9S. 23d. 28m. gives the argument of the tenth equation, 8S. 24d. 25m. whence the tenth equation is +10ſ.

Now add the ſum of ſuch of the ten equations as are additive +3m. 12ſ. to the mean longitude of the moon, 8S. 7d. 8m. 27ſ. gives 8S. 7d. 11m. 39ſ. and from thence ſubſtract the ſum of the other of the ten equations, which are ſubſtractive —10m. 51ſ. gives the longitude of the moon correct, 8S. 7d. 0m. 48ſ.

Equ. XI. and XII. Correct the mean anomaly of the moon, 9S. 23d. 28m. 24ſ. by adding the ſum of ſuch of the ten equations as are additive +3m. 12ſ. and ſubſtracting the ſum of the reſt, which are negative —10m. 51ſ. and further apply the double of the firſt equation, or —17m. 18ſ. gives the mean anomaly of the moon correct, or the argument of the eleventh equation of the centre, 9S. 23d. 3m. 27ſ.

From the longitude of the moon correct, 8S. 7d. 0m. 48ſ. ſubſtract the longitude of the ſun, 1S. 19d. 14m. 53ſ. gives the diſtance of the moon from the ſun correct, 6S. 17d. 45m. 55ſ. from which diſtance doubled, 1S. 5d. 31m. 50ſ. ſubſtract the mean anomaly of the moon correct, or argument the 11th, 9S. 23d. 3m. 27ſ. leaves the argument of the twelfth equation, the evection, 3S. 12d. 28m. 23ſ.

With the mean anomaly of the moon correct, 9S. 23d. 3m. 27ſ. take out the 11th equation of the centre, +5d. 38m. 39ſ. which, applied to the longitude of the moon correct, 8S. 7d. 0m. 48ſ. gives the longitude of the moon corrected by the equation of the centre, 8S. 12d. 39m. 27ſ.

F With

With argument 12th, 3S. 12d. 28m. 23ſ. take out the 12th equation, the evection, —1d. 19m. 3ſ. which applied to the longitude of the moon corrected for the equation of the centre, 8S. 12d. 39m. 27ſ. gives the longitude of the moon equated, 8S. 11d. 20m. 24ſ.

Equ. XIII. From the longitude of the moon equated, 8S. 11d. 20m. 24ſ. ſubſtract the true longitude of the ſun, 1S. 19d. 14m. 53ſ. gives the argument of the 13th equation, 6S. 22d. 5m. 31ſ. whence the 13th equation, the variation is + 29m. 7ſ. which, applied to the longitude of the moon equated, 8S. 11d. 20m. 24ſ. gives the longitude of the moon reckoned in her orb, 8S. 11d. 49m. 31ſ.

Equ. XIV. To the mean longitude of the moon's aſcending node, 1S. 11d. 21m. 12ſ. apply the firſt of the ten equations —8m. 39ſ, gives the longitude of the moon's aſcending node corrected, 1S. 11d. 12m. 33ſ. which ſubſtract from the longitude of the moon in her orb, 8S. 11d. 49m. 31ſ. and you have the argument of reduction and of latitude, 7S. 0d. 36m. 58ſ. which gives equ. 14, the reduction —6m. 6ſ. which, applied to the longitude of the moon in her orb, 8S. 11d. 49m. 31ſ. gives the true longitude of the moon in the ecliptic, 8S. 11d. 43m. 25ſ.

Latitude I. With the argument of latitude, 7S. 0d. 36m. 58ſ. take out latitude firſt, is 2d. 37m. 17ſ. South.

Latitude II. To argument the 9th, 11S. 22d. 6m. add argument the 13th, 6S. 22d. 5m. gives argument of latitude ſecond, 6S. 14d. 11m. whence latitude ſecond is 2m. 9ſ. S. The ſum of this and latitude firſt, 2d. 37m. 17ſ. S. as they are both of the ſame kind, gives the moon's true latitude,

titude, 2d. 39m. 26ſ. S. If theſe two latitudes had been of contrary denominations, that is to ſay, one north and the other ſouth, their difference would have been the true latitude of the moon, of the ſame kind with the greater latitude.

With argument 11th of equation of centre, 9 S. 23d. 3m. take out the equatorial parallax of the moon, 55m. 47ſ. With argument 12th of evection, 3 S. 12d. 28m. take out the firſt equation of parallax, +8ſ. and with argument 13th of variation, 6 S. 22d. 5m. take out the ſecond equation of parallax, +20ſ. which being both applied to the equatorial parallax, 55m. 47ſ. give the horiſontal parallax of the moon correct, 56m. 15ſ. From hence find the horiſontal ſemidiameter of the moon by table, page 23, 15m. 21ſ. to which add from the table in the ſame page, 3ſ. for the augmentation for the altitude of 12d. gives the apparent ſemidiameter of the moon, 15m. 24ſ.

From the table of the moon's hourly motion in page 24, with argument 11th of centre, take out the moon's hourly motion, 31m. 22ſ. With argument 12th of evection, take out the firſt correction of the hourly motion, +8ſ. With the mean diſtance of the moon from the ſun, uſed in making out the ſecond of the firſt ten equations, take out the ſecond correction of the hourly motion, +34ſ. Theſe both applied to 31m. 22ſ. give the true hourly motion of the moon, 32m. 4ſ.

An Example of the Calculation of the Sun's Longitude, May 9, 1762, at 12h. 34m. 19s. P. M. apparent Time, by the Meridian of the Royal Observatory at Greenwich.

Apparent Time.	Longitude of ☉ S.	°	′	″	Apog. of ☉ S.	°	′	″
1762.	9	10	6	28	3	8	49	35
May.	3	28	16	40				21
D. 9.		8	52	15				0
H. 12.			29	34				0
M. 34.			1	24				0
S. 19.				1				0
	1	17	46	22	3	8	49	55
Equ. ☉'s centre	+	1	28	38	1	17	46	22
Long. ☉ nearly	1	19	15	0	10	8	56	27
Equ. of time		—	3	54	Mean anomaly of ☉			
Corr. mot. of ☉			—	7	Arg. of equ. of ☉'s centre.			
True long. of ☉	1	19	14	53				

Apparent time	12	34	19	P. M.
Equation of time	—	3	54	
Mean time	12	30	25	P. M.

An

An Example of the Calculation of the Moon's Longitude, Latitude, Horisontal Parallax, Semidiameter, and hourly Motion, May 9, 1762, at 12 h. 30 m 25 s. P. M. mean Time by the Meridian of the Royal Observatory at Greenwich.

Mean Time.	Mean long. ☽ S.	°	′	″	Mean Anom. ☽ S.	°	′	″	Mean Long. ☊ S.	°	′	″
1762.	11	10	31	9	1	11	16	53	1	18	12	43
May.	4	21	10	3	4	7	47	54		6	21	16
D. 9.	3	28	35	15	3	27	35	6		0	28	36
H. 12.		6	35	18		6	31	57			1	35
M. 30.			16	28			16	20				4
S. 25.				14				14		— 6	51	31
	8	7	8	27	9	23	28	24	1	11	21	12

	S.	°	′	″
Sum of ten equat.		—	7	39
Long. ☽ corr.	8	7	0	48
Equat. XI. of cen.	+	5	38	39
	8	12	39	27
Equ. XII. evect.	—	1	19	3
Long. ☽ Eq.	8	11	20	24
Equ. XIII. variat.		+	29	7
Long. ☽ in orb	8	11	49	31
Equ. 14 Reduct.		—	6	6
Long. ☽ in eclipt.	8	11	43	25

	S.	°	′	″
Long. of ☉	1	19	14	53
Dist. ☽ a ☉	6	17	53	
Doub. dist. ☽ ☉	1	5	46	

Arguments of the ten first equations.

		S.	°	′	First ten equ. + ′	″	− ′	″
Mean Anom. ☉.	I.	10	8	56			8	39
2 ☽ ☉ + Arg. 1.	II.	11	14	42	0	14		
2 ☽ ☉ — Arg. 1.	III.	2	26	50			1	2
Arg. 2. — Me. An. ☽.	IV.	1	21	14	1	24		
Arg. 3. — Me. An. ☽.	V.	5	3	22		33		
2 ☽ a ☉ + Me. An. ☽.	VI.	10	29	14				46
2 ☽ a ☊ — Me. An. ☽.	VII.	3	28	6		51		
Me. An. ☽ — Arg. 1.	VIII.	11	14	32				11
Long. ☊ — long. ☉.	IX.	11	22	6				13
☽ a ☉ — Me. An. ☽.	X.	8	24	25		10		
Sums					3	12	10	51
						+	3	12
Sum of ten equations						—	7	39

	°	′	″	
Lat. I.	2°	37′	17″	S.
Lat. II.		2	9	S.
True lat. ☽	2	39	26	S.

