

1 - OBSERVATIONS AROUND MERIDIAN TRANSIT OF A CELESTIAL BODY

Over a short time span - typically less than 1 hour - "**LAN**" (*Local Apparent Noon*) **observations** also called "**Near Noon**" **observations** can provide the Celestial Navigator with a quite reliable position.

Hence these "**LAN**" observations have long been studied and there is a quite important literature to cover them.

Practical steps - The Navigator first starts recording his sights: both UT's and Heights during a time span around the estimated time a celestial Body "crosses" his own meridian. The Navigator then processes his observations to derive his position.

Since most generally the Sun is the aimed Body hence these "**LAN**" or "**Near Noon**" acronyms.

99.99 % of the time **Upper Culminations** only are observed: heights first rise, then stay constant and eventually decrease.

2 - KEY POINTS TO REMEMBER

(2.1) - Plotting Heights vs. UT times on a graph yields a "**parabola type**" series of points open towards the bottom of the graph with a symmetry axis parallel to the Heights scale.

(2.2) - Within the limits of quality of the observations for both a **Fixed Latitude Observer** and a **constant Declination Body** such "**parabola type**" is exactly symmetrical relatively to the **Time of Body Transit (UT tran)**, i.e. Body crossing the Observer's meridian. Apparent Culmination **H culm** occurs exactly at such **UT tran**. Hence from Body GHA and Declination at **UT tran** the Navigator very easily gets his own Latitude and Longitude.

(2.3) - Within the same observations quality limits, for a **moving Observer** and/or a **changing Declination Body**, Culmination most often does not coincide with Body Transit. **UT culm** becomes different from **UT tran**. Nonetheless the symmetry axis of such [new] "observation curve" remains quite parallel to the Heights scale.

For all practical purposes this "new curve" can be considered as simply "shifted" from the previous one depicted in **(2.2)** through a **most often negligible N/S translation** and through an **often quite sizeable E/W translation**.

(2.4) - Whichever solution is subsequently carried out, all solving methods require to accurately pinpoint - both for Time and Height - the top/summit of such actual "observation curve", i.e. both UT culm and H culm.

(2.5) - From the available data - i.e. **UT culm** and **H culm** - the motions effects earlier mentioned in **(2.3)** can be taken in account and corrected for, in order to recover both **UT tran** and **H tran**. The Navigator is back to an optimum configuration to derive his own LatLon as per **(2.2)** here-above.

Such classical corrections formulae are listed **SECTION 3** here-after.

(2.6) - And finally to close this important list - which otherwise could easily be extended almost *ad infinitum* - the random errors standard deviation (SDEV) comes to play as follows in the final determination of the "**LAN**" Position:

- SDEV directly **degrades** Latitude accuracy by a ratio very close from 1 to 1. And:
- SDEV directly **degrades** Longitude accuracy by a much higher ratio: up to 5 to 1 or even 8 or 10 to 1.
- The "lower" the **H culm**, the bigger the Longitude accuracy degradation for one given SDEV.

3 - CLASSICAL CORRECTIONS FROM CULMINATION TO TRANSIT

With the following abbreviations:

- **Lat** : Observer's Latitude
- **Dec** : Body Declination
- **NS** : Observer's North Speed in knots (negative for South speeds). And:
- **μDec** : Body Declination Hourly change in Arc Minutes (algebraic values too) :

(3.1-a) - To correct from (UT culm) into (UT tran) for **UPPER CULMINATIONS** :

$$(UT\ culm - UT\ tran)\ in\ seconds\ of\ time = (48 / \pi) * (\tan\ Lat - \tan\ Dec) * (\mu Dec - NS) - (1a)$$

(3.1-b) - To correct from (UT culm) into (UT tran) for **Circumpolar Bodies LOWER CULMINATIONS** :

$$(UT\ culm - UT\ tran)\ in\ seconds\ of\ time = (48 / \pi) * (\tan\ Lat + \tan\ Dec) * (\mu Dec - NS) - (1b)$$

(3.2) - Finally to correct from H culm into H tran :

$$(H\ culm - H\ tran)\ in\ arc\ Minutes = \frac{1}{2} (\mu Dec - NS) * (UT\ culm - UT\ tran)\ in\ hours - (2a)$$

On a practical stand-point and in almost all literature (see 3.3.5 here-under):

$$H\ culmination = H\ transit - (2b)$$

(3.3) - Notes

(3.3.1) - Formula (1a) can be regarded as the **FUNDAMENTAL FORMULA** for all classical NAL solving methods.

Formulae (1a) and (1b) are established for the Sun with a GHA rate of 15°/hour. Even so they are **1st order formulae** only with respect to Time. For this reason they can be in error by up to 15% in extreme cases even for the Sun. They can also be used for the Planets and the Stars without significant extra fine tuning required.

