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THE

British MARINER'S GUIDE.

CONTAINING,

Complete and Easy Instructions

FOR THE

Discovery of the LONGITUDE at Sea and Land, within a Degree, by Observations of the Distance of the Moon from the Sun and Stars, taken with HADLEY'S Quadrant.

To which are added,

An APPENDIX, containing a Variety of interesting Rules and Directions, tending to the Improvement of Practical Navigation in general.

And a Sett of correct

ASTRONOMICAL TABLES.

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M DCC LXIII.

TO

THE RIGHT HONOURABLE

The LORDS COMMISSIONERS

For Executing

The Office of LORD HIGH ADMIRAL of GREAT BRITAIN and IRELAND,

This TREATISE,

Designed for the Benefit of the British Navigation,

Is most humbly Inscribed,

BY

Their LORDSHIP'S

Very Obedient,

And Devoted

Humble Servant,

NEVIL MASKELYNE.

PREFACE.

EVERY attempt towards the clearer aſcertainment of the Longitude at ſea will readily be acknowledged as a point of great importance and utility to a maritime and commercial nation. Daily experience ſhews the wide uncertainty of a ſhip's place, as inferred from the common methods of keeping a reckoning, even in the hands of the ableſt and moſt careful navigators. Five, ten, or even fifteen degrees, are errors which no one can be ſure he may not fall into in the courſe of long voyages. The water is ſo fluid an element, and ſo eaſily put in motion by the force of the wind, or the action of the ſun and moon upon it, and theſe motions are ſo readily propagated to great diſtances, that it is not to be wondered at, that hitherto no means have been found out of allowing for the effects of ſuch variable and irregular cauſes.

During the courſe of my voyage to and from St. Helena, I made frequent obſervations of the diſtance of the moon from the ſun and fixed ſtars, in order to determine our longitude; and can from ſuch experience venture to anſwer, that this method, carried into practice, will, (without diſparagement to the labours and inventions of others) bring the longitude to great nearneſs.

I here preſent to the public the precepts neceſſary to be followed, whether in making the obſervations, or the calculations; being for the moſt part ſuch as I have practiſed myſelf: and they will, I hope, be found much more eaſy and conciſe than any yet propoſed. In particular, I flatter myſelf, that my two rules for clearing the obſervations of the effects of refraction and parallax, will appear to reduce to great ſimplicity that part of the computation, which was before the moſt tedious and perplexed.

In order that I may not leave the diligent mariner in want of any thing eſſential to his computations, I have added correct tables of the ſun and moon. The lunar equations are taken from the late Mr. Mayer's printed tables, which I have reaſon to think, applied to careful obſervations, will determine the longitude al-

ways

ways within about a degree, and generally within half a degree. The reader will alſo find here many other tables, uſeful on this, as well as on many other occaſions, which might not eaſily be procured elſewhere; particularly, a copious table of the longitudes of places determined from aſtronomical obſervations, or inferred by their bearings and diſtances from neighbouring places ſo determined; many of which are deduced from the obſervations of the late tranſit of the planet Venus over the ſun, or from obſervations made on that occaſion; to which are added their latitudes. The longitudes are reckoned from the meridian of the Royal Obſervatory at Greenwich, which is only five minutes thirty-ſeven ſeconds of longitude to the eaſt of St. Paul's Church in London. The tables of the ſun and moon are likewiſe adapted to the ſame meridian; and the two examples of the calculation of the longitude, the one diſperſed through the book, as an example of the ſeveral rules, and the other placed entire towards the end of the Appendix, are therefore made from the ſame meridian.

Having compriſed in ſeven Chapters the general proceſs for finding the longitude by obſervations of the moon, I thought it would be of ſervice to add an Appendix, containing a variety of intereſting rules and directions, uſeful in particular caſes, or tending to the improvement of practical navigation in other branches. A deſire of making the treatiſe more complete hath led me to the extenſion of the latter part of it beyond the limits I at firſt propoſed ; but ſhould that extenſion give it greater utility, as I flatter myſelf it will, I ſhall not think my pains miſemployed about it, nor my reader, I hope, deem it ſuperfluous. In particular, I conceive, that the Problem propoſed and explained in page 61, for finding the longitude from three cotemporary obſervations only, which is new, may on many occaſions be of great uſe.

I foreſee but one material objection that can be made to the uſefulneſs of this method of finding the longitude at ſea, namely, the difficulty and nicety of the calculations. But then I flatter myſelf, that the rules I have laid down, demanding only care in the computer, and requiring no particular knowledge of ſpherical trigonometry, or the projection of the ſphere, as moſt of the methods which have been formerly propoſed do, will, in a great meaſure, obviate this

this objection. However, I own, it is rather to be wished, that such parts of the computations as conveniently can, were made previously at land by capable persons, and published, from time to time, for the common benefit of navigation; especially, as not only the much greater, but also the nicest part of the calculations is of this kind: for if the moon's longitude and latitude were but computed for every twelve hours in the year, and thence her distance from the sun, or a proper star on each side of her, was calculated carefully for every six hours, and the computations published a convenient space of time before-hand, I do not see why the longitude might not be as universally found at sea by this method, as the latitude is at present.

Since the book has been printed off, I find I have omitted two particulars, the observance of which will conduce to the more accurate determination of the longitude; the one respecting the observations, and the other the calculations, which I shall here deliver, and desire the reader will pay a proper regard to.

I have mentioned, in the second Chapter, the necessity of giving a sweep with the quadrant, so ſo as to make the ſtar ſeem to deſcribe a tangent line to the moon's limb; but forgot in the ſame place to add the neceſſity of obſerving this contact as near the middle of the teleſcope, or field of view, as poſſible, which is as eſſential a circumſtance as the other; for, unleſs a ſweep be given with the quadrant, the diſtance of the ſtar may be taken from ſuch a part of the moon's limb as is neither the neareſt nor remoteſt; and if the contact be not obſerved near the middle of the field of view, the rays by which the moon and ſtar are ſeen, will not paſs parallel to the plane of the inſtrument, and conſequently the angle meaſured will not be exact.

The other particular, of which I deſire to advertiſe the reader, reſpecting the calculations, is a ſecond ſmall correction neceſſary to be made to the obſerved diſtance of the moon's limb from the ſun or a ſtar on account of parallax.

In the rule delivered in Chap. V. for the correction of parallax, which is deduced from the fluxions of the ſpherical triangle, formed between the zenith, the moon, and the ſun or ſtar, as well as in moſt other methods that have been propoſed for this purpoſe, the perpendicular dropt from the apparent place of the moon upon the great circle joining the true places of the moon and ſun, or ſtar, is taken inſtead of the ſmall arch of a parallel circle, deſcribed from the ſun or ſtar as a pole. This rule therefore may ſometimes err a minute, if the diſtance of the moon from the ſun or ſtar be thirty degrees, and more, if the diſtance be leſs; but is eaſily corrected as follows.

Call the principal effect of parallax, found by Chap. V. the parallax in diſtance; and with the moon's apparent altitude, and horiſontal parallax, take the parallax in altitude out of the Table, page 26. Then, to the conſtant logarithm 0.941 add the logarithm of the ſum of the parallaxes in altitude and diſtance, the logarithm of the difference of the ſame parallaxes, and the cotangent of the apparent diſtance of the ſtar from the moon's centre; the ſum, deducting 13 from the index, is the logarithm of the number of ſeconds required, being the ſecond correction of parallax, and is always to be added to the diſtance of the ſtar from the moon's limb, firſt corrected for refraction, and the principal effect of refraction, as by Chap. V. except

except the diſtance exceeds 90 degrees, in which caſe it is to be ſubſtracted.