Horisontal parallax of ☽

		′	″
Arg. XI.		55′	47″
Arg. XII.		+	8
Arg. XIII.		+	20
Hor. par. of ☽		56	15
Hor. semidiam. ☽		15	21
Augm. for alt.	12°	+	3
App. semidiam. ☽		15	24

Hourly motion of ☽

	′	″
Arg. XI.	31′	22″
Arg. XII.	+	8
Mean dist. ☽ a ☉	+	34
True hor. mot. of ☽	32	4

	S.	°	′	″
Mean anomaly of ☽	9	23	28	24
	9	23	20	45
Double first equation ——		—	17	18
Anom. ☽ corr. Arg. XI.	9	23	3	27
Dist. ☽ a ☉ corr. ——	6	17	45	55
Doub. dist. ☽ a ☉ corr.	1	5	31	5[illegible]
Arg. XII. of evection ——	3	12	28	23
Arg. XIII. of variation	6	22	5	31
Mean long. of ☊ ——	1	11	21	12
Equat. I. correction of ☊		—	8	39
Long. ☊ corr. ——	1	11	12	33
Arg. of lat. and reduction	7	0	36	58
Arg. IX. + arg. XIII. is arg. lat. II.	6	14	11	

C H A P. V.

To reduce the obſerved Diſtance of the Moon's Limb from a Star, or from the Sun's Limb to the apparent Diſtance of the Centres, by allowing the Moon's Semidiameter, or the Sum of the Semidiameters of the Sun and Moon; and to clear the apparent Diſtance of the Centres of the Effects of Refraction and Parallax.

1. *To correct the obſerved Diſtance by the Moon's Semidiameter, or the Sum of the Semidiameters of the Sun and Moon.*

CORRECT the obſerved diſtance of the moon's limb from a ſtar by allowing the error of the adjuſtment of the quadrant; and to the diſtance thus corrected add the moon's apparent ſemidiameter, increaſed according to her altitude, if the limb obſerved was that neareſt to the ſtar; but ſubſtract the ſemidiameter, if the limb obſerved was that fartheſt from the ſtar. This gives the apparent diſtance of the moon's centre from the ſtar. But ~~from~~ to the obſerved diſtance of the ſun and moon's neareſt limbs, firſt corrected for the error of the quadrant, ~~ſubſtract~~ add the ſum of the ſemidiameters of the ſun and moon; this gives the apparent diſtance of the centres of the ſun and moon. The ſemidiameter of the ſun is found from table in page 5, and the ſemidiameter of the moon is found from her parallax by table in page 23; as is alſo the augmentation for the degree of altitude. In this example, the mean of the three obſerved diſtances of the ſtar from the moon's remoteſt

moteſt limb was 51d. 40m. 40ſ. to which adding the error of the adjuſtment of the quadrant, 3m. 20ſ. gives the apparent diſtance correct, 51d. 44m. 0ſ. from which ſubſtracting the moon's ſemidiameter, 15m. 25ſ. becauſe the obſerved limb was that furtheſt from the ſtar, leaves 51d. 28m. 35ſ. the apparent diſtance of the ſtar from the moon's centre.

2. *To clear the apparent Diſtance of a Star from the Moon's Centre, or the apparent Diſtance of the Centres of the Sun and Moon, from the Effect of Refraction.*

Correct the obſerved altitude of the ſtar by the error of adjuſtment, if any; and ſubſtract the dip of the horiſon of the ſea, and you have the apparent altitude of the ſtar. From the obſerved altitude of the moon's upper or lower limb, firſt corrected for the error of the quadrant, if any, ſubſtract the dip, and add 16m. if the lower limb was obſerved, but ſubſtract 16m. if the upper limb was obſerved, and you have the apparent altitude of the moon's centre. If the diſtance of the moon was taken from the ſun, add the difference of the dip and 16m. to the obſerved altitude of the ſun's lower limb, corrected for the error of the quadrant if any, and you have the apparent altitude of the ſun's centre. To the apparent altitudes of the ſtar, and of the centres of the ſun and moon, add three times the refractions belonging to thoſe altitudes reſpectively, found from the table in page 1, and you have the altitudes of the ſtar, ſun, and moon increaſed, the complements of which to 90 are their zenith diſtances diminiſhed.

Now add together the logarithm tangents of half the ſum and half the difference of the diminiſhed zenith diſtances of of the moon and star, or of the sun and moon. The sum, abating 10 from the index, is the tangent of arc the 1st.

To the tangent just found add the cotangent of half the apparent distance of the star from the moon's centre, or the centres of the sun and moon; the sum, abating 10 from the index, is the tangent of arc the 2d.

Then to the tangent of double the first arc add the constant logarithm, 2.0569, and from the sum substract the sine of double the second arc, leaves the logarithm of the number of seconds expressing the effect of refraction required, which added always to the apparent distance of the star from the moon's centre, or of the centres of the sun and moon, gives the distance cleared of refraction.

Example of the Computation of the Effect of Refraction.

	°	′
Mean of three observed altitudes of the star	24	52
Error of adjustment — —	0	0
Substract dip — —	—	4
Add three times refraction for altitude 25°	+	6
Altitude of the star increased —	24	54
Complement to 90 is zenith distance of star diminished	65	6
Mean of three observed altitudes of the moon's lower limb	12	18
Error of quadrant — —	0	0
Substract dip — —	—	4
Add semidiameter of the moon —	+	16
Add three times the refraction for altitude	+	13
Altitude of the moon's centre increased —	12	43
Compl. to 90 is zenith distance of the moon diminished	77	17
Zenith distance of star diminished —	65	6

Half

Half ſum 71° 11′ tangent ——	10.4675
Half difference 6° 5′ tangent ——	9.0276
Tangent of arc the firſt, 17° 22′ ——	9.4951
Cotan. half apparent diſtance of the ſtar from the moon's centre 25° 44′ ——	10.3170
Tangent of arc the ſecond, 32° 58′ ——	9.8121
Double arc the firſt, 34° 44′ tang. ——	9.8409
Conſtant logarithm —— ——	2.0569
Sum —— ——	11.8978
Double arc ſecond, 65° 56′ ſine ſubſtract.	—9.9605
Logarithm of 86″, the effect of refraction	1.9373
Apparent diſtance of the Virgin's Spike from the moon's centre —— ——	51° 28′ 35″
Add the effect of refraction ——	+ 1 26
Apparent diſt. of the ſtar from the moon's centre, cleared of refraction ——	51 30 1

3. *To clear the apparent Diſtance of a Star from the Moon's Centre, or of the Centres of the Sun and Moon of the Effect of Parallax.*

From the obſerved altitudes of the ſtar, and the moon's upper or lower limb, or the ſun's lower limb, firſt corrected for error of quadrant, ſubſtract the ſum of the dip and refraction from tables in page 1, and add further 16m. to the altitude of the ſun or moon's lower limb, or ſubſtract 16m. from the altitude of the upper limb, gives the true altitude of the ſtar, and of the centres of the ſun and moon; the complements of which to 90d. are their true zenith diſtances.

Then

Then add together into one ſum the tangents of half the ſum and half the difference of the true zenith diſtances, and the cotangent of half the apparent diſtance of the ſtar from the moon's centre, or of the centres of the ſun and moon cleared from refraction; the ſum, abating 20 from the index, is the tangent of arc the ſecond correct, (which is nearly the ſame with arc the ſecond, found above in computing refraction) the ſum of this, and half the apparent diſtance of the ſtar from the moon's centre, if the zenith diſtance of the moon is greater than that of the ſtar; or their difference, if the zenith diſtance of the moon is leſs than that of the ſtar, gives arc the third. To the tangent of arc the third add the coſine of the apparent zenith diſtance of the moon's centre, cleared of refraction, and the logarithm of the moon's horiſontal parallax, expreſſed in minutes and decimals, the ſum, abating 20 from the index, is the logarithm of the number of minutes expreſſing the effect of parallax, in augmenting or contracting the diſtance of the moon from a ſtar, which always ſubſtract from the apparent diſtance of the ſtar from the moon's centre, or of the centres of the ſun and moon, cleared of refraction, except arc the third ſhould be the difference of arc the ſecond, and half the diſtance of the moon and ſtar; and at the ſame time, arc ſecond be greater than the ſaid half diſtance of the moon and ſtar, or ſun, in which caſe add the effect of parallax to the apparent diſtance cleared of refraction, gives the true diſtance of the ſtar from the moon's centre, or of the centres of the ſun and moon, cleared both of refraction and parallax.

	°	′
Mean of three obferved altitudes of ftar by quadrant	24	52
Error of quadrant — —	0	0
Subftract fum of dip 4′ and refraction 2′	—	6
True altitude of the ftar —	24	46
The complement to 90 is the true zenith dift. of ftar	65	14

Mean of three obferved altitudes of the moon's lower limb	12	18
Error of quadrant — —	0	0
Subftract fum of dip 4′ and refraction 4′	—	8
Add femidiameter of the moon —	+	16
True altitude of the moon's centre —	12	26
Compl. to 90 is the true zenith diftance of the moon	77	34
Zenith diftance of ftar — —	65	14

Half fum of zenith diftances	71°	24′	tang.	10.4730
Half difference of zenith diftances	6	10	tang.	9.0336
Half dift. of ftar from the moon's centre	25	45	cotang.	10.3166
Sum, tangent of arc fecond correct	33	39		9.8232

Sum of arc 2d and ½ dift. of moon fr. ftar	59	24	tang.	10.2281
Moon's zenith diftance —	77	34	cofine	9.3330
Moon's horifontal parallax	56′ 15″=56′,25		log.	1.7501
Logarithm of effect of parallax	20,48 =20′ 29″			1.3112

This by the rule is here to be fubftracted from 51d. 30m. 1f. the apparent diftance of the ftar from the moon's centre, cleared of refraction, which gives 51d. 9m. 32f. the true diftance of the ftar from the moon's centre, cleared both of refraction and parallax.