(3.3.2) - For the Moon (Ref 3) recommends increasing the results of (1a) by 7%. It seems a reasonable starting point.

(3.3.3) - Formulae (1a) and (1b) explicitly assume the **North Speed to stay constant over the full time of the Observations, a constraint governing all types of solutions methods.**

(3.3.4) - Formula (1b) has been very rarely if ever published because the opportunities for such Observations are quite infrequent (e.g. circumpolar bright Stars during sufficiently long twilights).

(3.3.5) - Formula (2a) is a **2nd order formula** with respect to Time and as such it yields results accurate to always better than 0.01'.

However its corrective terms are so small - always inferior to 0.3' - that it has been very rarely if ever published too.

It nonetheless remains the mathematical justification for formula (2b) here-above.

4 - METHODS TO DERIVE UT culm AND H culm

Accurately pinpointing both **UT culm** and **H culm** remain **essential steps** in the course of the LAN fix process.

A quick "historical review" here-after about some successive significant methods shows their advantages and drawbacks.

In **SECTION 5**, one specific example is treated with these methods for subsequent comparisons between them.

4.0 - AN OPTIONAL EARLY DATA PROCESSING STEP: *THE FICTITIOUS NAVIGATOR AND CELESTIAL BODY METHOD*.

4.0.1 - Description of the Fictitious Navigator and Fictitious Celestial Body method.

4.0.1.1 - Let us imagine that at some specific moment during the observations process, the Real World Navigator is “overlapped” by an extra Fictitious Navigator who is to keep exactly the same E/W speed, but has no N/S Speed. While they are to “observe” at the same Times, both Navigators will stay at exactly the same Longitudes.

At this very same time, let’s assume that the Real World Body is also overlapped by an extra Fictitious constant Declination Body with which keeps exactly the same GHA’s as the Real World Body.

Both the Fictitious Navigator and the Fictitious Body fulfill the conditions of **Section (2.2)** here-above. Therefore **for the Fictitious Navigator who keeps “observing” his own Fictitious Celestial Body Heights: $UT\ culm = UT\ tran$.**

At Time (**T1**) of the first observation, the Real World Height (**H1**) and the Fictitious Body Height (**H’1**) are exactly the same. From that instant they start gradually diverging from one another.

4.0.1.2 - For the entire time-span of Observations and if we make the assumption - which is a quite valid one - that the Cosines of the Body Azimuths always remain *almost exactly* equal to *Unity* then both **the Real World Heights and the Fictitious Heights keep diverging from one another at a rate almost exactly equal to $(\mu Dec - NS)$.**

Hence, from the *Real Word Heights set* it is possible to derive a *Fictitious Heights set* showing a “curve” expected to be exactly symmetrical with respect to its own *UT culm*.

4.0.1.3 - With this *FICTITIOUS NAVIGATOR AND FICTITIOUS CELESTIAL BODY METHOD* which is no more than a handy optional data pre-treatment step, the LAN is solved as follows:

$UT\ tran = UT\ culm$ of the Fictitious Heights set, and $H\ tran = H\ culm$ of the Real World Heights set.

4.0.2 - This optional Method looks quite attractive for all graph paper methods as it frees the Navigator from recourse to **Formula (1a)** since **it immediately yields $UT\ tran$!**

Nonetheless, and especially when dealing with a high number of observations (e.g. **Ref 3**), it requires a meticulous tally of the ever changing corrections to the individual Heights. This weakness can be regarded as a *rather subtle and unnecessary complication which may easily and entirely ruin the claimed simplicity of such graph paper methods.*

4.1 - EQUAL HEIGHTS METHOD (Ref 1)

By the 1930’s both the French Navy (**Ref 1**) and the UK Royal Navy (**Ref 2**) among others published the first “LAN type” method which advocates recording **equal heights sufficiently before and after culmination, and to monitor and record maximum height.**

Formulae (1a) and (3) were then altogether published in order to solve for such LAN fixes.

$UT\ culm$ is taken as the average of all corresponding equal heights times, and

$H\ culm$ is taken as equal to the maximum height recorded.

4.1.1 - Advantages

4.1.1.1 - A definite simplicity of use.

4.1.1.2 - Can be used under degraded material conditions (no electricity ...). Hence it can be considered as an Emergency method to-day.

4.1.2. - Drawbacks

The observation time-spans are subject to strict constraints addressed in dedicated literature, e.g. :

4.1.2.1 - Given random observation errors, any efficient *UT culm* recovery Method requires some of the Real World Heights to be observed **sufficiently far away from *UT culm***, i.e. with **sufficient increasing / decreasing rates, typically 3°/hour or more**. Otherwise, if too close from culmination, or if the [Real World] *H culm's* are too low - typically under @ 50°/60° for most methods - **insufficient rates** eventually translate into **unacceptable geometric dilution of position (GDOP)**. Hence the Longitude errors mentioned in **(2.6)**.

