

AF MANUAL 51-40
VOLUME I



AIR
NAVIGATION

DEPARTMENT OF THE AIR FORCE

Sextants and Errors of Observation

A celestial LOP is determined by measuring the altitude of a celestial body with a sextant. The sextants now in general use are of two types: (1) marine sextants, which measure altitude above the visible horizon, and (2) aircraft bubble sextants, which measure altitude above the artificial horizon; that is, above the celestial horizon as established by a leveling device (the bubble). The marine sextant is not used for aerial navigation and, consequently, is not discussed in this manual.

Sextants are subject to certain errors that must be compensated for when determining LOPs. Some of these errors are corrected by adjusting the H_s or H_c , and others are corrected by adjusting the LOP or assumed position. Most of the material presented in this chapter is devoted to the explanation of the causes of these errors and the necessary corrections.

THE BUBBLE SEXTANT

The aircraft bubble sextant measures altitude above a horizontal plane established by a bubble. The Air Force uses several types of bubble sextants, all of which are indirect sighting. This means the navigator does not look directly toward the celestial body, but always looks in a horizontal direction as shown in figure 15-1. The image of the body is reflected into the field of view when the field prism is set at the correct angle. In the bubble sextant, the bubble and body are visible in the same field of view.

Accurate observations can be obtained from the various bubble sextants by using proper collimation techniques, and by using the proper size bubble. Collimation is effected when the body is placed in the center of the bubble, or horizontally

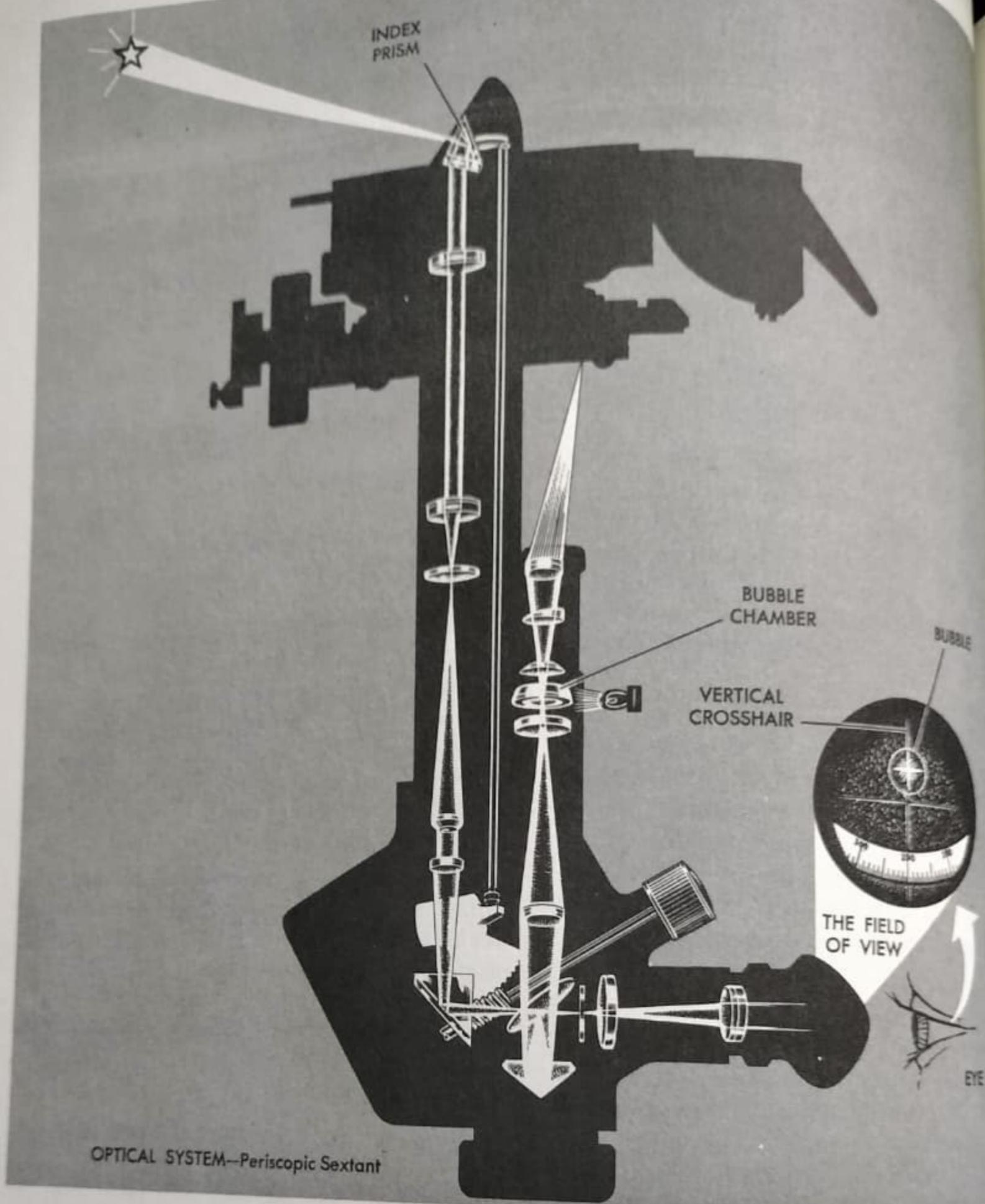
centered alongside the bubble if it is the opaque type. For greatest accuracy, the bubble should be in the center of the field. The error will be small if the bubble is anywhere on the vertical line of the field, as long as it does not touch the top or bottom of the bubble chamber. Figure 15-2 shows examples of correct and incorrect collimation.

