210908 - LatLon Near Noon methods comparison

1 - OBSERVATIONS AROUND MERIDIAN TRANSIT OF A CELESTIAL BODY

Over a short time span - typically less than 1 hour - "***LAN***" (*L*ocal *A*pparent *N*oon) ***observations*** also called "***Near Noon***" ***observations*** can provide the Celestial Navigator with a quite reliable position.

Hence these "***LAN***" observations have long been studied and there is a quite important literature to cover them.

Practical steps - The Navigator first starts recording his sights: both UT's and Heights during a time span around the estimated time a celestial Body "crosses" his own meridian. The Navigator then processes his observations to derive his position.

Since most generally the Sun is the aimed Body hence these "***LAN***" or "***Near Noon***" acronyms.

99.99 % of the time ***Upper Culminations*** only are observed: heights first rise, then stay constant and eventually decrease.

2 - KEY POINTS TO REMEMBER

(2.1) - **Plotting Heights vs. UT times on a graph** yields a "***parabola type***" series of points open towards the bottom of the graph with a symmetry axis parallel to the Heights scale.

(2.2) - Within the limits of quality of the observations for both a ***Fixed Latitude Observer*** and a ***constant Declination Body*** such "***parabola type***" is exactly symmetrical relatively to the *Time of Body Transit (**UT tran**)*, i.e. Body crossing the Observer's meridian. Apparent Culmination ***H culm*** occurs exactly at such ***UT tran***. Hence from Body GHA and Declination at ***UT tran*** the Navigator very easily gets his own Latitude and Longitude.

(2.3) - Within the same observations quality limits, for a ***moving Observer*** and/or a ***changing Declination Body***, Culmination most often does not coincide with Body Transit. ***UT culm*** becomes different from ***UT tran***. Nonetheless the symmetry axis of such [new] "observation curve" remains quite parallel to the Heights scale.

> For all practical purposes this "new curve" can be considered as simply "shifted" from the previous one depicted in **(2.2)** through a **most often negligible N/S translation** and through an **often quite sizeable E/W translation**.

(2.4) - Whichever solution is subsequently carried out, all solving methods require to accurately pinpoint - both for **Time and Height - the top/summit of such *actual* "observation curve", i.e. both *UT culm* and *H culm***.

(2.5) - From the available data - i.e. ***UT culm*** and ***H culm*** - the motions effects earlier mentioned in (2.3) can be taken in account and corrected for, in order to recover both ***UT tran*** and ***H tran*** . The Navigator is back to an optimum configuration to derive his own LatLon as per **(2.2)** here-above.

> Such classical corrections formulae are listed **SECTION 3** here-after.

(2.6) - And finally to close this important list - which otherwise could easily be extended almost *ad infinitum* - the random errors standard deviation (SDEV) comes to play as follows in the final determination of the "LAN" Position:

- SDEV directly ***degrades*** Latitude accuracy by a ratio very close from 1 to 1. And:

- SDEV directly ***degrades*** Longitude accuracy by a much higher ratio: up to 5 to 1 or even 8 or 10 to 1.

- The "lower" the ***H culm***, the bigger the Longitude accuracy degradation for one given SDEV.

3 - CLASSICAL CORRECTIONS FROM CULMINATION TO TRANSIT

With the following abbreviations:

- ***Lat*** : Observer's Latitude
- ***Dec*** : Body Declination
- ***NS*** : Observer's North Speed in knots (negative for South speeds). And:
- ***µDec*** : Body Declination Hourly change in Arc Minutes (algebraic values too) :

(3.1-a) - To correct from (UT culm) into (UT tran) for UPPER CULMINATIONS :

(UT culm - UT tran) in seconds of time = (48 / π) * (tan Lat - tan Dec) * (µDec - NS) **- *(1a)***

(3.1-b) - To correct from (UT culm) into (UT tran) for Circumpolar Bodies LOWER CULMINATIONS :

(UT culm - UT tran) in seconds of time = (48 / π) * (tan Lat + tan Dec) * (µDec - NS) **- *(1b)***

(3.2) ***- Finally to correct from H culm into H tran :***

(H culm - H tran) in arc Minutes = ½ (µDec - NS) * (UT culm - UT tran) in hours **- *(2a)***

On a practical stand-point and in almost all literature *(see **3.3.5** here-under) :*

H culmination = H transit **- *(2b)***

(3.3) - <u>Notes</u>

(3.3.1) - *Formula (1a)* can be regarded as the FUNDAMENTAL FORMULA for all classical NAL solving methods.

