A Table of the Length of the Knots of the Log-line for different Glasses.

Seconds of Glass.	Length of Knots.	
	Feet.	In.
24	40	0
25	41	8
26	43	4
27	45	0
28	46	8
29	48	4
30	50	0
31	51	8
32	53	4
33	55	0
34	56	8
35	58	4
36	60	0

The true length of the knots of the log-line here delivered, was first laid down by our countryman, Mr. Richard Norwood, in his Seaman's Practice, in the year 1636. For having carefully measured the distance from London to York, and reduced it to the arch of the meridian, and having also found the difference of latitude of those two cities by astronomical observations, he thence inferred a degree of a great circle of the earth to contain 367200 English feet, or about 69½ English statute miles, instead of 60, which

 was

was the common eſtimation before his time. Hence he concludes a geographical mile, or minute of a great circle of the earth, which is $\frac{1}{60}$ of a degree, to contain 6120 feet, and therefore, that the length of the knots of a log-line for a half-minute glaſs ſhould be 51 feet, which is the ſame part of a geographical mile, or 6120 feet, as half a minute is of an hour, namely, $\frac{1}{120}$th. But he adds, becauſe it is ſafer to have the reckoning rather before the ſhip, than after it; therefore 50 feet may be taken as the proper length of each knot, which I have here accordingly followed.

Before Mr. Richard Norwood's accurate menſuration, the length of the knots of the log-line was made 42 feet for a half-minute glaſs; and, notwithſtanding the meaſures of the degrees of the earth, as well as daily experience ſhew it to be too ſhort, yet it is uſed by many to this day. This diviſion of the log-line was founded on a ſuppoſition, that a degree of a great circle of the earth conſiſts of 60 miles, each containing 5000 Engliſh feet; whence a minute of a great circle, or a geographical mile, ſhould contain 5000 feet; and the 120th part of 5000 feet, or $41\frac{2}{3}$, or, in round number, 42 feet, ſhould therefore be the length of each knot. But as a degree of a great circle of the earth is found to contain $69\frac{1}{2}$ Engliſh miles, the length of the knots of the log-line ought to be increaſed to 50 or 51 feet.

I ſhall only add, that though many, through prejudice, ſtill retain the erroneous diviſion of 42 feet, yet I could never hear one tolerable argument brought in defence of it. For my own part, I can affirm, that I found the true diviſion of 50 feet here propoſed, to anſwer very nearly to the obſervations of the ſhip's latitude, both in my voyage to St. Helena, and in my voyage on my return, when without the

the tropics; for within the tropics, and in the courſe of the trade winds, there are conſtant and conſiderable currents, which ſeem to tend in the ſame direction with thoſe winds, and moſt probably derive their origin from them. On the other hand, what journals of ſhips I have had an opportunity to inſpect, which have made long runs to the eaſtward, and have uſed too ſhort a diviſion of the log-line, I have found to reckon the longitude 10 or 12 degrees too far to the eaſtward upon the making of land, as one might naturally expect from the cauſe of error here pointed out.

Directions for obſerving the Beginning and Ending of an Eclipſe of the Moon, and an Immerſion or Emerſion of the Satellites of Jupiter.

THOUGH the method of finding the longitude by diſtances of the moon from the ſun and fixed ſtars is both ſufficiently exact for general uſe, and may be equally uſed at ſea and land, yet it cannot but be an advantage to have it in our power to practiſe every method of finding the longitude, whether of a ſhip at ſea, or of any port at land.

The beginning and ending of an eclipſe of the moon is very eaſily obſerved; and being compared with the times calculated, taken from an exact ephemeris, will determine the longitude of a ſhip at ſea, where the obſervation is made within a degree; the difference between the apparent times of the obſervations at the ſhip, and thoſe calculated in the ephemeris, ſhewing the longitude of the ſhip in time, reckoned from the meridian to which the computations of the ephemeris are adapted.

The obſervations muſt be made by the help of a watch, corrected from an altitude of the ſun or a ſtar, according to the rules given in the preceding treatiſe. But if an eclipſe of the moon ſhould be obſerved at any place at land, whoſe longitude we deſire to know, this may be found more exactly by comparing the obſervation with a correſponding one, made at ſome other place at land, whoſe longitude is known.

The

The beginning and ending of an eclipſe of the moon may be obſerved very well with the naked eye; but if any teleſcope is uſed, it muſt be one that has a great deal of light, and does not magnify much. Thoſe called night glaſſes may be very conveniently uſed for this purpoſe, in which caſe I ſhould adviſe placing a circular aperture at the object end, about half or $\frac{1}{3}$ of the ſize of the common aperture.

The ſhadow begins to come upon the moon on the eaſt ſide, and goes off on the weſt ſide. The firſt obſcuration that comes upon the moon, called the penumbra, is not perfectly dark; but after a few minutes, the true and darker ſhadow will begin to make its appearance upon the moon's limb, in the place which lies immediately behind the penumbra. At the ending of the eclipſe the true ſhadow goes off firſt, and the penumbra follows it. If the obſerver notes the time of the appearance and diſappearance of the penumbra, or falſe ſhadow, as well as of the darker and true ſhadow, it will appear that he has not miſtaken the former for the latter, as they are apt to do, who are not uſed to the obſervation.

If the eclipſe be total, the immerſion into, and emerſion out of, total darkneſs, are two other obſervations to be made, and will equally ſerve to determine the longitude.

The immerſions and emerſions of Jupiter's ſatellites afford the beſt general method known for determining the longitudes of places at land, and could they be likewiſe obſerved at ſea, as there is room to hope they may, by the help of the marine chair, the longitude might thence be determined, as often as ſuch obſervations were obtained.

A re-

A refracting telescope of ten feet, or a reflecting telescope of one foot at least, will be necessary for observing these eclipses, though a refractor of 15 feet, and a reflector of 18 inches, or two feet, would answer still better.

In general, the immersions only are visible, before the opposition of Jupiter to the sun; and, after the opposition, the emersions only, the body of Jupiter hiding the other phænomenon. This is always the case with respect to the first satellite; but sometimes the immersions and emersions of the other satellites, especially the remoter ones, are both visible. The immersions and emersions before the opposition happen on the west side of Jupiter, as after the opposition they happen on the east side: only if an astronomical refracting telescope be used, consisting of two convex glasses, the appearances will be inverted, and the eclipses will seem to happen on the east side before the opposition, and on the west side after the opposition.

At the immersions and emersions, which happen within a few days of Jupiter's opposition to the sun, the satellites seem to touch his body; but at all other times they fall into, or come out of the shadow at some distance from his limb, which is greater, the further he is from the opposition, and the nearer to the quadrature with the sun. Thus the greatest distance of the 1st satellite from Jupiter's limb, at the eclipses, in the quadratures, is a little more than a semidiameter of Jupiter; of the 2d, not quite two semidiameters; of the 3d, near three semidiameters; and of the 4th, five semidiameters. But as it is useful for distinguishing the satellite which is to immerge into the shadow from the rest, or as a direction whereabouts to look for the satellite which is to emerge out of the shadow, to know nearly the distance

diſtance it ſhould be at from Jupiter's limb, I have added the following Table for this purpoſe, which may always be depended upon within a tenth part of the whole, the general form of it not admitting of a greater degree of preciſion, on account of the excentricity of the orbits of Jupiter and the earth.

It ſhould be carefully noted, that the apparent diſtances in this table are thoſe at the regular eclipſes, namely, at the immerſions before the oppoſition, and the emerſions after the oppoſition; but at the emerſions which are viſible before the oppoſition, and at the immerſions which are viſible after the oppoſition, the diſtances of the ſatellites from Jupiter's limb will be leſs than is ſet down in the table, by a quantity which bears the ſame proportion to Jupiter's diameter, as the duration of the eclipſe does to the longeſt durations of the eclipſes when the ſatellites are in the nodes. The greateſt durations of the eclipſes of the firſt, ſecond, third and fourth ſatellites are 2h. 16m. 2h. 54m. 3h. 34m. and 4h. 48m. reſpectively.

A Table of the apparent Diſtance of the Satellites of Jupiter from his Limb at the time of the Eclipſes, for every tenth Day of Jupiter's Diſtance from the Oppoſition or Conjunction.

