ASTRONOMICAL REFRACTION for heights between °0 and -1° UNDER THE ASTRONOMICAL HORIZON

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Astronomical Refraction Tables - from the **Éphémérides Nautiques** (**EN)** or the **Nautical Almanac** (**NAL**) - publish "standard" Refractions for heights only above the ***Astronomical Horizon*** (*perpendicular to the Local Vertical*). However, from a sufficient altitude it is sometimes possible to observe a Celestial Body at some negative altitude, i.e. under the ***Astronomical Horizon*** as, for example, just above the ***Maritime Refracted Horizon (MRH)*** at sea.

Based solely upon the existing Refraction Tables for positive heights and through sensible and hopefully *reasonable* considerations the following Method *augments* these Tables with Astronomical Refractions for heights between 0° and -1° under the Astronomical Horizon.

One should nonetheless keep in mind that - even under steady atmospheric conditions matching the "standard" parameters used in the relevant Theories - *the computed **Astronomical Refractions** become more and more scattered with decreasing heights*. For example, for **h = 0°** the Standard Astronomical Refraction is listed as **-33,8'** in the **EN** (1982 edition) and as **-34.5'** in the ***pre-2004*** **NAL** (1983 Edition). **At the Astronomical Horizon level this already represents a 0.7' scatter** which can only be expected to keep increasing further under this Horizon.

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(1) - Let us consider the following:

A is a given Celestial Body observed at height h'_a *with* $-1° \leq h'_a \leq 0°$ under the Astronomical Horizon. And:
R_{ha} is the tabular Standard Refraction for height $h_a = -h'_a$. And finally:
R_o is the tabular Standard Refraction for height de 0° (when Celestial Body on the Astronomical Horizon).

(2) - Let us also consider a ***Heights Offset Scale*** under the Astronomical Horizon such as *Body A* height becomes 0°.

2.1 - In such ***Heights Offset Scale*** the Astronomical Horizon height is h_a at which Refraction is R_o.
For one same value h_a of the height variable: in the Astronomical Horizon referenced heights scale Refraction is R_{ha}, while in the ***Offset Scale*** Refraction is R_o. *Let* "λ_a" be the R_o/R_{ha} ratio.

2.2 - If we assume - *as a non-unreasonable assumption to be checked further down the line* - that **in each *Offset Scale* specific to each h_a value, this same "λ_a" ratio remains valid/applicable to all intermediate heights between 0° and h_a under the Astronomical Horizon,** then *Body A* Refraction $R'_{(A)}$ is equal to R_o^2/R_{ha}.

2.3 - On the other hand, the on-station temperature and its atmospheric pressure (QFE) generally depart from the standard atmosphere parameters. Hence a *Daily Correction Factor "μ"* applies to the tabular Astronomical Refractions. It is computed as follows: $\mu = (P/P_o) * (273+T_o)/(273+\Theta)$, which for both the **EN** and the **NAL** translates into: $\mu = (P_{mb}/1013)*(283)/(273+\Theta_{°C})$.

2.4 - Finally for a Celestial Body A seen at a height h'_a under the Astronomical Horizon:

$$(2.4) \qquad R'_{(A)} = \mu\lambda_a R_o = \mu R_o^2/R(-h'_a)$$

(3) - Check for consistency with the Astronomical Refraction Tables

It is important to check that our §2 *"proportionality hypothesis"* is consistent with the Refraction Tables. One same height ***h'*** under the Astronomical Horizon can indeed be found in *various **Offset Scales** with each one having its own "λ"*. We need to ascertain that the observed scatter of all ***R'*** values derived from one same ***h'*** in different ***Offset Scales*** is ***consistent with the scatter*** acknowledged for the Tables (**EN**, **NAL**, …) as per their ***following differences***:

EN and NAL Standard Refractions (+10°C - 1013mb / 760 mm Hg)	h	EN	NAL	(NAL-EN) differences
The **EN** publish Astronomical Refractions with 4 significant digits (0,01') and at 10' intervals.	*1°00'*	-24,37'	-24.3'	***- 0.07'***
	50'	-25,66'	-25.63'	***- 0.03'***
	40'	-27,05'	-27.13'	***+ 0.08'***
The **NAL** publishes these Refractions with 3 digits (0.1') at 3' intervals.	*0°30'*	-28,55'	-28.7'	***+ 0.15'***
To minimize the size of our spreadsheet we keep the **EN** 10' interval.	*20'*	-30,17'	-30.47'	***+ 0.30'***
When necessary the **NAL** refractions are interpolated with	*10'*	-31,92'	-32.40'	***+ 0.38'***
4 significant digits (0.01') *to minimize subsequent round-off errors.*	*0°00'*	-33,80'	-34.5'	***+ 0.70'***

*Note: The **EN** and **NAL** Celestial Navigation data published to generally the nearest 0.1' (round-off at 0.05').*
For h > 30' there is no significant difference here between the EN and NAL Standard Astronomical Refractions.