Formulae (1a) and (1b) are established for the Sun with a GHA rate of 15°/hour. Even so they are ***1st order formulae*** only with respect to Time. For this reason they can be in error by up to 15% in extreme cases even for the Sun. They can also be used for the Planets and the Stars without significant extra fine tuning required.

(3.3.2) - For the Moon ***(Ref 3)*** recommends increasing the results of ***(1a)*** by 7%. It seems a reasonable starting point.

(3.3.3) - Formulae (1a) and (1b) explicitly assume the North Speed to stay constant over the full time of the Observations, a constraint governing all types of solutions methods.

(3.3.4) - *Formula (1b)* has been very rarely if ever published because the opportunities for such Observations are quite infrequent (e.g. circumpolar bright Stars during sufficiently long twilights).

(3.3.5) - *Formula (2a)* is a ***2nd order formula*** with respect to Time and as such it yields results accurate to always better than 0.01'.

However its corrective terms are so small - always inferior to 0.3' - that it has been very rarely if ever published too.

It nonetheless remains the mathematical justification for formula *(2b)* here-above.

4 - METHODS TO DERIVE *UT culm* AND *H culm*

Accurately pinpointing both *UT culm* and *H culm* remain **essential steps** in the course of the LAN fix process.

A quick "historical review" here-after about some successive significant methods shows their advantages and drawbacks.

In **SECTION 5**, one specific example is treated with these methods for subsequent comparisons between them.

4.0 - AN OPTIONAL EARLY DATA PROCESSING STEP: *THE FICTITIOUS NAVIGATOR AND CELESTIAL BODY METHOD*.

4.0.1 - Description of the Fictitious Navigator and Celestial Body method.

Let'us imagine that at some specific moment during the observations process, the Real World Navigator is "overlapped" by an extra Fictitious Navigator who is to keep exactly the same E/W speed, but has no N/S Speed. While they are to "observe" at the same Times, both Navigators will stay at exactly the same Longitudes.

At this very same time, let's assume that the Real World Body is also overlapped by an extra Fictitious constant Declination Body with which keeps exactly the same GHA's as the Real World Body.

Both the Fictitious Navigator and the Fictitious Body fulfill the conditions of **(2.2)** here-above**. Hence for the Fictitious Navigator who keeps "observing" Fictitious Heights : *UT tran = UT culm*.**

At Time (T1) of the first observation, the Real World Height (H1) and the Fictitious Body Height (H'1) are exactly the same. From that instant they start gradually diverging from one another.

For the entire time-span of Observations and if we make the assumption - which is a quite valid one - that the Cosines of the Body Azimuths always remain ***almost exactly*** equal to ***Unity*** then both **the Real World Heights and the Fictitious Heights keep diverging at a rate *almost exactly* equal to *(μDec -NS).***

Hence, from the Real Word Heights set, it is possible to devise a Fictitious Heights set whose "curve" is expected to be exactly symmetrical with respect to its own ***UT culm***.

With this ***FICTITIOUS NAVIGATOR AND CELESTIAL BODY METHOD*** which is no more than a handy optional data pre-treatment step, the LAN is solved as follows:

UT tran = UT culm of the Fictitious Heights set, and H tran = H culm of the Real World Heights set.

Notes* :** This optional Method looks quite attractive for all graph paper methods as it frees the Navigator from recourse to **Formula (1a)**. ***It immediately yields UT tran !

Nonetheless, and especially when dealing with a high number of observations (e.g. **Ref 3**), it requires a meticulous tally of the ever changing corrections to the individual Heights. This weakness can be regarded as a ***rather subtle and unnecessary complication which may easily and entirely ruin the claimed simplicity of such graph paper methods***.

Nonetheless, the idea behind it - *Real World and Fictitious Heights diverging rate equal to* ***(μDec - NS)*** - is one the 2 keys to the very clever ***stunning Method (4.3.2)*** depicted here-after.

4.0.2 - Difference between Real World Heights and Fictitious Heights Increasing/decreasing rates

4.0.2.1 - The 2nd key to ***Method (4.3.2)*** deals with the increasing/decreasing rates comparison between the Real World Heights and the Fictitious Heights.

Any efficient ***UT culm*** recovery requires some of the Real World Heights to be observed with sufficient increasing / decreasing rates. Otherwise**, insufficient rates** (e.g. if too close from culmination, or if [Real World] ***H culm's*** are too low, typically below @ 60° for most methods) eventually translate into **unacceptable geometric dilution of position (GDOP)**. Hence the Longitude errors mentioned in **(2.6)**.