In the example diſperſed through the book, by way of illuſtration of the rules, the apparent diſtance of the ſtar from the moon's centre was 51d. 30m. the parallax in diſtance was found 20½m. in Chap. V. and the moon's horiſontal parallax being 56¼m. and her apparent altitude 12d. 26m. the parallax in altitude, by the table, page 26. is 55 m.

Conſtant logarithm	—	—	0.941
Sum of par. in alt. and diſt.	55′+20′=75	logarithm	1.875
Diff. of par. in alt. and diſt.	55′−20′=35	logarithm	1.544
App. diſt. of ſtar from moon's centre	51d. 30m.	cotang.	9.900
Sum, abating 13 from the index, is		—	1.260

The logarithm of 18ſ. the ſecond correction of parallax, which, added to 51d. 9m. 32ſ. the true diſtance of the ſtar from the moon's centre, cleared of refraction, and the principal effect of parallax, at the bottom of page 43, gives 51d. 9m. 50ſ. the true and abſolute diſtance of the ſtar from the moon's centre. If the computation in Chap. VI. be repeated with this diſtance correct, the difference of the moon and ſtar's longitudes

longitudes will come out 18 ſ. greater than was found in page 48, or 51d. 12 m. 23 ſ. whence the longitude of the moon inferred from obſervation ſhould be 8 S. 11d. 44 m. 13 ſ. and the difference between this longitude and that computed from the tables, ſee Chap. VII. will be 47 ſ. which, multiplied by $28\frac{1}{10}$, gives 1321 ſeconds, or 22 minutes, by which the ſhip is weſt of the account.

I have applied this correction to the obſervations made in my voyage home from St. Helena to England, though it has in none of them exceeded half a minute ; the reaſon of which is, that I never took the diſtances of the moon from ſtars under 25 or 30 d. but rather choſe them at the greateſt diſtances, even up to 90 d.

In my voyage out this correction was leſs neceſſary, moſt of the obſervations being made either about or under the latitude of 30 degrees, where the ſtars riſing and falling more nearly perpendicular, than in higher latitudes, the angle of poſition at the moon, contained between the two great circles paſſing from thence, one thro' the ſtar, and the other thro' the zenith, will be ſmaller, and conſequently the correction in queſ-

 tion,

tion, being as the ſquare of the ſine of that angle, will be ſmaller alſo.

In ſtrictneſs, a ſecond correction ought to be applied on account of refraction, as well as of parallax; but as this can only be neceſſary when the moon or ſtar are very near the horiſon, namely under three degrees of altitude, and the obſervations ought never to be taken ſo low, on account of the variability and irregularity of the refraction itſelf, this correction may be ſafely omitted, as of little conſequence in practice.

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OF

The BRITISH MARINER's GUIDE to the Difcovery of the LONGITUDE at Sea and Land.

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THE

THE

BRITISH MARINER'S Inſtructions

For FINDING the

LONGITUDE

BY SEA AND LAND.

CHAP. I.

Of the INSTRUMENTS *neceſſary to be uſed in making the Obſervations.*

ALL the inſtruments neceſſary for determining the longitude by obſervations of the moon, are a Hadley's quadrant, and a tolerably good watch. The quadrant ſhould be of about eighteen inches radius, and, if not made all of braſs, ſhould be framed of well-ſeaſoned wood, with a braſs arch and index. The Vernier's ſcale muſt be made to carry the ſubdiviſions to ſingle minutes; and the eye may itſelf ſubdivide ſtill nearer to a fraction of a minute, eſpecially if aſſiſted with a magnifying glaſs.

If the inſtrument be made a ſextant, in which caſe it will meaſure 120 degrees, it will be very convenient for meaſuring the diſtance of the ſun and moon for two or three days after the firſt, and before the laſt quarter, which cannot be done if it be made, as they uſually are, to meaſure only

90 degrees. And the obſervation of the diſtance of the ſun from the moon cannot be rendered too frequent, being more eaſily, and, in general, more exactly obſerved, than the diſtance of the moon from a ſtar in the night: beſides which, it is attended with another very great advantage, that the time of the day may be then directly aſcertained by an altitude of the ſun, taken by an aſſiſtant at the very inſtant of the meaſurement of the diſtance of the ſun from the moon, without depending upon the going of a watch for ſome hours, as will be generally neceſſary, where the diſtance of the moon from a ſtar is taken in the night.

The motion of the hand alone will not be exact and ſteady enough for moving the index in theſe nice obſervations of the meaſurement of the diſtance of the moon from the ſun or a ſtar. Therefore it will be neceſſary to perform this motion by means of a ſcrew; for, the quadrant being often to be held in a very inclined plane, it will be often impracticable, on account of the motion of the ſhip, to make the obſervation inſtantaneouſly; but, by means of the ſcrew, the index may be moved by gentle degrees, and each time after altering the ſcrew, the quadrant muſt be turned by the hand round the viſual ray, going to the ſtar as an axis, ſo as to make the moon's limb paſs by the ſtar, till at laſt, in paſſing by, it exactly coincides with it.

Beſides the two dark glaſſes commonly applied to theſe inſtruments, there muſt be added a third, much lighter than the others, in order to take off the exceſſive brightneſs of the moon, and the glare upon the horizon glaſs, which always attends it, except when the moon's creſcent is ſmall, or her light is accidentally weakened by a thin cloud.

The

The speculums, as well as dark glasses, must be ground with their surfaces truly plane, and parallel to each other; for the doing of which, I understand, that some of the optical instrument makers are now masters of, a particular and infallible method. The performance of many quadrants, otherwise very exact, is spoiled by an imperfection in this particular; and therefore too much care cannot be taken about it.

The contact of a star with the moon's limb will be rendered much more discernible by the help of a small telescope, magnifying two or three times. This must be contrived to have a motion parallel to itself, (like a parallel ruler) by which it may be brought nearer to, or carried farther from, the plane of the quadrant; and the moon, which is seen always by reflection, will appear more or less bright, according as more or less of her light is received by the telescope from the quicksilvered part of the object speculum; or, by raising the telescope still higher, she will be seen entirely by reflection from the unsilvered part of the speculum, which is generally the most convenient. The object glass of the telescope may be made to slide in its tube, in order to adjust it to the most perfect distinctness, for the eye of the observer.