TABLES (A) - "Brute Force" Method

Éphémérides Nautiques (EN)				
***Échelle Décalée de Hauteurs* : h'a = -1°**				
h = *0°00'* : Astre à l'Horizon		***h' = *1°00'* : Astre à l'Horizon***		
h	R	*h'*	*R'*	*λ=R'/R*
2°00'	-18,39'	*2°00'*	*-24,37'*	*1,33*
50'	-19,21'	*50'*	*-25,66'*	*1,34*
40'	-20,09'	*40'*	*-27,05'*	*1,35*
1°30'	-21,04'	*1°30'*	*-28,55'*	*1,36*
20'	-22,07'	*20'*	*-30,17'*	*1,37*
10'	-23,18'	*10'*	*-31,92'*	*1,38*
1°00'	**-24,37'**	****1°00'****	***-33,80'***	***1,39***
50'	-25,66'	*50'*	*-35,92'*	*1,40*
40'	-27,05'	*40'*	*-38,14'*	*1,41*
0°30'	-28,55'	*0°30'*	*-40,54'*	*1,42*
20'	-30,17'	*20'*	*-43,14'*	*1,43*
10'	-31,92'	*10'*	*-45,96'*	*1,44*
0°00'	-33,80'	*0°00'*	*-49,01'*	*1,45*

pre-2004 Nautical Almanac (NAL)				
***Heights Offset Scale* : h'a = -1°**				
h = *0°00'* : Body at Horizon		***h' = *1°00'* : Body at Horizon***		
h	R	*h'*	*R'*	*λ=R'/R*
2°00'	-18.3'	*2°00'*	*-24.3'*	*1.33*
50'	-19.1'	*50'*	*-25.63'*	*1.34*
40'	-20.0'	*40'*	*-27.13'*	*1.36*
1°30'	-20.9'	*1°30'*	*-28.7'*	*1.37*
20'	-22.00'	*20'*	*-30.47'*	*1.39*
10'	-23.10'	*10'*	*-32.40'*	*1.40*
1°00'	**-24.3'**	****1°00'****	***-34.5'***	***1.42***
50'	-25.63'	*50'*	*-36.78'*	*1.43*
40'	-27.13'	*40'*	*-39.34'*	*1,45*
0°30'	-28.7'	*0°30'*	*-42.05'*	*1.46*
20'	-30.47'	*20'*	*-45.10'*	*1,48*
10'	-32.40'	*10'*	*-48.44'*	*1.49*
0°00'	-34.5'	*0°00'*	*-52.10'*	*1,51*

The basis of this Paper is the concept of the "***Heights Offset Scale***" in which *for the same values of the variables **h** and **h'*** we compare the ***λ=R'/R*** ratios.

On both Spreadsheets to the left we see the *very regular increases of **λ***.

This ***"Brute Force" Method*** computes the missing ***R'*** values from the missing ***λ*** values assumed to follow the *same unchanged growth pattern* (*bottom 2 rightmost columns in small print*).

EN and NAL results summarized in both Tables (A) here-under

ÉPHÉMÉRIDES NAUTIQUES (EN) ***Table étendue de Réfractions Astronomiques (A)*** ***[0°, -1°] sous l'Horizon Astronomique***			
	h'	***R'***	***Dispersion standard***
Conditions standard ***P = 760 mm Hg, T = +10°C***	***0°00'***	***-33,8'***	***0,7'***
	-10'	***-35,9'***	***1,1'***
Dispersion standard *obtenue par différence entre les valeurs calculées pour les **ÉPHÉMÉRIDES NAUTIQUES** et pour le **NAUTICAL ALMANAC***	***-20'***	***-38,1'***	***1,2'***
	-0°30'	***-40,5'***	***1,5'***
	-40'	***-43,1'***	***2,0'***
	-50'	***-46,0'***	***2,4'***
	-1°00'	***-49,0'***	***3,1'***

pre-2004 NAUTICAL ALMANAC (NAL) ***Astronomical Refractions Augmented Table (A)*** ***[0°, -1°] under the Astronomical Horizon***			
	h'	***R'***	***Standard scatter***
Standard conditions ***P = 1,013 hPa, T = +10°C***	***0°00'***	***-34.5'***	***0.7'***
	-10'	***-37.0'***	***1.1'***
Standard scatter *obtained through differences between values computed for the **NAUTICAL ALMANAC** and for the **ÉPHÉMÉRIDES NAUTIQUES***	***-20'***	***-39.3'***	***1.2'***
	-0°30'	***-42.0'***	***1.5'***
	-40'	***-45.1'***	***2.0'***
	-50'	***-48.4'***	***2.4'***
	-1°00'	***-52.1'***	***3.1'***

Such ***Tables (A) nonetheless*** rely upon just only ***h'a = -1°***. The following "***More in-depth***" Method accommodates all tabular ***h'*** values for comparisons between them in order to derive ***Tables (B), believed to bring some improvement***.