Bubble size affects the accuracy of a sextant observation. The ideal situation is to have a small bubble for ease in determining the center. However, too small a bubble is sluggish, and it is better to have a bubble that is active. Experience has shown that best results can be obtained with a bubble approximately one and a half times the apparent diameter of the sun or moon.

The field prism is geared to an altitude scale so that when the body is collimated the altitude can be read from the scale. An averaging mechanism is also incorporated which allows the navigator to take a series of observations over a period of time. This is more accurate than relying on a single observation.

ERRORS OF SEXTANT OBSERVATION

If collimation of the body with the bubble and reading the sextant were all that had to be done, celestial navigation would be simple. This would mean that LOPs accurate to within one or two miles could be obtained without any further effort. Unfortunately, considerable errors are encountered in every sextant observation made from an aircraft. A thorough understanding of the cause and magnitude of these errors, as well as the proper application of corrections to either the H_c or H_s , will help minimize their effect. (Any correction applied to the H_s may be applied instead to



OPTICAL SYSTEM—Periscopic Sextant

Figure 15-1. Body is Not Sighted Directly

H_c with a reverse sign.) Accuracy of celestial navigation is, therefore, dependent upon thorough application of these corrections, together with proper shooting techniques.

The errors of sextant observation may be classified into four groups: (1) parallax error, (2) refraction errors, (3) instrument errors, and (4) acceleration errors.

Parallax Error

Parallax in altitude is the difference between the altitude of a body above a bubble horizon at the surface of the earth, and its calculated altitude above the celestial horizon at the center of the earth. All H_c 's are given for the center of the earth. If the light rays reaching the center

