An interesting feature resulting from the Astronomical/Geographic coordinates system used on Ellipsoids

A current thread started by M. Frank E. Reed (*https://navlist.net/triangle-equator-FrankReed-oct-2025-g57872*) keeps reminding us that Latitudes for Navigation are defined according to their Astronomical (*not* their Geocentric) meaning.

- To best describe and represent the existing world all Maritime Charts have long been using various ellipsoids (e.g. WGS84 from now on) instead of the spherical canvases formerly used to reference coordinates in the remote past.

- Nonetheless Navigators have stuck to their traditional long range Navigation computations tools using relatively simple spherical trigonometry formulae instead of their very complex ellipsoidal trigonometry counterparts.

- This widely accepted *inconsistency* between 2 distinct references used by Navigators - Ellipsoids for Nautical Charts vs. Spheres for Navigation computations - shows curious results albeit none of them being a dramatic one.

An example among others: *there are sizeable differences between accurately observed and recorded values of the horizontal separation between two given points seen from a third one* (e.g. through *a Sextant or* a *Cercle de Borda*) *and the results derived from processing their Ellipsoid referenced coordinates through Spherical trigonometry.*

In the following practical example, a Navigator in position A is assumed to closely measure the horizontal angular separation between 2 distant flagpoles B and C. For the sake of simplicity, we assume that flagpole B is exactly North of the Navigator. This situation then all boils down to the Navigator measuring the actual Azimuth of flagpole C.

We then need some reliable and accurate mathematical tool adapted to Ellipsoids to compute real world observations.

- One first idea is to use the ***Vincenty's Formulae***. But these are not the exact mathematical tools required here since they compute departure azimuths of the [least distance] "Geodesics" towards targets. A quite different mathematical problem indeed, although these Formulae should yield outstanding results for short distances (e.g. < 200 NM). *Why not inventing some new and specific tool dedicated to directly computing Azimuths on Ellipsoids?*

- *Let us then rig the Navigator's local vertical axis with a rotating plan and move it until it contains the "target", at which time we can record its exact local Azimuth.* This ***3D computation mathematical tool*** exactly suits our specific needs. It is totally independent from any geodesic considerations whatsoever. It also has the unique advantage - not shared by the ***Vincenty's formulae*** - of very easily accommodating all and any Targets/Observers altitudes changes.

Let us then work the *following example* : *Let's build three simple towers with flagpoles at the top that we can observe with sextants from a distance. The flagpoles will be positioned by GPS at the following locations:*

A: 0°00' N, 50°00' W (0 m) *B: 0°06' N, 50°00' W (0 m)* *C: 0°06' N, 49°52' W (0 m)*

Computed results from flagpole A to flagpole C:

(1) - Mid-Latitude straight line plan computation (1 NM = 1,852 meters): *D = 18,520.000 m Az = 053°07'48.369"*

(2) - "Full" Rhumb line computation on a sphere (1 NM = 1,852 meters): *D = 18,519.994 m Az = 053°07'48.322"*

(3) - Great Circle computation on a sphere (1 NM = 1,852 meters): *D = 18,519.994 m Az = 053°07'48.177"*

(4) - WGS84 ***Vincenty's Formulae*** : *D = 18,508.625 m Az 053°18'52.566"*

(5) - WGS84 ***Vertical Axis 3D Method*** : *D = 18,508.625 m Az 053°18'52.566"*

As expected we observe here that for such short distances involved and so near from the Equator:

- (1) , (2) and (3) yield extremely close results. And:

- To the precision of the digits published *(4) and (5) yield identical Azimuths, **in excess of 11' from (1), (2) and (3) Azimuths**.*