4.0.2.2 - By definition such "**Sufficient rates"** mentioned here are to be far greater than ***(μDec - NS)*** so that in some applications the latter maybe considered as negligible. Under such cases we can safely assume the following:

During the time-period when the Real World Heights increase/decrease at such sufficient rates - typically equal or superior to 3°/hour - the slopes of the Real World and of the Fictitious Heights tangents can be considered as having the same value for some reasonably short period.

4.1 - EQUAL HEIGHTS METHOD (Ref 1)

By the 1930's both the French Navy and the UK Royal Navy (among others) published the first "LAN type" method which advocates recording **equal heights sufficiently before and after culmination, and to monitor and record maximum height**.

Formulae (1a) and (3) were then altogether published in order to solve for such LAN fixes.

UT culm is taken as the average of all corresponding equal heights times, and

H culm is taken as equal to the maximum height recorded.

4.1.1 - Advantages

4.1.1.1 - A definite simplicity of use.

4.1.1.2 - Can be used under degraded material conditions (no electricity ...). Hence it can be considered as an Emergency method to-day.

4.1.2. - Drawbacks

The observation time-spans are subject to strict constraints addressed in dedicated literature, e.g. :

4.1.2.1 - Given random observation errors, the rising/decreasing equal altitude heights must be taken **sufficiently before/after *UT culm*** to benefit from adequate time-spans with sufficient height variation rates vs. time. This is required in order to not degrade ***UT culm*** determination (See **4.0.2.1**).

4.1.2.2 - **On the contrary**, such equal altitudes must be taken **sufficiently close from *UT culm*** so that the "Observation curve" ***odd powers*** in Time do not start becoming significant. In other words such "curve" must be considered as acceptably symmetrical around ***UT culm***.

4.1.2.3 - Especially with the graph paper methods, the Navigator needs to process data sufficiently symmetrical (see 4.4.2.2.2). Accordingly the higher the *H culm* the closest all observations must all gather around *UT culm*. This constraint also governs - to variable extents - all kinds of solution methods.

4.1.2.4 - As an adverse consequence there can be configurations with only a **reduced observation window** before and after ***UT culm*** during which the Navigator is to take a minimum of 3 rising/decreasing heights in order to average at least 3 pairs of observation times.

4.1.2.5 - Within the hard constraints of **4.1.2.3** such equal Heights should - ideally - be equally spaced between them in order to best minimize the effects of random observation errors.

4.1.2.6 - All constraints here-above require more or less adequate uninterrupted observations conditions.

4.2 - EARLY CALCULATOR STATISTICAL METHODS (Ref 2)

Taking advantage of the recently introduced hand held Calculators, and by the late 1970's some Pioneers (e.g. **Ref 2**) quickly came out with statistical programs permitting adequate treatments of LAN's.

4.2.1 - Advantages

4.2.1.1 - **Statistical methods offer a definite advantage over graph paper methods in terms of results accuracy.**

This is definitely the main reason why they have become quite successful and popular.

4.2.1.3 - Such Statistical methods greatly alleviate some of the graph paper methods constraints here-above.

Constraints 4.1.2.1 and 4.1.2.2 become irrelevant. Any [not too large] unexpected gap in the observations schedule should bring no adverse consequence as long as at least one Height close to ***H culm*** can be recorded. If sufficient observations are made anytime within the time span permitted by **constraint 4.1.2.3** then the Navigator can get a quite decent LAN fix.

4.2.1.2 - Some simplicity of use still requiring attention in data entering (caveats in **4.2.2.1** and **4.2.2.2**)

4.2.2 - Drawbacks

4.2.2.1 - Lot of attention required to key data in. About 15 keystrokes are required per one single observation, not counting environmental data. Repeatedly and carefully checking for typing errors before entering data may become utterly painstaking. This step must be performed since *garbage in, garbage out*.

4.2.2.2 - It was often difficult if not impossible to correct for typing errors when data already entered. A solid example to run (e.g. **Ref 3**) can require up to over 400 [uninterrupted] individual keystrokes.

4.2.2.3 - For lack of space a number of early software versions simply used **Formula (1a)** itself which prevented them from adequately processing Moon observations. Re: Note **(3.3.2)** .

4.2.2.4 - There is no built-in real safeguard against using LAN Software outside the strict limitations of **constraint 4.1.2.3**.

4.2.2.5 - Relying too much onto 100% computations may lead to oversights or even blunders otherwise quite visible with graph paper methods. Lack of situation awareness may quickly bring Safety concerns.

4.2.2.6 - Cannot be considered as an Emergency use method to rely on.