4.1.2.2 - **On the contrary**, such equal altitudes must be taken **sufficiently close from *UT culm*** so that the "Observation curve" **odd powers** in Time do not start becoming significant. In other words such "curve" must always be considered as acceptably symmetrical around *UT culm*.

4.1.2.3 - **Especially with the graph paper methods, the Navigator needs to process data sufficiently symmetrical (see 4.4.2.2.2). Accordingly the higher the *H culm* the closest all observations must all gather around *UT culm*. This constraint also governs - to variable extents - all kinds of solution methods.**

4.1.2.4 - As an adverse consequence there can be configurations when only a **reduced observation window** is available before and after *UT culm* during which the Navigator is to take a minimum of 3 rising/decreasing heights in order to average at least 3 pairs of observation times.

4.1.2.5 - Within the hard constraints of **4.1.2.3** such equal Heights should - ideally - be equally spaced between them in order to best minimize the effects of random observation errors.

4.1.2.6 - All constraints here-above require more or less adequate uninterrupted observations conditions.

4.2 - EARLY CALCULATOR STATISTICAL METHODS (Ref 2)

Taking advantage of the recently introduced hand held Calculators, and **by the late 1970's** some Pioneers (e.g. **Ref 2**) quickly came out with statistical programs permitting adequate treatments of LAN's.

4.2.1 - Advantages

4.2.1.1 - **Statistical methods offer a definite ACCURACY ADVANTAGE over graph paper methods.**

This is definitely the main reason why they have become quite successful and popular.

4.2.1.2 - Such Statistical methods greatly alleviate some of the graph paper methods constraints here-above.

Constraints 4.1.2.1 and 4.1.2.2 become irrelevant. Any [not too large] unexpected gap in the observations schedule should bring no adverse consequence as long as at least one Height close to *H culm* can be recorded. If sufficient observations are made anytime within the time span permitted by **constraint 4.1.2.3** then the Navigator can get a quite decent LAN fix.

4.2.1.3 -The statistical Methods offer some simplicity of use still requiring attention in the data loading sequences (caveats in **4.2.2.1** and **4.2.2.2**)

4.2.2 - Drawbacks

4.2.2.1 - Lot of attention required to key data in. About 15 keystrokes are required per one single observation, not counting environmental data. Repeatedly and carefully checking for typing errors before entering data may become utterly painstaking. This step must be performed since *garbage in, garbage out*.

4.2.2.2 - It was often difficult if not impossible to correct for typing errors when data already entered. A solid example to run (e.g. **Ref 3**) can require up to over 400 [uninterrupted] individual keystrokes.

4.2.2.3 - For lack of space a number of early software versions simply used **Formula (1a)** itself which prevented them from adequately processing Moon observations. Re: Note **(3.3.2)** .

4.2.2.4 - There is no built-in real safeguard against using LAN Software outside the strict limitations of **constraint 4.1.2.3**.

4.2.2.5 - **Relying too much onto 100% computations may lead to oversights or even blunders otherwise quite visible with graph paper methods. Lack of situation awareness may quickly bring Safety concerns.**

4.2.2.6 - Cannot be considered as an Emergency use method to rely on.

4.3 - JAMES N. WILSON'S METHOD (Ref 3) and its further "Russian" IMPROVEMENT

4.3.1 - STANDARD J.N. WILSON'S METHOD (Spring 1985)

By the mid-1980's Calculators were taking advantage over the graphical methods. Any new such method yet to be invented would be worth of consideration *ONLY IF* it could significantly improve the *UT culm* determination process. And, *lo and behold, in 1985* James N. Wilson came up with one such graphical method to determine *UT culm*.

Within the limitations of **4.1.2.1** and **4.1.2.2** **Wilson's Method** requires 3 sets of observations:

- Rising heights,
- Culminating height(s), and
- Decreasing heights

The culminating heights yield *H culm* through visual interpolation on a graph.

The *rising Heights* are drawn on a colored graph and then visually approximated by a *straight colored line*.

On that same graph, and using an extra and overlapping time scale the *decreasing Heights* are drawn *in different color* and visually approximated too by a *straight line* so that both the rising line and the descending line intersect one another about halfway on the graph. They both cut at a given **Intersection Height**.

The Sun Height reaches this same **Intersection Height** value - *whichever it may be* - at a **Time T1** on the *ascending Heights branch* and at a **Time T2** on the *descending Heights branch*. Like for the previous **Equal Heights Method** (in **Section 4.1**): $UT\ culm = \frac{1}{2} * (T1 + T2)$. This Method shows a very clever use of overlapping Offset Time scales.