Diſt. of Jupiter from oppoſition of ⊙.	Diſt. of the ſatellites from Jupiter's limb at the eclipſes, in ſemidiameters of Jupiter and decimal parts.				Diſt. of Jupiter from conjunction with ⊙.	Diſt. of the ſatellites from Jupiter's limb at the eclipſes, in ſemidiameters of Jupiter and decimal parts.			
Days.	1ſt ſat.	2d ſat.	3d ſat.	4th ſat.	Days.	1ſt ſat.	2d ſat.	3d ſat.	4th ſat.
10	0, 20	0, 33	0, 50	0, 85	10	0, 15	0, 25	0, 35	0, 55
20	0, 40	0, 66	1, 05	1, 66	20	0, 30	0, 45	0, 70	1, 25
30	0, 60	0, 95	1, 50	2, 65	30	0, 40	0, 67	1, 05	1, 70
40	0, 75	1, 20	1, 90	3, 35	40	0, 55	0, 90	1, 40	2, 50
50	0, 90	1, 40	2, 25	3, 95	50	0, 70	1, 00	1, 80	3, 20
60	1, 00	1, 60	2, 50	4, 40	60	0, 80	1, 25	2, 00	3, 50
70	1, 05	1, 70	2, 66	4, 70	70	0, 90	1, 40	2, 25	3, 95
80	1, 10	1, 75	2, 75	4, 85	80	1, 00	1, 55	2, 45	4, 33
90	1, 10	1, 75	2, 75	4, 85	90	1, 05	1, 66	2, 60	4, 60
100	1, 10	1, 70	2, 70	4, 80	100	1, 10	1, 75	2, 70	4, 80

The diſtance of the ſatellites from Jupiter's limb in the foregoing table, are to be meaſured either in a line with Jupiter's equator, or longer axis, or in a line parallel thereto; or, which is the ſame thing, to his belts; for the ſatellites generally appear a little to the north or ſouth of this line.

During the ſpace of ſix years, while Jupiter is paſſing from the aſcending to the deſcending nodes of the ſatellites, as from the beginning of 1760 to the beginning of 1766, the ſatellites are eclipſed to the north of the equator, departing to the greateſt diſtance from thence at the middle of this period. Thus, at this preſent time, or the beginning of 1763, the 1ſt ſatellite is eclipſed $\frac{1}{7}$th of Jupiter's ſemidiameter, and the 2d about half his ſemidiameter to the north of his equator, and the 3d almoſt in a line with his north pole; the 4th goes north of his pole, and therefore at preſent is not eclipſed at all, nor will be all this year.

For the like ſpace of ſix years more, while Jupiter is paſſing from the deſcending to the aſcending nodes of the ſatellites, as from the beginning of 1766 to the beginning of 1772, the ſatellites will be eclipſed to the ſouth of Jupiter's equator, departing to the greateſt diſtance from thence at the middle of this period, as in 1769. And thus every ſix years the eclipſes happen alternately to the north and ſouth of Jupiter's equator.

The firſt and ſecond ſatellites are moſt uſeful for finding the longitudes of places, the tables of the firſt never erring more than 1m. of time, and thoſe of the 2d not more than 4m. Therefore, if the time of the total immerſion or firſt emerſion of either of theſe ſatellites, corrected from an altitude of the ſun or a ſtar, according to the rules in Chap. III.

III. of the preceding treatiſe, be compared with the time ſet down in an exact ephemeris, the difference will ſhew the difference of longitude in time from the meridian of the ephemeris, which is turned into degrees and minutes, at the rate of 15d. to 1h. and 15m. to 1m. of time, &c. But if a correſponding obſervation can be obtained, made in any place whoſe longitude is known, the longitude will be found independently of any error of the tables.

I ſhall conclude this ſubject with pointing out a method of taking altitudes at land with Hadley's quadrant, whence the latitude may be found, and the time of the day computed, as from obſervations made at ſea.

Prepare ſome ſhallow veſſel, and pour water, quickſilver, or, which would be ſtill better, ſome thick oil or treacle into it, and place it near ſome window within a room, ſheltered as much as poſſible from the wind, and move the index of the quadrant till the ſun or ſtar in the heavens is brought down by reflection to appear conjoined to the reflected image of the ſun or ſtar in the veſſel, holding a dark glaſs cloſe before the eye, in order to darken both images of the ſun. Then half the angle that the index points at, if the quadrant is rightly adjuſted ; or the half of the numbers on the quadrant, firſt corrected for the error of the adjuſtment, if any, is the apparent altitude of the ſun or ſtar. From which ſubſtract the refraction, according to table, page 1, and you have the true altitude, the complement of which to 90d. is the zenith diſtance, with which compute the latitude or time in the common manner. The quadrant may be adjuſted, or the error of the adjuſtment found by any ſtreight object, which is not leſs than a quarter of a mile off. If the altitude of the ſun or ſtar be above 45d. they muſt be taken by the back obſervation.

To find the Latitude from the apparent Meridian Altitude of a Star obſerved at Sea.

FROM the apparent altitude of the ſtar obſerved above the horiſon of the ſea, ſubſtract the ſum of the dip and refraction, according to the tables in page 1, and you have the true altitude of the ſtar, the complement of which to 90d. is the zenith diſtance; with which, and the ſtar's declination, taken out of table page 2 and 3, find the latitude in the very ſame manner as you would do from the ſun.

EXAMPLE.

May 9, 1762, at night, I obſerved the meridian altitude of the Virgin's Spike 30d. 48½m. I ſubſtract the ſum of the dip 4m. and refraction 2m. which leaves the true altitude of the Virgin's Spike 30d. 48½m. and therefore the zenith diſtance is 59d. 17½m. The declination of the Virgin's Spike, by table, page 2, is 9d. 54m. S. which, ſubſtracted from the zenith diſtance, 59d. 17½m. leaves the latitude 49d. 23½m. N. Near an hour afterwards, by the meridian altitude of Arcturus, 61d. 6m. I found the latitude only 1m. greater. The ſhip had not made any northing or ſouthing between the obſervations, and the latitude by account carried on from the noon but one before, for the noon before it was cloudy, was 49d. 17m.

The example here given ſhews the great uſe that may often be made of this method of finding the latitude; for we had no obſervation of the ſun for the latitude on the noon before, whence our latitude by account muſt be very uncertain;

tain; and at day-break, a few hours after finding the latitude from the altitudes of theſe two ſtars, we diſcovered the Scilly Iſles.

To find the Latitude from the apparent Meridian Altitude of the Moon obſerved at Sea.

From the obſerved meridian altitude of the moon's lower limb, ſubſtract the ſum of the dip and refraction, by tables, page 1. and add 16m. for the moon's ſemidiameter; or, from the obſerved altitude of the moon's upper limb ſubſtract the ſum of the dip, refraction, and 16m. for the moon's ſemidiameter; or, from the obſerved altitude of the moon's centre ſubſtract the ſum of the dip and refraction only; gives the altitude of the moon's centre correct, to which add the parallax of altitude, taken out of table, page 26, gives the true altitude of the moon's centre, the complement of which to 90d. is the moon's zenith diſtance. The preſent horiſontal parallax of the moon, with which the table is to be entered, may be taken out of an ephemeris; or, if one containing it be not at hand, the mean horiſontal parallax, 57m. may be uſed, as the error ariſing from hence will never exceed 4m.

Next, the moon's preſent declination muſt be found. For this purpoſe, firſt take the time of the moon's paſſing the meridian out of an ephemeris, and the daily difference of the ſame; and ſay, as 24h. more the ſaid difference, is to the longitude of the ſhip, reckoned from the meridian of the ephemeris, turned into time by table, page 3 and 4, ſo is the ſame daily variation to a number of minutes, which add to the time of the moon's paſſing the meridian in the ephemeris, if you are to the weſt of that meridian, or ſubſtract if you are to the eaſt of it, and you have the correct time of of the moon's paſſing the meridian of the ſhip. To this add or ſubſtract your longitude from the meridian of the ephemeris, turned into time, according as you are to the weſt or eaſt of that meridian, and you have the time of the moon's paſſing the meridian of the ſhip, reduced to the time of the meridian of the ephemeris. Then ſay, as 24 hours is to the time juſt found, ſo is the moon's daily variation of declination, taken out of the ephemeris, to the variation of declination from the noon of the ephemeris to the preſent time nearly; which add to the moon's declination in the ephemeris for the noon before, if the declination is increaſing, or ſubſtract if it is decreaſing, and you have the moon's preſent declination very nearly. This may be further corrected in the following manner. Take out of the ephemeris the moon's declination for the noon but one before, the noon before, the noon after, and the noon but one after; take the ſum of the firſt and laſt declinations, and alſo the ſum of the two middle declinations; and take the difference of theſe two ſums, and $\frac{1}{4}$th of the difference, and ſay, as 24 hours is to the correct time of the moon's paſſing the meridian, reduced to the meridian of the ephemeris found above; ſo is $\frac{1}{4}$th of the difference of the two ſums to a firſt number. Say again, as 24h. is to the correct time of the moon's paſſing the meridian, reduced to the meridian of the ephemeris, ſo is the firſt number, juſt found, to a ſecond number: take the difference of the firſt and ſecond numbers, and add it to the moon's declination found nearly above, if the ſum of the two middle declinations, found above, is greater than the ſum of the two extreme declinations; elſe ſubſtract it, and you have the moon's true declination; with which and the moon's zenith diſtance, found above from obſervation, find the latitude as from the ſun by the common rules.

The correction of the moon's declination here exhibited may ſometimes amount to 10m. when the declination is great, which may ſeem too much to be neglected; but if the declination is under 10d. the correction will be inconſiderable, and need not be applied at all.