4.3 - JAMES N. WILSON'S METHOD (Ref 3) and its further "Russian" IMPROVEMENT

4.3.1 - STANDARD J.N. WILSON's METHOD (1985)

By the 1980's and with the wide recognition and use of ***Formulae (1a) and (3)***, **any new graphical method then attempting to solve for LAN could bring added value *ONLY IF* it could significantly improve *UT culm* determination**.

And, *lo and behold*, in 1985 James N. Wilson came with one such improvement method to determine ***UT culm***.

Within the limitations of 4.1.2.1 and 4.1.2.2 **Wilson's Method** requires 3 sets of observations:

- Rising heights
- Culminating height(s)
- Decreasing heights

The culminating heights yield ***H culm*** through visual interpolation on a graph.

The ***rising heights*** are drawn on a colored graph and then visually approximated by a ***straight colored line***.

On that same graph, and using an extra and overlapping time scale the ***decreasing heights*** are drawn ***in different color*** and visually approximated by a ***straight line*** so that both rising and descending lines cut one another about halfway on the graph. The two different overlapping UT Times scales (one for the ***increasing heights*** and one for the ***decreasing heights***) can be read for this same intersection height - whichever it may be - and the mean value of the averaged intersection UT's is ***UT culm***. This is a very clever use of overlapping Time scales.

A nicely devised Abacus (***Appendix 1***) enables to visually derive the value of ***K = (48 / π) * (tan Lat -/+ tan Dec)*** .

With ***K*** and ***Formula (1a/1b)*** the Navigator computes (***UT tran - UT culm).***With ***Formula (3)*** he computes ***H tran***.

4.3.2 - IMPROVED WILSON's METHOD

And - cherry on the cake - a Russian Navigator subsequently came out with a very **clever improvement to Wilson's Method without even having recourse to the Classical formula** **(1a)** .

It relies on exactly the same principle as the **Fictitious Navigator and Celestial Body method (4.0.1)** but without any of its potential traps.

This **stunning Improvement** is better explained in an example **5.7.2** here-after.

4.4 - ENHANCED CALCULATORS / COMPUTERS STATISTICAL METHODS

In the past decades, significant improvements have been made onto Navigation Computers / Calculators. Both the hardware and the software of LAN programs have been significantly improved and are more User friendly.

Reliable and accurate **Long Term Almanacs** are now routinely part of the proposed software and are sourced onto the very best recent developments by mainly the ***French Bureau des Longitudes*** (***VSOP xxxx*** , ***ELP-xxxxx*** Solar/Planetary and Lunar analytical theories) and the ***JPL*** from the USA (***DExxx*** numerical integrations).

4.4.1 - Advantages and drawbacks

We could now state that most of the drawbacks identified in **4.2.2.1 to 4.2.2.4** have now vanished. And for these very good reasons using modern software to solve LAN's has become quite popular among the Community.

As for their drawbacks, we should still consider that the caveats addressed in **4.2.2.5 and 4.2.2.6** do remain relevant.

4.4.2 - Software Contributors

Among the many software Publishers a special mention is given to two highly respected Contributors to NavList who have made their own Navigation Software - including their LAN Software - available to the Community:

- ***Andres Ruiz González from Kingdom of Spain***
  - *https://sites.google.com/site/navigationalalgorithms/about*
  - *navigationalalgorithms@gmail.com* . And:

- ***Peter Hakel from the USA***
  - *http://www.navigation-spreadsheets.com/*

Their LAN Software belongs to this Enhanced Software category with their many useful refinements such as nice displays of the observations set. Such enhancements start bringing back the **Navigator's situation awareness** much closer to the real world. Hence both Software greatly avoid the trap mentioned in **4.2.2.5** .

Their LAN Software is believed to use a refined regression Parabola (2^{nd} order to Time) in which the exact Body GHA rates are accurately computed, which makes them fully valid for any celestial Body, including the Moon.

4.4.2 - Higher order Regressions with respect to Time (June 1989)

4.4.2.1 - One way to alleviate **constraint 4.1.2.3** is to implement Higher order Regressions with respect to Time in order to broaden the "time-window" during which observations can validly be made around ***UT culm***. Such methods can also ***easily accommodate lower H culm's*** - as low as 20° - vs. a minimum of 50° or even 60° generally accepted for many LAN methods.

4.4.2.2 - However, even if assuming that all changing elements (Observer's Latitude and Longitude, Body GHA and Declination) vary strictly linearly with respect to Time - *i.e. all second order derivatives strictly equal to zero* - ***High order Regressions Methods are to face 3 adverse environmental cumulated constraints:***

4.4.2.2.1 - With respect to the Time powers the **number of terms of the relevant differential functions grows faster than the number of terms of the Pascal's Triangle**. Quickly the task becomes quite cumbersome.