 N. B.

N. B. If the altitudes of the moon and ſtar are both more than 10d. they may be uſed in theſe computations of refraction and parallax, as taken from the quadrant, only allowing the error of the adjuſtment, if any, and the dip, and the moon's ſemidiameter, to reduce the altitude of the limb to that of the centre; and arc the ſecond may be uſed for arc the ſecond correct, without any ſenſible error, which will ſave ſome trouble in calculation. Four places of figures beſides the index, will be ſufficient in theſe computations.

CHAP.

CHAP. VI.

From the true Diſtance of the Centres of the Sun and Moon reduced from the Obſervation, and the Moon's Latitude found from the Tables, to compute the Difference of Longitude between the Sun and Moon, and thence to find the Moon's true Longitude; or, from the true Diſtance of the Star from the Moon's Centre reduced from the Obſervation, the Star's Latitude, and the Moon's Latitude, found from the Tables, to compute the Difference of Longitude between the Moon and Star; and thence to find the Moon's true Longitude.

1. *From the true Diſtance of the Centres of the Sun and Moon, reduced from the Obſervation, and the Moon's Latitude found from the Tables, to compute the Difference of Longitude between the Sun and Moon, and thence to find the Moon's true Longitude.*

TO the coſine of the diſtance of the centres of the ſun and moon reduced from the obſervation, add 10 in the place of the index, and from the ſum ſubſtract the coſine of the moon's latitude, the remainder is the coſine of the difference of longitude between the ſun and moon, which, added to the ſun's longitude found by the tables, if the moon is to the eaſt of the ſun, or ſubſtracted from the ſun's longitude, if the moon is to the weſt of the ſun, gives the moon's true longitude.

EX-

EXAMPLE.

Let us ſuppoſe the true diſtance of the centres of the ſun and moon reduced from an obſervation to be 73d. 37m. 13ſ. and the moon's latitude, found by the tables, to be 4d. 23m. 36ſ.

Coſine of diſtance of moon from ſun	73° 37′ 13″	9.4502521
Add to index	—— ——	10.
		19.4502521
Subſtract coſine of the moon's latitude		9.9987220
		9.4515301

Coſ. of dif. of long. of ſun and moon 73° 34′ 14″=2S. 13° 34′ 14′

Suppoſe the ſun's longitude to have been computed by the tables 7S. 15d. 22m. 46ſ. and the moon to be to the weſt of the ſun, the difference 2S. 13d. 34m. 14ſ. ſubſtracted from the ſun's longitude, 7S. 15d. 22m. 46ſ. leaves 5S. 1d. 48m. 32ſ. the moon's true longitude.

2. *From the true Diſtance of a Star from the Moon's Centre, reduced from the Obſervation, the Star's Latitude, and the Moon's Latitude, found by the Tables, to compute the Difference of Longitude between the Moon and Star, and thence to find the Moon's true Longitude.*

If the latitudes of the moon and ſtar are both of the ſame denomination, that is to ſay, both north or both ſouth, their complements to 90d. are their diſtances from the neareſt pole of the ecliptic: or if their latitudes are of different denominations, ſubſtract one from 90d. it does not matter which, and add the other to 90d. which gives their diſtances from one of the poles of the ecliptic, which I ſhall call their polar diſtances.

Now

Now take half the difference of the polar diſtances, and half the diſtance of the moon and ſtar reduced from the obſervation. Then add together theſe four logarithms, the ſine of the half diſtance increaſed by the ſaid half difference, the ſine of the half diſtance leſſened by the ſaid half difference, the arithmetical complement of the ſine of the polar diſtance of the ſtar, and the arithmetical complement of the ſine of the polar diſtance of the moon. Half the ſum of theſe four logarithms is the ſine of an arc, which doubled is the difference of longitude between the moon and ſtar; this added to the ſtar's longitude, if the moon is eaſt of the ſtar, or ſubſtracted from the ſtar's longitude, if the moon is weſt of the ſtar, gives the true longitude of the moon in the ecliptic.

In making theſe calculations ſeven places of figures muſt be uſed, beſides the index, and proportion muſt be made for ſeconds, except Gardiner's logarithms be uſed, out of which the logarithms may be taken at ſight, for the neareſt ten ſeconds. The ſtar's latitude and longitude muſt be taken out of table in page 4, and the latter muſt be corrected by table in page 5, and alſo by adding 50¼ ſ. for every year after 1763.

EXAMPLE.

	°	′	″
Diſt. of moon from ſtar, reduced from obſervation	51	9	32
Latitude of the Virgin's Spike ——	2	2	9 S.
Latitude of the moon ——	2	39	26 S.
Polar diſtance of the Virgin's Spike ——	87	57	51
Polar diſtance of the moon ——	87	20	34
Difference of polar diſtance ——	0	37	17
Half difference —— ——	0	18	38
Half diſtance of the moon from the Virgin's Spike, reduced from the obſervation ——	25	34	46

Half

	°	′	″		
Half dist. of moon from star + ½ diff.	25	53	24	sine	9.6401282
Half dist. of moon from star — ½ diff.	25	16	8	sine	9.6302925
Pol. dist. of Virgin's Spike	87°	57′	51″	ar. compl. sin.	0.0002739
Pol. dist. of the moon	87	20	34	ar. compl. sin.	0.0004672
Sum					19.2711618
Half sum, sine of 25° 36′ 2½″					9.6355809
Doubled is	51	12	5	diff. of long. of moon and star.	

	S.	°	′	″
Longitude of the Virgin's Spike in table, page 4	6	20	32	1
Add correction for May 9, from table, page 5th				+ 38
Subst. 50″ because the year precedes 1763 by 1 year				— 50
Longitude of star correct	6	20	31	49
To which add the diff. of long. of moon and star found above, 51° 12′ 5″ (because moon is east of star) =	1	21	12	5
True long. of moon inferred from the observation	8	11	43	54

CHAP.

CHAP. VII. and Laſt.

To find the Error of the Ship's Reckoning, and conſequently the true Longitude of the Ship.

TAKE the difference of the moon's longitude, as inferred from the obſervations by the laſt Chapter, and the moon's longitude computed from the tables by Chap. IV. and, if the moon's diſtance was obſerved from a ſtar, ſay, by the rule of proportion, as the hourly motion of the moon is to the ſaid difference, ſo is 900m. to the error of the ſhip's account in minutes. But, if the diſtance of the moon was obſerved from the ſun, inſtead of the hourly motion of the moon, uſe the difference of the hourly motions of the ſun and moon. The hourly motion of the ſun is contained in table, page 5. Or the error of the account may be found more briefly thus: With the hourly motion of the moon, or the difference of the hourly motions of the ſun and moon, according as the moon's diſtance was obſerved from a ſtar or the ſun, take out a number, called the multiplier, from table, page 25, with which multiply the difference of the moon's longitude inferred from obſervation, and that computed; the product is the error of the ſhip's account of longitude. And if the longitude of the moon computed by the tables, is greater than that inferred from the obſervation, the ſhip is to the eaſt of the longitude by account; but, if leſs, the ſhip is weſt of the longitude by account. When the ſhip is eaſt of the longitude by account, the error of the account is to be added to her eaſt longitude by account, or ſubſtracted from her weſt longitude by account; but when the ſhip is weſt of the longitude by account, the ſaid error is to be ſubſtracted from her eaſt longitude

gitude by account, and added to her weſt longitude by account. The ſaid correction may be applied either to the ſhip's longitude by account, on the noon preceding the obſervation, or to the ſame carried on to the next noon, or to any intermediate time, and will give the true longitude of the ſhip at the time propoſed; which was the thing to be found from all the obſervations and calculations.

In the example, the moon's true longitude, inferred from the obſervation, is 8S. 11d. 43m. 54ſ. and the moon's longitude, computed from the tables, from the time aſſumed at Greenwich, according to the ſhip's reckoning, is 8S. 11d. 43m. 25ſ. The difference is 29ſ. and ſay, by the rule of proportion, as 32m. 4ſ. the hourly motion of the moon, found after the computation of the moon's place, is to 29ſ. the ſaid difference, ſo is 900m. to 14m. the error of the ſhip's account. Or, more briefly, multiply 29ſ. by $28\frac{1}{10}$ the multiplier, taken out of table, page 25, with the hourly motion of the moon, 32m. 4ſ. the product, 815ſ. or 14m. nearly, is the error of the ſhip's reckoning; and the moon's longitude computed from tables, being leſs than that inferred from obſervations reduced, the ſhip is weſt of account, and her reckoning by account on the preceding noon having been 7d. 44m. weſt of Greenwich, her true longitude at that time was 7d. 44m.+14m.=7d. 58m. weſt of Greenwich.

