Stars and Planets occultations by the Moon
Deriving positions for given Immersion and Emersion times

Solving for positions from a NavList quiz giving immersion and emersion times

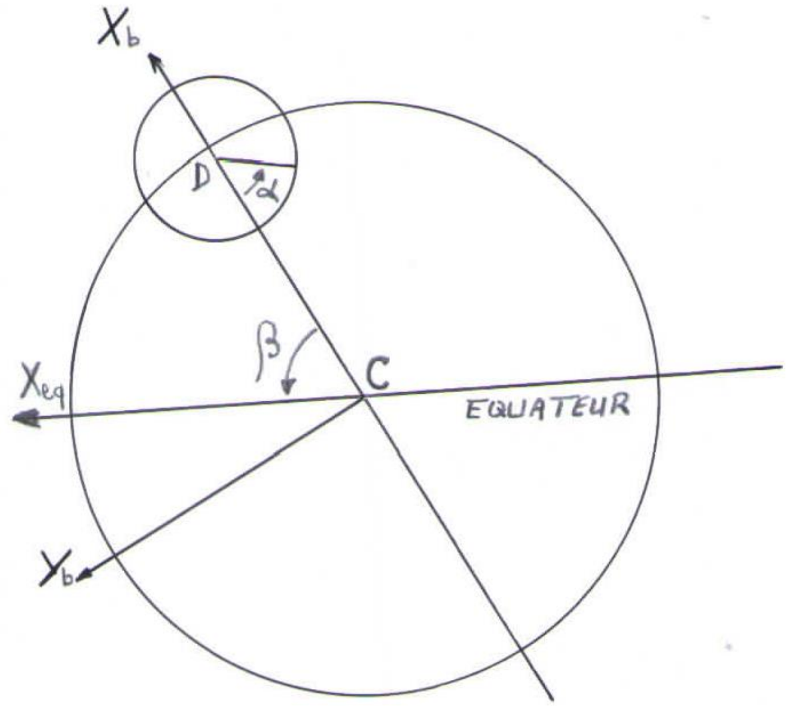
"You're in the Western US. You see Zubenelgnubi occulted by the moon on 22 Feb 2022 at 11 5 48 UT. Can you get a line of position? It reappears at 12 27 22. Where are you?" By Mr. Dave Walden

The numerical results given here come from the following Ephemeris: ***INPOP Numerical Integration*** *(TT-UT = + 69s)*, ***HIPPARCOS 2 Star catalog*** with ***IAU 2006 (P03) PRECESSION*** and ***IAU 2000A NUTATION***.
Earth Equatorial Semi-Diameter = 6,378.137 km, "perfect sphere" Moon Semi-Diameter = 1,738 km

For any given time, the occulted positions curve - if any - lie at the intersection of the cylinder obtained through projecting the Moon shape from the occulted Body onto the Earth globe.

The proposed solution involves 2 main steps:

(1) - Assume a perfectly spherical Earth, no refraction and occulted Body at an infinite distance. Closely approximate the intersection of the applicable Immersion and Emersion curves.
(2) - From such "approximate" intersection, take in account Body Parallax, Earth Oblateness, Refraction and Observer's Altitude to derive the "real world" position fulfilling the requested Immersion and Emersion times.



First Approximation

(UT=11h05m48s adjacent figure to the left)

On the "Besselian" plan containing the Earth Center "***C***" and perpendicular to the occulted Body direction draw a spherical Earth with Radius = 1 and depict "**D**" as the Moon Center projection. With axes X_b and Y_b as shown and with Z_b aiming at the occulted Body, this plan intersects the Equatorial plan along the X_{eq} axis. With "α" as a computation variable we can determine all intersection curve points (x_b,y_b,z_b) since they belong to both the "Moon cylinder" $\sqrt{\lambda_b^2+\mu_b^2}$*=Moon Radius* and to the Earth Surface ($x_b^2+y_b^2+z_b^2=1$). Through a "β" rotation around Z_b, X_b overlaps X_{eq}. Then a ($\pi/2-\delta_{Body}$) rotation around X_{eq} puts us in the Equator plan, and a [GHA_{Aries} - (ARA_{Body} - $\pi/2$)] rotation around the North Pole yields our Intersection Curve usual Lat/Lon coordinates.

Starting computations CD and β check values :

UT = 11h05m48s, CD = 0.959986 and β = 62.703413° UT = 12h27m22s, CD = 0.632452 and β = 118.871710°

Next Page Step (1) : From User chosen "α" increments it is possible to unfold a list of successive positions of the Occultation Curves for both UT times. Some careful search shows that there are 2 matching different coordinates.
Next Page Step (2) : Refined "α" increments narrow results down to 2 reasonable "Approximate Positions".
Next page Steps (3) : Bracket each "Approximate Position" (2 such positions for each UT).

Refined approximations : Follow *Next Page Steps (4), (5), (6)* and *(7)*.

