Influence of tilt angles onto Moon refracted Diameters

By Antoine M. Couëtte Jan. 17th, 2022

Frank E. Reed published a quiz here about a "Lunar" GMT determination from a Moon Picture (© Fred Espenak). From this plate angular distances can be estimated ***(1)*** : through a "*scaling coefficient*" from its only measurable Moon Diameter, or ***(2)*** : from a *pixel count* "*Plate scale*" determined elsewhere from its (18) visible stars.

After showing how much their tilt angles can change MOON Refracted Diameters, this paper also shows one method to derive the best angular value of the Diameter being used and solves for GMT through various methods.

Starting Data

05 Jan 2022 TT-UT=+69.0s UT = 01h11m15.0s N31°55.8' / W109°07.2' Alt = 1,450m QFE=850mb T=+10°C

(1) - *Apparent Equatorial Coordinates of date* : **HD 202284** (Point "A") Dec= -21°12.4' GHA= 163°18.8'

HD 202672 (Point "B") Dec= -21°09.7' GHA= 162°38.2' *Angular separation and relative orientation: **38.0 NM / 085.9°***

(2) - *Refracted Horizontal Coordinates of date* (Hs : Height as read in a zero error Bubble Sextant and Az : Azimuth) :

HD 202284 Hs= 15°48.4' Az= 231°46.6' **HD 202672** Hs= 16°17.2' Az= 231°21.0' *yielding : **37.9 NM / 040.5°***

Note: 0.1 NM smaller separation in (2) because refracted images are "dragged upwards" to one same Zenith point.

Plate measurements (*see next page sketch*)

From picture measure AB = 162 mm, hence ***scaling coefficient "k" = 37.9 / 162 NM/mm = 0.234 NM/mm*** .

From AB as its Diameter draw Circle (Ce). From ***k*** convert : Δ Dec= 2.7 NM → 11.5mm and ΔHs= 28.8 NM → 123 mm .

Given relative Declinations and Heights, on each correct side of AB, draw points "E" and D" on Circle (Ce) as follows :

BE = Δ Dec = 11.5 mm (Star to North Pole, i.e. *Declinations scale*). Line *EA is the GHA scale*. And:

BD = ΔHs = 123 mm (Star to Zenith, i.e. *Sextant Heights scale*). Horizontal line *DA is the Azimuth scale*.

From center of Moon "C" draw horizontal line KCL parallel to DA. From "A" draw line ACP and AP= Far Limb distance.

Through the "Moon horns tips" draw line MCN which defines the only Moon Diameter to be practically measured.

From point "A", draw any "upwards" line not intercepting the Moon itself.

This line intercepts KCL in point "F" representing ***Fictitious Star* HD3*** at the same altitude as the Moon Center "C".

This same line intercepts MCN in "H" representing ***Fictitious Star* HD4*** on the "horns tips" extended line.

With **HD 202284** *2000.0 Dec = -21°17'41.819"* and *RA = 21h15m14.796s*
Measure the differential coordinates from **HD 202284** to **HD3*** and from **HD 202284** to **HD4***.

Perpendicularly onto line AE project point "F" into point "G" and point "H" into point "I".

GF = 15.5 mm, hence Δ Dec (**HD3*** - **HD 202284**) = 15.5 * ***k*** = + 4'36" → **HD3*** *Dec = -21°13'06"*

IH = 44.5 mm, hence Δ Dec (**HD4*** - **HD 202284**) = 44.5 * ***k*** = + 10'24" → **HD4*** *Dec = -21°07'18"*

AG = 39 mm, hence Δ RA (**HD3*** - **HD 202284**) = 39 mm * ***k*** * (1 / cos Dec) * 4 = + 39,2 s → **HD3*** *RA = 21h15m54.0s*

AI = 91 mm, hence Δ RA (**HD4*** - **HD 202284**) = 91 mm * ***k*** * (1 / cos Dec) * 4 = + 91,6 s → **HD4*** *RA = 21h16m46.4s*