Or if it be required to find the longitude of the ſhip at the time of taking the diſtance of the moon from the ſtar, add 14m. to the longitude by account at that time, which was 6d. 54m. weſt; whence the true longitude of the ſhip at that time was 6d. 54m.+14m.=7d. 8m. weſt of Greenwich. Or if it be propoſed to find the longitude of the ſhip on the following noon, or the noon of May 10, add 14m. to to the longitude by account carried on to that time, which is 6d. 26m. weſt; whence the true longitude, May 10, at noon, was 6d. 26m.+14m.=6d. 40m. weſt of Greenwich.

The reaſon why the error of the ſhip's reckoning is ſo ſmall in this example, is becauſe it was corrected by an obſervation the night before.

At ſun-riſe we ſaw the Scilly iſlands bearing N.E.b.N. by the compaſs, or 14d. E. of the true north, and diſtant by eſtimation about ſix leagues; and we had made 40½m. of departure =1d. 3m. of longitude to the eaſtward ſince noon; therefore our longitude by account was 6d. 41m. W. But, by the obſervations of the moon, 14m. is to be added to correct the longitude by account; therefore the true longitude at this time is 6d. 41m.+14m.=6d. 55m. W. But by the bearing and eſtimated diſtance, the middle of the Scilly iſlands muſt be 7m. of longitude to the eaſt of the ſhip; and therefore their longitude, as deduced from the obſervations of the moon, is 6d. 55m—7m.=6d. 48m. W.

At 9 A.M. the light-houſe on St. Agnes, one of the Scilly iſlands, bore N. 8½d. W. by the compaſs, diſtant by eſtimation 5 leagues; and we had made 47 miles departure =1d. 12m. of longitude to the eaſt ſince noon. Therefore, our longitude by account is 6d. 32m. weſt; or, corrected by the obſervation of the moon, 6d. 32m.+14m.=6d. 46m. W. But, by the bearing and eſtimated diſtance, St. Agnes light-houſe was 12m. of longitude to the weſt of the ſhip; therefore its longitude, deduced from the obſervations of the moon, is 6d. 46m.+12m.=6d. 58m. W. This longitude differs 10m. from the longitude of the middle of the Scilly iſlands, according to their bearing and eſtimated diſtance at ſun-riſe, tho' it ſhould ſeem, that it ought to agree nearly with it. But this ſmall difference may be eaſily accounted for, if it be conſidered, that theſe iſlands extend near half a degree from eaſt to weſt, and that the ſame part may not appear to be the middle of them at a diſtance at ſea, as does in the charts; not to mention our being in a tide's way, and the difficulty of eſtimating diſtances with exactneſs. But, which-ever of theſe determinations is taken, it will not differ above 18 or 20 minutes of longitude from the beſt charts, which make but 12 miles in this latitude.

Having had a good obſervation, May 10, at noon, determining the latitude of St. Agnes, I ſhall ſet it down, though it be a little deviation from my preſent ſubject. The latitude of the ſhip, by the ſaid obſervation, was 49d. 48m. N. and the light-houſe of St. Agnes at that time bore N. 38d. W. by the compaſs, diſtant by eſtimation five leagues; whence its latitude ſhould be 49d. 56m. N. which is 10m. more than is given to it in Whiſton's Chart of the Channel.

APPENDIX.

A P P E N D I X,

Containing ſome additional Rules, ſerving to find the Moon's right Aſcenſion and Declination, and thence to compute her Altitude; as alſo to compute the Altitude of the Star, ſuppoſing they were not obſerved: and alſo a particular Problem for finding the Longitude from three cotemporary Obſervations only.

AS it may ſometimes happen, though very rarely, that, at the obſervation of the diſtance of the moon from the ſtar, the altitude of the ſtar, and even that of the moon, cannot be taken with ſufficient certainty, I ſhall here ſubjoin the method of computing them.

In order to this it is neceſſary firſt to know their right aſcenſions and declinations, and alſo the right aſcenſion of the ſun at the time. The right aſcenſions and declinations of all the principal ſtars are contained in the table, page 2 and 3; the right aſcenſion of the ſun is to be found from his longitude, by table page 27 and 28. I now proceed to ſhew how the right aſcenſion and declination of the moon may be readily found with ſufficient exactneſs.

The Longitude and Latitude of the Moon being given, to find the Right Aſcenſion and Declination.

This may be done very expeditiouſly from Halley's or Senex's map of the zodiac, or any map of the like kind, containing circles of latitude with their parallels, and circles of declinations with their parallels; or even by a large globe, accurately conſtructed. But leſt the obſerver ſhould be unprovided with theſe helps, I offer him the two following eaſy eafy rules for this purpofe, which are fufficiently exact to find the moon's right afcenfion and declination true to two minutes.

PREPARATION.

Take out of the tables of the declination of the degrees of the ecliptic, page 26, the declination anfwering to the moon's longitude, and note whether it be north or fouth. Subftract three figns from the moon's longitude, and take out of the fame tables the declination correfponding, and alfo note whether it be north or fouth. Alfo take out of the tables of the right afcenfion of the degrees of the ecliptic, page 27 and 28, the right afcenfion anfwering to the moon's longitude.

RULE I.

To find the Moon's Declination.

To the fine of the moon's latitude add the cofine of the declination of that point of the ecliptic, whofe longitude is lefs than the moon's by three figns; the fum, abating 10 from the index, is the fine of an arc, the fum of which, and the declination of the point of the ecliptic belonging to the moon's longitude, provided that the moon's latitude and the faid declination are of the fame kind, that is to fay, both north, or both fouth; or their difference, if they are of contrary denominations, gives the true declination of the moon, of the fame kind with the greater.

RULE II.

To find the Moon's right Afcenfion.

To the fine of the moon's latitude add the fine of the declination of that point of the ecliptic, whofe longitude is lefs

lefs than the moon's by three figns, and from the fum fubftract the cofine of the declination of the point of the ecliptic anfwering to the moon's longitude, leaves the fine of an arc, which add to the right afcenfion of the point of the ecliptic belonging to the moon's longitude, if the moon's latitude is of the fame kind with the declination of the point of the ecliptic, whofe longitude is lefs than the moon's by three figns; or fubftract it if they are of different kinds; the refult is the true right afcenfion of the moon.

EXAMPLE.

Of the Computation of the Moon's Declination and right Afcenfion from the Longitude and Latitude given.

I fhall adapt the example to the time of the obfervations above recited, and fhall firft of all compute the moon's right afcenfion and declination, by which we fhall be enabled afterwards to compute the altitude of the moon, fuppofing it had not been obferved.

PREPARATION.

At the time of the obfervation of the diftance of the Virgin's Spike from the moon, above recited, the moon's longitude was computed by the tables 8S. 11d. 43m. and latitude 2d. 39m. S. leaving out the feconds. By, table p. 26, the declination anfwering to the moon's longitude, 8S. 11d. 43m. is 22d. 14m. S. Subftract three figns from the moon's longitude, leaves 5S. 11d. 43m. the declination anfwering to which is 7d. 11m. N. By table, page 27 and 28, the right afcenfion anfwering to the moon's longitude, 8S. 11d. 43m. is 250d. 11m.

To

To find the Moon's Declination.

Sine of the moon's latitude 2° 39′ S. ——	8.66497
Cofine of declination anfwering to longitude 5S. 11° 43′ or moon's longitude, 8 S. 11° 43′—3 S. 7° 11′ N.	9.99658
Sine of 2° 38′ —— ——	8.66155

The fum of this and 22d. 14m. S. the declination anfwering to the moon's longitude, becaufe the moon's latitude and the faid declination are of the fame kind, is the moon's declination, 24d. 52m. S.

To find the Moon's right Afcenfion.

Sine of the moon's latitude 2° 39′ S.	8.66497
Sine of declination anfwering to longitude of the moon, — 3 figns, or 5 S. 11° 43′ 7° 11′ N.	9.09706
	17.76203
Subftract cof. of declination anfwering to the moon's longitude 22° 14′ ——	—9.96645
Sine of 0° 21′ —— ——	7.79558

Which fubftract from 250d. 11m. the right afcenfion anfwering to the moon's longitude, becaufe the moon's latitude, and the declination anfwering to the moon's longitude — 3 figns, are of contrary denominations, leaves the right afcenfion of the moon 249d. 50m.

RULE

RULE III.

To compute the apparent Altitude of the Moon or a Star, at the Time of the Obſervation of the Diſtance of the Moon from the Star.

Turn the apparent time, corrected by allowing the error of the watch, as found by the altitude of the ſun, into degrees and minutes, by table, page 3 and 4, and add it to the ſun's right aſcenſion, taken out of table, page 27 and 28, with the ſun's longitude, computed in Chap. IV. The ſum is the right aſcenſion of the mid-heaven nearly. Correct this by the longitude made by the ſhip ſince the taking of the altitude of the ſun in the afternoon, by which the watch was corrected, adding it, if the ſhip has gone to the eaſt, or ſubſtracting it, if ſhe has gone to the weſt, gives the right aſcenſion of the mid-heaven correct; the difference between which, and the right aſcenſion of the moon or ſtar, is their hourly angle, or diſtance from the meridian. If the moon or ſtar's declination is of the ſame denomination with the latitude of the ſhip, ſubſtract it from 90d. but, if it be of the contrary denomination, add it to 90d. and you have the diſtance from the elevated pole. To the coſine of the hourly angle add the cotangent of the latitude of the ſhip; the ſum, abating 10 from the index, is the tangent of arc the firſt, the difference of which, and the diſtance of the moon or ſtar from the elevated pole, is arc the ſecond. To the coſine of arc the ſecond add the ſine of the ſhip's latitude, and from the ſum ſubſtract the coſine of arc the firſt, give the ſine of the true altitude of the moon or ſtar. And to find the apparent altitudes, add the refraction from table, page 1, to the true altitude of the ſtar, and ſubſtract the difference between the refraction, from table, page 1, and the moon's parallax in altitude, from table, page 26, from the moon's true altitude: gives the apparent altitudes of the moon and ſtar.