Previous to all observations, but particularly that of the distance of the moon from the sun, or star, for finding the longitude, the observer should adjust his quadrant with the greatest care; a point of equal consequence with the exact observation of the distance of the moon from the star: This is so much the more necessary, as the adjustment is subject to alter sensibly, even from one day to another. The best object of all for this purpose is the horison of the sea, when clear, and the examination is made best without the telescope; only, if the observer is at all short-sighted, he ought to make use of a concave glass, properly suited to his eye, as he ought likewise to do in all other observations made without the telescope, as particularly in taking the altitudes of stars, or of the moon in the night. The observer should be careful always to examine the adjustment of his quadrant in the day-time, when the horison is to be seen clearly, and particularly in the afternoon when he expects to make an observation the ensuing night. Should he have failed of making this examination the afternoon before an observation, he may examine what the error was, if any, the next morning, or he may make this examination by means of the moon itself, in which case it will be best to use the telescope. Instead of fresh adjusting the quadrant before every observation, perhaps the following method of examining and allowing for the error of it, may be more advantageous. Turn the index of the quadrant till the horison of the sea, or the moon, or any other proper object, appears as one, by the union of the reflected image with the object seen directly; then the number of minutes, by which o on the index differs from o on the arch, is the error of the adjustment. If o on the index stands advanced upon the quadrant before, or to the left hand of o on the arch, that number of minutes is to be substracted from all observations; but if it stands off the arch behind, or to the right hand of o on the arch, it must be added to the observations. By examining the error of adjustment in this Manner, by at least three trials, and taking a medium of the results, one can scarce err above half a minute in determining the precise quantity of the error of the quadrant, whereas one may be mistaken a minute or more in adjusting the quadrant, which is but a single operation. The observer would do well to write down in a book the error of adjust-

ment

ment thus found, and which way it is to be applied, whether it is additive, or ſubſtractive.

Beſides this adjuſtment, which is familiar to every one who uſes the inſtrument, there is another, which is too little known, and too little regarded, the neglect of which is often of great prejudice in common uſe, but would entirely defeat the end of theſe nice obſervations : I mean, the ſetting of the little ſpeculum to a paralleliſm with the great one, by the ſcrews on the foreſide of the quadrant. The manner of doing it is this : Hold the plane of the quadrant parallel to the horiſon, and the index being brought near to o, if the horiſon of the ſea, ſeen by reflection in the little ſpeculum, is higher than the direct horiſon, ſeen by the ſide of it, unſcrew the neareſt ſcrew a little matter, and ſcrew up the oppoſite one, till the direct and reflected horiſons agree nearly. But if the reflected horiſon is lower than the true horiſon, unſcrew the ſcrew furtheſt from you, and ſcrew up the neareſt ſcrew, till the two horiſons agree nearly, taking care to leave the ſcrews both tight, by ſcrewing them up equally, if they are ſlack. If the reflected horiſon ſhould not differ more than five minutes from the true one, it will be of no ill conſequence. This adjuſtment, when once well made, will ſeldom want correction, provided the ſcrews are ſufficiently tight, and that they have not been touched. Nevertheleſs, for fear of any accidents, it will be proper to examine it, from time to time, in order to be aſſured that no material alteration has happened ; for a deviation of ſo ſmall a quantity as four or five minutes can be of no prejudice.

CHAP.

CHAP. II.

Of the Obſervations neceſſary to be made for finding the Longitude at Sea, and the Manner of making them.

THERE are four obſervations neceſſary to be made, according to the method here propoſed, in order to determine the longitude at ſea: The firſt is an altitude of the ſun, or ſome bright ſtar, for regulating a watch, by which the other obſervations are to be made, or, at leaſt, for determining exactly the error of its going. The next obſervation is, the diſtance of the moon's enlightened limb from the ſun, or a ſtar. The third and fourth obſervations are, the altitudes of the moon and the ſun, or the ſtar, from which the moon's diſtance is obſerved, to be taken by two obſervers aſſiſting the perſon who takes the diſtance of the moon from the ſun, or a ſtar, at the very inſtant, or, at the utmoſt, within a minute of the time he gives notice that he has compleated his obſervation. At the ſame inſtant, or at moſt within a quarter of a minute, and before the obſervers attempt to read off the degrees and minutes from their quadrants, ſomebody muſt note the hour, minute, and quarter part of a minute of the watch, regulated as mentioned before. Only, if the diſtance of the moon be taken from the ſun, the altitude of the ſun, taken at the ſame inſtant, will determine the time without the uſe of a watch; and therefore the firſt obſervation mentioned above will not be neceſſary. But if it be propoſed to obſerve the diſtance of the moon from a ſtar in the night, the altitude of the ſun will be neceſſary for regulating the watch, and ſhould be previouſly taken in the afternoon, not ſooner than two o'clock; but the

the nearer he is to being due weſt, the better; only care ſhould be taken, that their altitudes be not much under five degrees, on account of the uncertainty of refraction near the horiſon. Or, if an altitude of the ſun was not taken the afternoon before the obſervation of the diſtance of the moon from the ſtar, it might be taken the next morning after, for a good common watch would not vary above a minute in that interval of time, which produces an error only of a quarter of a degree of longitude.

Could the altitude of the bright ſtars be taken with certainty in the night, there would be no occaſion to depend upon the going of a watch at all. In fine ſummer nights, or when there is any ſmall appearance of twilight, thoſe obſervations may be made very well: and on ſuch occaſions, to make them uſeful for deriving the time, the ſtars muſt be ſo choſen, as not to be leſs than two hours from the meridian: the nearer they are to the prime vertical, or to being due eaſt or weſt, the more exact they will be for this purpoſe. But in general, the altitude of ſtars admit of too much uncertainty for the purpoſe of finding the time, though exact enough for the uſes intended to be derived from the third and fourth obſervations, in computing the effect of refraction and parallax. The altitude of the ſun's lower limb muſt be obſerved, as in the meridian obſervation, except the air ſhould be foggy, and the ſun's limbs ill defined, in which caſe it is better to take the altitude of his centre.

In order to take the diſtance of the moon from the ſun, the obſerver muſt look at the moon directly through the unfoiled part of the little ſpeculum, and having firſt let down one of the dark glaſſes uſed in taking the meridian altitudes, and having likewiſe brought the plane of the quadrant nearly into the arch of a great circle, going through the

the ſun and moon, he muſt ſhift the index gradually, and move or ſweep the arch of the quadrant alternately to the right hand and left, till he ſees the ſun and moon in view together, and has brought their neareſt limbs almoſt into contact. Let him then ſcrew down the plate to the arch, containing the ſcrew which is deſigned to give a more minute and regular motion to the index, and while he moves the index by the turning of the ſcrew, or immediately after, he muſt turn the quadrant round the viſual ray, going to the moon as an axis, ſo as to make the ſun ſeem to paſs by the moon's limb. Theſe operations are to be repeated till the neareſt limbs of the ſun and moon, in their apparent paſſage by each other, exactly touch one another outwardly at their neareſt approach, without cutting one into the other, which compleats the obſervation.

But, in order to take the diſtance of the moon from a ſtar, the obſerver muſt look at the ſtar directly, and holding the plane of the quadrant ſo as to paſs nearly through it and the moon, he muſt move the index (juſt as in the obſervation of the diſtance of the moon from the ſun) till the ſtar appears almoſt in contact with the moon's enlightened limb, whether it be the neareſt to, or furtheſt from, the ſtar. If the moon's enlightened limb be that furtheſt from the ſtar, the moon's reflected image muſt be brought beyond the ſtar. Only if a teleſcope be uſed of the ſort which ſhews objects upſide down, it muſt be carefully noted, that when the enlightened limb is really neareſt the ſtar, the moon will appear as if ſhe were brought beyond it; and when the enlightened limb is really furtheſt from the ſtar, the moon will appear as if ſhe were only brought to it. Before the full, the weſtern limb is the enlightened limb; and after the full, the eaſtern one; though this may be judged of certainly by the eye, within a day of the full on each ſide.