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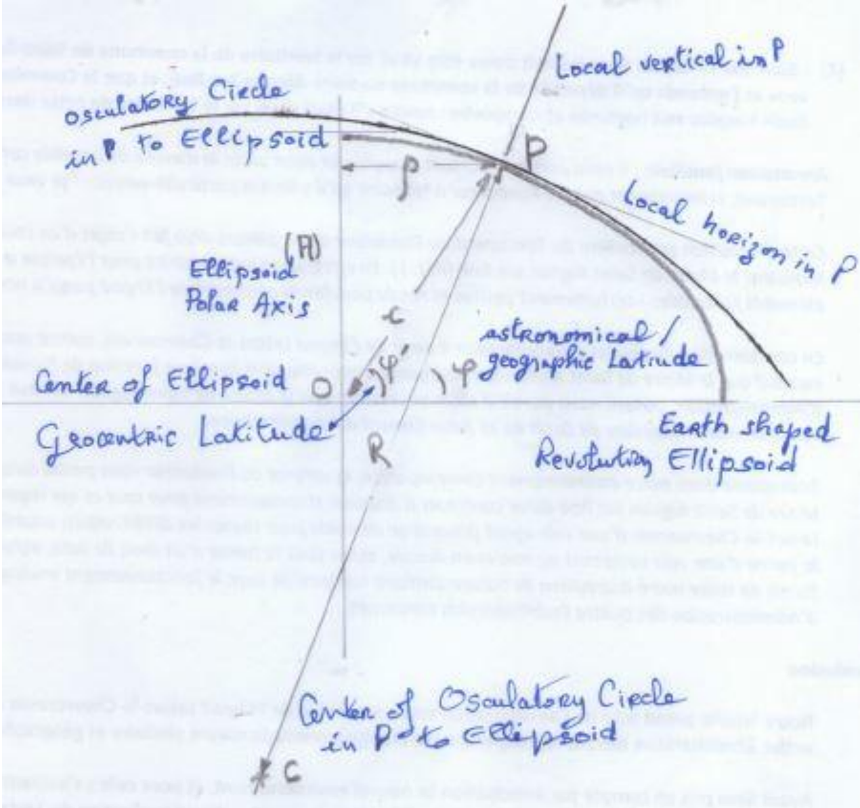
An interesting challenge from M. Frank E. Reed here: *"https://navlist.net/triangle-equator-FrankReed-oct-2025-g57899"*

"Find yourself some targets for observation at ranges similar to the scenario in my original post at any latitude, calculate the sextant angles between targets using the plain latitudes and longitudes from GPS devices (or mapping intended to be used with GPS devices --meaning anything in the modern world), and you will discover comparably large errors in the angles [except at one latitude... anyone?]."

If any existing solution, we are to find Latitudes on the Ellipsoid were the length of a Degree of Longitude divided by the length of a Degree of Latitude exactly matches the same ratio for a Sphere. Let us then take a look at the following sketch:



From a point P on the surface of an Ellipsoid:

- The length of a degree at the astronomical/ geographic Latitude φ is proportional to the length of the radius "R" of the Osculatory Circle to the Ellipsoid.
- The Length of degree of Longitude is similarly proportional to the length of its distance ρ to the North-South axis (P)

On a point of a Sphere, we know that this same ratio ρ/R is exactly equal to cos φ

Let us then compare these ρ/R ratios obtained on a Sphere and on WGS84 to see whether they are some existing Latitudes where they are equal.

Let us define S = ρ/R (sph.) / ρ/R (ell.)
which simplifies into:
S = cos φ / ρ/R (ell.)

We then obtain the following results for a few specific values of the Latitudes:

Latitudes	Sphere	WGS 84			S
	ρ/R (Sph.) = cos φ	ρ km	R km	ρ/R (ell.)	S = cos φ / ρ/R (ell.)
0°	1.000 000	*6,378.137*	6,335.439	1.006 739	0.993 306 = *1 - 1/149.4*
45°	0.707 107	4,517.591	6,367.382	0.709 490	0.996642
54°46'50"	0.576 710	3,686.578	*6,378.137*	0.578 002	0.997 763
70°	0.342 020	2,187.928	6,392.033	0.342 290	0.999 212
88°	0.034 899	223.342	6,399.515	0.034 900	0.999 992
89°59'	0.000 291	1.862	6,399.594	0.000 291	1.000 000
90°	0°	0	6,399.584	0	*(1.000 000)*

We can therefore conclude that *only very close to either the North or the South Pole* can we expect that from flagpole A both flagpole C Azimuths calculated by the Great Circle and the 3D Methods should show some difference much smaller than the 11' observed at the Equator. Nonetheless the Rhumb Line results will likely differ very significantly from them both.

As an example : from *flagpole A at N89°54'00/W090°00'00/0 m* to *flagpole C at N89°52'00/W000°00'00/0 m*

(1) - "Full" Rhumb line computation on a sphere (1 NM = 1,852 meters): *D = 20,560.880 m Az = 100°22'42.135"*

(2) - Great Circle computation on a sphere (1 NM = 1,852 meters): *D = 18,519.994 m Az = 053°07'48.597"*

(3) - WGS84 ***Vincenty's Formulae*** : *D = 18,615.656 m Az = 053°07'48.590"* ***almost identical to Az(2) this time*** .

(4) - WGS84 ***Vertical Axis 3D Method*** : *D = 18,615.656 m Az = 053°07'48.595"* ***almost identical to Az(2) this time*** .

Only some quite [very] limited academic interest here.

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