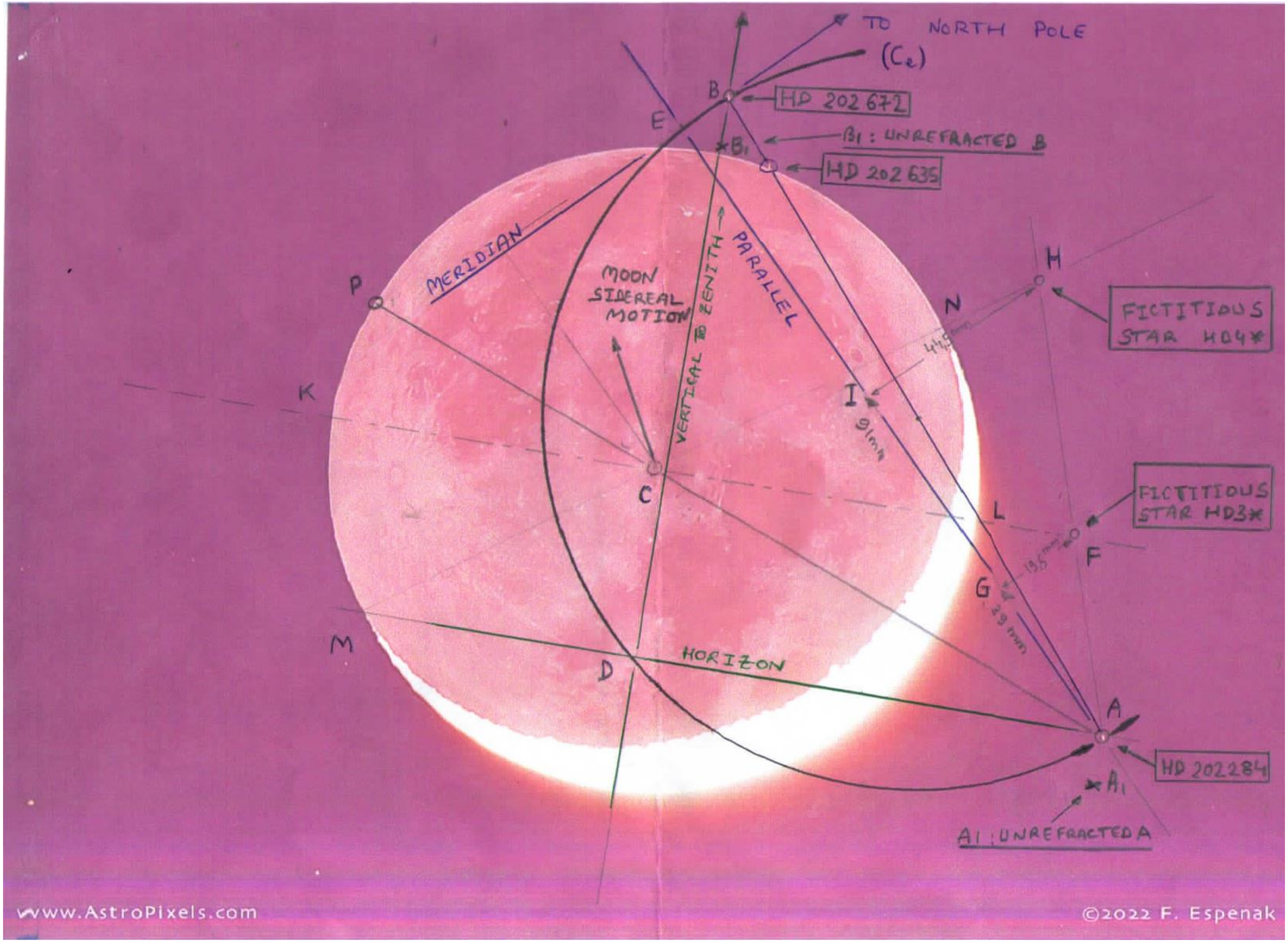
Various refracted and unrefracted Diameters with different tilt angles (assuming "*spherical*" Moon radius = 1,738 km)											
		With Refraction (QFE=850 hPa, T = +10° C)					Without Refraction				
	I	**II**	**III**	**IV**	**V**	**VI**	**VII**	**VIII**	**IX**	**X**	**XI**
		Hs	Az	Far Limb	Near Limb	/ Diam.	Hs	Az	Far Limb	Near Limb	/ Diam.
1	☾ UL	15°74.275'	231.3°	☾ (UL - LL) = Vertical Diam. = 32.652' (with Refr.)			15°71.513'	231.3°	☾ (UL - LL) = Vertical Diam. = 32.750' (without Refr.)		
2	☾ LL	15°41.623'	231.3°				15°38.763'	231.3°			
3	☾Center	15°57.9'	231.3°				15°55.1'	231.3°			
4	**HD 202284**	**15°48.4'**	**231.8°**	**43.256'**	**10.518'**	**32.738'**	**15°45.6'**	**231.8°**	**43.278'**	**10.528'**	**32.750'**
5	HD 202635	16°14.1'	231.4°	32.799'	0.144'	32.655'	16°11.4'	231.4°	32.894'	0.144'	32.750'
6	HD 202672	16°17.2'	231.3°	35.618'	2.967'	32.651'	16°14.5'	231.3°	35.725'	2.975'	32.750'
7	HD3*	15°58.1'	231.7°	38.445'	5.695'	32.750'	15°55.3'	231.7°	38.450'	5.700'	32.750'
8	HD4*	16°10.8'	231.6°	38.186'	5.471'	MN=32.715'	16°08.0'	231.6°	38.299'	5.480'	32.749'

Note - *As expected:* ***close values for*** *II/3 and II/7*, *and* ***identical values for*** *VI/1 and VI/6*, *and for VI/7 and XI.*

Solving for GMT with **HD 202284** Far Limb Distance (AP) through various methods

- With "scaling factor" : MN = 139.4 mm = 32.715'. Since AP = 184.2 mm, then **AP = 43.23' → GMT = 01h11m11.0s .**
- Hybrid method : MN= 728.3px and AP= 966.9px Since MN = 32.715' then **AP = 43.43' → GMT = 01h11m 48.8s .**
- From "Plate scale" with 2.691"/px . Since AP= 966.9 px then **AP = 43.37', yielding → GMT = 01h11m37.2s .**

Overall conclusion : The "Plate scale" GMT determination is probably the most reliable one since its scaling involved 18 stars measurements. The other ones are derived from only one (uneasy) Diameter used for scaling.



TO NORTH POLE
(C_2)
B
HD 202 672
E
B_1
B_1 : UNREFRACTED B
HD 202 635
MERIDIAN
P
MOON SIDEREAL MOTION
VERTICAL TO ZENITH
PARALLEL
H
N
FICTITIOUS STAR HD4*
K
I
91mm
44.5mm
C
L
FICTITIOUS STAR HD3*
F
13.5mm
G
29mm
M
D
HORIZON
A
HD 202284
A_1
A1 : UNREFRACTED A
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