The

The lighteſt dark glaſs will generally be requiſite in theſe obſervations ; and the ſcrew being applied for moving the index, muſt be turned gradually, and the quadrant at the ſame time, or immediately after, turned round the viſual ray, going to the ſtar as an axis, ſo as to make the moon's limb ſeem to paſs by the ſtar ; which operation muſt be repeated till the middle of the ſtar (for it will ſometimes appear of a ſenſible breadth) ſhall, at the moon's neareſt approach, appear to paſs exactly upon her limb, without entering further within it. The obſervation being thus compleated, whether of the diſtance of the ſun and moon's limbs, or of a ſtar from the moon's limb, the obſerver has nothing to do but to read the degrees, minutes, and fraction of a minute, which he muſt eſtimate by his eye, off the quadrant ; unleſs, which will be more convenient, he aſſiſts his ſight with a ſmall magnifying glaſs, of one and a half or two inches focus.

The following cautions to be uſed in reading off the obſervations may not be ſuperfluous. To hold the quadrant with the arch towards you, and the center directly from you, and to place your eye fully oppoſite, and nearly perpendicular to that part of the arch where the nonius appears to coincide with the diviſions ; holding alſo the magnifying glaſs, if you uſe one, ſtraight before you, and parallel to the line of the diviſions, and taking care to have a full and true light, equally illuminating the part you look at, both to the right hand and to the left.

It may be proper to take notice, that it will be neceſſary to invert the quadrant, that is to ſay, to hold it with its face and centre downwards, when the object, whether it be the moon or the ſtar, which is to be ſeen directly, is to the right hand of the other object in the heavens; which is to be ſeen by reflection. This will be made familiar by a little experience.

It has been obſerved above, that the two other obſervations, namely, the altitudes of the moon and ſun, or ſtar, muſt be taken by two other obſervers, at the ſame inſtant, or at leaſt within a minute of the taking of the diſtance of the moon from the ſun, or ſtar. The altitude of the moon's upper or lower limb muſt be taken according to the illumination of her diſk. A little practice will make it eaſy to take the altitude of the upper limb, which is difficult at firſt, as the moon's upper limb, at her loweſt deſcent, muſt be made to ſweep the horiſon above her. But in dark nights it is better not to attempt to give the ſweep at all, but to hold the quadrant as nearly perpendicular to the horiſon as you can judge by your eye. Further, theſe obſervations of altitudes in the night will be made better, as the horiſon will be ſeen much clearer, by removing the little hole uſed in moſt quadrants to limit the ſight, or by looking by the ſide of it, which will produce no error that will be here of any prejudice.

It may be expected that I ſhould ſay ſomething with reſpect to the degree of exactneſs that may be attained in theſe ſeveral obſervations; I ſhall therefore freely deliver my opinion in this reſpect, which I think myſelf ſufficiently warranted in doing, from the repeated obſervations I have taken on board of ſhip, as well as on ſhore, without ever finding myſelf deceived. " That the altitudes of the ſun " may be taken with good quadrants to one, or, at the moſt, " two minutes, if the horiſon be tolerably clear; therefore, " the time may generally be found by the ſun, within a " quarter, or at moſt, half a minute, even in theſe great " northern latitudes, which is a degree of exactneſs ſuffi-

" cient for the purpose. That the distance of the moon
" from the sun, or a star, may be taken with good qua-
" drants, well rectified, to about a minute, when the sea is
" not very rough; but even in rougher weather, if it does
" not approach to a storm, I apprehend, a careful observer
" cannot well be mistaken above two minutes in the ob-
" servation, which produces an error only of about a de-
" gree in the determination of the longitude; and even
" then, by taking a medium of several observations, he may
" arrive at a still greater exactness. The altitudes of the
" moon and a star, taken in the night, I own, admit of
" much greater degrees of error; but then, fortunately, it
" is not required to have these altitudes with the same ex-
" actness as is necessary in the other observations; since an
" error of a whole degree in them will seldom be of more
" prejudice, than an error of a single minute in measuring
" the distance of the moon from a star; therefore, ten or
" twenty minutes can be of no great prejudice; and a case
" scarce ever occurred in so many observations, where the
" altitudes could not be taken within these limits. But
" should a case occur in practice, as it rarely will, that they
" cannot be properly observed, a rule is given at the end
" for computing them."

It has been here supposed, that the observer is acquainted with some of the principal stars by sight, and also, that he knows which of them he ought to chuse, in order to measure the distance of the moon from; for every star is not proper for this purpose.

The following directions will instruct him in these particulars. The table in page 4, contains a catalogue of twelve stars of the first and second magnitude, which are all that he will ever have need of, and therefore all that he ought ever to uſe for this purpoſe, being choſen neareſt to the zodiac. Out of theſe he muſt ſelect one in this manner, either to the eaſt or weſt of the moon, according as circumſtances will admit.

Let him find the moon's longitude and latitude from any common almanack, or ephemeris, at the time he intends to make his obſervations; or, if he is deſtitute of an ephemeris, he muſt compute the moon's place to the given time, reduced to the meridian of Greenwich, which is that of the tables, by adding at the rate of an hour for every fifteen degrees he is to the weſt, or ſubſtracting, if to the eaſt of it, (which muſt alſo be done, if he makes uſe of an ephemeris). In making this computation, he may neglect the equation of time, the ten ſmall equations, the equation of Anomaly, and the node, and the reduction to the ecliptic; and only take in the equations of the centre, evection, and variation: he may alſo neglect the correction of the latitude, called latitude 2d. Having obtained the moon's longitude and latitude nearly, if he wants a ſtar to the eaſt of the moon, he muſt look in Table, p. 4. for one having a greater longitude than the moon; or if he wants a ſtar to the weſt of the moon, he muſt look for one having a leſs longitude than the moon, and muſt ſo ſelect it, that its longitude may differ at leaſt ten degrees from that of the moon; how much more, does not ſignify; and that the difference of their latitudes, if they are both north, or both ſouth, or the ſum of their latitudes, if one is north, and the other ſouth, may not exceed the limits of the table in page 4. Should he not be able to find a ſtar within the limits of the table, which will hardly ever be the caſe, let him chuſe one which approaches neareſt to them. I ſhould adviſe always the uſing one of theſe ſeven ſtars of the catalogue, Aldebaran, Pollux, Regulus, the Virgin's Spike, Antares,

Antares, Fomalhaut, or Markab, in preference to any of the remaining five ſtars, unleſs the former ſhould very much exceed the limits of the table in page 4. as, being brighter, they will be eaſier found, and alſo their contact with the moon's limb better diſcerned than that of dimmer ſtars.

An eaſy method now offers itſelf for the obſerver to find the ſtar in the heavens, ſhould he be unacquainted with it by ſight. He has nothing to do but to ſet the index of his quadrant to the number of degrees expreſſing the difference of the longitudes of the moon and ſtar, and direct his ſight either to the eaſt or weſt of the moon, according as the longitude of the ſtar is greater or leſs than that of the moon; and having found the moon upon the little ſpeculum, let him give a ſweep with the quadrant to the right and left, and he will find the ſtar he ſeeks for nearly in a line perpendicular to the line of the moon's horns, or longer axis, or, which is the ſame thing, in the line of the moon's ſhorter axis produced.