TABLES (B) - More "in-depth" Method

In this more ***"in-depth" Method*** - also relying on ***Formula (2.4)*** here-above - for each of the 2 Standard Refraction Tables (**EN** and **NAL**) and starting from each **h'a** and related **λa** values we are to compute all intermediate ***R'*** refraction values obtained with this same **λa** factor. The "***δ max***" right column lists the maximum differences between such ***R'*** values derived from the successive **λa** factors resulting from other **h'a** values.

The rightmost column here-under gives the mean values rounded to the nearest 0.1' of the extreme R' values obtained from the successive h'a values.

ÉPHÉMÉRIDES NAUTIQUES (EN) Conditions standard: P = 760 mm Hg, T = +10°C, TABLE ÉTENDUE A [0°, -1°]															
λa = 1,39		**λa = 1,32**		**λa = 1,25**		**λa = 1,19**		**λa = 1,12**		**λa = 1,07**		**λa = 1,00**		***EN***	
h'	*R'*	*h'*	*R'*	*h'*	*R'*	*h'*	*R'*	*h'*	*R'*	*h'*	*R'*	*h'*	*R'*	***h'***	***R'***
0°00'	***-33,80'***	***0°00'***	***-33,80'***	***0°00'***	***-33,80'***	***-0°00'***	***-33,80'***	***-0°00'***	***-33,80'***	***-0°00'***	***-33,80'***	***-0°00'***	***-33,80'***	***0°00'***	***-33,8'***
-10'	*-35,59'*	*-10'*	*-35,63'*	*-10'*	*-35,67'*	*-10'*	*-35,72'*	*-10'*	*-35,79'*	***-10'***	***-35,79'***	***δ** max :*	*0,20'*	***-10'***	***-35,7'***
-20'	*-37,52'*	*-20'*	*-37,61'*	*-20'*	*-37,70'*	*-20'*	*-37,79'*	***-20'***	***-37,87'***				*0,35'*	***-20'***	***-37,7'***
-30'	*-39,60'*	*-30'*	*-39,74'*	*-30'*	*-39,89'*	***-30'***	***-40,02'***						*0,42'*	***-0°30'***	***-39,8'***
-40'	*-41,84'*	*-40'*	*-42,05'*	***-40'***	***-42,23'***								*0,39'*	***-40'***	***-42,0'***
-50'	*-44,27'*	***-50'***	***-44,52'***										*0,25'*	***-50'***	***-44,4'***
-1°00'	***-46,88'***												*0,00'*	***-1°00'***	***-46,9'***

pre-2004 NAUTICAL ALMANAC (NAL) Standard conditions: P=1,013 hPa, T = +10°C, AUGMENTED TABLE FOR [0°, -1°]															
λa = 1.42		λa = 1.35		λa = 1.27		λa = 1.20		λa = 1.13		λa = 1.06		λa = 1.00		NAL	
h’	R’	h’	R’	h’	R’	h’	R’	h’	R’	h’	R’	h’	R’	h’	R’
0°00’	**-34.50’**	**0°00’**	**-34.50’**	**0°00’**	**-34.50’**	**-0°00’**	**-34.50’**	**-0°00’**	**-34.50’**	**-0°00’**	**-34.50’**	**-0°00’**	**-34.50’**	**0°00’**	**-34.5’**
-10’	-36.39’	-10’	-36.52’	-10’	-36.50’	-10’	-36.63’	-10’	-36.74’	**-10’**	**-36.74’**	**δ max :**	0.35’	**-10’**	**-36.6’**
-20’	-38.52’	-20’	-38.63’	-20’	-38.75’	-20’	-38.95’	**-20’**	**-39.06’**				0.54’	**-20’**	**-38.8’**
-30’	-40.75’	-30’	-41.02’	-30’	-41.28’	**-30’**	**-41.47’**						0.72’	**-0°30’**	**-41.1’**
-40’	-43.26’	-40’	-43.61’	**-40’**	**-43.87**								0.61’	**-40’**	**-43.6’**
-50’	-46.00’	**-50’**	**-46.44’**										0.44’	**-50’**	**-46.2’**
-1°00’	**-48.98’**												0.00’	**-1°00’**	**-49.0’**