***Final results*:**

Target values : Immersion at 11h05m48s and Emersion at 12h27m22s

Position 1 : N32°00.8'/W108°58.7' (within CONUS), Immersion at 11h05m47,9s and Emersion at 12h27m21,8s

Position 2 : N24°45.1'/W112°58.8' (outside CONUS), Immersion at 11h05m48,2s and Emersion at 12h27m21,9s

(1) - First Computation (spherical Earth, no refraction)

1st Approximate Position 1						1st Approximate Position 2					
α	+N/-S	+W/-E	α	+N/-S	+W/-E	α	+N/-S	+W/-E	α	+N/-S	+W/-E
Immersion : 11h05m48s			Emersion : 12h27m22s			Immersion : 11h05m48s			Emersion : 12h27m22s		
9.0	24.1	113.6				35.0	31.0	109.2	253.0	32.6	108.3
10.0	24.3	113.3	276.0	25.2	112.8	36.0	31.3	109.1	254.0	32.3	108.6
11.0	**24.5**	**113.1**	**277.0**	**24.8**	**112.9**	**37.0**	**31.7**	**109.0**	**255.0**	**32.0**	**108.8**
12.0	**24.7**	**112.9**	**278.0**	**24.5**	**113.0**	**38.0**	**32.1**	**108.9**	**256.0**	**31.7**	**109.1**
13.0	24.9	112.7	279.0	24.2	113.1	39.0	32.4	108.8	257.0	31.4	109.3
14.0	25.1	112.5	280.0	23.8	113.2	40.0	32.8	108.7	258.0	31.1	109.5

(2) - Second computation (spherical Earth, no refraction)

2nd Approximate Position 1						2nd Approximate Position 2					
α	+N/-S	+W/-E	α	+N/-S	+W/-E	α	+N/-S	+W/-E	α	+N/-S	+W/-E
Immersion : 11h05m48s			Emersion : 12h27m22s			Immersion : 11h05m48s			Emersion : 12h27m22s		
11.2	24.5	113.1	277.8	24.6	113.0	37.8	32.0	108.9	255.5	31.8	109.0
11.3	**24.5**	**113.0**	**277.9**	**24.5**	**113.0**	**37.7**	**31.9**	**108.9**	**255.4**	**31.9**	**108.9**
11.4	**24.5**	**113.0**	**278.0**	**24.5**	**113.0**	**37.6**	**31.9**	**108.9**	**255.3**	**31.9**	**108.9**
11.5	24.6	113.0	**278.1**	**24.5**	**113.0**	**37.5**	**31.9**	**108.9**	**255.2**	**31.9**	**108.9**
11.6	24.6	113.0	278.2	24.4	113.0	37.4	31.8	109.0	255.1	32.0	108.9

(3) - Bracketing Approximate Positions

Bracketing Approximate Position 1		Bracketing Approximate Position 2	
P1-NW: N24°40’/W113°10’	**P1-NE**: N24°40’/W112°50’	**P2-NW**: N32°04’/W109°04’	**P2-NE**: N32°04’/W108°44’
P1-SW: N24°20’/W113°10’	**P1-SE**: N24°20’/W112°50’	**P2-SW**: N31°44’/W109°04’	**P2-SE**: N31°44’/W108°44’

(4) - Real World computation (Ellipsoid and refraction)
Actual Immersion and Emersion times at Bracketing Positions

Position 1		Position 2	
P1-NW **I** : 11h05m40,7s **E** : 12h26m58,7s	**P1-NE** **I** : 11h06m06,9s **E** : 12h27m47,5s	**P2-NW** **I** : 11h05m36,5s **E** : 12h27m08,5s	**P2-NE** **I** : 11h06m14,4s **E** : 12h27m43,2s
P1-SW **I** : 11h06m09,2s **E** : 12h27m14,8s	**P1-SE** **I** : 11h06m35,0 **E** : 12h28m04,2s	**P2-SW** **I** : 11h05m46,0 **E** : 12h27m35,9s	**P2-SE** **I** : 11h06m23,8s **E** : 12h28m11,0s

(5) - Interpolation to determine coordinates for Immersion : 11h05m48s and Emersion : 12h27m22s

Position 1	Position 2
11h05m48s Immersion at: N24°40’ / W113°04.4’ and N24°20’ / W113°26.4’	11h05m48s Immersion at: N32°04’ / W108°57.9’ and N31°44’ / W109°02.9’
12h27m22s Emersion at: N24°40’ / W113°00.5’and N24°20’ / W113°07.1’	12h27m22s Emersion at: N32°04’ / W108°56.2’ and N31°44’ / W109°11.9’

(6) - Coordinates for given Immersion : 11h05m48s and Emersion : 12h27m22s

Position 1	Position 2
N24°45.1’/W112°58.8’	**N32°00.8’/W108°58.7’**

(7) - Final Cross check vs. given Immersion : 11h05m48s and Emersion : 12h27m22s

Position 1	Position 2
Immersion at 11h05m48,2s **Emersion at 12h27m21,9s**	**Immersion at 11h05m47,9s** **Emersion at 12h27m21,8s**