EXAMPLE.

Of the Computation of the apparent Altitudes of the Moon and Star.

The apparent time was found 12h. 5m. 35ſ. at the end of Chap. III. which, turned into degrees and minutes by table, page 3 and 4, is 181d. 24m. And, by table, page 27 and 28, the ſun's right aſcenſion anſwering to his longitude, 1S. 19d. 15m. computed in Chap. IV. is 46d. 47m. The ſum is 228d. 11m. the right aſcenſion of the mid-heaven nearly; to which add 17m. of longitude, which the ſhip has made to the eaſt, ſince the altitude of the ſun in the afternoon, by which the watch was corrected, gives 228d. 28m. the right aſcenſion of the mid-heaven correct.

To find the apparent Altitude of the Virgin's Spike.

The declination of the Virgin's Spike, by table, page 2, is 9d. 55m. S. whence the latitude of the ſhip being north, the diſtance of the ſtar from the elevated pole is 99d. 55m. The right aſcenſion of the Virgin's Spike, by table, page 2, is 198d. 10m. the difference of which, and 228d. 28m. the right aſcenſion of the mid-heaven, is the hourly angle of the Virgin's Spike, 30d. 18m.

Coſine of hourly angle of Virgin's Spike 30° 18′		9.93621
Cotan. of latitude of ſhip 49° 23′	——	9.93329
Tan. of arc the firſt 36° 31′	——	9.86950

The

The difference between which and 99d. 55m. the diſtance of the Virgin's Spike from the elevated pole, gives arc the ſecond, 63d. 24m.

Coſine of arc the ſecond, 63° 24′	—	9.65104
Sine of latitude of ſhip, 49° 23′	—	9.88029
		19.53133
Subſtract coſine of arc the firſt, 36° 31′		—9.90508
Sine of true altitude of the Virgin's Spike, 25° 1′		9.62625

Add 2m. of refraction at this altitude, by table, page 1, gives apparent altitude of the Virgin's Spike, 25d. 3m. It was obſerved 24d. 52m. or taking off 4m for the dip, 24d. 48m.

To find the apparent Altitude of the Moon's Centre.

The declination of the moon has been computed above, 24d. 52m. S. therefore, the latitude of the ſhip being north, the diſtance of the moon from the elevated pole is 114d. 52m. The right aſcenſion of the moon has been computed above, 249d. 50m. the difference of which, and 228d. 28m. the right aſcenſion of the mid-heaven, is 21d. 22m. the hourly angle of the moon.

Coſine of hourly angle of the moon, 21° 22′		9.96907
Cotan. of latitude of the ſhip, 49° 23′	—	9.93329
Tangent of arc the firſt, 38° 37′	—	9.90236

The difference between which, and 114d. 52m. the diſtance of the moon from the elevated pole, gives arc the ſecond, 76d. 15m.

Cofine of arc the fecond, 76° 15	—	9.37600
Sine of latitude of the fhip, 49° 23′	—	9.88029
		19.25629
Subftract cofine of arc the firft, 38° 37′		—9.89284
Sine of the true altitude of the moon, 13° 21′		9.36345

From which fubftract the difference of 4m. the refraction, by table, page 1, and 55m. the parallax in altitude, by table, page 26, gives the apparent altitude of the moon, 12d. 30m. The altitude of the moon's lower limb was obferved 12d. 18m. from which fubftracting 4m. for the dip, and adding 16m. for the moon's femidiameter, the apparent altitude of her centre from the obfervation is 12d. 30m. which happens to agree exactly with that computed.

A PRO-

A PROBLEM.

To determine the Longitude at Sea or Land from three cotemporary Obſervations only, namely, the Diſtance of the Moon's Limb from a Star, and the apparent Altitudes of the Moon and Star, provided the Moon be not leſs than two Hours diſtant from the Meridian.

IT has been ſaid in Chap. II. that four obſervations are generally neceſſary to be made, in order to find the longitude, according to the method deſcribed in the following chapters; namely, an altitude of the ſun, or a bright ſtar, in order to regulate a watch, by the time of which all the obſervations are noted down; the diſtance of the moon's limb from the ſun, or a proper ſtar, and the apparent altitudes of the ſame ſtar, and of the moon's upper or lower limb. It has, however, been remarked in the ſame place, that the neceſſary obſervations are reduced to three, when the diſtance of the moon from the ſun is obſerved; the altitude of the ſun, taken at the ſame inſtant, being ſufficient to determine the time, without the uſe of any other altitude of the ſun, or a ſtar, and even without the aid of a watch. For the ſame reaſon, the altitude of the ſtar, from which the diſtance of the moon's limb is meaſured, being taken at the ſame inſtant, will alſo determine the time with ſufficient exactneſs, provided the horiſon be tolerably enlightened, and the ſtar be at leaſt two hours from the meridian, without requiring the altitude of the ſun, or any other ſtar, and even without the uſe of a watch.

For

For the very ſame reaſon, the altitude of the moon likewiſe, taken at the ſame inſtant with the obſervation of her diſtance from the ſtar, will ſerve equally to determine the time, provided ſhe be at leaſt two hours from the meridian, and the horiſon be ſufficiently enlightened, ſo that her altitude may be taken with ſome exactneſs. But as the moon's right aſcenſion and declination, which are both neceſſary to be known in this caſe, are continually varying, and cannot be aſſumed, near enough, from the eſtimated time of the day, and the longitude of the ſhip by account, as the declination of the ſun may, this caſe requires another proceſs, and demands a particular method of computing the longitude, diſtinct from that explained in the preceding pages, which hath not, that I know of, been yet thought of, or, at leaſt, not recommended to the public for this purpoſe. I have therefore propoſed it as a problem, and now proceed to give the ſolution. I ſhall only firſt remark, that this method, as it requires only the three eſſential obſervations taken at the ſame inſtant, ſo it may often be of very great uſe, eſpecially where the watch, which the obſervers uſe, is ſo indifferent as not to be depended upon within one, or even two minutes, for the ſpace of ſome hours; and that, in many caſes, it may be full as exact as the general method, which I have hitherto propoſed, as the altitude of the moon, eſpecially if not very high, may be often taken at ſea in the night, by the help of her own light illuminating the horiſon, to almoſt as great a degree of exactneſs, as the altitude of the ſun in the day-time; whereas the altitudes of ſtars, which do not lie in the ſame vertical with the moon, cannot be taken with the ſame certainty, the horiſon under them being not enlightened in the ſame manner.

The

The following Steps must be observed in the Solution of the Problem proposed.

1. Assume the apparent time of the observation as near as you can, which may be done very well, within half an hour, either by common watches, or the glasses of the ship; only take care to allow for the ship's change of longitude, at the rate of four minutes for every degree you have made since they were last corrected, adding this correction if you have been going eastward, or substracting it if you have been going westward. To the time so assumed, add or substract your longitude from Greenwich, turned into time, by table, page 3 and 4, according as you are to the west or east of Greenwich. To this time compute the sun's longitude, and the moon's longitude, latitude, &c. as in Chap. IV.

2. From the three given observations, the distance of the star from the moon, and the altitudes of the moon and star, and the moon's horisontal parallax, find the true distance of the star from the moon's centre, by correcting the observed distance of the star from the moon's limb, for the moon's semidiameter, and for refraction and parallax, according to Chap. V.

3. With the true distance of the star from the moon's centre thus found, and the moon's latitude computed, find the difference between the longitudes of the moon and star, and thence the moon's true longitude, according to Chap. VI.

4. From the moon's longitude thus found, and latitude computed above, find her right ascension and declination, by the two rules after Chap. VII. with the help of the tables, page 26, 27, and 28; and from the sun's longitude, computed

puted above, find his right aſcenſion, by table, page 27 and 28. From the obſerved altitude of the moon's lower limb ſubſtract the ſum of dip and refraction; add 16m. for the moon's ſemidiameter, and add the parallax in altitude, taken out of table, page 25, with the moon's apparent altitude and horiſontal parallax: or from the obſerved altitude of the moon's upper limb, ſubſtract the ſum of dip, refraction, and 16m. for the moon's ſemidiameter, and add the parallax in altitude, and you have the true altitude of the moon's centre, the complement of which to 90d. is the zenith diſtance of the moon. With this, and the moon's declination, and the latitude of the ſhip at the time, find the moon's diſtance from the meridian, according to Chap. III. which add to, or ſubſtract from, the moon's right aſcenſion, found above, according as ſhe is to the weſt or eaſt of the meridian, gives the right aſcenſion of the mid-heaven, from which ſubſtract the ſun's right aſcenſion, found above; the remainder, converted into time by table, page 3 and 4, is the apparent time required. In ſtrictneſs, the time thus found ſhould be corrected at the rate of 1ſ. upon every 6m. of the difference between the ſaid time and the time aſſumed, to be ſubſtracted if the firſt is greateſt, and added if the ſecond is greateſt. The computation is to be performed according to Chap. III.