It will be proper to compute the apparent time of the moon's paſſing the meridian of the ſhip before-hand, in order to be prepared for the obſervation of the meridian altitude, the rule for which is given in the directions above. For the ſame reaſon it would be neceſſary to compute the apparent time of a ſtar's paſſing the meridian before-hand, when it is propoſed to obſerve its meridian altitude for finding the latitude. This is done by ſubſtracting the preſent right aſcenſion of the ſun from the right aſcenſion of the ſtar; the remainder, turned into time by table, page 3 and 4, is the apparent time of the ſtar's paſſing the meridian.

EXAMPLE.

Of the Calculation of the Latitude from the meridian Altitude of the Moon's Centre obſerved at Sea, March 5, 1762, *at Night*, 61d. 0½m.

I ſubſtract the ſum of the dip 4m. and refraction ½m. there remains 60d. 56m. the altitude of the moon's centre corrected; to which I add 29m. the moon's parallax in altitude, the horiſontal parallax of the moon being at this time 60m. and I have the true altitude of the moon's centre 61d. 25m. whence the zenith diſtance is 28d. 35m.

By the Abbé de la Caille's ephemeris for ten years, the moon paſſes the meridian at Paris, March 5, at 7h. 58m. P. M.

P. M. and March 6, at 8h. 58m. P.M. whence the daily difference is 60m. The longitude of the ſhip was 17d. 1m. weſt of Greenwich, or 19d. 21m. weſt of Paris, which, turned into time, is 1h. 17m. Therefore ſay, as 24h.+60m. or 25h. is to 1h. 17m. :: ſo is 60m : to 3m. nearly, which added to 7h. 58m. the time of the moon's paſſing the meridian at Paris, becauſe the ſhip is in longitude weſt from Paris, gives 8h. 1m. P.M. the time of the moon's paſſing the meridian of the ſhip. To this add 1h. 17m. the longitude reckoned from Paris in time, the ſum is 9h. 18m. the time of the moon's paſſing the meridian of the ſhip, reduced to the time of the meridian of Paris. Then ſay, as 24h. is to 9h. 18m. ſo is 73m. the moon's daily variation of declination, from March 5 to March 6, by the ephemeris, to 28m. which ſubſtract from 27d. 20m. the moon's declination March 5 at noon, by the ephemeris, becauſe the declination is decreaſing, leaves 26d. 52m. the moon's declination nearly.

To correct it, take out of the ephemeris the moon's declination on the preceding noon, 27d. 20m. N. the following noon, 26d. 7m. N. the noon but one before, 26d. 47m. and the noon but one after, 23d. 4m. the ſum of the two middle declinations is 53d. 27m. and of the two extreme declinations, 49d. 51m. The difference is 3d. 36m. $\frac{1}{4}$th of which is 54m. Then ſay, as 24h. : is to 9h. 18m. the time of the moon's paſſing the meridian of the ſhip, reduced to Paris time :: ſo is 54m. : to a firſt number 21m. Again, as 24h. : is to 9h. 18m. : ſo is the firſt number 21m. : to a ſecond number 8m. The difference of the firſt number 21m. and the ſecond number 8m. is 13m. which add to 26d. 52m. the moon's declination found nearly above, becauſe the ſum of the two middle declinations is greater than the ſum of the two extreme declinations, and you

you have the moon's true declination, 27d. 5m. N. This subſtracted from 28d. 35m. the moon's zenith diſtance, inferred above from the obſervation, leaves the latitude of the ſhip 1d. 30m. S. which is only 2m. leſs than the latitude by account, carried on from the meridian altitude of the ſun at noon.

EXAMPLE.

Of a Calculation of the Longitude from Obſervations of the Diſtance of the Moon from Antares, taken at Sea, May 13, 1762, *in the Morning, in Sight of the Lizard Lights.*

OBSERVATIONS.

May 13, 1762, at 1h. 47m. 54ſ. A. M. per watch, the apparent altitude of the bright ſtar of the Harp from the horiſon of the ſea was taken, 75d. 12m. The dip of the ſea is 5m. and the error of the quadrant is 2m. to be added to the obſervation. At this time the Lizard lights bore N. 63d. W. diſtant by eſtimation 13 miles. But the latitude of the Lizard being 49d. 57m. N. and its longitude 5d. 43m. weſt of the Royal Obſervatory at Greenwich, the latitude of the ſhip is 49d. 51m. N. and its longitude 5d. 25m. weſt of Greenwich.

Here follow five obſervations of the diſtance of Antares, or the Scorpion's Heart, from the moon's eaſtern and remoteſt limb, together with the time ſhewn by the watch, and the correſponding altitudes of the ſtar, and of the moon's lower limb, obſerved at the ſame time. There is 1m. 50ſ. to be added to the diſtances of Antares from the moon's limb, for the error of the adjuſtment of the quadrant. But the altitudes of the moon and ſtar are only to be corrected by ſubſtracting 4m. on account of the dip of the horiſon.

May 13, 1762. time per watch.			Obſ. diſt. of Antares fr. ☽'s remoteſt limb.			Obſ. alt. of ☽'s lower limb.		Obſ. alt. of Antares.	
H.	M.	S.	D.	M.	S.	D.	M.	D.	M.
1	55	45	43	45	25	8	49	11	58
2	0	50	43	46	35	9	9	11	45
2	5	15	43	48	10	9	26	11	18
2	11	5	43	51	0	9	49	11	0
2	16	30	43	53	40	10	2	10	44
Mean of times.			Mean diſt.			Mean alt. of ☽'s lower limb.		Mean alt. of Antares.	
2	5	53	43	48	50	9	27	11	21

Computation of the Time by the Altitude of the bright Star of the Harp. *See* Chap. III.

Its right aſcenſion at this time, by table, page 3, is 277d. 13m. and its declination 38d. 34m. N.

	°	′
Obſerved altitude of bright ſtar of the Harp	75	12
Add for error of quadrant — —	+	2
Subſtract 5 for the dip, the refraction being o	—	5
True altitude of ſtar — —	75	9
Subſtract from — — —	90	0
True zenith diſtance of ſtar — —	14	51
Diſt. of ſtar from elevated or north pole $=90°-38° 34'=$	51	26
Colatitude $=90°-49° 51'=$ — —	40	9
Sum — — —	106	26
Half ſum — —	53	13

Arith.

	°	′	
Arith. compl. of fine of colatitude	40	9	0 19058
Arith. compl. of fine of polar diftance	51	26	0.10686
Sine of half fum —colat.=53°13′— 40° 9′=	13	4	9.35427
Sine half fum—pol. dift.=53°13′— 51°26′=	1	47	8.49304
Sum of four logarithms			18.14475
Half fum			9.07237
Is fine of	6°	47′	
		2	
	13	34	
Doubled is horary angle, or diftance of Lyra from the meridian to the eaft; therefore fubftract it from the right afcenfion of Lyra	277	13	
Gives right afcenfion of mid-heaven	263	39	

The watch being about half an hour too flow for the fun, and the time fhewn by the watch at taking the altitude of Lyra, being 1h. 47m. 54f. the apparent time is about 2h. 18m. A.M. or, in aftronomical reckoning, May 12, 14h. 18m. P.M. to this add 22m. for 5d. 25m. of longitude weft of Greenwich, gives the apparent time reduced to the meridian of Greenwich nearly, May 12, 14h. 40m. P.M. to which time the fun's longitude by computation is 1S. 22d. 13m. Hence, by table, page 27, the fun's right afcenfion is 49d. 47m. which fubftracted from 263d. 39m. the right afcenfion of the mid-heaven found above, leaves 213d. 52m. for the apparent time in parts of the equator, which, by table, page 3 and 4, gives the apparent time 14h. 15m. 28f. But the time by the watch was 13h. 47m. 54f. therefore the watch is 27m. 34f. too flow for the fun; which is confirmed by feveral other altitudes of Lyra, and of other ftars, and alfo by altitudes of the fun taken in the afternoon before, and the morning after the obfervation of the diftance of the moon from Antares.

The ſhip did not go above half a mile between the obſervation of the altitude of Lyra, and the mean time of the obſervation of the diſtances of the moon from Antares. The mean of the times of thoſe obſervations by the watch is 2h. 5m. 53ſ. to which, 27m. 34ſ. being added, by which the watch is too ſlow for the ſun, gives 2h. 33m. 27ſ. A.M. or, May 12, 14h. 33m. 27ſ. P.M. in aſtronomical reckoning, for the apparent time of the mean of the diſtances of Antares from the moon. To this add 21m. 40ſ. for 5d. 25m. longitude reckoned weſt of Greenwich, turned into time, and we have the apparent time reduced to Greenwich, May 12, 14h. 55m. 7ſ. P.M.

Hence computing, according to Chap. IV. the ſun's longitude is 1S. 22d. 14m. 7ſ. the equation of time —4m. 0ſ. to be ſubſtracted from the apparent time at Greenwich, 14h. 55m. 7ſ. leaves 14h. 51m. 7ſ. the mean time at Greenwich, to which computing, according to Chap. IV. the moon's longitude comes out 9S. 20d. 9m. 3ſ. her latitude 4d. 55m. 17ſ. ſouth; the horiſontal parallax 54m. 42ſ. the apparent ſemidiameter of the moon, augmented according to her altitude, 14m. 58ſ. the hourly motion of the moon 30m. 5ſ. and the multiplier of the difference between the moon's longitude computed, and that inferred from the obſervations, $29\frac{9}{10}$.