4.4.2.2.2 - The ***even order terms*** are much more significant than the ***odd order terms***. ***Note****: this makes sense since we are dealing with rather symmetrical curves (see **constraint 4.1.2.3**)*. As a consequence **any significant improvement** to a given level statistical Regression **requires differentiating 2 steps further**.

4.4.2.2.3 - Successive higher order Regressions widen the "time-window" **by smaller and smaller increments**.

In spite of such cumulative obstacles it is believed that a 6th order Regression with respect to Time is a reasonable trade-off since it doubles the size of the valid time-window of a 2nd order regression.

4.5 - FOLDED PAPER SHEET METHOD (Ref 4)

Back to graphic determinations: given the added benefits of the **Wilson's Method (Section 4.3)** one could have regarded as almost impossible to invent any further significant improvement to the graphical ***UT culm*** determination process.

Nonetheless ***Frank E. Reed*** founder and moderator of the NavList Forum *http://fer3.com/arc/* invented such clear cut improvement with his "**Folded semi-transparent sheet of paper method**" and deserves full credit for it.

In its simplest version (not using Option 4.0), use a sheet of semi-transparent paper to carefully graph ***Heights vs. UT times*** for all observations. Manually draw a curve to smoothly approximate as many observations points as possible.

Gently start folding the graph alongside a parallel to the heights scale and hold its two "half pages" against ambient light. Look through the 2 sheets for both "branches" of the curve. Do not hard fold this graph yet but carefully fine tune the 2 branches positions so that they overlap one another as much as possible. Then hard fold your sheet of paper exactly alongside a parallel to the heights scale in order to keep this best superimposition in visible memory. This hard folded axis gives you ***UT culm***. And the corresponding height gives you ***H culm***.

Wilson's Abacus (Appendix 1) and ***Formula (1a)*** then enable to solve for ***UT tran***. ***Formula (2b)*** solves for ***H tran***.

4.5.1 - Advantages and drawbacks

Among all the graph methods earlier mentioned the "Folded semi-transparent sheet of paper method" if and when used without option (4.0) ranks out as the best one in terms of simplicity, ease of use and reliability.

It is only subject - to a lesser extent than all other methods - to the limits of **constraint 4.1.2.3 .**

Other than that, and contrarily to the **Equal Altitudes** and to **Wilson's** methods, it can accommodate in particular:

- Reasonable gaps in the observation series. Nonetheless at least one observation near ***H culm*** still required.

- And it can also easily accommodate unequally spaced Observations with respect to Time.

It also fully qualifies as a Back up / Emergency method.

5 - ONE EXAMPLE FOR COMPARING THE PREVIOUS METHODS

Let us review the following example submitted by ***Frank E. Reed*** to NavList.

http://fer3.com/arc/m2.aspx/Compare-Methods-LatLon-Near-Noon-FrankReed-aug-2021-g51028

Date: May 22, 2022
Observer: Sailing 4 kts at True heading 185° on surface. **Current** : 2 kts to true East.
SUN sights, UT / uncorrected Altitudes:
1 - 16:17:20 / 72°28.5' **2 -** 16:22:40 / 72°44.5' **3 -** 16:36:45 / 73°04.0' **4 -** 16:39:15 / 73°04.5' **5 -** 16:51:25 / 72°49.0' **6 -** 16:56:20 / 72°35.5'
All Altitudes corrections forced to +13.0'

5.1 - PRELIMINARY RESULTS

5.1.1 - **This exercise permits crosschecking various LAN Software in a thorough and meaningful way**.

- Since all altitude corrections are imposed, all users are processing exactly the same set of observations.

- Hence any difference on end results can be attributed to only:
  - Differences in the Ephemeris used. And/or :
  - Differences in the number of significant digits published. And/or :
  - Differences in the processing software.

5.1.2 - Whenever applicable **extra-digits** are published here-under to permit more in-depth comparisons.

5.1.3 - **Method 5.2** here-under uses the following Ephemeris for the SUN (***VSOP 2013***).