A nicely devised Abacus (**Appendix 1**) enables to visually derive the value of $K = (48 / \pi) * (\tan Lat -/+ \tan Dec)$.

With *K* and **Formula (1a/1b)** the Navigator computes (*UT tran - UT culm*). With **Formula (3)** he computes *H tran*.

4.3.2 - IMPROVED WILSON'S METHOD

And - cherry on the cake - a Russian Navigator subsequently came out with a very **clever improvement to Wilson's Method without even having recourse to the Classical formula (1a)**.

4.3.2.1 - The result of **Section 4.0.1.2** - i.e. *the Real World and Fictitious Heights are diverging from one another at a rate equal to ($\mu Dec - NS$)* - is the *1st* key used in solving this **Improved Wilson's Method** which does not show the weakness mentioned in **Section 4.0.2** .

4.3.2.2 - The *2nd* key to this **Improved Wilson's Method** deals with the rates differences between the Real World Heights and the Fictitious Heights. If we assume - which we may very well do - that **during reasonably short periods**, e.g. up to a few minutes of time, we can safely ignore the effect of ($\mu Dec - NS$) onto the Navigator's position, then we may consider that **the slopes of the Real World Heights tangents are almost identical to the slopes of the Fictitious Heights tangents** (See **Example 5.7.2** here-after).

4.4 - ENHANCED CALCULATORS / COMPUTERS STATISTICAL METHODS

In the past decades, significant improvements have been made onto Navigation Computers / Calculators. Both the hardware and the software of LAN programs have been significantly improved and are more User friendly.

Reliable and accurate **Long Term Almanacs** are now routinely part of the proposed software and are sourced onto the very best recent developments by mainly the **French Bureau des Longitudes** (*VSOP xxxx* , *ELP-xxxxx* Solar/Planetary and Lunar analytical theories) and the **JPL** from the USA (*DExxx* numerical integrations).

4.4.1 - Advantages and drawbacks

We could now state that most of the drawbacks identified in **4.2.2.1 to 4.2.2.4** have now vanished. And for these very good reasons using modern software to solve LAN's has become quite popular among the Community.

As for their drawbacks, we should still consider that the caveats addressed in **4.2.2.5 and 4.2.2.6** do remain relevant.

4.4.2 - Software Contributors

Among the many software Publishers a special mention is given to two highly respected Contributors to NavList who have made their own Navigation Software - including their LAN Software - available to the Community:

- **Andres Ruiz González from Kingdom of Spain**
 - o <https://sites.google.com/site/navigationalgorithms/about>
 - o navigationalgorithms@gmail.com . And:
- **Peter Hakel from the USA**
 - o <http://www.navigation-spreadsheets.com/>

Their LAN Software belongs to this Enhanced Software category with their many useful refinements such as nice displays of the observations set. Such enhancements start bringing back the **Navigator's situation awareness** much closer to the real world. Hence both Software greatly avoid the trap mentioned in **4.2.2.5** .

Their LAN Software is believed to use a refined regression Parabola (2nd order to Time) in which the exact Body GHA rates are accurately computed, which makes them fully valid for any celestial Body, including the Moon.

4.4.2 - Higher order Regressions with respect to Time (*June 1989*)

4.4.2.1 - One way to alleviate **constraint 4.1.2.3** is to implement Higher order Regressions with respect to Time in order to broaden the "time-window" during which observations can validly be made around **UT culm**. Such methods can also **easily accommodate lower H culm's** - as low as 20° - vs. a minimum of 50° or even 60° generally accepted for many LAN methods.

4.4.2.2 - However, even if assuming that all changing elements (Observer's Latitude and Longitude, Body GHA and Declination) vary strictly linearly with respect to Time - *i.e. all second order derivatives strictly equal to zero* - **High order Regressions Methods are to face 3 adverse environmental cumulated constraints:**

4.4.2.2.1 - With respect to the Time powers the **number of terms of the relevant differential functions grows faster than the number of terms of the Pascal's Triangle**. Quickly the task becomes quite cumbersome.

4.4.2.2.2 - The **even order terms** are much more significant than the **odd order terms**. **Note:** *this makes sense since we are dealing with rather symmetrical curves (see **constraint 4.1.2.3**)*. As a consequence **any significant improvement** to a given level statistical Regression **requires differentiating 2 steps further**.

4.4.2.2.3 - Successive higher order Regressions widen the "time-window" **by smaller and smaller increments**.

In spite of such cumulative obstacles it is believed that a 6th order Regression with respect to Time is a reasonable trade-off since it doubles the size of the valid time-window of a 2nd order regression.

4.5 - FOLDED PAPER SHEET METHOD (Ref 4)

Back to graphic determinations: by the late 20th Century and given the great improvements brought by the **Wilson's Method (Section 4.3)** one could regard as definitely impossible to significantly simplify the **UT culm** determination process through any innovative and new 100% graphical method.