The next thing to be done is to allow for the ſemidiameter of the moon, refraction, and parallax, in order to reduce the obſerved diſtance of the ſtar from the moon's limb, to the true diſtance of the ſtar from the moon's centre, according to Chap. IV.

1. To

1. To 43d. 48m. 50ſ. the mean of the 5 obſerved diſtances of Antares from the moon's remoteſt limb, I add 1m. 50ſ. for the error of the adjuſtment of the quadrant, and ſubſtract 14m. 58ſ. the apparent ſemidiameter of the moon; whence the apparent diſtance of the ſtar from the moon's centre is 43d. 35m. 42ſ.

2. To allow for refraction and parallax, the obſerved altitude of the moon's centre being near 10d. and that of the ſtar being above 10d. I make uſe of them as taken from the quadrant, only ſubſtracting 4m. for the dip, according to the remark at the end of Chap. V. which makes the calculation ſomewhat more compendious, as follows:

From 9d. 27m. the mean of the five obſerved altitudes of the moon's lower limb, I ſubſtract 4m. for the dip of the horiſon, and add 15m. for the moon's ſemidiameter; whence the apparent altitude of the moon's centre is 9d. 38m. and the apparent zenith diſtance 80d. 22m. and from 11d. 21m. the mean of the obſerved altitudes of the ſtar, ſubſtracting 4m. for the dip. there remains 11d. 17m. the apparent altitude of the ſtar; whence the apparent zenith diſtance of the ſtar is 78d. 43m.

Half ſum of zenith diſtances of the moon and ſtar 79° 32′	tangent	10.7334
Half differences of zenith diſtances 0° 49′	tangent	8.1539
Tangent of arc the firſt, 4° 25′	——	8.8873
Half apparent diſtance of ſtar from the moon's centre, 21° 48′ —— ——	cotan.	10.3980
Tangent of arc the ſecond, 10° 55′	——	9.2853

Sum

The ſhip did not go above half a mile between the obſervation of the altitude of Lyra, and the mean time of the obſervation of the diſtances of the moon from Antares. The mean of the times of thoſe obſervations by the watch is 2h. 5m. 53ſ. to which, 27m. 34ſ. being added, by which the watch is too ſlow for the ſun, gives 2h. 33m. 27ſ. A.M. or, May 12, 14h. 33m. 27ſ. P.M. in aſtronomical reckoning, for the apparent time of the mean of the diſtances of Antares from the moon. To this add 21m. 40ſ. for 5d. 25m. longitude reckoned weſt of Greenwich, turned into time, and we have the apparent time reduced to Greenwich, May 12, 14h. 55m. 7ſ. P.M.

Hence computing, according to Chap. IV. the ſun's longitude is 1S. 22d. 14m. 7ſ. the equation of time —4m. 0ſ. to be ſubſtracted from the apparent time at Greenwich, 14h. 55m. 7ſ. leaves 14h. 51m. 7ſ. the mean time at Greenwich, to which computing, according to Chap. IV. the moon's longitude comes out 9S. 20d. 9m. 3ſ. her latitude 4d. 55m. 17ſ. ſouth; the horiſontal parallax 54m. 42ſ. the apparent ſemidiameter of the moon, augmented according to her altitude, 14m. 58ſ. the hourly motion of the moon 30m. 5ſ. and the multiplier of the difference between the moon's longitude computed, and that inferred from the obſervations, $29\frac{9}{10}$.

The next thing to be done is to allow for the ſemidiameter of the moon, refraction, and parallax, in order to reduce the obſerved diſtance of the ſtar from the moon's limb, to the true diſtance of the ſtar from the moon's centre, according to Chap. IV.

1. To

1. To 43d. 48m. 50ſ. the mean of the 5 obſerved diſtances of Antares from the moon's remoteſt limb, I add 1m. 50ſ. for the error of the adjuſtment of the quadrant, and ſubſtract 14m. 58ſ. the apparent ſemidiameter of the moon; whence the apparent diſtance of the ſtar from the moon's centre is 43d. 35m. 42ſ.

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Half ſum of zenith diſtances of the moon and ſtar 79° 32′	tangent	10.7334
Half differences of zenith diſtances 0° 49′	tangent	8.1539
Tangent of arc the firſt, 4° 25′	—	8.8873
Half apparent diſtance of ſtar from the moon's centre, 21° 48′ — —	cotan.	10.3980
Tangent of arc the ſecond, 10° 55′	—	9.2853

Sum

Sum of arc 2d and half diſtance of moon from ſtar 32° 43′	— —	tang.	9.8078
Moon's zenith diſtance 80° 22′	—	coſine	9.2236
Moon's horiſontal parallax 54′ 42″=54′, 7		log.	1.7380
Logarithm of effect of parallax 5′, 88=5′ 53″			0.7694

To be ſubſtracted by the rule, page 42, from the apparent diſtance of the ſtar from the moon's centre.

Computation of Refraction.

Double arc the firſt, 8° 50′	—	tangent	9.1914
Conſtant logarithm	—	—	2.0569
Sum	—	—	11.2483
Double arc the ſecond, 21° 50′		ſine ſubſtract.	—9.5704
Logarithm of 47″, the effect of refraction	—		1.6779

From 43d. 35m. 42ſ. the apparent diſtance of the ſtar from the moon's centre, ſubſtract the effect of parallax —5m. 53ſ. and add the effect of refraction, +47ſ. and you have the true diſtance of Antares from the moon's centre very nearly, 43d. 30m. 36ſ. There remains only to be applied the little correction of the effect of parallax, the rule for finding of which is given at the end of the Preface.

The principal effect of parallax juſt found, 5m. 53ſ. I call the parallax in diſtance. With the altitude of the moon's centre, 9d. 38m. and the horiſontal parallax, 55m. take the moon's parallax in altitude, 54m. out of table, p. 26.

Sum of parallax in altitude, and parallax in diſt. 60′	log.	1.778
Difference of parallax in alt. and parallax in diſt. 48′	log.	1.681
App. diſt. of ſtar from the moon's centre 43° 36′	cotan.	10.021
Conſtant logarithm — —		0.941
Sum, abating 13 from the index —		1.421

is

is logarithm of little correction of effect of parallax, 26ſ. to be added to 43d. 30m. 36ſ. the true diſtance of Antares from the moon's centre, found nearly above; whence the true and abſolute diſtance of Antares from the moon's centre is 43d. 31m. 2ſ.

Next, from the diſtance here found, 43d. 31m. 2ſ. and the moon's latitude computed from the tables, 4d. 55m. 17ſ. S. compute the difference of longitude between the ſtar and the moon, and thence the moon's true longitude, according to Chap. VI.

	°	′	″
Diſtance of Antares from the ſouth pole of the ecliptic, by table, page 4, is ——	85	27	25
Diſtance of the moon from ſouth pole of the ecliptic	85	4	43
Difference of polar diſtances ——	0	22	42
Half difference — —	0	11	21
Half true diſtance of moon from Antares found above	21	45	31

Half diſt. of moon from ſtar + ½ diff. 21° 56′ 52″ ſine	9.5725944
Half diſt. of moon from ſtar — ½ diff. 21 34 10 ſine	9.5654093
Polar diſt. of Antares 85° 27′ 25″ ar. compl. of ſine	0.0013666
Polar diſt. of the moon 85 4 43 ar. compl. of ſine	0.0016041
Sum —— ——	19.1409744
Half ſum, is ſine of 21° 50′ 10″ ——	9.5704872

Doubled is 43 40 20 diff. of long. of moon and Antares.

	S.	°	′	″
Longitude of Antares in table, page 4, ——	8	6	26	59
Subſt. 50″ becauſe the year precedes 1763 by 1 year				— 50
Add correction for May 13, from table, page 5				+ 53
True longitude of Antares ——	8	6	27	2
To which add the difference of longitude of the moon and Antares found above (becauſe the moon is to the eaſt of the ſtar) = 43° 40′ 20″ =	1	13	40	20

True

True longitude of moon inferred from obfervations	9	20	7	22
Longitude of moon computed from tables to time affumed at Greenwich ——	9	20	9	3
Longitude of moon computed from tables, greater than that inferred from obfervations			1	41
Multiply by $29\frac{9}{10}$ the multiplier from table, p. 25,			29	$\frac{9}{10}$
			°	′
Product fhews how much the fhip was eaft of the affumed longitude, according to Chap. VII. ——			0	50 E
But the longitude of the fhip, according to its bearing and diftance from the Lizard lights, was affumed weft of Greenwich —— ——			5	25 W
Therefore the true longitude of the fhip by thefe obfervations is —— ——			4	35 W
And the longitude of the Lizard, being 18′ weft of that of the fhip, is —— ——			4	53 W

Which differs 0° 50′ from the true longitude affigned to it in the tables.