In order to attain the greater degree of exactneſs, it will be better to repeat the obſervations, till at leaſt three diſtances of the moon from the ſtar, and their correſponding altitudes, are obtained ; but the more that are taken the better. The ſum of all the obſervations of the diſtances, divided by the number of them, is to be regarded as the diſtance at the middle time, which is in like manner obtained by adding together all the times ſhewn by the watch, and dividing their ſum by the number, as before. In like manner, the mean of the correſpondent altitudes muſt be found. But the interval of time between the extreme obſervations muſt not exceed half an hour ; if it does, each obſervation muſt be cleared of refraction and parallax ſeparately, and the medium of the diſtances muſt be taken, after they are thus corrected.

corrected. Or the obſervations, if numerous, may be divided into ſeveral ſets, each included within the limits of half an hour, and the medium of each being taken, and cleared of refraction and parallax, their medium may be taken as the true and correct diſtance at the mean of all the times.

I ſhall now recite a ſet of actual obſervations, taken by myſelf, and the officers of the ſhip, upon our coming into the Britiſh channel, on our return from St. Helena, in the night preceding the morning on which we made the Scilly iſlands; by computations from which, I ſhall exemplify each precept as I go along, and laſtly deduce the true longitude of the ſhip at the time. But I muſt here give a general caution once for all, and deſire the obſerver to take particular notice, that the time made uſe of in the following computations is according to aſtronomical reckoning, which begins the day from the preceding noon. The denomination of the day at noon is the ſame in the nautical, in the civil, and in the aſtronomical account; but as it is the cuſtom of mariners to entitle the day's works in their journals by the noon at which the day's work ends, theſe computations are to be made according to the day of the month with which the day's work begins; or, in other words, the computations falling in with the time of any day's work in the log-book, muſt be made from the day of the month, which is leſs by one than the title of the journal. Thus, in the following example, the obſervations fall in with the time of the day's work in the log-book, which is entitled May 10; but in the computations muſt be reckoned to belong to May 9, and to ſuch hour, minute, and ſecond, as they follow the noon of May 9; ſo that if they happen after midnight, they are not to be reckoned 1, 2, 3, &c. in the morning, but 13, 14, 15, &c. hours, counting the hours up up to 24 hours, or the next following noon. In the example, the altitude of the ſun taken in the afternoon is to be entitled May 9, 1762, 5h. 55m. 26ſ. P.M. by the watch. The mean of the three diſtances of the moon from the Virgin's Spike, May 9, 11h. 33m. 42ſ. P. M. by the watch. The altitude of the ſun the following morning, May 9, 20h. 31m. 30ſ. P. M. by the watch.

Obſervations made May 9, 1762, on Board of the Warwick Eaſt-Indiaman, upon coming into the Channel, on our Return from St. Helena to England, a few Hours before we made the Scilly Iſlands.

May 9, 1762, time per watch, 5h. 55m. 26ſ. P.M. Apparent altitude of the ſun's lower limb from the horiſon of the ſea was taken 8 deg. 35½m. There is 3½m. to be added for the error of the adjuſtment of the quadrant. The height of the eye above the ſurface of the ſea was about 25 feet. Our latitude at this time, by allowing for the ſhip's run ſince noon, was 49d. 2m. N. and our longitude, by account, 7d. 11m. Weſt of the Royal Obſervatory at Greenwich.

Intending to take the diſtance of the moon from ſome ſtar about midnight, I turn 7d. 11m. our longitude reckoned from Greenwich, into time, by the table in page 3, which produces 28m. 44ſ. which I add to 12 hours, becauſe we were in weſt longitude, gives 12h. 28m. 44ſ. or 12½h. P. M. nearly by the meridian of Greenwich, to which time I ought to find the moon's longitude and latitude by any ephemeris, calculated for this meridian, or that of London. But ſuppoſing I had no ephemeris at hand, I muſt then make a rough calculation, according to the directions given in page 12, as follows.

Mean

	Mean Long. of ☉ S.	°	′	Long. ☉'s Apog. S.	°	′	
1762.	9	10	6	3	8	49	
May.	3	28	17				
D. 9.	0	8	52				
H. 12.	0	0	30				
M. 30.	0	0	1				
	1	17	46	—3	8	49	
Equa. of ☉'s centre	+	1	28	1	17	46	
Long of ☉	1	19	14	10	8	57	Mean An. ☉

	Mean Long. ☽ S.	°	′	Mean An. ☽ S.	°	′	Long. ☊ S.	°	′
1762.	11	10	31	1	11	17	1	18	13
May.	4	21	10	4	7	48	0	6	21
D. 9.	3	28	35	3	27	35	0	0	28
H 12.	0	6	35	0	6	32	0	0	1
M. 30.	0	0	16	0	0	16	—	6	50
	8	7	7	9	23	28	1	11	23
Eq. of centre	+	5	37	Arg. XI. of centre.			S.	°	′
	8	12	44	Arg. XII. of evection			3	12	18
Evection	—	1	19	Arg. XIII. of variation			6	22	11
	8	11	25						
Variation		+	28						
☽'s Long.	8	11	53						
☽'s ☊	1	11	23						
Arg. of Lat.	7	0	30						
☽'s Lat.		2	37 S.						

With the sun's longitude, 1 S. 19d. 14m. I take the equation of time out of the table, p. 9, — 3 m. 54 s. which ought to be substracted from 12 h. 30 m. but being so small, I neglect it in this rough calculation.

The

The moon's long. being thus found to be 8 S. 11 d. 53 m. and her latitude 2 d. 37 m S. I find, by the help of the tables in page 4, that the Virgin's Spike is a proper ſtar to compare the moon with, their difference of longitude being 51 d. 22 m. and their difference of latitude being only 5 m. the Virgin's Spike having leſs longitude, and conſequently being to the weſt of the moon. But as the full moon is paſt, the eaſtern limb is the enlightened one, which being moſt remote from the Virgin's Spike, 16 m. muſt be added to 51 d. 22 m. for the moon's ſemidiameter, which gives 51 d. 38 m. for the diſtance which the moon's limb will have from the Virgin's Spike, which is exact enough to ſet the index of the quadrant to for obſervation, and thence to find the ſtar, if it was not known.

Sequel of the Obſervations.

1762, May 9. Time by the watch, 11 h. 26 m. 45 ſ. P. M. Diſtance of the moon's eaſtern and remoteſt limb from the Virgin's Spike was meaſured 51 d. 38 m. by the Hadley's quadrant; but $3\frac{1}{2}$ m. is to be added for the error of adjuſtment. At the ſame time, the apparent altitude of the moon's lower limb from the horiſon of the ſea was obſerved 11 d. 56 m. and the apparent altitude of the ſtar 25 d. 40 m, both including the dip, about 4 m. but free from any error of the adjuſtment of the quadrant.

In like manner, at 11 h. 32 m. 35 ſ. and 11 h. 41 m. 45 ſ. P. M. by the watch, the apparent diſtances of the moon's limb from the Virgin's Spike, by the quadrant, were 51 d. 40 m. and 51 d. 44 m. the apparent altitudes of the moon's lower limb from the horiſon of the ſea, 12d. 18m. and 12d. 39m. and thoſe of the ſtar 25d 0m. and 23d. 57m.

Though the altitude of the ſun taken in the afternoon might be ſufficient for correcting the watch, yet for regulating it with greater certainty, another altitude of the ſun's lower limb was taken the next morning at 8h. 31m. 30ſ. or rather adding 12 hours, at 20d. 31m. 30ſ. P. M. and was found 42d. 22½m. There is 2½m. to be added for the error of the quadrant. The height of the eye above the ſea was about 17 feet. Our latitude at that time was 49d. 30m. N. and we had made 39m. of longitude to the eaſt, ſince the altitude of the ſun taken in the afternoon.