Nonetheless **Frank E. Reed** founder and moderator of the NavList Forum <http://fer3.com/arc/> invented such clear cut improvement with his "**Folded semi-transparent sheet of paper method**" and deserves full credit for it.

In its simplest version (not using Option 4.0), use a sheet of semi-transparent paper to carefully graph **Heights vs. UT times** for all observations. Manually draw a curve to smoothly approximate as many observations points as possible.

Gently start folding the graph alongside a parallel to the heights scale and hold its two "half pages" against ambient light. Look through the 2 sheets for both "branches" of the curve. Do not hard fold this graph yet but carefully fine tune the 2 branches positions so that they overlap one another as much as possible. Then hard fold your sheet of paper exactly alongside one parallel to the heights scale in order to keep this best superimposition visible. This hard folded axis gives you **UT culm**. And the corresponding height gives you **H culm**.

Wilson's Abacus (Appendix 1) and **Formula (1a)** then enable to solve for **UT tran**. **Formula (2b)** solves for **H tran**.

4.5.1 - Advantages and drawbacks

Among all the graph methods earlier mentioned the "Folded semi-transparent sheet of paper method" if and when used without option (4.0) ranks out as the best one in terms of simplicity, ease of use and reliability.

It is only subject - to a lesser extent than all other methods - to the limits of **constraint 4.1.2.3**.

Other than that, and contrarily to the **Equal Altitudes** and to **Wilson's** methods, it can accommodate in particular:

- Reasonable gaps in the observation series. Nonetheless at least one observation near **H culm** still required.
- And it can also easily accommodate unequally spaced Observations with respect to Time.

It also fully qualifies as a Back-up / Emergency method.

5 - ONE EXAMPLE FOR COMPARING THE PREVIOUS METHODS

Let us review the following example submitted by **Frank E. Reed** to NavList.

<http://fer3.com/arc/m2.aspx/Compare-Methods-LatLon-Near-Noon-FrankReed-aug-2021-q51028>

Date: May 22, 2022

Observer: Sailing 4 kts at True heading 185° on surface. **Current** : 2 kts to true East.

SUN sights, UT / uncorrected Altitudes:

1 - 16:17:20 / 72°28.5' **2** - 16:22:40 / 72°44.5' **3** - 16:36:45 / 73°04.0' **4** - 16:39:15 / 73°04.5' **5** - 16:51:25 / 72°49.0' **6** - 16:56:20 / 72°35.5'

All Altitudes corrections forced to +13.0'

5.1 - PRELIMINARY RESULTS

5.1.1 - This exercise permits crosschecking various LAN Software in a thorough and meaningful way.

- Since all altitude corrections are imposed, all users are processing exactly the same set of observations.

- Hence any difference on end results can be attributed to only:
 - Differences in the Ephemeris used. And/or :
 - Differences in the number of significant digits published. And/or :
 - Differences in the processing software.

5.1.2 - Whenever applicable **extra-digits** are published here-under to permit more in-depth comparisons.

5.1.3 - **Method 5.2** here-under uses the following Ephemeris for the SUN (**VSOP 2013**).

SUN	GHA	DEC	Distance, SD and μ Dec.
16:00 UT	60°49.56'	N + 20°28.18'	Distance : 1.012305 UA,
17:00 UT	75°49.51'	N + 20°28.66'	SD : 15.80' μ Dec : + 0.4862 '/h

5.1.4 - **Speed Made Good**: North Speed (**NS**) -3.9848 kt , Speed to the East : +1.6514 , i.e. **4.3134 kt/157.4898 °**

5.1.5 - (μ Dec - NS) = + 4.4710 '/h

5.1.6 - From the starting data, the following **approximate values** are readily derived: **Lat = N 37.2°** and **Dec = N 20.5°**

5.1.7 - From 5.1.6 and **Wilson's Abacus (Appendix 1)**, **K = 5.9**

5.1.8 - From 5.1.5 and 5.1.7: (**UT culm - UT tran**) = + 26.4 s (**Reminder: Wilson's Abacus** used here as per 5.1.7)

5.1.9 - From 5.1.5, 5.1.8 and **Formula (2)**, (**H culm - H tran**) = 0.0164'. As expected in 3.3.5 this is a totally negligible difference.

5.2 - RESULTS OF 6th ORDER LAN SOFTWARE (see 4.4.2)

UT tran = 16h37m30.7s and LAN fix at N37°11.3' / W070°12.2'

UT culm - UT tran = + 26.15 s (vs. +26.4s at 5.1.8)

H culm - H tran = 0.0164' (vs. 0.0164' at 5.1.9)

SDEV = 0.23 NM (0.8' on Lon)

Method Validity time span (16h11m - 17h04m : OK)

Many Body Fix Method : At the LAN time get a Many Body Fix at N37°11.3' / W070°12.2'.