SUN	GHA	DEC	Distance, SD and µ Dec.
16:00 UT	60°49.56'	N + 20°28.18'	Distance : 1.012305 UA,
17:00 UT	75°49.51'	N + 20°28.66'	SD : 15.80' ***µDec : + 0.4862 '/h***

5.1.4 - **Speed Made Good**: North Speed ***(NS) -3.9848 kt*** , Speed to the East : +1.6514 , i.e. **4.3134 kt/157.4898 °**

5.1.5 - **(µDec - NS)** = **+ 4.4710 '/h**

5.1.6 - From the starting data, the following **approximate values** are readily derived: **Lat = N 37.2°** and **Dec = N 20.5°**

5.1.7 - From **5.1.6** and **Wilson's graph** (**Appendix 1**), **K = 5.9**

5.1.8 - From **5.1.5** and **5.1.7**: **(UT culm - UT tran) = + 26.4 s** (***Reminder***: **Wilson's** graph used here as per **5.1.7)**

5.1.9 - From **5.1.5**, **5.1.8** and **Formula** **(2)**, **(H culm - H tran) = 0.0164'**. As expected in **3.3.5** this is a totally negligible difference.

5.2 - RESULTS OF 6th ORDER LAN SOFTWARE (see **4.4.2**)

UT tran = 16h37m30.7s *and* ***LAN fix*** *at* ***N37°11.3' / W070°12.2'***
UT culm - UT tran *=+ 26.15 s (vs. +26.4s at* ***5.1.8****)*
H culm - H tran *= 0.0164' (vs. 0.0164' at* ***5.1.9****)*
SDEV *= 0.23 NM (0.8' on Lon)*
Method Validity time span (16h11m - 17h04m : OK)

Many Body Fix Method *: At the LAN time get a* ***Many Body Fix*** *at* ***N37°11.3' / W070°12.2'.***
This ***Many Body Fix*** *is only 34 meters - i.e. less than 120 ft - from* ***LAN fix*** *and get* ***SDEV*** *= 0.22 NM.*

17h Fix *at* ***N37°09.8' W070°11.4'***

Note : The 6th order LAN method is expected to be the most accurate of all LAN methods addressed here.

- In particular the **LAN Fix** and the **Many Body Fix** are almost identical (less than 0.022 NM apart).
- The **SDEV's** in both cases are almost identical too: 0.22' vs. 0.23'.
- These comparisons constitute an meaningful and independent check of this LAN method.

5.3 - RESULT OF THE EQUAL ALTITUDES METHOD (see **4.1**)

Due to its "Equal Heights" constraint this Method cannot support the current example.

5.4 - RESULTS PUBLISHED BY PETER HAKEL (See **4.4.2**)

Peter Hakel readily published his own results here and as follows:

http://fer3.com/arc/m2.aspx/Compare-Methods-LatLon-Near-Noon-PeterHakel-aug-2021-g51029

UT tran = 16h37m29s *and* ***LAN fix*** *at* ***N37°10' / W070°11.4'***
UT culm - UT tran *= not published*
H culm - H tran *= not published*

Many Body Fix Method *: At the LAN time get a* ***Many Body Fix*** *at* ***N37°11' / W070°12'.***

17h Fix *at* ***N37°10' W070°12'***

These results are very close from the ones obtained in 5.2.

5.5 - RESULTS OBTAINED BY THE FOLDED PAPER SHEET METHOD (See **4.5**)

These results, using the **simplest version of this method**, ***(not using Option 4.0)***, were published here and as follows:

http://fer3.com/arc/m2.aspx/Compare-Methods-LatLon-Near-Noon-Couëtte-sep-2021-g51044

UT culm = 16h38m00s
From ***Formula 1a : UT culm - UT tran*** *= +26.3 s (vs.+26.4s at* ***5.1.8*** *and +26.15s at* ***5.2****)*
H culm - H tran *= not published*
UT tran = 16h37m34s *and* ***LAN fix*** *at* ***N37°11.0' / W070°11.0'***

17h Fix *at* ***N37°09.5' / W070°10.9'***

The Folder Paper sheet method works very well too. Results within half of mile from 5.2 .

5.7 - RESULTS OBTAINED BY THE WILSON'S METHOD (See **4.3**)

5.7.1 - STANDARD WILSON'S METHOD (See 4.3.1)

This example carries the minimum number of Observations to permit using Wilson's Method since Observations **1,2** and then **5,6** are just sufficient in number to permit defining the increasing and decreasing Heights straight lines.

Note : Incidentally since these both Height lines are uniquely and simply passing through 2 well defined points for each, it is even possible to give results just from algebraic computations, but this is not the name of the game here.