5. Take the difference between this time and the time at firſt aſſumed, and, by the rule of proportion, as 60m. of time is to this difference, ſo is the moon's hourly motion (found after computing the moon's place) to a number of minutes and decimals of a minute, which add to, or ſubſtract from, the moon's longitude computed to the aſſumed time, reduced to Greenwich, according as the time found by the altitude of the moon, exceeds or falls ſhort of the time aſſumed, and you have the longitude of the moon agreeable to the tables, at the time found by the altitude of

2 the

the moon, reduced to Greenwich, according to the ſhip's account.

6. By comparing this longitude of the moon, reſulting from computation, with that inferred from the obſervation, find the error of the ſhip's account, and thence the true longitude of the ſhip, according to Chap. VII.

EXAMPLE.

Of the Determination of the Longitude from three cotemporary Obſervations of the Diſtance of a Star from the Moon's Limb, and of the Altitudes of the Moon and Star, according to the Problem.

I ſhall make uſe of ſome of the ſame obſervations as I employed in the illuſtration of the general method.

May 9, 11h. 33m. 42ſ. by the watch, the diſtance of the Virgin's Spike from the moon's remoteſt limb was meaſured 51d. 40m. 40ſ. but 3m. 20ſ. is to be added for the error of the adjuſtment of the quadrant. At the ſame inſtant, the altitude of the ſtar from the apparent horiſon of the ſea was taken, 24d. 52m. and the altitude of the moon's lower limb above the ſame, 12d. 18m. both free from any error of the quadrant, but including the dip and refraction. The latitude of the ſhip was 49d. 23m. and the longitude by account, 6d. 53m. weſt of Greenwich.

Article I. I ſhall aſſume the time to be 11h. 40m. P.M. the longitude from Greenwich, at this time, by account, is 6d. 53m. W.=27m. 32ſ. of time, by table, page 3 and 4; this, added to the time aſſumed, 11h. 40m. P.M. gives the aſſumed time reduced to Greenwich, 12h. 7m. 32ſ. P.M.

K Hence

Hence the ſun's longitude, computed by the tables, is 1 S. 19d. 13m. 48ſ. The equation of time — 3m. 54ſ. the mean time at Greenwich, by aſſumption, 12h. 3m. 38ſ. P. M. Hence the moon's longitude, computed by the tables, is 8 S. 11d. 29m. 6ſ. the latitude 2d. 38m. 17ſ. S. the moon's horiſontal parallax 56m. 15ſ. horiſontal ſemidiameter 15m. 21ſ. and hourly motion 32m. 4ſ.

Art. II. From the obſervations given, the true diſtance of the ſtar from the moon's centre is found, cleared of refraction and parallax, 51d. 9m. 32ſ. in Chap. V.

Art. III. With the true diſtance of the ſtar from the moon's centre, 51d. 9m. 32ſ. and the moon's latitude, 2d. 38m. 17ſ. S. find the difference between the longitudes of the moon and ſtar, and thence the moon's true longitude, according to Chap. VI. making the computation, the difference of longitude between the moon and ſtar will come out 51d. 12m. 4ſ. differing only 1ſ. from what was found in Chap. VI. though the moon's latitude is taken above a minute leſs in this computation than it was in that; the reaſon of which is, that the latitudes of the moon and ſtar are both of the ſame kind, and ſo nearly agree together. But even if the difference of their latitudes is greater, provided it does not exceed the limits of the table in page 4, an error of 1m. 2m. or even 3m. in the moon's latitude, (which we may poſſibly be ſometimes ſubject to in this computation, whether from the imperfection of the tables in this reſpect, or an error in the aſſumption of the time) can be of no prejudice in the reſult.

The longitude of the Virgin's Spike, by table, page 4, corrected by table, page 5, and alſo by ſubſtracting 50ſ. is 6S. 20d. 31m. 49ſ. to which add the difference of longitude between

between the moon and ſtar, found above, 51d. 12m. 4ſ. =1S. 21d. 12m. 4ſ. becauſe the moon was eaſt of the ſtar, gives the true longitude of the moon inferred from the obſervation, 8S. 11d. 43m. 53ſ.

Art. IV. From the moon's longitude, juſt found, 8S. 11d. 44m. and latitude computed before, 2d. 38m. S. find her right aſcenſion and declination by the two rules after Chap. VII. with the help of the tables, page 26, 27, and 28. The moon's declination will be found 24d. 51m. S. within a minute, as it is in the example to the rules, and her right aſcenſion, as in the example there, 249d. 50m. With the ſun's longitude, 1S. 19d. 14m. find his right aſcenſion, by table, page 27 and 28, 46d. 46m. From the obſerved altitude of the moon's lower limb, 12d. 18m. ſubſtract the ſum of dip 4m. and refraction 4m. and add 16m. for the moon's ſemidiameter; alſo add the parallax, 55m. anſwering to the altitude, 12d. 18m. taken out of table, page 26, with the moon's horiſontal parallax, 56m. hence the true altitude of the moon's centre is 13d. 21m. and her zenith diſtance, 76d. 39m. Her declination being found, 24d. 51m. S. and the latitude being 49d. 23m. N. the diſtance of the moon from the elevated pole is 90d.+24d. 51m.= 114d. 51m. and the colatitude 90d.—49d. 23m.= 40d. 37m.

Computation of the apparent Time from the obſerved Altitude of the Moon. Vide Chap. III.

	°	′
Zenith diſtance of the moon — —	76	39
Diſt. of the moon from the elevated pole	114	51
Colatitude — —	40	37
Sum of zenith diſt. of moon, polar diſt. and colatitude	232	7
Half ſum — —	116	3

	°	′	
Arith. compl. of fine of colatitude	40	37	0.18642
Arith. compl. of fine of pol. dift. of moon	114	51	0.04219
Sine of half fum —colat.=116°3′— 40°37′=	75	26	9.98581
Sine half fum—pol. dift.=116°3′—114°51′=	1	12	8.32103
Sum of four logarithms ——			18.53545
Half fum, fine of 10° 40½′ ——			9.26772

	°	′
Doubled is dift. of moon from meridian	—21	21
Which fubftract from the moon's right afcenfion, the moon being eaft of the meridian, ——	249	50
Leaves right afcenfion of mid-heaven ——	228	29
Subftract the fun's right afcenfion, found above,	—46	46
Remainder —— ——	181	43

	h.	m.	f.
Apparent time, by table, page 3 and 4,	12	6	52 P. M
But this exceeding 11h. 40m. P. M. the time affumed, by 27m. which contains 6m. four times, fubftract ——			4
Apparent time correct from altitude of moon	12	6	48 P. M.

Art. V. The difference between the apparent time thus found, 12h. 6m. 48f. and the time affumed, 11h. 40m. being 26m. 48f. fay, as 6cm. : 26m. 48f. : : 32m. 4f. (hor. mot. ☽) : 14m. 20f. which add to 8S. 11d. 29m. 6f. the moon's longitude, computed from the tables, becaufe the time computed from the altitude of the moon exceeds the time affumed, gives the longitude of the moon from computation correct, 8S. 11d. 43m. 26f.

Art.

Art. VI. The longitude of the moon juſt found, 8S. 11d. 43m. 26ſ. is leſs than that inferred from the obſervation in Art. III. 8S. 11d. 43m. 53ſ. by 0m. 27ſ. which comes out the ſame within 2ſ. as in the example in Chap. VII. The error of the ſhip's account will therefore be found, as in the ſequel of that Chapter, 14m. the ſhip being ſo much to the weſt of her account; whence her true longitude, at any aſſigned time between the noons of May 9 and 10, is eaſily found, as is exemplified in the latter part of the ſame Chapter.

A conciſe

A concise Method of finding the Latitude, from two observed Altitudes of the Sun, with the Interval of Time given by a common Watch.

THE common practice of navigation, being incapable of giving the longitude with any certainty, requires, on that account, a more exact knowledge of the latitude; and it is generally supposed, that this can be sufficiently determined, at all times, from meridian observations of the sun. It may indeed be thus inferred, to more than a sufficient degree of exactness, by the use of that invaluable instrument, the Hadley's quadrant, if no clouds or fogs intervene at the time. But what are we to do in this latter case? The meridian observation being only that of a single instant, or, at the most, of a few minutes, if a cloud should happen to come at that time, even though it pass off again presently, it may either prevent the observation, or, at least, render it very doubtful; for, without the observer can see the sun both rise and fall, or, at least, find him continue in a manner stationary for some minutes, he cannot be sure that he has obtained the meridian or greatest altitude.