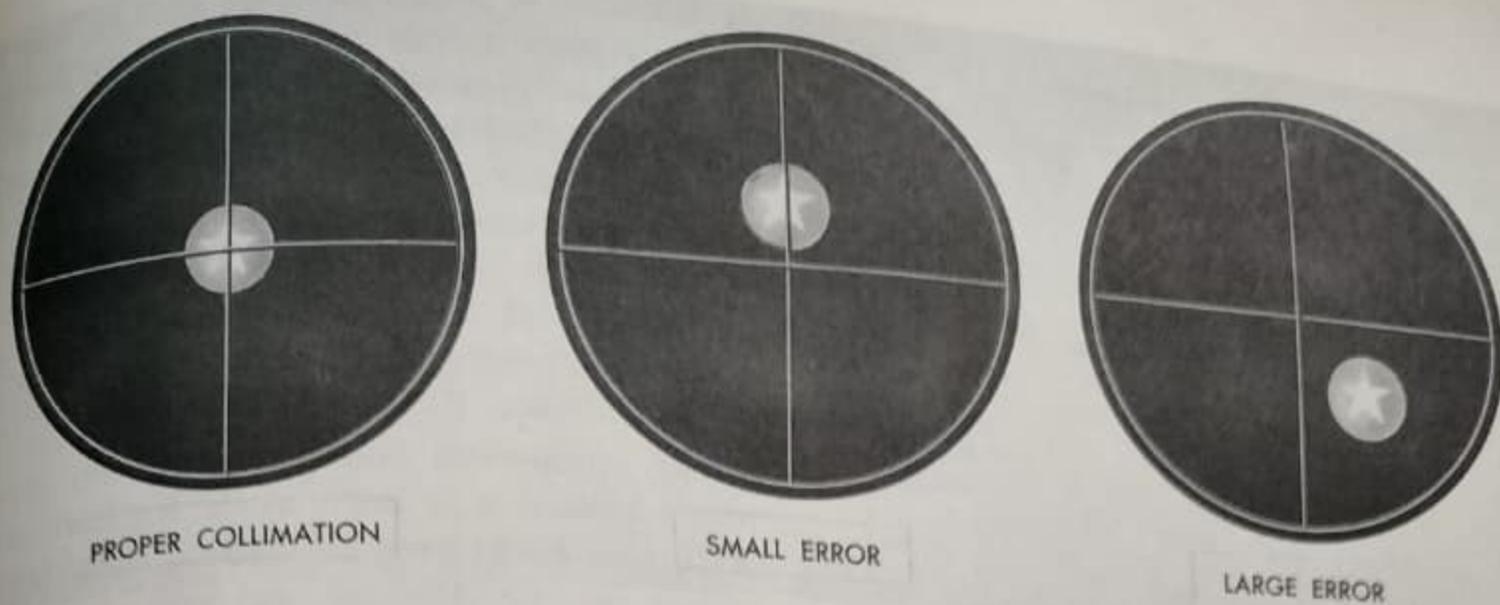


Figure 15-2. Correct and Incorrect Collimation

from a celestial body are parallel, the body has the same altitude at both the center and the surface of the earth. For most celestial bodies, therefore, parallax is negligible for purposes of navigation.

CORRECTION FOR THE MOON. This does not apply to the moon because it is so close to the earth that its light rays are not parallel. The parallax of the moon may be as great as 1° ; thus, when observing the moon, a parallax correction must be applied to the Hs. Figure 15-3 shows that the moon appears at a lower altitude from the surface of the earth than it would appear from the center of the earth; therefore, the correction is always plus. The amount of this correction varies with the altitude and with the distance of the moon from the earth. The correction varies from day to day because the distance of the moon from the earth varies. On each daily sheet in the *Air Almanac*, there is a table of the moon's parallax for all observed altitudes.

SEMIDIAMETER CORRECTION. Another correction found on the daily pages of the *Air Almanac* is the semidiameter correction, which is the apparent radius of the sun or moon. It is needed when shooting the upper or lower limb of the moon or the sun.

It is more likely to occur on observations of the moon, because when the moon is not full (completely round), the center is difficult to estimate. Therefore, the navigator observes either the upper or lower limb and applies the semidiameter correction listed in the almanac on the lower right-hand side of the daily pages. If the upper limb is observed, subtract the correction