This Many Body Fix is only 34 meters - i.e. less than 120 ft - from LAN fix and get SDEV = 0.22 NM.

17h Fix at N37°09.8' W070°11.4'

Note : The 6th order LAN method is expected to be the most accurate of all LAN methods addressed here.

- In particular the **LAN Fix** and the **Many Body Fix** are almost identical (less than 0.022 NM apart).
- The **SDEV's** in both cases are almost identical too: 0.22' vs. 0.23'.
- These comparisons constitute a meaningful and independent check of this LAN method.

5.3 - RESULT OF THE EQUAL ALTITUDES METHOD (see 4.1)

Due to its "Equal Heights" constraint this Method cannot support the current example.

5.4 - RESULTS PUBLISHED BY PETER HAKEL (See 4.4.2)

Peter Hakel readily published his own results here and as follows:

<http://fer3.com/arc/m2.aspx/Compare-Methods-LatLon-Near-Noon-PeterHakel-aug-2021-q51029>

UT tran = 16h37m29s and LAN fix at N37°10' / W070°11.4'

UT culm - UT tran = not published

H culm - H tran = not published

Many Body Fix Method : At the LAN time get a Many Body Fix at N37°11' / W070°12'.

17h Fix at N37°10' W070°12'

These results are very close from the ones obtained in 5.2.

5.5 - RESULTS OBTAINED BY THE FOLDED PAPER SHEET METHOD (See 4.5)

These results, using the **simplest version of this method**, (*not using Option 4.0*), were published here and as follows:

<http://fer3.com/arc/m2.aspx/Compare-Methods-LatLon-Near-Noon-Couette-sep-2021-q51044>

UT culm = 16h38m00s

From Formula 1a : UT culm - UT tran = +26.3 s (vs. +26.4s at 5.1.8 and +26.15s at 5.2)

H culm - H tran = not published

UT tran = 16h37m34s and LAN fix at N37°11.0' / W070°11.0'

17h Fix at N37°09.5' / W070°10.9'

The Folder Paper sheet method works very well too. Results within half of mile from 5.2 .

5.7 - RESULTS OBTAINED BY THE WILSON'S METHOD (See 4.3)

5.7.1 - STANDARD WILSON'S METHOD (See 4.3.1)

This example carries the minimum number of Observations to permit using **Wilson's Method** since Observations (1,2) and (5,6) are just sufficient in number to permit defining the increasing and decreasing Heights straight lines.

Note : Incidentally since these both Height lines are simply passing through 2 well defined points for each, it is even possible to give results just from algebraic computations, but this is not the name of the game here.

The first result of this method was first published by Tony Oz here:

<http://fer3.com/arc/m2.aspx/Compare-Methods-LatLon-Near-Noon-TonyOz-sep-2021-q51042>

On his **Wilson's paper graph** Tony Oz carefully derived *UT culm* as the average of 16h20m56s and 16h54m56s to get:

UT culm = 16h37m56.0 s

Algebraic result (see Note just above): UT culm = 16h37m56.6s

From this result by Tony Oz and with the **Wilson's Abacus (Appendix 1)** we get:

UT culm - UT tran = + 26.4 s and UT tran = 16h37m29.6s (vs. 16h37m30,7s in 5.2)