CHAP.

C H A P. III.

To compute the apparent or ſolar Time from the obſerved Altitude of the Sun, or a known Star, and thence to find the apparent Time of the Obſervation of the Diſtance of the Moon from the Sun or Star.

THE firſt thing to be done in computing the longitude from the obſervations given, is to find the hour of the obſervation of the diſtance of the moon from the ſun or ſtar. In order to this, we muſt find the error of the watch, that is to ſay, how much it is too faſt or too ſlow, by the altitude of the ſun or bright ſtar taken for this purpoſe. From the obſerved altitude of the ſun's lower limb, ſubſtract the ſum of the dip and refraction, taken out of the tables in page 1, and to the remainder add 16 minutes for the ſun's ſemidiameter, and you have the true altitude of the ſun's centre. Or, if the ſun's upper limb was obſerved, ſubſtract the ſum of the dip, refraction, and 16 min. for the ſun's ſemidiameter; but if the altitude of his centre was taken, ſubſtract the ſum of the dip and refraction. Subſtract the true altitude of the ſun thus found from 90 deg. and you have his true zenith diſtance.

In like manner the obſerved altitude of a ſtar muſt be leſſened by the ſum of the dip and refraction, and the complement to 90 is the true zenith diſtance. Take the declination of the ſun out of the tables, which are to be found in all books of navigation, which declination being computed for the noon of ſome fixed meridian, ſuppoſe London,

 muſt

muſt be corrected by allowing both for your longitude from thence, and the diſtance of the time from noon eſtimated nearly. The beſt way is to add your longitude from London or Greenwich, turned into time, to the hour of the day, if you are to the weſt of it; or ſubſtract it, if you are to the eaſt of it, which gives the true hour at Greenwich; and ſay, as 24 hours is to this number of hours, ſo is the ſun's daily variation of declination from noon to noon, to a number of minutes, which add to the ſun's declination in the table at noon, if it is increaſing, or ſubſtract if it is decreaſing, and you will obtain the ſun's true declination required.

The ſtar's declinations are found in the table, page 2 and 3. The ſun or ſtar's declination, if of the ſame denomination with the latitude of the place, ſubſtract from 90 deg. but if of a different denomination, add to 90 deg. and you have the diſtance of the ſtar from the pole of the world, which is above the horiſon. Find alſo the latitude of the ſhip at the taking the altitude of the ſun or ſtar, by allowing for the ſhip's run from the latitude determined by the laſt meridian obſervation, and take its complement to 90, or the colatitude. Now add together the zenith diſtance, polar diſtance, and colatitude, and take half the ſum; then add together the arithmetical complement of the ſine of the colatitude, the arithmetical complement of the ſine of the polar diſtance, the ſine of the half-ſum found above, leſſened by the colatitude, and the ſine of the ſaid half-ſum, leſſened by the polar diſtance; half the ſum of theſe four logarithms is the ſine of an arc, which doubled gives the true diſtance of the ſun or ſtar from the meridian. Five places of logarithms will be ſufficient in this computation. The diſtance of the ſun from the meridian turned into time, at the rate of an hour for every 15 degrees, and 1 minute of time

time for every 15 minutes, and 1 ſecond of time for 15 ſeconds, which is expeditiouſly done by the table in p. 3 and 4. gives the apparent time of the day, if it be the afternoon; or, ſubſtracted from 12 hours, gives the apparent time of the day, if it be forenoon.

To find the apparent time from the altitude of a ſtar, ſubſtract its diſtance from the meridian from its right aſcenſion (taken out of table page 2 and 3) if it be to the eaſt of the meridian, or add it to the right aſcenſion, if it be to the weſt of the meridian; the difference or ſum is the right aſcenſion of the mid-heaven. To the time of the day eſtimated nearly, add your longitude from Greenwich, turned into time, if you are to the weſt of Greenwich; or ſubſtract the longitude turned into time, if you are to the eaſt of Greenwich, gives the time reduced to the meridian of Greenwich; to which compute the ſun's longitude, according to the directions in the beginning of the next chapter, and hence find his right aſcenſion by the table, page 27 and 28, making a proportion for the minutes of the ſun's longitude above the next inferior degree. Now ſubſtract the ſun's right aſcenſion from the right aſcenſion of the mid-heaven, borrowing 360 degrees, if neceſſary; the remainder turned into time, by table in page 3 and 4, is the apparent time, reckoned from the preceding noon.

The difference between the apparent time thus found, whether by the altitude of the ſun or a ſtar, and the time ſhewn by the watch, at the inſtant of taking the ſaid altitude, ſhews how much the watch is too faſt or too ſlow for apparent time, which difference being applied as a correction to the time ſhewn by the watch, when the diſtance of the moon from the ſtar was taken, being added thereto if the

the watch is too ſlow, or ſubſtracted therefrom if it is too faſt, gives the apparent time of the obſervation.

Example of a Computation of the apparent Time from the Altitude of the Sun, recited above, taken May 9, 1762, 5h. 55m. 26ſ. P. M. *per Watch*

	°	′
Altitude of the ſun's lower limb by the quadrant	8	$35\frac{1}{2}$
Add. on account of the error of the quadrant	+	$3\frac{1}{2}$
Altitude of the ſun's lower limb corrected	8	39
Subſtract ſum of dip 5′, and refraction 6′	—	11
True altitude of the ſun's lower limb ——	8	28
Add ſemidiameter of the ſun ——	+	16
	8	44
Subſtract from —— ——	90	0
True zenith diſtance of the ſun ——	81	16
Diſtance of the ſun from the elevated or north pole, $=90°-17°\ 28'$ his declination at that time	=72	32
Complement of the latitude $=90-49°\ 21'$	=40	39
Sum of zen. diſt. of ſun. pol. diſt. of ſun, and colat.	194	27
Half the ſum —— ——	97	$13\frac{1}{2}$

Ar. compl. ſin. colat.	40.39	0.18613
Ar. compl. ſin. pol. diſt.	72.32	0.02050
Sin. of $\frac{1}{2}$ ſum — colat. $=97°\ 13\frac{1}{2}'-40°\ 39'$	=56.34	9.92148
Sin. of $\frac{1}{2}$ ſum—pol. diſt. $=97\ \ 13\frac{1}{2}-72\ \ 32$	$=24.41\frac{1}{2}$	9.62090
Sum of four logarithms ——		19.74901
Half ſum —— ——		9.87450

	°	′
Is ſine of	48°	$30\frac{1}{2}'$
		2
Sun's horary angle	97	1

This,

This, by table, page 3 and 4, gives the apparent time, 6 h. 28 m. 4 f. P. M. The time by the watch was 5 h. 55 m. 26 f. P. M. Hence the watch is 32 m. 38 f. too flow for apparent time.

I fhall now give an example of the computation of the time from the altitude of the Virgin's Spike, recited above, taken May 9, 1762, at 11 h. 26 m. 45 f. P. M. per watch, at the time of taking the firft diftance of the moon from the Virgin's Spike. But this is to be looked upon only as an example of the method of calculation, the time determined from hence not being to be depended on fo exactly as that from the altitudes of the fun, for want of a better horifon. The latitude of the fhip at that time was 49 d. 23 m. N. and fhe had made $17\frac{1}{2}$ m. of longitude to the eaft, fince the altitude of the fun in the afternoon. The right afcenfion of Spica Virginis is at this time 198 d. 10 m. 37 f. and its declination 9 d. 54 m. 45 f. S.