In the foregoing obfervations, from whence the longitude of the fhip has been computed, the fea was almoft perfectly calm, the motion thereof being very gentle and regular; fo that it was rather an advantage to the obfervations in caufing the ftar to fweep the moon's limb, than of any prejudice. I alfo determined the error of the adjuftment of the quadrant very exactly by means of the moon, particularly by meafuring her longeft diameter feveral times with the reflected and direct images brought alternately to contrary fides, with refpect to each other; reading the numbers off the quadrant, in one cafe, in the ufual manner to the left hand of the beginning of the divifions, and, in the other cafe, to the right hand of the fame; and taking half the difference of the two meafures of the moon's diameter thus found for the error of the adjuftment; which method of finding the fame I look upon to be as certain as any, efpecially near the time of the full moon, being more exact than by bringing the two images of the moon into one, and not less so than by the horison of the sea. The night also was very fine, and there was a good twilight, which afforded me the opportunity of taking the altitudes of several stars pretty exactly; from the comparison of all which together, as well as with the altitudes of the sun taken in the evening before, and in the morning after the observations of the distance of the moon from Antares, I could be very certain of the time of the night at the ship.

Extract of the Journal of a Voyage from England to St. Helena in the Prince Henry East India Ship, corrected by Obſervations of the Diſtance of the Moon from the Sun and fixed Stars.

TUESDAY, January 20, 1761, at noon, we took our departure from the Lizard Point, which bore then full north, diſtant by eſtimation 21 miles, allowing its longitude to be 5d. 14m. weſt from London, and its latitude 49d. 27m. north.

Days of the month.	Latitude per obſervation at noon. ° ′	Long. Weſt of London per comm. reckon. ° ′	Long. weſt from obſer. of ☽, reduced to neareſt noon. ° ′	Longit. by ☽ weſt of comm. reckoning. ° ′	Number of obſervations taken of the diſtance of the moon's E. or W. limb from the ſun or ſtars to the eaſt or weſt.
Feb. 10	16 49 N	27 33	30 22	2 49	WL 8 from the ſun to the W
11	14 3	26 47	29 22	2 35	WL 10 from the ſun W
15	5 10	22 44	23 39	0 55	WL 2 from Regulus E
19	1 42	22 44	23 35	0 51	EL 5 from Pollux W
28	9 6 S	26 2	29 44	3 42	EL 8 from the ſun E
Mar. 9	24 9	25 55	30 7	4 12	WL 2 from Aldebaran E
10	25 51	24 32	29 32	5 0	WL 1 from Aldebaran E
13	29 49	22 19	27 55 27 44	5 36 5 25	WL 8 from the ſun W WL 1 from Regulus E
15	30 8	22 8	26 52	4 44	WL 4 from Regulus E
17	30 39	18 58	24 50 25 5	5 52 6 7	WL 3 from Pollux W WL 3 from the Virgin's Spike E
18	31 1	18 47	24 0 24 33	5 13 5 46	WL 4 from Pollux W WL 4 from the Virgin's Spike E
19	31 37	17 7	21 41	4 34	WL 3 from Pollux W
20	32 4	15 27	20 19 22 4	4 52 6 37	WL 3 from Pollux W WL 1 fr. Antares E taken in haſte
25	31 23	5 57	12 4	6 7	EL 3 from the Virgin's Spike W
26	30 51	4 17	9 55	5 38	EL 1 from the Virgin's Spike W
29	30 32	1 13	6 49	5 36	EL 3 from the ſun E

April

April 6, 7 A. M. we came to an anchor in the harbour before James fort at St. Helena, when the longitude, by the common reckoning, was 1d. 28m. eaſt of London; but from my laſt obſervation of the diſtance of the moon from the ſun, March 29, was 4d. 16m. weſt of London. The longitude of St, Helena I find, by my obſervations of the eclipſes of Jupiter's ſatellites, to be 5d. 49m. weſt of Greenwich, or 5d. 44m. weſt of London. Therefore the error of the ſhip's common reckoning is 7d. 12m. eaſterly. But my account, deduced from the diſtances of the moon from the ſun, ſeven days before we made the iſland, differs only 1d. 28m. from the true longitude. Many reckonings kept on board the ſhip were no leſs than ten degrees erroneous.

Extract of the Journal of a Voyage from St. Helena to England, in the Warwick East India Ship, corrected by Observations of the Distance of the Moon from the Sun and fixed Stars.

FRIDAY, Feb. 19, 1762, at 10 A.M. we got under sail; and at noon took our departure from the island of St. Helena.

Feb. 25, at noon we made the island of Ascension, and at 6¼ P.M. the Sugar-loaf point of Ascension bore S. W. distance by estimation 12 miles. By the ship's reckoning we made 6d. 50m. difference of longitude from St. Helena to Ascension, though the real difference, from astronomical observations, is 8d. 10m. so that we were set 1d. 20m. to the westward: we were also set 25m. in latitude to the northward in the same time.

Took a fresh departure from Ascension, allowing it to lie, as by astronomical observations, in longitude 13d. 59m. west of the Royal Observatory at Greenwich, and in latitude 7d. 57m. S.

The next night, by a mean of three distances of the moon from Aldebaran, I found my longitude 22m. less than according to my departure, and distance run from that time; hence, by this observation, the longitude of Ascension should be 22m. less than assumed in my departure from thence, or 13d. 37m. west of Greenwich. But, as the longitude found by the observations of the moon, with Mayer's printed tables, is liable to an error, sometimes of a whole a whole degree or more, this difference of 22m. is rather to be regarded as a proof of the exactneſs of this method of determining the longitude, than to be uſed to correct the aſſumed longitude, which may be ſuppoſed better eſtabliſhed by aſtronomical obſervations of a more delicate nature made on ſhore.

Extract of the Journal of the Remainder of the Voyage from our Departure taken from the Iſland of Aſcenſion to our Arrival at Plymouth.

N. B. The latitudes and longitudes of the following table belong to the noon of the days to which they correſpond, the longitudes deduced from the obſervations of the moon having been reduced always to the noon immediately following them, by means of the ſhip's reckoning, except towards the end of the table, where the hour of the obſervations is ſet down, in which caſe the latitudes and longitudes are ſuch as correſpond to that hour. The bearings and diſtances of the land are alſo ſet down at the times of theſe latter obſervations, or as near to them as poſſible, in order to ſhew how near the longitudes deduced from the obſervations of the moon, agree both with one another, and alſo with the known longitudes of the Scilly Iſles, the Lizard, and Plymouth, which may be found very accurately from the tables of the latitudes and longitudes of places at the end of the tables.

Days

Days of the month.	Latitude observed.	Longitude West of Greenwich by common reckoning.	Longitude West, by observations of the moon.	Longit. by ☽ West of comm. reckoning.	Number of observations taken of distance of the moon's E. or W. limb from the sun or stars to the east or west.
	° ′	° ′	° ′	° ′	
Mar. 9, Noon	0 24 N	18 14 W	21 24	3 10	WL 4 from Antares to E
			21 44	3 30	WL 3 fr. Virgin's Spike E
			21 19	3 5	WL 3 from Pollux W
18, ditto	5 19	22 53	27 19	4 26	E L 3 from the sun E
28, ditto	15 28	29 14	36 10	6 56	WL 3 from Aldebaran E
29, ditto	17 3	29 45	37 1	7 16	WL 2 from Pollux E
Apr. 8, Noon	25 25	32 46	41 46	9 0	WL 2 from Pollux W
17, ditto	32 50	28 41	38 28	9 47	E L 2 from the sun E
28, ditto	46 34	14 56	22 9	7 13	WL 3 from Pollux E
			22 17	7 21	WL 1 from Regulus E
29, ditto	46 54	14 7	21 39	7 32	WL 1 from Regulus E
May 3, Noon	49 17	9 23	15 33	6 10	WL 1 fr. Virgin's Spike E
			16 59	7 36	WL 1 from Pollux W
6, ditto	48 30	4 27	11 41	7 14	WL 3 from Regulus W
			11 45	7 18	WL 2 from Antares E
8, 10h. P. M.	48 19	2 6	8 54	6 48	E L 3 fr. Virgin's Spike W
9, Midnight	49 23	0 11 W	7 16	7 5	E L 3 fr. Virgin's Spike W
10, 9h. A. M.	49 43	0 11 E	6 54	- - -	St. Agnes light-house bore N. 28, W. dist. 5 leagues.
11, 1h. A. M.	49 52	0 22	6 5	6 27	E L 4 fr. Virgin's Spike W Scilly lights bore W. N. W. ½ W. distant 4 leagues.
13, 2½h. A. M.	49 50	1 53	4 35	6 28	E L 5 from Antares E Lizard lights bore N. 63 W. distant 13 miles.
14, 9h. A. M.	49 59	2 30	3 58		Ediston light-house bore N. 22 E. dist. 3 miles.

At night, came to an anchor in Plymouth Sound, and the next morning went on shore.