To prevent these inconveniencies, the mathematicians have proposed a problem, to find the latitude from two observed altitudes of the sun, with the interval of time measured by a common watch. By this method, provided the two altitudes can be obtained between the hours of nine in the morning and three in the afternoon, with a proper interval of time between them, the latitude may be determined nearly, as exact as by the meridian observation; and

as

as in cloudy weather it affords an incomparably greater chance of ſucceſs, than the meridian obſervation only, it cannot but be of the greateſt uſe, perhaps next in value to the method of finding the longitude. It is therefore to be hoped, that every diligent mariner will practice this improvement, as occaſions may require, eſpecially as it is attended with very little trouble, either on the part of the obſervations or the computations. I need not obſerve, how nearly the art of practical navigation would be perfected by the addition of this more general method of finding the latitude, to the method of finding the longitude, explained in the preceding treatiſe.

The following method of calculation is taken, for the moſt part, from a paper of Dr. Pemberton's in Part II. of the 51ſt Volume of the Philoſophical Tranſactions for 1760, and is more expeditious and convenient, than that by Harriſon's logarithmic ſolar tables, publiſhed in 1759, as it requires, beſides the common tables of logarithms, only a table of logarithmic verſe-ſines, and a table of natural ſines, both of which are to be found in Sherwin's exact tables of logarithms, which ought alſo to be uſed in the computations of the longitude, as being the moſt exact and compleat tables extant.

I have ſubjoined ſome cautions that ought to be uſed in the choice of the times of the obſervations, to enſure the ſucceſs of the computations.

PREPARATION.

If both the altitudes of the ſun were taken in the morning, or both in the afternoon, ſubſtract the time ſhewn by the watch at the firſt obſervation, from the time ſhewn by the

the watch at the ſecond obſervation: but if one was taken in the forenoon, and the other in the afternoon, add 12 hours to the latter, from which ſubſtract the time of the forenoon obſervation; the difference is the elapſed time. Turn the elapſed time into parts of a circle, at the rate of 15d. to 1h. and 15m. to 1m. of time, &c. or more briefly, by the table, page 3, and to the degrees and minutes thus found, add the longitude made by the ſhip in the interval between the obſervations, if it has been ſailing eaſterly, or ſubſtract it, if it has been ſailing weſterly; the ſum, or difference, is the elapſed angle, of which take the half.

Correct both altitudes of the ſun in the following manner. From the obſerved altitude of the ſun's lower limb, corrected for the error of the quadrant, if any, ſubſtract the ſum of the dip of the ſea and refraction, by tables, page 1, and add 16m. for the ſun's ſemidiameter: or from the obſerved altitude of the ſun's centre, corrected for error of quadrant, if any, ſubſtract the ſum of the dip and refraction, and you have the true altitude of the ſun's centre; the complement of which to 90d. is the zenith diſtance of the ſun.

N. B. The meridian altitudes of the ſun ought always to be corrected in this manner for computing the latitude; and in very hazy weather, when the limbs of the ſun are very ill defined, it is more exact to obſerve the altitude of the centre.

Take the ſun's declination for the day at noon, as in computing the latitude from a meridian obſervation; by correcting, as in that caſe, the declination contained in your ephemeris, or printed tables, for the difference of longitude from London, to which thoſe tables are adapted. Laſtly, caſting up your log-book, find the latitude of the ſhip by account

at

at noon, which is the ſuppoſed latitude, to be made uſe of in the following calculations, in order to find the true latitude.

Now add together the ſine of half the ſum of the zenith diſtances, the ſine of half the difference of the zenith diſtances, the arithmetical complement of the coſine of the ſun's declination, the arithmetical complement of the coſine of the ſuppoſed latitude, and the arithmetical complement of the ſine of the half-elapſed angle; the ſum of theſe five logarithms, abating ten from the index, is the ſine of the half time from noon, or the middle time between the two obſervations from noon, in parts of a circle. The difference of this and half the elapſed angle, is the diſtance of the obſervation of the greateſt altitude from noon, whether it be before or after noon.

Next add together the verſed ſine of the diſtance of the greateſt altitude from noon, the coſine of the ſun's declination, and the coſine of the ſuppoſed latitude; the ſum of theſe three logarithms, abating 23 from the index, is the logarithm of a number, which, added to the natural ſine of the greateſt obſerved altitude of the ſun's centre, corrected as above for dip, refraction, &c. taken out of Sherwin's tables, is the natural ſine of the ſun's meridian altitude; whence the meridian altitude of the ſun is found by the ſame tables, and the latitude is thence found in the uſual manner.

N. B. If a table of natural ſines was uſed, conſiſting only of ſix or five places of figures, when greateſt, inſtead of ſeven, as Sherwin's, then, inſtead of abating 23 from the index of the ſum of the three logarithms above, 24 or 25 muſt be abated.

If the diſtance of the obſervation of the greateſt altitude from noon, found by the calculation in parts of a circle, be turned into time, at the rate of 1h. to 15d. and 1m. of time to 15m. &c. or, more briefly, by table, page 3, and be compared with the time given by the watch, it will ſhew how much the watch is too faſt or too ſlow for apparent time.

EXAMPLE.

Of the Computation of the Latitude from two obſerved Altitudes of the Sun, with the intermediate Time meaſured by a Watch.

Jan. 22, 1761, at 9h. 24m. 21ſ. A.M. by a watch, the apparent altitude of the ſun's lower limb from the horiſon of the ſea, was obſerved 13d. 44m. and at 10h. 56m. 38ſ. 20d. 16m. There is 2m. to be ſubſtracted for the error of the adjuſtment.

PREPARATION.

Subſtract 9h. 24m. 21ſ. the time of the firſt obſervation, from 10h. 56m. 38ſ. the time of the ſecond obſervation, the difference, 1h. 32m. 17ſ. is the elapſed time. This turned into parts of a circle, at the rate of 1h. to 15d. and 15m. to 1m. of time, &c. or, more briefly, by the reverſe of table, page 3, gives 23d. 4m. from which ſubſtract 3m. of longitude, which the ſhip has made to the weſtward, in the interval between the obſervations, leaves 23d. 1m. the elapſed angle, half of which is 11d. 30m.

From 13d. 44m. the firſt altitude of the ſun obſerved, ſubſtract 2m. for the error of the quadrant, and likewiſe

ſubſtract

ſubſtract the ſum of the dip 4m. and refraction 4m. according to tables, page 1, and add 16m. for the ſun's ſemidiameter, gives the true altitude of the ſun's centre, 13d. 50m. the complement of which to 90d. or 76d. 10m. is the zenith diſtance of the ſun.

In like manner, from 20d. 16m. the ſecond obſerved altitude of the ſun's lower limb, ſubſtract 2m. for the error of the quadrant, and ſubſtract the ſum of the dip 4m. and refraction 2m. and add 16m. for the ſemidiameter of the ſun, gives 20d. 24m. the true altitude of the ſun's centre, the complement of which to 90d. or 69d. 36m. is the zenith diſtance of the ſun.

The ſun's declination this day at noon at London is 19d. 34m. S. but being 6d. 40m. weſt of London, and the ſun's daily variation of declination decreaſing, being 14m. ¼m. ſhould be ſubſtracted on this account, which is of no conſequence.

I ſhall ſuppoſe the latitude to be 48d. 30m. N.

Calculation of the Latitude from two obſerved Altitudes, with the intermediate Time meaſured by a Watch.

Half ſum of the zenith diſtances	72° 53′	ſine	9.98032
Half difference of zenith diſtances	3 17	ſine	8.75795
Sun's declination, 19° 34′ arithmet. compl. of coſine			0.02583
Suppoſed latitude, 48° 30′ arithmet. compl. of coſine			0.17873
Half elapſed angle, 11° 30′ arithmet. compl. of ſine			0.70034
Sine of angle of middle time from noon = 26° 5′			9.64317

 The

The difference of this and half elapfed angle, 11° 30′ is diftance of greateft altitude from noon, 14° 35′ verfed fine			8.50809
Sun's declination, 19° 34′ cofine			9.97417
Suppofed latitude, 48° 30′ cofine			9.82126
			28.30352
Subftract from index			23
Logarithm of natural number	201150		5.30352
Natural fine of greateft altitude of fun's centre — 20° 24′ by Sherwin's tables	3485720		
	3686870		
Sum is natural fine of the fun's meridian altitude	° 21	′ 38	
Add fun's declination	19	34 S.	
Complement of latitude	41	12	
Latitude	48	48	
It was obferved	48	45	

Diftance of greateft altitude from noon found above, 14° 35′ in time, by table, page 3, 58m. 20f. whence the time is 11h. 1m. 40f. A. M. But the time of the watch was 10h. 56m. 38f. Hence the watch is 5m. 2f. too flow for apparent time.

The following Cautions are proper to be ufed in choofing out the Times of the Obfervations.

1. The two obfervations muft be always taken between 9 in the morning, and 3 in the afternoon; but the nearer they are to noon, the better, provided there be a fufficient interval between them. The following directions will fhew what interval is proper.

2. If both obfervations are in the forenoon, the interval of time between them muft not be much lefs than half the diftance of the firft obfervation from noon.

3. If

3. If both obſervations are in the afternoon, the interval between them muſt not be much leſs than the diſtance of the firſt obſervation from noon.

4. If one obſervation is in the forenoon, and the other in the afternoon, the interval of time between them muſt not exceed $4\frac{1}{2}$ hours.