	°	′
Obferved altitude of Spica	25	40
Subftract fum of dip 4′, and refraction 2′	---	6
True altitude of Spica ——	25	34
	90	
Subftract from 90 leaves true ——	64	26
Zenith diftance of Spica Virginis, diftance of Spica from elevated or north pole = 90 + 9° 55′ = ——	99	55
Colatitude =90---49° 23′ =	40	37
Sum of zenith diftance of the ftar, polar diftance of the ftar, and the colatitude ——	204	58
Half fum —— ——	102	29

Ar,

Ar. compl. ſin. colatitude	——	40.37	0.18642
Ar. compl. ſin. pol. diſt.	——	99.55	0.00654
Sin. half ſum --- colat.	=102.29---40.37=61.52		9.94540
Sin. half ſum --- pol. diſt.	=102.29---99.55= 2.34		8.65110
Sum of four logarithms	——		18.78946
Half ſum	—— ——		9.39473
is ſine of	—— ——	14° 22½′	
		2	
Doubled, is horary angle,		28 44½	
or diſtance of Spica Virginis from the meridian to the weſt; therefore, add it to the right aſcenſion of Spica Virginis		198 10½	
Gives right aſcenſion of mid heaven		226 55	

To the eſtimated time, which is exactly midnight, or 12h. P.M. add 28m. for about 7d. of longitude weſt of Greenwich, gives the time reduced to the meridian of Greenwich, 12h. 28m. or 12½h. P.M. to which time compute the ſun's longitude, which comes out 1S. 19d. 15m. Hence the ſun's right aſcenſion, by table, page 27 and 28, is 46d. 47m, which ſubſtract from 226d. 55m. the right aſcenſion of the mid-heaven, found above, leaves 180d. 8m. for the apparent time in parts of the equator, which, by table, page 3 and 4, gives the apparent time 12h. 0m. 32ſ. But the time ſhewn by the watch was 11h. 26m. 45ſ. hence the watch is 33m. 47ſ. too ſlow for apparent time. By the altitude of the ſun taken in the afternoon, the watch was then 32m. 38ſ. too ſlow for apparent time; but ſince that time, the ſhip having made 17½m. of a degree of longitude towards the eaſt = 1m. 10ſ. of time; the watch ought to be ſo much ſlower than it was in the afternoon, or 33m. 48ſ. too ſlow for apparent time, with which the obſervation of the Virgin's Spike happens to agree exactly.

Com-

Computation of the Time from the Altitude of the Sun recited above, taken May 9, 20h. 31m. 30s. P.M. *by the Watch.*

	°	′	
Observed altitude of the sun's lower limb	42	22½	
Add error of quadrant ———	+	2½	
Altitude corrected for error of quadrant	42	25	
Substract sum of dip 4′, and refraction 1′	—	5	
True altitude of the sun's lower limb	42	20	
Add for semidiameter of the sun	+	16	
True altitude of the sun's centre	42	36	
Substract from ———	90	0	
True zenith distance of the sun	47	24	
Colatitude $=90°-49°\ 30'=$	40	30	
Distance of the sun from elevated or north pole $90°-17°\ 39'=$	72	21	
Sum ——— ———	160	15	
Half sum ——— ———	80	7½	
Ar. compl. sin. colat. ———	40.30		0.18746
Ar. compl. sin. pol. dist. ———	72.21		0.02094
Sin. half sum — colat. $=80.7\frac{1}{2}-40.30=$	39.37½		9.80465
Sin. half sum — pol. dist. $=80.7\frac{1}{2}-72.21=$	7.46½		9,13124
Sum ——— ———			19.14429
Half sum ——— ———			9.57214
is sine of 21° 55½′			
2			

Doubled, is 43 51 = horary angle, or distance of the sun from the meridian to the east; and turned into time by table in page 3 and 4, = 2h. 55m. 24s. which, substracted from 12, leaves 9h. 4m. 36s. A.M. apparent time, or rather 21h. 4m. 36s. P.M. reckoned from the preceding noon. The watch was 8h. 31m. 30s. hence the watch is 33m. 6s. too slow for apparent time.

By the altitude of the ſun in the afternoon before, the watch was 32m. 38ſ. too ſlow for apparent time ; therefore it ſeems to have loſt 28ſ. ſince that time, in 14 hours and a half; but the ſhip has made 39m. of longitude to the eaſt, =2m. 36ſ. of time, in the interval ; and conſequently the watch ought to have appeared to have loſt 2m 36ſ. of time ; but it appears to have loſt only 28ſ. therefore the watch has really got the difference between 2m. 36ſ. and 28ſ. or 2m. 8ſ. in 14½ hours, which agrees very well with the reſult of all the obſervations made of late, by which, when the change of meridians is allowed for, it gets at the rate of about 3¼m. in a day upon ſolar time.

As the altitude of the Virgin's Spike was not taken to compute the time from, nor could be had ſufficiently exact to determine it certainly within a minute, through the obſcurity of the horiſon, I ſhall deduce the error of the watch from the altitudes of the ſun. By the firſt altitude of the ſun, at 5h. 55m. 26ſ. P. M. by watch, the watch appears to have been then 32m. 38ſ. too ſlow for apparent time, which might be added to the mean of the three times ſhewn by the watch at the taking of the three diſtances of the moon from the Virgin's Spike, to find the apparent time correct.

But as the watch appears, as well by the two altitudes of the ſun above recited, as by all the obſervations made of late, after allowing for the change of the meridians, to be gaining at the rate of about 3¼ m. per day upon ſolar time, I will ſuppoſe it to have got at the ſame rate between the taking of the altitude of the ſun in the afternoon, and the time of the taking the diſtance of the moon from the ſtar. The three times by the watch, at the taking of the three diſtances of the moon's limb from the Virgin's Spike, were,

11h.

11h. 26m. 45ſ. 11h. 32m. 35ſ. 11h. 41m. 45ſ. the ſum of which, divided by 3, is 11h. 33m. 42ſ. the mean time of the three obſervations by the watch. The interval of time between 5h. 55m. 26ſ. the time between the taking of the ſun's altitude in the afternoon and this is about $5\frac{1}{2}$h. in which time the watch will get 45ſ. at the rate of $3\frac{1}{4}$m. per day. But at 5h. 55m. 26ſ. P. M. by the watch, it was 32m. 38ſ. too ſlow for apparent time; and at 11h. 33m. 42ſ. having got 45ſ. it will be then only 31m. 53ſ. too ſlow for apparent time, which, added to 11h. 33m. 42ſ. gives the apparent time of the obſervations of the diſtance of the moon's limb from the Virgin's Spike, May 9, 12h. 5m. 35ſ. P. M.

C H A P. IV.

To compute the Sun's Longitude, the Moon's Longitude, Latitude, horisontal Parallax, Semidiameter, and hourly Motion, at any given Time, by the Meridian of Greenwich.

