ASSUMED POSITION METHOD

Development of the Formulae

The spherical triangle used in navigation is shown projected on the plane of the horizon in Figure 1. The parts of the triangle are lettered as follows:

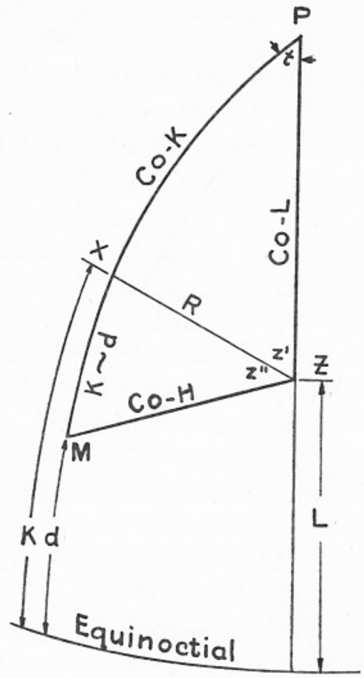

FIG. 1

P. Pole.
Ƶ. Zenith of observer. Azimuth (angle PƵM) is also called Ƶ.
Z′. Part of Ƶ in the "time triangle."
Z″. Part of Ƶ in the "altitude triangle."
M. Heavenly body observed.
L. Latitude of observer.
d. Declination of body M.
t. Meridian angle of body M.
H. Altitude of body M.
R. Perpendicular let fall from Ƶ on PM.
X. Intersection of R with PM.
K. Arc from X to the equinoctial. This is an auxiliary part introduced to facilitate solution.

The following formulae have been derived:

From triangle PƵX

$$1.\ \csc R = \csc t \sec L$$

$$2.\ \csc K = \frac{\csc L}{\sec R}$$

$$3.\ \csc Z' = \frac{\sec K}{\sec L}$$

From triangle ƵMX

$$4.\ \csc H = \sec R \sec (K \sim d)$$

$$5.\ \csc Z'' = \frac{\csc (K \sim d)}{\sec H}$$

The unknown elements of triangle PƵX (the time triangle) have been precomputed for integral degree values of t and L and tabulated in Table I, which is a spherical traverse table for solution of right spherical triangles. The essential angles and functions from formulae (1), (2), and (3) may thus be obtained from Table I. The arrangement of Table I is similar to that in a number of previous tables, except that it has been tabulated in columns for latitude instead of for t and that it includes data for all latitudes from 0° to 90°. This improved tabulation has made it possible to take the data from Table I for a number of star sights at one opening of the book.

Table II is the same table that the author used in a previous method of navigation which solves the astronomical triangle from the D. R. position. Since formulae (4) and (5) are in terms of cosecants and secants, this table has been arranged in parallel columns of cosecants (A column) and secants (B column) multiplied by 100,000. The tabulation of functions every half minute throughout the table has obviated the necessity for interpolation.

D. R. Position Method

Since Table II is in terms of cosecants and secants, it can also be used for solution of the astronomical triangle by the author's earlier method.

METHOD OF SOLUTION

Typical solutions for the sun and stars are shown, with two sights illustrating the use of the A *Almanac.* The following steps are taken to compute the altitude and azimuth in one combined solutio

1. Find the Greenwich hour angle (GHA) in degrees and minutes.
2. Apply to the GHA an assumed longitude (less than 30′ from the D. R. longitude) to obtain a loc hour angle of an integral number of degrees. From this, determine the meridian angle (t) east or we from 0° to 180°.
3. Enter in form declination to nearest tenth of a minute and assumed latitude (integral degree within 30′ of D. R. latitude.
4. Correct H_s to obtain H_o to nearest tenth of a minute.
5. Enter Table I with t and the assumed latitude. Lat. is across the top of the table. t is found the sides of the table, 1° to 90° at the left and 90° to 179° at the right. The column tabs at the top g with the left hand values of t, the column tabs at the bottom go with the right hand values of t.
6. In column B is found the secant of the perpendicular, R, of the astronomical triangle, in colum Z' is found the value of Z' in degrees and tenths of degrees, and in column K is found the value of K i degrees and minutes. *If t is greater than 90°, $K = 180° - K'$.* For example, in Problem II, K = 180° − 73° 25′.2 = 106° 34′.8.
7. Record B, K, and Z' in the proper spaces in the form.
8. ***Give K same name as latitude.*** **Combine K with declination to obtain (K∼d), adding K an d if *different* names and subtracting the smaller from the larger if *same* name.**
9. Enter Table II with (K∼d) and take from the B and A columns the *nearest tabulated function Do not interpolate.*
10. Add the B function of (K∼d) to the B function previously taken from Table I as shown on form
11. With the function thus obtained, enter A column and take angle H_c and the corresponding B function from Table II.
12. The difference between the computed (H_c) and the observed (H_o) altitudes is the altitude differ ence, a; a is measured from the assumed position *toward* the body when H_o is greater than H_c, an *away* when smaller. **The line of position is plotted from the assumed position defined by the assume longitude and the integral degree of assumed latitude.**
13. To compute Z'', subtract the A and B functions as shown in the form and with the A functio thus obtained, enter the A column and take Z'' from Table II. *Z'' never exceeds 90°.*
14. **To obtain Z, *always add Z' and Z'', except when K is same name and less than declination o when K is greater than 90°, in which case subtract the smaller from the larger angle.* Z is measure from the elevated pole 0° to 180° east or west to the body.**