Remarks

Remarks on the foregoing Extracts of the two Journals.

I have laid before my readers the foregoing extracts of the journals of my two voyages, each containing a ſeries of longitudes of the ſhip, deduced from obſervations of the moon from time to time, compared with a reckoning kept in the common manner; both as proofs of the exactneſs of this method of finding the longitude at ſea, and alſo as examples which I hope to ſee imitated by navigators in future voyages. For tho' it is not very material for them to know their longitude very accurately, while they are in the middle of an ocean, and at a great diſtance from their intended port, but more particularly neceſſary when they are approaching it within the diſtance of 30 or 20, or fewer degrees: yet I ſhould rather recommend it to them, to determine their longitude by obſervation from time, ſuppoſe once in a week, ten days, or a fortnight; not only as it will render them more expert both in making the obſervations and calculations, againſt the time of their approach to their port, when they will have moſt occaſion to depend upon them; but alſo as they will by this means enrich their journals with materials, from whence, at leiſure, very uſeful inferences may be drawn, reſpecting currents, the variation of the compaſs, or other curious particulars.

The exactneſs with which the longitude may be found by theſe obſervations, may be ſhewn from the foregoing abridged journals by three different proofs. The firſt and principal one is from the near agreement of the longitude inferred by obſervations made within a few days or hours of making land, with the known longitude of ſuch land.

A

A ſecond proof may be drawn from the near agreement of the longitudes of the ſhip, inferred from obſervations made on a great many different days near to one another, when connected together by the help of the common reckoning.

A third argument, and, in my opinion, as cogent as any in proof of the certainty of this method of finding the longitude, may be deduced from the near agreement of the longitudes of the ſhip, deduced from obſervations of ſtars on different ſides of the moon, taken on the ſame night.

With reſpect to the firſt and principal proof, in my voyage out, I made the iſland of St. Helena within $1\frac{1}{3}$ degrees of its true ſituation, tho' by cloudineſs of weather, and the proximity of the moon to the ſun, I had not had an obſervation of the moon within eight days before: but in my voyage home, I found the Scilly Iſles by obſervations, which I took in the night a few hours before we made them, within 12 miles of their true ſituation, as I have ſhewn by the computation which I have diſtributed through the book, as an example of the rules. But if the ſecond correction of parallax be applied, as is done at the end of the Preface, (and as is alſo done with reſpect to the journal of my voyage from St. Helena home) the obſervations will agree ſtill nearer, or rather in a manner perfectly with the true ſituation of the Scilly iſles.

By obſervations taken the next night, being in ſight of the Scilly lights, I infer the longitude of St. Agnes light-houſe to be 6d. 23m. weſt of the Royal Obſervatory at Greenwich, which is only 51m. leſs than its true longi-
tude

tude. Alſo by obſervations made May 8, at night, thirty hours before we made the Scilly Iſles, allowing for the ſhip's run in the mean time, the longitude of St. Agnes light-houſe comes out 6d. 49m. or only 25m. leſs than its true longitude. In like manner, by obſervations made May 13, in the morning, in ſight of the Lizard lights, the longitude of the Lizard point comes out 4d. 53m. weſt of Greenwich, or 50m. leſs than its true longitude. The only land which we ſaw in coming home was the Iſland of Aſcenſion, and I was not able to get one obſervation of the moon in the ſpace of the ſhort run from St. Helena thither, owing to clouds and the change of the moon happening in the intermediate time; but the next night after we had paſſed Aſcenſion, I had ſome very good obſervations, which gave me the longitude of the ſhip, agreeing within 22 minutes of a degree with my account, kept from the departure I took from Aſcenſion, whoſe longitude I aſſumed, as it has been determined from aſtronomical obſervations made on ſhore; or, allowing the ſhip's run backwards to the iſland of Aſcenſion, its longitude might be inferred from the ſaid obſervations, and would differ only 22 minutes of a degree from its true longitude. As therefore in none of theſe ſix inſtances, the longitude found from the obſervations of the moon differs a degree from the true longitudes of the places, except in the firſt inſtance, where the run of the ſhip is depended upon for eight days, and where the error of the reckoning, corrected by the laſt obſervations of the moon, gives the longitude of St. Helena only 1½ degrees different from its true ſituation, I think I have ſufficient foundation to ſuppoſe, that the longitude may be always found at ſea by this method, within the compaſs of a degree, or a very little more, which anſwers to about 40 geographical miles in the latitude of the Engliſh channel.

I obſerved, that a ſecond proof of the certainty of this method of finding the longitude, might be deduced from the near agreement of the longitudes of the ſhip, determined from obſervations made on a great many different days near to one another, when connected together by means of the ſhip's run. For this purpoſe I deſire the reader particularly to inſpect the ſeries of longitudes of the ſhip, deduced from obſervation, being no leſs than 12 in number, in the compaſs of 11 days, between March 9th and 20th, 1761, in my voyage out to St. Helena. Setting aſide any currents, as we were then in a pretty open ſea, and ſuppoſing the ſhip's run to be rightly allowed for from the log in the common account, during this time, the ſame difference ought to have been found between all the longitudes by account, and the correſponding longitudes deduced from the obſervations of the moon; or the ſame error of the common account ought to have reſulted from all the twelve obſervations. But as there is, as might be ſuppoſed, ſome difference between them, I take the mean of all the twelve errors of account, which is 5d. 20m. by which we were really more to the weſt than the common account made us; and, comparing each particular error of account with this quantity, the difference between them, in any of the twelve obſervations, ſcarce exceeds a degree; whence I think myſelf intitled to ſuppoſe, that the longitude was deduced truly from every one of the obſervations, within the compaſs of a degree, or not much more.

I have ſet down in the following table the error of the common account, and the difference between 5d. 20m. the mean quantity of it, and each particular error of account; which, except in the firſt and laſt obſervations, dces not exceed three quarters of a degree.

The

The laſt obſervation, which differs moſt from the medium, was taken in ſome haſte, on account of the poſition of the ſails of the ſhip, which did not allow me an uninterrupted view of the ſtar; nevertheleſs, as I was tolerably ſatisfied with the obſervation at the time, I do not think proper to reject it.

1761 March	Error of account. °	′	Diff. from 5d. 20m. mean error °	′
9	4	12	—1	8
10	5	0	—0	20
13	5	36	+0	16
	5	25	+0	5
15	4	44	—0	36
17	5	52	+0	32
	6	7	+0	47
18	5	13	—0	7
	5	46	+0	26
19	4	34	—0	46
20	4	52	—0	28
	6	37	+1	17
Mean error of account,	5	20		

The latter part of the extract of the journal of my voyage home to England affords a ſimilar proof of the conſiſtency of obſervations with each other in determining the longitude of the ſhip on ſeveral nights near to one another. Having ſufficiently ſatisfied myſelf of the certainty of theſe obſervations for finding the longitude, by the great number I made in my voyage out to St. Helena, in the month of March, 1761, I only thought it neceſſary in my voyage home to take ſo many from time to time, at the diſtance of a week or ten days from one another, as was ſufficient to correct

 the

the reckoning, without leaving room for any conſiderable error to creep into it. But towards the end of my voyage, at the latter end of April, and during the firſt part of the month of May, till our arrival at Plymouth, I omitted no opportunity of making obſervations that I could gain; and from the 28th of April to the 13th of May, that is to ſay, in the compaſs of 15 days, I was ſo fortunate as to obtain no leſs than 11 different ſets of obſervations, all which ſhew the error of the ſhip's reckoning the ſame, within the compaſs of 1d. 26m. for the greateſt error of the ſhip's account from one of the obſervations May 3, is 7d. 36m. and the leaſt error of account is from the other obſervation on the ſame day, 6d. 10m. But to ſet this point in a fuller light, I have, in the following table, made a like compariſon of theſe obſervations with the common reckoning, as I have done above with reſpect to the obſervations I made in my voyage to St. Helena between March 9th and 20, 1761.

1762 April	Error of account. °	′	Difference from mean error. ′
28	7	13	+ 12
	7	21	+ 20
29	7	32	+ 31
May 3	6	10	— 51
	7	36	+ 35
6	7	14	+ 13
	7	18	+ 17
8	6	48	— 13
9	7	5	+ 4
11	6	27	— 34
13	6	28	— 33
Mean error of account,	7	1	

In

In this comparifon the greateft difference between the mean error of account and the feveral particular errors of account, is under one degree; and therefore it feems probable, that the greateft error made in determining the longitude from any of thefe eleven obfervations, does not much exceed this quantity: efpecially, fince the greateft difference between the two errors of account which differ moft widely from each other, is only one degree, twenty-fix minutes.