The "***δ max***" maximum differences do not exceed *0,42’* for the **EN** and *0.72’* for the **NAL**. As such they remain under the **EN**/**NAL** scatter, *0.7’ at h = 0°*, observed at the Astronomical Horizon. From each rightmost column the successive ***R’*** differences keep smoothly increasing. Between both Tables the ***R’*** differences for one same given height ***h'a*** also keep increasing smoothly from 0.7’ at h = 0° to reach 2.1’ at h = -1°, a *non-unrealistic* threefold increase here.

Our §2 “proportionality hypothesis” can then be reasonably considered as a valid one.

One numerical example

On Nov. 1st 2005 from *Allauch* - QFE = 983 mb and Θ = +16°C - a magnificent picture was made of the Sun setting behind the *Pic du Canigou* with its ***Center at 0.7’ above the Pic*** (*http://canigou.allauch.free.fr/Refract_atm.htm*).

We can then compute the ***Sun Refracted center at 25.1’ under the Astronomical Horizon***. It is also at 6.0’ above the **Maritime Refracted Horizon MRH** (vs. 5.4’ measured from a **different MRH derived by the Author**, *a non-significant difference here*). The observed Sun Azimuth *time-clocks* this picture at *UT = 16h30m52s*, when the ***geodetic Unrefracted Sun Center was at 1°03.29’ under the Astronomical Horizon***. From the difference (-63,3’ + 25,1’):

The observed Sun Center Refraction R’ was then at -38.2’

Daily Correction Factor*: μ (**EN**) = μ (**NAL**) = (983/1013)(283/289). With **μ = 0.9502** and **h'a = -25.1’** :*

With the **EN** Augmented Refraction ***Table (B)*** *here-under* get: ***R’a(EN) = -36.8’ +/- 1.5’, 1.4’ from the observation***

With the **NAL** Augmented Refraction ***Table (B)*** *here-under* get: ***R’a(NAL) = -38.0’ +/- 1.5’, 0.2’ from the observation***

While this example certainly is not a definite validation proof of this “More in-depth” Method, it is not - and by far - a counter-example either.

We are therefore submitting here-under ***Augmented Astronomical Refraction Tables (B)*** for both the **EN** and the **NAL** with *the extra specific column indicating the standard scatter between both sources as an important factor*.

It would be also quite instructive to check these results against the supposedly existing “Very Low Heights Refractions Tables” used probably onboard SEAPLANES and almost certainly onboard AIRSHIPS in the XX^th Century.

ÉPHÉMÉRIDES NAUTIQUES (EN) Table étendue de Réfractions Astronomiques (B) [0°, -1°] sous l’Horizon Astronomique			
	h’	***R’***	***Dispersion standard***
Conditions standard P = 760 mm Hg, T = +10°C	***0°00’***	***-33,8’***	***0,7’***
	-10’	***-35,7’***	***0,9’***
Dispersion standard	***-20’***	***-37,7’***	***1,1’***
*obtenue par différence entre les valeurs calculées pour les **ÉPHÉMÉRIDES NAUTIQUES** et pour le **NAUTICAL ALMANAC***	***-0°30’***	***-39,8’***	***1,3’***
	-40’	***-42,0’***	***1,6’***
	-50’	***-44,4’***	***1,8’***
	-1°00’	***-46,9’***	***2,1’***

pre-2004 NAUTICAL ALMANAC (NAL) Astronomical Refractions Augmented Table (B) [0°, -1°] under the Astronomical Horizon			
	h’	***R’***	***Standard scatter***
Standard conditions P = 1,013 hPa, T = +10°C	***0°00’***	***-34.5’***	***0.7’***
	-10’	***-36.6’***	***0.9’***
Standard scatter	***-20’***	***-38.8’***	***1.1’***
*obtained through differences between values computed for the **NAUTICAL ALMANAC** and for the **ÉPHÉMÉRIDES NAUTIQUES***	***-0°30’***	***-41.1’***	***1.3’***
	-40’	***-43.6’***	***1.6’***
	-50’	***-46.2’***	***1.8’***
	-1°00’	***-49.0’***	***2.1’***

Conclusion: The **EN** / **NA**L scatter significantly increases with decreasing heights, and we are not investigating for heights lower than -1° under the Astronomical Horizon here. Actually ***Tables (A) and Tables (B) are not fundamentally different in this respect***. *The **EN Table (A)** is almost identical to the **NAL Table (B)**, both having our personal preference.*

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