TO the apparent time of the obſervations of the diſtance of the moon from the Virgin's Spike, which has been found above to be, May 9, 12h. 5m. 35ſ. P. M. add 28m. 44ſ. of time, which, by table, page 3 and 4, anſwers to 7d. 11m. of longitude weſt from Greenwich, by account when the altitude of the ſun was taken in the afternoon, by which the watch was corrected. The ſum is 12h. 34m. 19ſ. the apparent time at Greenwich, according to the ſhip's reckoning. To this time the ſun and moon's longitude, latitude, &c. muſt be computed. If the ſhip had been eaſt of Greenwich, the longitude by account turned into time, muſt have been ſubſtracted from the apparent time. It may be thought, the apparent time of the obſervation of the diſtance of the moon from the ſtar ought to be reduced to the time at Greenwich, by the longitude of the ſhip by account at that time; but as the watch, corrected by the altitude of the ſun, muſt continue to ſhew the time according to the meridian where that obſervation was made, the longitude of the ſhip's account at that time muſt be uſed.

1. *To find the Sun's Longitude.*

To the ſun's mean longitude for the beginning of the year, add his mean motions to the beginning of the month; and likewiſe for the day of the month, hour, minute, and ſecond of time given, according to the tables in page 6, the ſum is the mean longitude of the ſun at the given time. Only in leap year, in the months of January and February, compute for the day of the month preceding the given day. In like manner, to the longitude of the ſun's apogee for the beginning of the year, add his motion to the beginning of the month, and for days. The ſum ſubſtracted from the ſun's mean longitude, found above, is his mean anomaly, with which take out the equation of the centre from page 7 and 8; this added to, or ſubſtracted from, his mean longitude, gives his true longitude nearly. With this take out the equation of time from the table in page 9, and find by the table in page 6, the motion of the ſun anſwering to that number of minutes and ſeconds of time, and add it to, or ſubſtract it from, the ſun's longitude found nearly above, according to the ſign of the equation of time, gives the true longitude of the ſun.

In the example following, the mean longitude of the ſun is 1 S. 17d. 46m. 22ſ. and the longitude of his apogee 3 S. 8d. 49m. 57ſ. which being ſubſtracted from the former, leaves 10 S. 8d. 56m. 25ſ. the mean anomaly of the ſun, with which take the equation of the ſun's centre out of table, page 8, +1d. 28m. 38ſ. to be added to the mean longitude of the ſun, 1 S. 17d. 46m. 22ſ. whence the true longitude of the ſun nearly is 1 S. 19d. 5m. 0ſ. With this take the equation of time from table, page 9, —3m. 54ſ. to which the motion of the ſun correſponding by table, page 6, is —7 ſ. to be ſubſtracted from the true longitude of the ſun found nearly, (becauſe the equation of time is ſubſtractive) leaves 1 S. 19d. 14m. 53ſ. the true longitude of the ſun.

2. To

2. *To compute the Moon's Longitude, Latitude, &c.*

To the apparent time at Greenwich, by which the ſun's longitude was computed, add or ſubſtract the equation of time, according to its ſign, and you have the mean time at Greenwich by account; to which time the moon's longitude, latitude, &c. muſt be computed by the tables.

N. B. In ſtrictneſs the equation of time ought to be made uſe of which belongs to the ſun's longitude, at the time when the altitude of the ſun or ſtar was taken, by which the watch was corrected; and not that at the time of the obſervation of the diſtance of the moon from the ſtar: but as the interval of theſe times need never exceed 12 hours, the error hence ariſing can never exceed a quarter of a minute of time, which is of no conſequence in this caſe.

In the example the apparent time at Greenwich by account is 12h. 34m. 19ſ. P. M. from which the equation of time, —3m. 54ſ. being ſubſtracted, leaves 12h. 30m. 25ſ. P. M. the mean time at Greenwich by account; to which time compute the moon's longitude, latitude, &c. in the following manner.

From the tables in page 10 and 11, take out the mean longitude of the moon, mean anomaly of the moon, and mean longitude of the aſcending node for the beginning of the year, and the mean motions to the beginning of the month, and for the days, hours, minutes, and ſeconds of the mean time laſt found. Add all the numbers belonging to the mean longitude together, and likewiſe all belonging to the mean anomaly together, and you have the mean longitude and mean anomaly of the moon at the given time. But

But from the mean longitude of the node for the beginning of the year subtract the sum of all the mean motions belonging to the node, and you have the mean longitude of the ascending node at the given time. Thus, in the example, the mean longitude of the moon comes out 8 S. 7d. 8m. 27s. the mean anomaly of the moon, 9 S. 23d. 28m. 24s. and the mean longitude of the ascending node, 1 S. 11d. 21m. 12s.

Next find the arguments of the ten first equations in the following manner.

Equ. I. The argument of the first equation is the mean anomaly of the sun, found above in computing the sun's longitude. This in the example is 10 S. 8d. 56m. Therefore the first equation, by table 1, of the moon, is — 8m. 39s.

Equ. II. From the mean longitude of the moon, 8 S. 7d. 8m. substract the true longitude of the sun, 1 S. 19d. 15m. gives the mean distance of the moon from the sun, 6d. 17m. 53s. double this distance is 1 S. 5d. 46m. to which add the mean anomaly of the sun, 10 S. 8d. 56m. gives the argument of the second equation, 11 S. 14d. 42m. whence the second equation is + 14 s.

Equ. III. From double the distance of the moon from the sun, 1 S. 5d. 46s. substract the mean anomaly of the sun, 10 S. 8d. 56m. gives the argument of the third equation, 2 S. 26d. 50m. whence the third equation is — 1m. 2s.

Equ. IV. From argument the second, 11 S. 14d. 42m. substract the mean anomaly of the moon, 9 S. 23d. 28m. gives the

the argument of the fourth equation, 1S. 21d. 14m. whence the fourth equation is +1m. 24ſ.

Equ. V. From argument the third, 2S. 26d. 50m. ſubſtract the mean anomaly of the moon, 9S. 23d. 28m. gives the argument of the fifth equation, 5S. 3d. 22m. whence the fifth equation is + 33ſ.

Equ. VI. To double the diſtance of the moon from the ſun, 1 S. 5d. 46m. add the mean anomaly of the moon, 9S. 23d. 28m. gives the argument of the ſixth equation, 10S. 29d. 14m. whence the ſixth equation is —46ſ.

Equ. VII. From the mean longitude of the moon, 8S. 7d. 8m. ſubſtract the mean longitude of the node, 1S. 11d. 21m. gives the mean diſtance of the moon from the node, 6S. 25d. 47m. from the double of which diſtance, or 1S. 21d. 34m. ſubſtract the mean anomaly of the moon, 9S. 23d. 28m. gives the argument of the ſeventh equation, 3S. 28d. 6m. whence the ſeventh equation is +51ſ.

Equ. VIII. From the mean anomaly of the moon, 9S. 23d. 28ſ. ſubſtract the mean anomaly of the ſun, 10S. 8d. 56m. gives the argument of the eighth equation, 11S. 14d. 32m. whence the eighth equation is —11ſ.

Equ. IX. From the mean longitude of the node, 1S. 11d. 21m. ſubſtract the true longitude of the ſun, 1S. 19d. 15m. gives the diſtance of the node from the ſun, or the argument of the ninth equation, 11S. 22d. 6m. whence the ninth equation is —13ſ.

Equ. X. From the diſtance of the moon from the ſun, 6S. 17d. 53m. ſubſtract the mean anomaly of the moon, 9S.