Accuracy

This method gives an accuracy of solution to within five-tenths of a minute in altitude withou interpolation. Greater accuracy is not required in practical sea and air navigation.

UTION

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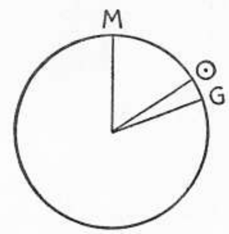
112° 46'

Problem I. At 7:31 a.m., on December 31, 1941, the *Pecos* was in D. R. position, latitude 42° 10′ 30″ S, longitude 68° 37′ 42″ W. Observed the sun on the prime vertical: W $7^h\ 31^m\ 05^s$, Watch error on zone time 11^s slow, H_s 35° 47′, Height of eye 27 feet. Time element computed by GHA method of *Nautical Almanac.*

	h m s
W	7– 31 –05
WE	s 0 –11
ZT	7– 31 –16
ZD	+5
GCT	12– 31 –16
Tab.	359°–15′.1
31^m	7 –45 .0
16^s	4 .0
GHA	7 –04 .1
aλ	69 –04 .1 W
LHA	298 –00 .0
t	62 E
aL	42 S
d	23 –06 .4 S
K	62 –27 .7 S
K∼d	39 –21 .3
H_c	35 –41 .5
H_o	35 –57 .0
a	Toward 15 .5

(31)



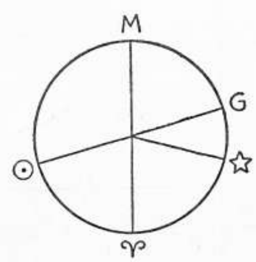
H_s	35°–47′.0
Corr	+ 9 .7
⊙Sub	+ 0 .3
H_o	35 –57 .0

B	**12227**			Z′	38°.5
B	11171	A	19779		
ADD					
A	23398	B	9035		
		SUBTRACT			
		A	10744	Z″	51 .3
		Z_n	090.2	Z	S 89 .8 E

Problem II. On May 15, 1941, about 7:34 p.m., the *Augusta* while in D. R. position, latitude 40° 43′ N, longitude 68° 30′ W, observed the star Vega as follows: W $7^h\ 34^m\ 14^s$, C–W $4^h\ 59^m\ 12^s$, Chronometer slow $1^m\ 01^s.2$. H_s 14° 45′ 40″. Height of eye 35 feet. IC 0′ 00″. Time element computed by the GST method of the *Nautical Almanac.*

	h m s
W	7– 34 –14
C–W	4– 59 –12
C	12– 33 –26
CC	s 1 –01 .2
GCT	0– 34 –27 .2
RA⊙+12	15– 33 –15 .6
Corr	0 –05 .7
GST	16– 07 –48 .5
RA☆	18– 34 –58 .4
GHA : t	21– 32 –50 .1
GHA : °	323°–12′.5
aλ	68 –12 .5 W
LHA	255 –00 .0
t	105 E
aL	41 N
d	38 –43 .7 N
K	106 –34 .8 N
K∼d	67 –51 .1
H_c	14 –57 .5
H_o	14 –36 .3
a	Away 21 .2

(16)



H_s	14°–45′.7
Corr	(–) 9 .4
H_o	14 –36 .3

B	**16462**			Z′	(–) 22°.2
B	42362	A	3329		
ADD					
A	58824	B	1497		
		SUBTRACT			
		A	1832	Z″	73 .5
		Z_n	051.3	Z	N 51 .3 E

K = 180° – K′ = 180° – 73° 25′.2 = 106° 34′.8.