The first result of this method was first published by Tony Oz here:

http://fer3.com/arc/m2.aspx/Compare-Methods-LatLon-Near-Noon-TonyOz-sep-2021-g51042

On his **Wilson's paper graph** Tony Oz carefully derived ***UT culm*** as the average of 16h20m56s and 16h54m56s to get:

UT culm = 16h37m56.0 s

*Algebraic result (see **Note** just above): UT culm = 16h37m56.6s*

From this result by Tony Oz's and the **Wilson's Abacus (Appendix 1)** we get:

UT culm - UT tran = + 26.4 s and ***UT tran = 16h37m29.2s*** *(vs. 16h37m30,7s in **5.2**)*

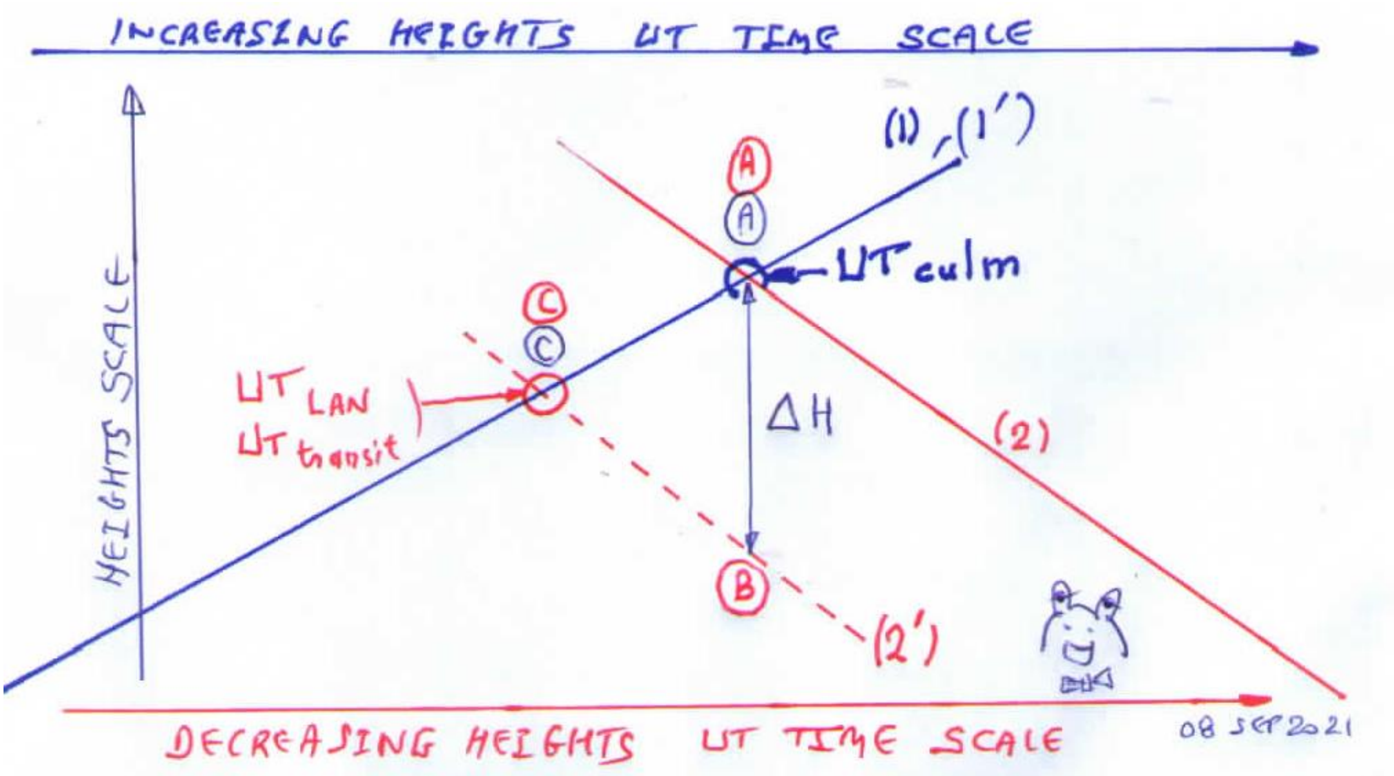
Taking the ***H culm*** average values we obtain: ***H tran = 73°17.25'***, which altogether with the Sun Ephemeris in **5.1.3** yields the following LAN position:

UT tran = 16h37m29.2s at fix N37°11.2'/ W 070°11.8' *(vs. N37°11.3' / W070°12.2' in **5.2**)*

Here again, we keep getting excellent results very close from our benchmark results.

5.7.2 - IMPROVED WILSON'S METHOD (See 4.3.2)

Starting with ***UT culm = 16h37m56.0 s*** let us take a close look at the **Wilson's graph** here-under.



Line (1) represents the **Real World increasing Heights**. On its own **UT scale** it intersects **(2)** in Ⓐ at **T1=16h20m56s**.

Line (2) represents the **Real World decreasing Heights**. On its own **UT scale** it intersects **(1)** in Ⓐ at **T2=16h54m26s**.