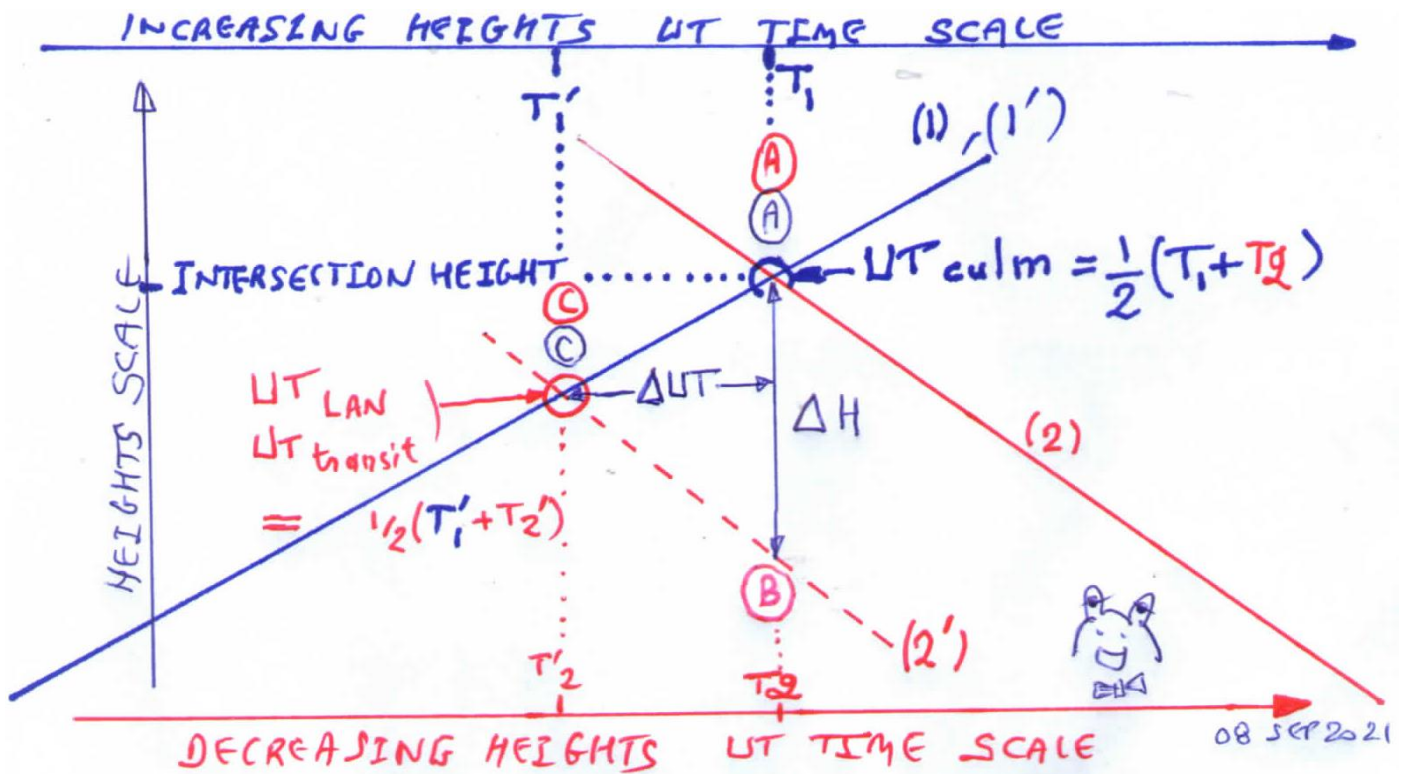
Taking the *H culm* average values we obtain: *H tran = 73°17.25'*, which altogether with the Sun Ephemeris in **5.1.3** yields the following LAN position:

UT tran = 16h37m29.2s at fix N37°11.2' / W 070°11.8' (vs. N37°11.3' / W070°12.2' in 5.2)

Here again, we keep getting excellent results very close from our benchmark results.

5.7.2 - IMPROVED WILSON'S METHOD (See 4.3.2)

Starting with *UT culm = 16h37m56.0 s* obtained just here-above let us take a close look at the **Wilson's graph**:



Line (1) represents the **Real World increasing Heights**. On its own **UT scale** it intersects **(2)** in **(A)** at **T1=16h20m56s**.

Line (2) represents the **Real World decreasing Heights**. On its own **UT scale** it intersects **(1)** in **(A)** at **T2=16h54m56s**.

These **identical points (A)/(A)** define **one same intersection Height** (*whichever it be*).

We have seen earlier in Section 4.3.1 that **$UT\ culm = \frac{1}{2} * (T1 + T2)$**

The Time elapsed between the moments when an **increasing Height** and a **decreasing Height** pass through this same **intersection Height value** (*again: of whichever value floats your boat*) is therefore equal to **T2 - T1 = 34 m**

From Section 4.0.1: **let's assume that at Time T1 both the Fictitious Navigator and Sun start coming to existence.**

Then by Section 4.3.2.2 - **During some reasonable time span around T1** the slope of the **Fictitious increasing Heights Line (1')** can be regarded as [almost] the same as the slope of the **Real World increasing Heights Line (1)**. Since at **T1** they both go through the same point **(A)** then we can then safely assume that **Line (1')** and **Line (1)** are identical.

From Section 4.3.2.1, between **T1** and **T2** and due to the Observer's and Sun **coupled N/S motion** the **Real World decreasing Heights Line (2)** is ending up "higher" at Time **T2** than the **Fictitious decreasing Heights Line(2')**.

It shows "higher" by an amount equal to **$\Delta H = (\mu Dec - NS) * (T2 - T1) = 4.4710 * 34/60 = 2.5\text{ NM}$** .

Since it has not been subject to this Observer's and Sun **coupled N/S motion**, at **T2** the **Fictitious decreasing Heights Line (2')** crosses [almost] exactly point **(B)** at 2.5 NM south of point **(A)**.

Back to Section 4.3.2.2 - **During some reasonable time span around T2**, the slope of the **Fictitious decreasing Heights Line (2')** can be regarded as [almost] the same as the slope of the **Real World decreasing Heights Line (2)**.

