Ray bending computation between finite distance points at low altitude

Reference Documents:

Ref1 : Andrew T. Young *https://aty.sdsu.edu/explain/atmos_refr/bending.html*
Ref2 : André Danjon Astronomie Générale - 1980 - §81 pp 161, 162

Both references indicate that actual values are difficult to predict under all possible meteorological conditions and that the following formulae are only approximate and apply to horizontal rays or nearly so for "standard" conditions.

1 - Ref1

1.1 - **Ref1** mentions the temperature vs. altitude lapse rate "γ" as a key factor governing ray bending radius R_t. With R_o being the Earth Radius and with γ in °K/m and, **Ref1** also indicates:

$$(1)\quad R_o/R_t = k \cong (0.034 - \gamma) / 0.154$$

1.2 - **Ref1** also lists the following "standard values":

1.2.1 - For the ***International Standard Atmosphere (ISA)*** $\gamma = 0.0065$ ***°K/m***. Hence $k \cong 0.179$ and $R_o/R_t \cong 5.6$

1.2.2 - For ***(adiabatic) free convection,*** $\gamma \cong 0.0106$ ***°K/m***. Hence $k \cong 0.152$ and $R_o/R_t \cong 6.6$.
Ref1 *also indicates here:*
Ref1 quote**: "***This is close to the condition of the atmosphere near the ground in the middle of the day, when most surveying is done; the value calculated is close to the values found in practical survey work*" ***Ref1 Unquote***

2 - Ref2

With V_e being the **Vertical elevation** in arc minutes of a point at distance **D** in **NM**, **Ref2** gives this simple formula:

$$(2)\quad V_e' = D_{NM} * k/2$$

3 - Application to the Horizon Dip at sea.

If no refraction, the sea Horizon Dip (in arc minutes) seen from a Navigator at height "h" (in meters) above the sea is:

$$(3)\quad Dip'_{Unrefracted} = 1.925 \sqrt{h_m}$$

Alongside the surface of a spherical Earth, the *curved distance* in **NM** between the Observer and the unrefracted horizon is equal to the unrefracted Dip in arc minutes, a value which is also extremely close to their *straight-in distance* in **NM**.

Hence we can use the **"upwards" additive term:** $V_e' = Dip'_{Unref} * k/2$ to correct Formula ***(3)*** into the refracted Dip**.**

Ref2 indicates that for all practical purposes, for Celestial Navigation, k = 0.16 . Since $Dip_{Refracted} = Dip_{Unrefracted}\,(1 - k/2)$,

$$(4)\quad Dip'_{Refracted} = 1.771 \sqrt{h_m}$$

which is ***exactly*** the quantity tabulated in the **French Éphémérides Nautiques** for the Horizon Dip at sea.

4 - Numerical application:

From ***Allauch*** (*N43°31'08"/ E005°29'10"/310m AMSL)* $Dip'_{Unrefracted}$ *= 33.9'* and $Dip'_{Refracted}$ *= 31.2'* . From there the unrefracted ***Pic du Canigou*** (*N42°31'08"/ E002°27'24"/2,784m AMSL*) is *142 **NM*** away at *38.6'* below the local horizon.

The **Vertical elevation** due to refraction being equal to **11.4'**, the refracted ***Pic du Canigou*** can then be seen from ***Allauch*** at 27,2' below the local horizon at @ 4.0' above the refracted maritime horizon, e.g. at *4.3' on Nov. 1st, 2005* with a picture of the setting Sun taken at 16h30m47.6s UT in Azimuth 250.7520° just behind ***Pic du Canigou***.

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