5. If the ſun's meridian zenith diſtance be conſiderably leſs than the latitude of the place, then the obſervation muſt be taken proportionably nearer to noon. Thus, if the latitude be double the ſun's meridian zenith diſtance, the firſt of two obſervations taken in the forenoon muſt not be before $9\frac{1}{2}$ A.M. nor the ſecond before $10\frac{3}{4}$ A.M. or the firſt of two taken in the afternoon, muſt not be after $1\frac{1}{4}$ P.M. nor the ſecond after $2\frac{1}{2}$ P.M. Or if one obſervation be taken in the forenoon, and the other in the afternoon, that in the forenoon muſt not be taken before $9\frac{1}{2}$ A.M. nor muſt the interval between that and the afternoon obſervation be above $3\frac{1}{2}$ hours. If the latitude be three times the ſun's meridian zenith diſtance, the firſt of two obſervations taken in the forenoon muſt not be before 10 A.M. nor the ſecond before 11 A.M. or the firſt of two taken in the afternoon muſt not be after 1 P.M. nor the ſecond after 2 P.M. Or, if one be taken in the forenoon, and the other in the afternoon, that in the morning muſt not be before 10 A.M. and the interval between them muſt not be above 3 hours. If the latitude be five times the ſun's meridian zenith diſtance, the firſt of two obſervations taken in the forenoon muſt not be before $10\frac{1}{2}$ A.M. nor the ſecond before $11\frac{1}{4}$ A.M. or the firſt of two taken in the afternoon muſt not be after $0\frac{3}{4}$ P.M. nor the ſecond after $1\frac{1}{2}$ P.M. or, if one be taken in the forenoon, and the other in the afternoon, the morning one muſt not be before $10\frac{1}{2}$ A.M. nor the

interval between them be above $2\frac{1}{4}$ hours. If the latitude be twelve times the ſun's meridian zenith diſtance, the firſt of two obſervations taken in the forenoon muſt not be before 11 A. M. nor the ſecond before $11\frac{1}{2}$ A.M. or the firſt of two taken in the afternoon muſt not be after $0\frac{1}{2}$ P.M. nor the ſecond after 1 P. M. or, if one be taken in the forenoon, and the other in the afternoon, the morning one muſt not be before 11 A.M. nor the interval between them be above $1\frac{1}{2}$ hours. But if the ſun's meridian zenith diſtance be ſtill leſs in proportion to the latitude, it will be neceſſary to take the obſervations ſo much nearer to the noon, that the advantage of the method will be very much diminiſhed.

If the above cautions are uſed, the latitude computed will be at leaſt five times nearer the truth than the latitude ſuppoſed; and therefore the error of the latitude computed will not be above $\frac{1}{4}$th of the difference between them; from whence a judgment may be formed, whether it will be neceſſary or not to repeat the calculation over again with the latitude found, inſtead of the latitude firſt ſuppoſed.

Cautions the 2d, 3d, and 4th, are given to prevent the unavoidable errors in taking the altitudes of the ſun, or in the meaſure of the elapſed angle by the watch, from producing a greater error than themſelves in the latitude computed. They are founded upon theſe two theorems; 1. That the error produced in the latitude computed is to the error in either of the altitudes, as the ſine of the ſun's azimuth at the taking of the other altitude, to the ſine of the difference, or ſum of the azimuths, according as they are of the ſame or different ſides of the meridian. 2. That the error in the latitude computed is to the error in the meaſure of the interval of time, turned into parts of the equator, as the product

product of the ſines of the two azimuths, and the coſine of the latitude, to the product of the ſquare of the radius, and the ſine of the difference, or ſum of the azimuths.

The error in the meaſure of the interval of time by a common watch, is moſt to be apprehended; but where a watch is uſed, whoſe rate of going is very regular, there the cautions 2d, 3d, and 4th, need not be obſerved ſo very ſtrictly, and a leſs interval of time may be admitted between the obſervations, when made both in the afternoon, or both in the forenoon; or a longer interval may be admitted between one obſervation made in the forenoon, and the other made in the afternoon. The time of the watch ſhould be eſtimated to $\frac{1}{4}$ m. or leſs if it can be done, if it has no ſecond hand.

Caution 5th, is neceſſary to be obſerved, in order that the error in the ſuppoſed latitude may be diminiſhed five times in the latitude computed, and is taken from the eſtimation of the error of the calculation, made in the abovementioned paper in the Philoſophical Tranſactions.

I have only one remark more to add, that the latitude thus found is that at noon, provided the change of the ſhip's latitude between the obſervations and noon be tolerably uniform, and the meridian ſun be not very near the zenith; the error ariſing in the calculations from the neglect of the ſhip's change of latitude, in this caſe, very nearly compenſating itſelf, and carrying on the latitude to the time of noon, as will be found upon trial. Therefore it is only neceſſary to allow for the ſhip's change of longitude between the obſervations, as is done in the precepts above.

Some

Some Remarks on the proper Length of the Log-Line.

AS the defire of imparting my beft affiftance to the diligent mariner was my motive to undertake the foregoing work, fo the fame reafon induces me to recommend to his ferious notice the great importance of the adjuftment of the log-line to its true length. Until this be done generally, not only will our fhips be incapable of making the beft dead reckonings poffible in the common method, which will, for many reafons, be always proper, notwithftanding the introduction of the method of finding the longitude, but alfo it will be impoffible to make the reckonings of any two fhips, even when failing the fame voyage, and in company with each other, to agree tolerably together; for while one fhip has a line of 42 feet between knot and knot to a glafs of 28 feconds; another, one of 42 feet to a half-minute glafs; and another one of 48 feet to a glafs of 28 feconds; all which proportions are very commonly ufed, their accounts muft differ as much from one another, as moft of them do from the truth; for, as only one can be right, all the reft muft confequently be wrong.

It is not difficult to fee that the want of a fixed ftandard in this particular carries a very evil influence along with it upon the practice of navigation, fince a fhip making any particular voyage hereby lofes the greateft part of the advantage which it might otherwife receive from the journals of fhips who have gone the fame tract before. For, of what great ufe can it be to me, to be informed, that a fhip has made a certain number of degrees between any two ports, as long as I am uncertain whether its log-line was divided

divided at the rate of 42 or of 50 feet to a half-minute glaſs? I am here left in an uncertainty to the amount of $\frac{1}{5}$ or $\frac{1}{6}$ of the length of my whole voyage; for I am uncertain, whether I ought to make the ſame quantity of longitude as that ſhip did, or $\frac{1}{5}$ or $\frac{1}{6}$ more or leſs.

It may perhaps be urged, that the difference of longitude between any two places may be taken from books of navigation and correct charts, without recurring to ſhip's journals at all. To this I anſwer, that though the longitudes of all places could be found in theſe books or charts to a ſufficient exactneſs, yet, unleſs my own log-line be properly divided, I cannot expect to make a true and correct reckoning. But I muſt alſo obſerve, that the longitudes of places are far from being laid down generally in books and charts, with that degree of exactneſs that it was to be wiſhed they were, for the benefit of navigation. Thoſe longitudes only, of places which have been found from aſtronomical obſervations, or which have been referred to other places, ſo ſettled, by accurate trigonometrical ſurveys, are to be looked upon as certain. The longitudes of all other places, not ſo ſettled, muſt depend either on the uncertain maps of countries not at all, or, at leaſt, not well ſurveyed; or on the journals of ſhips, the uncertainty of both which, eſpecially in conſiderable diſtances, is too apparent to need any pointing out. As therefore the difficulty of procuring aſtronomical obſervations to be made in all parts of the globe will always oblige us to depend upon the runs of ſhips, or maps of countries, for the longitudes of many places, it is plain, that the preſent uncertain manner of dividing the log-line as well prevents us from knowing, with that certainty we otherwiſe might, the longitudes of thoſe places, which can be known only from ſhips runs, as alſo from keeping a correct reckoning, even when we know with certainty the longitudes both of the port we depart from, and that we are bound to.

There is another argument, which adds much ſtrength to the foregoing ones, and greatly enforces the neceſſity of a uniform and correct length of the log-line on board of all ſhips; that in many parts of the ocean, eſpecially between the tropics and near moſt headlands, there are conſiderable currents, which muſt conſequently introduce a freſh error into the reckoning; and if this error ſhould happen to combine with that already produced by a wrong length of the log-line, as it may, as well as not, it is not eaſy to ſay how far the total error of the reckoning might go, or to what inconveniencies or dangers the ſhip might be expoſed on that account. But if the juſt and proper length of the log-line was uſed on board of all ſhips, they would be then liable only to the errors of the currents themſelves; and even theſe, as far as they are conſtant and regular, might be found out and aſcertained from the journals of ſeveral ſhips, which would then agree much nearer with one another.

After inſiſting ſo ſtrongly on the neceſſity of adjuſting the knots of the log-line to a proper length, it is time that I ſhould mention what that length is. This, by the teſtimony of all mathematicians, is 50 or 51 feet to a glaſs which runs out in 30 ſeconds, or half a minute. But if the glaſs take a longer or ſhorter time to run out, the diſtance of the knots ſhould be made longer or ſhorter in the ſame proportion, according to the following Table.

A Table