Therefore the **Fictitious decreasing Heights Line (2')** can be represented by a line parallel to the **Real World decreasing Heights Line (2)** and crossing point **(B)**.

Reverting to **Section 4.0.1.1**, the **Fictitious increasing Heights Line (1')** intersects the **Fictitious decreasing Heights Line (2')** at the **identical point ©/©** at times **T'1** on **Line (1')** and **T'2** on **Line (2')** with their mean values at **UT tran**. Our final result is therefore:

$$UT\ tran = \frac{1}{2} * (T'1 + T'2)$$

NOTE: Such intersection is drawn in just a few seconds, much less than the time required for reading these lines.

From Tony's Oz graph which is meticulously drawn, we get $\Delta UT = T1 - T'1 = +26.0s$, and then obtain :

$$UT\ tran = 16h37m30s \text{ (vs. } 16h37m30,7s \text{ in } 5.2)$$

Algebraic result (With UT culm = 16h37m56.6s, see 5.7.1): UT culm - UT tran = 26,54s . Hence UT tran = 16h37m30.1s

Once more, we have got a UT tran determination very close from our benchmark results.

5.7.2.1 - CHECKS AND REMARKS

5.7.2.1.1 - The **Cosines of the Sun Azimuths** always stay between 0.96 and 1 with an average value of 0.98. Hence **assumption (4.0.1.2)** is quite valid.

5.7.2.1.2 - Let us estimate the **consequences of neglecting (μ Dec - NS) over 26 s**, i.e. **0.03 NM** (Re: **Section 4.0.1.2**):

* **Point ©** is obtained on **descending Line (2') backwards** from **©** and it **needs a correction of 0.03 NM to the North**. **Line (2')** has a "**steeper descending slope**" than **Line (2)** in which the Real World Navigator is going southwards to the Sun which in turn is going northwards to him.

* Likewise **Point ©** is obtained on **ascending Line (1') backwards** from **©** and it also **needs the same correction of 0.03 NM to the North**. **Line (1')** has a "**less steep ascending slope**" than **Line (1)** in which, again, the Real World Navigator is going southwards to the Sun which in turn is going northwards to him.

So the new intersection of both "**corrected Line (1')**" and "**corrected Line (2')**" remains unchanged Time-wise.

Finally, neglecting (μ Dec - NS) brings no appreciable change to the Navigator's Longitude.

ACKNOWLEDGMENTS AND **FINAL CONCLUSION**

Thanks to **Frank E. Reed** for having brought back onto the table this subject of the various LAN solving methods.

Thanks to you **Greg Rudzinski** for having so kindly directed my attention towards **Ref 3**.

And very special thanks to you too ... **Tony Oz** for having unveiled to my bewildered and amazed eyes this Russian Navigator's stunning method of **5.7.2** !

Could you not fall in love with such a beautiful method ???

You have made my day, Tony.

REFERENCES

(1) - **HAUTEURS CIRCUMZÉNITHALES CORRESPONDANTES**, published by the **French Naval Academy (1934)**

(2) - **ADMIRALTY NAVIGATION MANUAL** (near page 152), by the **HM Stationary Office Vol III (1938)**

(3) - **CALCULATOR NAVIGATION** by **Mortimer Rogoff**, printed by **NORTON Publishers ISBN 0-393-03192-6 (May 1979)**

(4) - **POSITION FROM OBSERVATION OF A SINGLE BODY**, by **James N. Wilson** in **Journal of the Institute of Navigation V.32 N.1 (Spring 1985)**

(5) - **NAVLIST FORUM ARTICLE** by **Frank E. Reed** <http://fer3.com/arc/m2.aspx/Latitude-Longitude-Noon-Sun-FrankReed-jun-2005-w24178> at <http://fer3.com/arc/>. **Note:** This Article describes **F.E. Reed's full "Folded Paper Method"** which advocates pre-treating data as described in **Section 4.0**. A simpler version without pre-treatment is described and demonstrated here with excellent end-results believed to be identical.

Au Roc Saint Luc en Pissotte, le 08 Septembre 2021 - Antoine M. "Kermit" Couëtte

APPENDIX 1

Source: *POSITION FROM OBSERVATION OF A SINGLE BODY* by James N. Wilson
Journal of the Institute of Navigation V.32 N.1 (Spring 1985)

Determination of the quantity $K = (48 / \pi) * (\tan Lat - \tan Dec)$ - See Formula 1a in Section 3.1-a.

Example: with LAT = N 37.2° and Dec N 20.5° get K = 5.9 (exact value 5.885)

Hint: For the very infrequent use of Formula (1b) in Section 3.1-a: reverse the signs rule here-under.