On this occafion I muft confefs, that notwithftanding the many and frequent proofs I have had of the certaintyand exactnefs of thefe kind of obfervations, and tho' I am fenfible I fpared no pains in making thofe I am treating of with all the care in my power; yet I am not a little furprifed myfelf at finding fo near an agreement of fo many with one another, particularly becaufe feveral of them are fingle ones, and the time, in fome of them, was determined from altitudes of the ftars in the night. But I would not be underftood by this to mean to arrogate any peculiar merit or fkill to myfelf in making thefe obfervations, which others may not equally attain with the fame care and experience; fince I am fatisfied, from the near agreement of many obfervations, taken by the officers of the Warwick, with my own, taken at the fame inftant, that mariners properly inftructed in making the obfervations, as I flatter myfelf they will find themfelves to be by means of my fecond Chapter, and moderately exercifed in the practice of them, which is their own part, will be able to determine their longitude, by only adhering ftrictly to my rules, within the limits I have already mentioned, namely, about a degree.

The

The third proof of the certainty of the obſervations of the moon for finding the longitude at ſea, which may be deduced from the foregoing extracts of my journals, is what reſults from the near agreement of the double longitudes of the ſhip deduced from diſtances of the moon from ſtars on different ſides of her, taken on the ſame night; for all the moſt probable kinds of error, that can be ſuppoſed in the obſervations, whether ariſing from a faulty diviſion of the limb of the inſtrument, a refraction in the ſpeculums or dark glaſſes, a wrong allowance for the correction of the adjuſtment, or a ſuppoſed bad habit of eſtimating the contact of a ſtar with the moon's limb too open, affecting the computation of the longitude from ſtars on different ſides of the moon different ways; their ill effect, if any, muſt be ſenſible in the reſult. But ſince in all the inſtances of the double longitudes thus determined, contained in the journal, the difference is very ſmall, and ſuch as fully juſtifies what I have already advanced, with reſpect to the degree of exactneſs with which the longitude may be determined at ſea, it is evident, that by good inſtruments and careful obſervations, theſe errors may be ſo far reduced, as to be of very little conſequence.

I have one more remark to add, reſpecting the journal of my voyage from St. Helena to England, which I flatter myſelf will not be thought foreign to the purpoſe; that April 20th and 21ſt, 1762, while we were croſſing the parallels of latitude of the weſtern iſlands, we ought, by the common reckonings of the ſhip, to have been ſailing thro' the midſt of them, which could not have happened without our, at leaſt, ſeeing ſome of them: but this is eaſily reconciled from the obſervations of the moon, by which it appears, that the ſhip

ſhip was at that time nine or ten degrees farther to the weſt than the common reckonings made it; ſo that if through diſtreſs of weather, or other accidents, we had wanted to make any of thoſe iſlands, we could not have done it by the common reckonings; but it cannot be doubted, after the proofs that have been already given, that we might have readily and certainly found them by means of the longitude of the ſhip, corrected by the obſervations of the moon, if we had wanted it.

An Account of the Principles and Authority upon which the following Tables depend.

THE depreſſion of the horiſon of the ſea, contained in the firſt table, page I. I have made leſs by one tenth part, than it is ſtated in the common tables, deduced from the known dimenſions of the earth; having found, both from theory and experiments, this correction to be neceſſary on account of the refraction of the rays of light in paſſing thro' the atmoſphere.

The next table, which is new, ſhewing the depreſſion of the water's edge of land at a given diſtance, I have computed from the known dimenſions of the earth, paying alſo a regard to the effect of refraction. I apprehend it may be very uſeful when the horiſon is land-locked on the ſide it may be wanted to obſerve; ſince, if the diſtance of the land adjoining to the water can be eſtimated nearly, the quantity of depreſſion taken out of this table may be applied, inſtead of the depreſſion of the horiſon in the former table.

The next table is computed from a rule formerly communicated to me by the late Dr. Bradley, and deduced by him from his obſervations, according to the principles contained in a paper of the late Mr. Simpſon's, in his Mathematical Diſſertations. The rule is, that the refraction at any altitude is to 57ſ. the mean refraction at the altitude of 45d. as the tangent of the apparent zenith diſtance, leſſened by three times the refraction, taken nearly out of any common table of refraction : is to the radius. This is adapted to an altitude of Fahrenheit's thermometer of 50d. and to an altitude of the barometer of $29\frac{6}{10}$ inches : and in this

3 ſtate

ſtate of the air has been found by Dr. Bradley to anſwer very nearly to obſervation quite down to the horiſon. In other ſtates of the air, he found the refraction to be to the quantity contained in this table, in a ratio compounded of the direct ratio of the altitude of the barometer to $29\frac{6}{10}$ inches, and in a reciprocal ratio of the altitude of the thermometer, increaſed by 350 to the number 400. As ſailors have occaſion to correct their altitudes only to ſingle minutes, I thought it would be more convenient for their uſe to ſet down the degrees and minutes of altitude, which anſwer to the even minutes of refraction, than to give the table in the common manner. Refraction cauſing the heavenly bodies to appear higher than they really are, all altitudes obſerved ought to be diminiſhed by the numbers contained in the table; but particularly the altitudes of the ſun, taken for computing his azimuth, in order to find the variation of the compaſs from obſervation, being generally very near the horiſon, ought to be ſo corrected. From the effect of refraction alſo, the ſun appearing to be in the horiſon, when he is really half a degree below it; though he is really in the horiſon, when his center appears at the altitude of 29 minutes above it; therefore the proper time for taking an amplitude is not when the ſun appears in the horiſon, but when his centre appears 29 minutes high, or his lower limb 15, or his upper limb 43 minutes high: but the apparent altitude of the ſun above the viſible horiſon of the ſea ſhould be greater than theſe numbers by the quantity of the dip of the horiſon, taken out of the firſt table.

The catalogue of right aſcenſions and declinations of ſtars, page II. and III. is ſelected from the Abbe de la Caille's obſervations, and is deſigned both for finding the time and the latitude, by altitudes taken in the night.

The next table, page III. and IV. ſerves to turn degrees and minutes of the equator into time, and the contrary, at the rate of 15d. to 1h. and 15m. to 1m. of time, &c. Thus the longitude from Greenwich by account is frequently turned into time, and *vice verſâ*, the apparent time in Rule 3d of the Appendix, is turned into degrees and minutes of the equator.

The table of limits, which the difference or ſum of the moon and ſtars latitudes ſhould not exceed, ought to be carefully attended to in chuſing out a proper ſtar for obſerving the diſtance of the moon from, *Vide* page 12, Chap. II. The principal reaſon for this is, that the moon's latitude, uſed in the computations of the 6th Chapter, may ſometimes err three minutes: for the latitude, as found from the tables, is ſometimes liable to an error of 2 minutes; and, if the ſhip's account be out 5d. this will make an error of 20 minutes in the aſſumed time at Greenwich, and may therefore ſometimes occaſion an error of a minute more in computing the moon's latitude. But if the ſtar be choſen agreeable to the table of limits, an error even of three minutes in the moon's latitude will not occaſion above 15ſ. error in the reſult of the computations of Chap. VI. in finding the moon's true longitude from the obſervations.

The catalogue of the longitudes and latitudes of twelve zodiacal ſtars, page IV. proper for obſerving the diſtance of the moon from, in order to determine the longitude, is taken from the beſt modern obſervations.

In the table, page V. calculated for every tenth day in the year, I have included the ſemidiameter and hourly motion of the ſun, and have thrown together into one ſum the increaſe

increaſe of the ſtar's longitude, owing to the preceſſion of the equinoxes from the beginning of the year, and the aberration in longitude. But as this ſum was ſometimes negative, I have in thoſe caſes added the greateſt negative quantity to every number in its reſpective table, and have ſubſtracted the ſame from the epochs of longitude at the bottom of page IV. ſo that the corrections of page V. are always additive.

The mean motions of the ſun and moon are ſet down agreeable to tables of them, which I received as a preſent from Mr. Gael Morris, being ſuch as he had conſtructed himſelf from the compariſon of a great number of the late Dr. Bradley's obſervations. The equation of time is alſo taken from the ſame tables.

The equation of the ſun's centre, and the tables of the equations of the moon, latitude, and parallax, are taken from Mr. Mayer's tables, as printed in the ſecond volume of the Gottingen Acts, excepting two ſlight alterations, which I have made in order to ſave a little trouble in the calculations, without hurting their exactneſs. One of the alterations I have made is by leaving out the annual equation of the mean anomaly and node, and making the firſt equation of the moon ſerve inſtead of it. This alteration, I own, may ſometimes occaſion a difference of 14ſ. in the computation of the moon's longitude; but then I find, that in Mr. Mayer's manuſcript tables, which I have ſeen, the value of the annual equation of the mean anomaly is increaſed ſo as to approach towards the value of double the firſt equation. The other alteration which I have made, is in ſubſtituting a more eaſy manner of taking out the argument of latitude the ſecond, inſtead of Mr. Mayer's, which can never make a difference of more than 8ſ. from the ſame taken out with

Mr. Mayer's argument. The alteration I have made in taking out equation the firſt of the moon for the annual equation of the node, may alſo occaſion a difference in the moon's latitude of 5ſ. But both theſe differences are of no conſequence, ſince the moon's latitude computed by theſe tables is ſometimes liable to err three minutes; and this being of no prejudice, if the ſtar be properly choſen, a ſmaller difference of a few ſeconds cannot be worth regarding.

