

Using Chauvenet's notations:

- h': apparent height of the center of the Moon
- H': apparent height of the center of the Sun
- d': apparent distance between the centers of the Moon and the Sun

and adapting these notations in the following way to be used here:

- $[\delta]h \rightarrow dh$
- $[\delta]H \rightarrow dH$
- $[\delta]d \rightarrow dd$

Assuming  $dH = 0$ , the equation (g) on page 404 become:

$$\begin{aligned} dd \sin(d' + dd/2) &= dh \sin(h' + dh/2) \cos d' / \cos h' - dh \sin(2H') / 2 \cos h' / \cos H' \\ \sin(2H') &= 2 \sin H' \cos H' \Rightarrow \\ dd \sin(d' + dd/2) &= dh \sin(h' + dh/2) \cos d' / \cos h' - dh \sin H' / \cos h' \end{aligned}$$

The angles  $dd$  and  $dh$  are small. Let's put  $S = \sin 1' = 0.00029$  if they are expressed in minutes ( $S = \sin 1''$  if they are expressed in seconds). The equation can be written:

$$dd (\sin d' + S dd \cos d'/2) = dh (\sin h' + S dh \cos h'/2) \cos d' / \cos h' - dh \sin H' / \cos h'$$

By developing and then reorganizing:

$$\begin{aligned} dd \sin d' + S dd^2 \cos d'/2 &= dh \sin h' \cos d' / \cos h' + S dh^2 \cos d'/2 - dh \sin H' / \cos h' \\ dd \sin d' + S dd^2 \cos d'/2 &= dh (\sin h' \cos d' - \sin H') / \cos h' + S dh^2 \cos d'/2 \end{aligned}$$

$$dh = HP \cos h' \Rightarrow$$

$$dd \sin d' + S dd^2 \cos d'/2 = HP (\sin h' \cos d' - \sin H') + S HP^2 (\cos h')^2 \cos d'/2$$

Dividing by  $\sin d'$ :

$$dd + S dd^2 \cot d'/2 = HP (\sin h' \cos d' - \sin H') / \sin d' + S HP^2 (\cos h')^2 \cot d'/2$$

By setting  $y = (\sin h' \cos d' - \sin H') / \sin d'$ :

$$dd + S dd^2 \cot d'/2 = HP y + S HP^2 (\cos h')^2 \cot d'/2$$

To the first order we have (see page 408):  $dd = HP y \Rightarrow dd^2 = HP^2 y^2 \Rightarrow$

$$dd = HP y + S HP^2 (\cos h')^2 \cot d'/2 - S HP^2 y^2 \cot d'/2$$

By factoring:

$$dd = HP y + S HP^2 [(\cos h')^2 - y^2] \cot d'/2$$

$$dd = HP \{y + S HP [(\cos h')^2 - y^2] \cot d'/2\}$$

Finally, since  $HP$  and  $dd$  are expressed in minutes:

$$dd = HP \{y + 0.000145 HP [(\cos h')^2 - y^2] \cot d'\}$$