

EXAMPLE: The Sumner lines of the last example being expressed by a single point and the direction, as given below, find the intersection.

A { 25° 40′ S. / 115 31 E. } Line runs ($\alpha =$) N. 39° W.

B { 25 25 S. / 115 33.5 E. } Line runs ($\beta =$) N. 18° E.

First bring second line up to Lat. 25° 40′ S. $\beta = 18°$; lat. $= +15'$; dep. $= -4.9$ m.; diff. long. $= -5'.4$; hence we have:

B′ { 25° 40′ S. / 115 28.1 E. } Line runs ($\beta =$) N. 18° E.

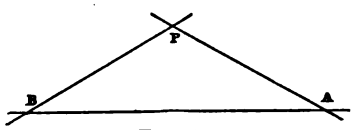


FIG. 53.

AB′ = 2′.9 = 2.6 miles.

B′ being west of A (fig. 53), and the lines through the two points running respectively NE. and NW., the intersection is north of both, east of B′, and west of A.

(*a*) By Table 5 B. $(90° - \alpha) = 51°$; $(90° + \beta) = 108°$. Ratio $0.88 \times 2.6 = 2'.3$ lat. $\alpha = 39°$, lat. $= 2'.3$, dep. $= 1.8$ m., diff. long. $= 2.0$. $\beta = 18°$, lat. $= 2'.3$, dep. $= 0.7$ m., diff. long. $= 0.8$.

Intersection:

2′.3 N. of 25° 40′ S. = 25° 37′.7 S.
2 .0 W. of 115 31 E. = 115 29 E. } check.
0 .8 E. of 115 28.1 E. = 115 28 .9 E. }

(*b*) By Table 2:

| | | | |
|---|---|---|---|
| Assuming lat | 4′ | 2′ | 2′.3 |
| Dep. for 39° | 3.2 | 1.6 | 1 .9 = 2′.1 |
| Dep. for 18° | 1.3 | 0.7 | 0 .7 = 0 .8 |
| Sum | 4.5 | 2.3 | 2 .6 = 2 .9 |

Intersection:

2′.3 N. of 25° 40′ = 25° 37′.7
2 .1 W. of 115 31 = 115 28 .9 } check.
0 .8 E. of 115 28.1 = 115 28 .9 }

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The following summary gives the various steps when the lines are each given by the coordinates of one point with the direction:

(*a*) *By Table 5 B.*

1. Write down lines as given.
2. If the points have not a common latitude, reduce one point to latitude of the other.
3. Write down difference of longitude and convert it to departure.
4. Draw rough sketch to illustrate direction of point of intersection.
5. Enter Table 5 B:

*Case I*, angles $(90°-\alpha)$ and $(90°+\beta)$ *or* $(90°-\beta)$ and $(90°-\alpha)$.

*Case II*, angles $(90°+\alpha)$ and $(90°+\beta)$ *or* $(90°-\beta)$ and $(90°-\alpha)$.

Take out ratio from second column, and multiply by departure between the two points; this gives difference of latitude of intersection from common latitude.

6. Find departure corresponding respectively to $\alpha$ and $\beta$ with this difference of latitude, and convert to difference of longitude; this gives differences of longitude to the point of intersection from the respective points of common latitude.

(*b*) *By Table 2.*

1. Write down lines as given.
2. If the points have not a common latitude, reduce one point to latitude of the other.
3. Write down difference of longitude and convert it to departure.
4. Draw rough sketch to illustrate direction of point of intersection.
5. Enter Table 2 at pages $\alpha$ and $\beta$; find by trial some latitude at which—

*Case I*, the sum of the corresponding departures equals the departure between the two points;

*Case II*, the difference of the corresponding departures equals the departure between the two points.

This difference of latitude, and these departures (converted into difference of longitude) give distance of point of intersection in latitude and longitude from the respective points of common latitude.

**390.** The modification of the methods for finding the intersection of two Sumner lines, where there is a run between the observations from which they are deduced, will be readily apparent. It is known that at the time of taking a sight the vessel is at one of the points of the Sumner line, but which of the various points represents her precise position must remain in doubt until further data are acquired. Suppose, now, that after an observation the vessel sails a given distance in a given direction; it is clear that while her exact position is still undetermined it must be at one of the series of points comprised in a line parallel to the Sumner line and at a distance and direction therefrom corresponding to the course and distance made good; hence, if a second sight is then taken, the position of the vessel may be found from the intersection of two lines—one, the Sumner line given by the second observation, and the other a line parallel to the first Sumner but removed from it by the amount of the intervening run.

Positions may be brought forward graphically on a chart by taking the course from the compass rose with parallel rulers, and the distance by scale with dividers. If the method given in article 383 be employed, runs in latitude and longitude must each be applied on their own scales, as explained in the description of the method. If one of the methods by computation be adopted, the point or points of the first line are brought forward by the traverse tables, using middle latitude sailing. The direction of a Sumner line as determined from the azimuth of the body always remains the same, whatever shift may be made in the position of the point by which the line is further defined.

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