Their intersection defines one same Height (*whichever floats one's boat*).

The Time elapsed between such Real World Heights is therefore equal to **T2 - T1 = 34m**

Reasoning from **Section 4.0.1**:

Let's assume that at Time T1 both the Fictitious Navigator and the Fictitious Sun start coming to existence.

Then by Section **4.0.2** - During some reasonable time span around **T1** the slope of the **Fictitious increasing Heights Line (1')** can be regarded as the same as the slope of the **Real World increasing Heights Line (1**). As they both share the same point Ⓐ/Ⓐ at **T1** then we can then safely assume that **Line (1')** and **Line (1)** are identical.

From **Section 4.0.1** again, between **T1** and **T2** and solely because of Observer's and Sun ***coupled N/S motion*** the **Real World decreasing Heights Line(2)** is ending up "higher" at Time **T2** than the **Fictitious decreasing Heights Line(2')** .

It shows "higher" by an amount equal to **(µDec - NS) * (T2 – T1) = 4.4710 * 34/60** = ΔH = 2.5 NM.

Since it has not been subject to this coupled N/S motion, at **T2** the **Fictitious decreasing Heights Line (2'**) crosses [almost] exactly point Ⓑ at 2.5 NM south of point Ⓐ.

Back to **Section 4.0.2** - During some reasonable time span around **T2**, the slope of the **Fictitious decreasing Heights Line (2'**) can be regarded as almost the same as the slope of **Real World decreasing Heights Line (2).**

Since the **Fictitious decreasing Heights Line (2'**) crosses point Ⓑ it can be represented by a line parallel to **Real World decreasing Heights Line (2)**.

Reverting to **Section 4.0.1** again, the intersection of the **Fictitious increasing Heights Line (1')** and of the **Fictitious decreasing Heights Line (2')** is at point Ⓒ/Ⓒ a point at which the average value of the Times read on each of both Time scales is equal to ***UT tran***.

<u>NOTE</u> : Such intersection can be drawn in just a few seconds, much less that the time required to read these lines.

From Tony's Oz graph which is meticulously drawn, we then derive:

UT tran = 16h37m30s *(vs. 16h37m30,7s in **5.2**)*

Algebraic result (With UT culm = 16h37m56.6s, see 5.7.1): UT culm - UT tran = 26,54s . Hence UT tran = 16h37m30.1s

Once more, we have got a UT tran determination very close from our benchmark results.

ACKNOWLEDGMENTS AND *FINAL CONCLUSION*

Thanks to ***Frank E. Reed*** for having brought back onto the table this subject of the various LAN solving methods.

Thanks to you ***Greg Rudzinski*** for having so kindly directed my attention towards **Ref 3**.

And very special thanks to you too … ***Tony Oz*** for having unveiled to my bewildered and amazed eyes this Russian Navigator's stunning method of ***5.7.2*** !

Could you not fall in love with such beautiful method ???

You have made my day, Tony.

REFERENCES

*(1) - **French Naval Academy "CIRCUMZENITHALES CORRESPONDANTES"** (1934). Similar course published by the **Admiralty** at same epoch.*

*(2) - **CALCULATOR NAVIGATION** by **Mortimer Rogoff*** by NORTON Publishers ISBN 0-393-03192-6

*(3) - **POSITION FROM OBSERVATION OF A SINGLE BODY*** by ***JAMES N. WILSON*** Journal of the Institute of Navigation V.32 N.1, Spring 1985

*(4) - **ARTICLE by Frank E. Reed*** http://fer3.com/arc/m2.aspx/Latitude-Longitude-Noon-Sun-FrankReed-jun-2005-w24178 *in* ***NavList Forum*** ***http://fer3.com/arc/*** <u>Note</u>*: This Article describes F.E. Reed's full "**Folded Paper Method**" which advocates pre-treating data as described in **Section 4.0** . A simpler version without pre-treatment is described and demonstrated here with excellent end-results believed to be identical.*

Au Roc Saint Luc en Pissotte, le 08 Septembre 2021 - Antoine M. "Kermit" Couëtte

APPENDIX 1

Source: Position from Observation of a single Body by JAMES N. WILSON
Journal of the Institute of Navigation V32 N1 , Spring 1985

Determination of the quantity $K = (48 / \pi) * (\tan Lat - \tan Dec)$ - See **Formula 1a** in **Section 3.1-a**.

Example: with LAT = N 37.2 ° and Dec N 20.5° get K = 5.9 (*exact value 5.885*)

Hint: For the very infrequent use of **Formula (1b)** in **Section 3.1-a**: reverse the signs rule her-under.