I have added, page XXIV. and XXV. three tables of the hourly motion of the moon, conſtructed by myſelf, which were much wanted, being not to be found in any books of tables that I have ſeen.

I have added a table of multipliers, page XXV. in order to ſave a little trouble in the computations: they are the quotients of 15d. or 900 minutes, divided by the hourly motion of the moon, or the difference of the hourly motions of the ſun and moon, which are the two firſt terms of the proportion preſcribed at the beginning of the 7th Chapter.

The table of the moon's parallax in altitude, page XXVI. is uſeful in computing the moon's apparent altitude, if it had not been obſerved, *vide* page 57 and 60; alſo in Article IV. in the reſolution of the problem in the Appendix, where the altitude of the moon is firſt corrected by parallax, in order to compute the time from: alſo in finding the latitude from the moon's meridian altitude: and alſo in computing the ſecond and ſmall correction of parallax, which was omitted in Chap. V. but is delivered at the end of the Preface.

The tables of the declination and right aſcenſion of the degrees of the ecliptic, page XXVI. XXVII. and XXVIII. are

are uſeful for finding the moon's declination and right aſcenſion, by the two rules in the appendix, page 54, from her longitude and latitude given: alſo for finding the ſun's right aſcenſion from his longitude, whether in computing the time from the altitude of a ſtar or the moon, or in computing the apparent altitude of the moon and ſtar from the apparent time given.

I have added the laſt table, containing the longitudes and latitudes of places that have been determined from aſtronomical obſervations, as a proper and neceſſary part of this work, and I hope it will appear an acceptable preſent to the public. As the method of finding the longitude by obſervations of the moon, or indeed any method of finding the longitude, can only agree with the true longitudes of places; and as thoſe determined from aſtronomical obſervations are the only ones that are abſolutely certain, ſo I would be underſtood to ſubmit the method of finding the longitude propoſed in this book to a trial only by a compariſon with the longitudes of places contained in the table; I mean, with reſpect to the degree of exactneſs which I have inferred from my own experience to be attainable by this method. But if the navigator, on making of land, ſhould find his account, deduced from obſervations of the moon, to differ much above a degree from the longitudes of places which are not contained in the table, but taken from other authorities, I hope he will be ſo candid as to ſuppoſe, that this greater ſeeming diſagreement of his obſervations with the land made, rather ariſes from ſome error in the longitude of the land, aſſumed from too inſufficient authority, than to any defect in a method which may ſeem from ſo many experiments to be ſufficiently proved.

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In order to render the foregoing table of longitudes and latitudes of places as compleat as possible, I have consulted all the best authorities I could find. In particular, I have borrowed all the places that are contained in the French Almanack, called the *Connoissance des temps*, for 1760, and in the explanation of the annual calculations of the same almanack, published in 1762, with very little alteration, only reducing them to the meridian of the Royal Observatory at Greenwich. For the longitudes of other places which I have here added, and which result from observations made in the latter end of the last, or the beginning of the present century, I refer the reader to the books or memoirs in which they are described, and which are to be supposed well known to geographers; but I shall give a brief account of the authorities upon which I give the longitudes of several places, deduced from more modern observations. Many of these are determined from the observations of the late transit of the planet Venus over the sun.

Thus I have deduced the longitude of Leaskard in Cornwall from the observation made there by the Reverend Mr. Haydon; that of the university of Cambridge from a mean of the observations made there by several ingenious gentlemen, tho' their modesty has prevented them from communicating them to the public; that of Exeter in Devonshire, from the observation of Mr. Chapple; that of Stalbridge in Devonshire, from the observation of the Rev. Mr. Bolton; and that of Wakefield in Yorkshire, from the observation of Mr. George Gargrave.

The observation of the Rev. Mr. Haydon is particularly valuable, both as it was made with good instruments, and as it serves, with the help of the maps of the county of Cornwall,

Cornwall, to determine very nearly the longitude of Plymouth, Falmouth, the Lizard Point, the Land's-end, and the Scilly Iſles; all which places are generally either the laſt that ſhips ſee in going out of the channel outward bound, or which they make firſt upon coming into the channel homeward bound.

The other places, whoſe longitudes are deduced from the late tranſit of Venus, are Leyden in Holland, from the obſervation of Mr. Profeſſor Lulofs, (and hence alſo that of the Hague and Rotterdam, by their known poſition to Leyden); alſo Drontheim in Norway, from the obſervation of Mr. George N. Holm; Carelſcroon and Landſcroon in Sweden, from obſervations made there, but I am not acquainted with the names of the obſervers; Tobolſki in Siberia, from the obſervation of M. L' Abbè de Chappe; and St. John's in Newfoundland, from the obſervation of Mr. Winthrop, who ſailed thither with proper inſtruments from New England for that purpoſe. The longitude of Fort St. George in the Eaſt Indies reſults from the obſervation of the tranſit of Venus made there by the Rev. Mr. Hirſt; that of Calcutta in Bengal, from the obſervation of Mr. Magee; and that of Iſlamabad from the obſervations of Meſſ. Knott and Plaiſted. For theſe four laſt obſervations we are indebted to the Directors of the Eaſt-India Company, who ſent out general inſtructions to all their ſettlements, particularly recommending this obſervation to ſuch as ſhould be capable of it.

The longitudes of the following places I have determined from obſervations of the eclipſes of Jupiter's firſt ſatellite, compared with correſponding ones made at Greenwich, or elſewhere; or, if not correſponding, yet happening within a few periods, and compared with the help of the tables. The longitude of the island of St. Helena I deduce from my own observations made there in the year 1761; the longitude of the Cape of Good Hope, from the observations made there by Mr. Charles Mason and Mr. Jeremiah Dixon in the same year. The longitude of the island of Rodrigues in the Ethiopic observation, from the observations of M. Pingré; and Casan in Russia, from two emersions, one of the first, and the other of the second satellite, observed there by M. L' Abbè de Chappe, in the year 1761. The longitude of Cape Francois I deduce from an emersion of the first satellite of Jupiter, observed there in the year 1745, by Don George Juan, one of the two Spanish mathematicians who went to Peru in conjunction with the French academicians, to measure the degrees of the meridian, which I compared with a computation which I have made of the same, from Mr. Wargentin's tables, published by M. De la Lande, not having possession of any observations made in England or elsewhere at that time. The longitudes of Stockholm, Hernosand, and Cajaneburg, all in Sweden, are taken from the authority of Mr. Wargentin. *Vide* papers 39th and 44th, Vol. LII. Part I. of the *Philosophical Transactions* for the year 1761. The same volume also contains most of the observations of the transit of Venus which have been made use of in determining the longitudes of the places already mentioned, which are inserted in the table.

The longitude of Cape Clear in Ireland was determined from eclipses of Jupiter's satellites, by the observers appointed by Mr. Whiston to survey the coasts of the channel. The longitude of the city of York was determined by Mr. Richard Norwood, from the same mensurations by which he inferred the length of the degrees of the meridian between London and York.

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I am obliged to Mr. James Short, F.R.S. for the latitude and longitude of Edinburgh, the latter resulting from the observations of the eclipses of Jupiter's satellites, made there formerly by himself: as also for the longitude of Port Royal in Jamaica, computed by himself from the observation of the external contact of Mercury with the sun's limb in the year 1753, taken by Mr. Macfarlane. The longitudes and latitudes of Louisbourg, Cape Sable, and the Straits of Fronsac, between Nova Scotia and the island of Cape Breton, are taken from M. Chabert's accurate observations. The situation of Achem, the north-west point of the island of Sumatra, and that of the island of Pulo Timon in the Gulf of Siam, I have taken from the authority of M. d' Aprés de Mannevillette, in his *Neptune Orientale*; as also the longitude of Manilla in the Philippine islands; the two first being accurately settled by him from their latitudes and bearings from Malacca, and the latter sufficiently determined from several short runs of ships to and from the island of Pulo Condor, which is settled from astronomical observations. The longitude of Callao in the South Seas depends on that of the city of Lima, of which it is the port, and which is known from astronomical observations.

The positions of Cape Lorenzo and Guaquil, likewise in the South Seas, are settled relatively to the city of Quito in Peru, by the trigonometrical mensurations of the gentlemen who measured the degrees of the meridian in Peru; therefore I have set down their longitudes agreeable to that of Quito contained in the tables, and the difference between the longitudes of those places and Quito, supposed in Don George Juan's accurate chart of the western coast of South America: the longitude of Quito contained in the tables, and which results from eclipses of Jupiter's first satellite,

 being

being much more certain than its longitude aſſumed in the chart from eclipſes of the moon.

Laſtly, the longitude of Panama in the South Seas is pretty accurately determined with reſpect to Porto Bello, lying on the other ſide of the Iſthmus, from which it was found to lie only 31 minutes of a degree to the weſt, by the eſtimation of the direction, and length of the way from the latter place to the former, made by the Spaniſh mathematicians who went to Peru to meaſure the degrees of the meridian.

THE END.