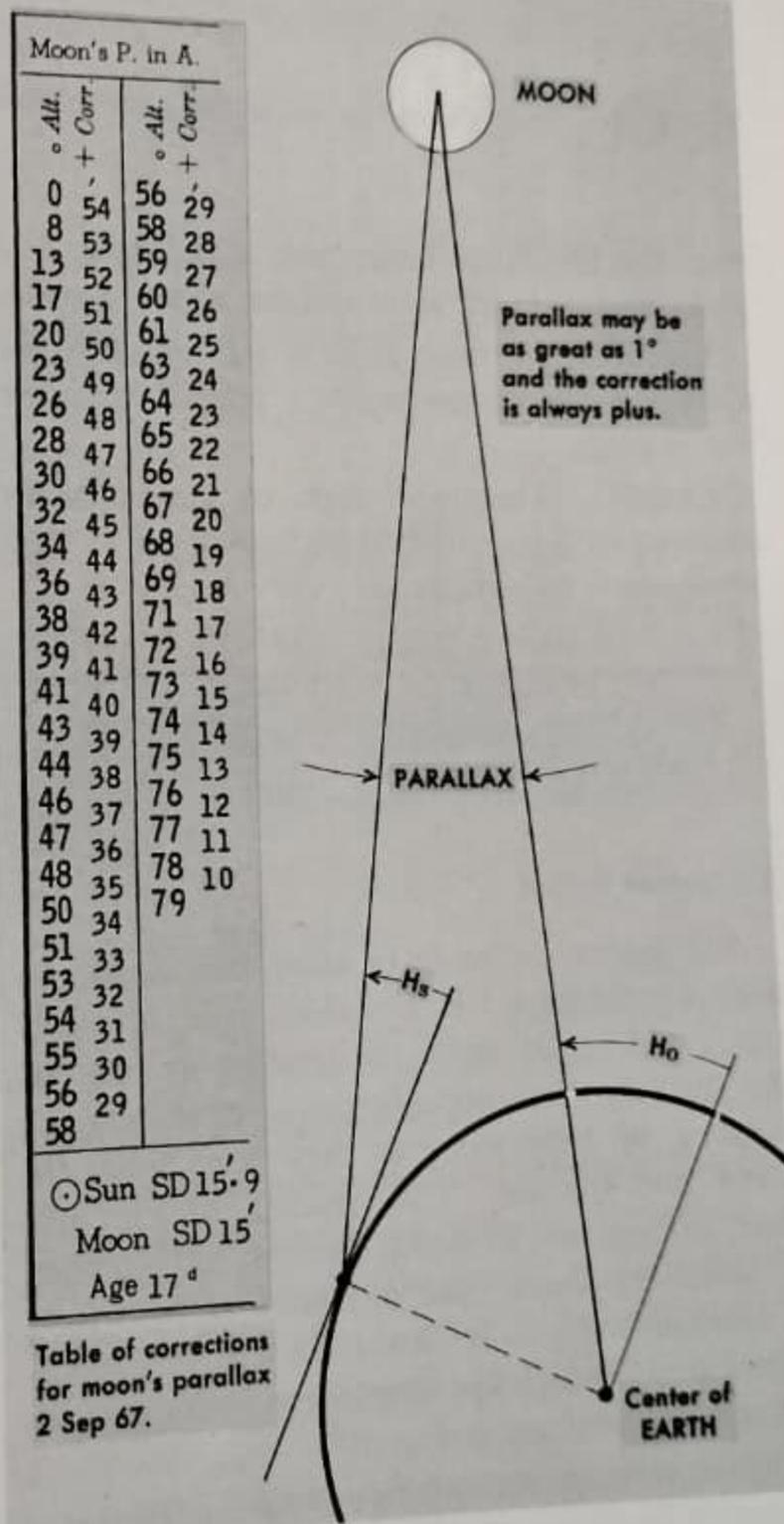


Figure 15-3. Correction for Moon's Parallax

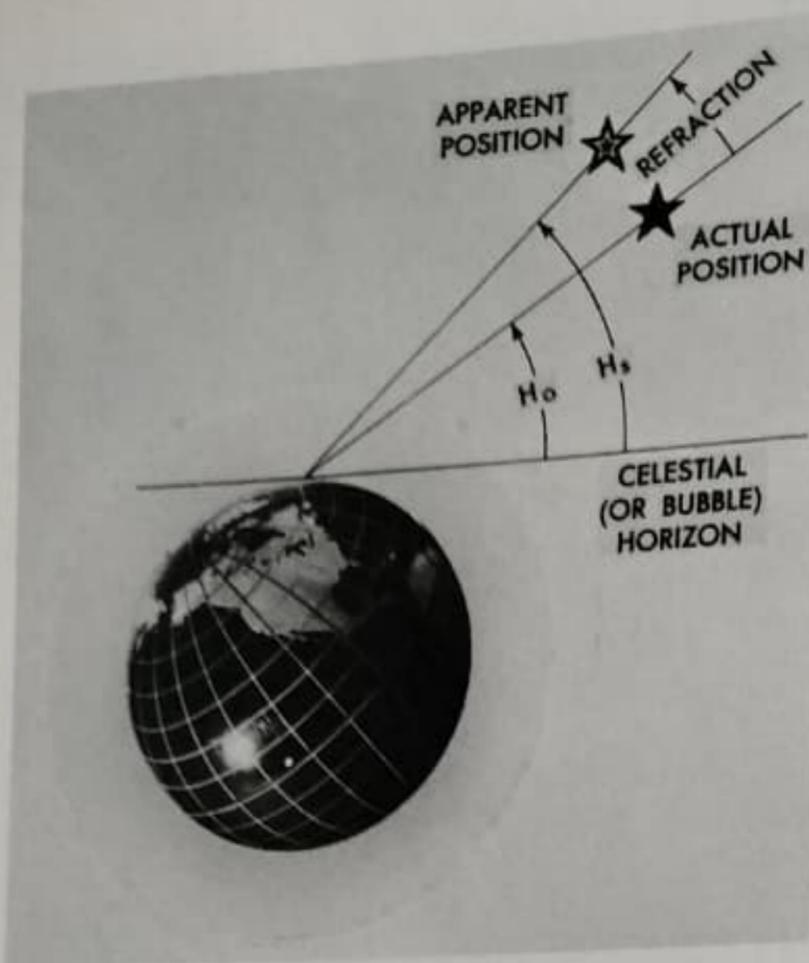


Figure 15-4. Error Caused by Atmospheric Refraction

from the H_s ; if the lower limb is observed, add the correction to the H_s or subtract it from the H_c .

Listed on the same page is the semidiameter correction for the sun, which is applied the same way if used.

Example. The upper limb of the moon as observed on 2 Sep 1966 at 30,000 feet is $33^\circ 41'$. Apply these corrections as:

H_s	$33^\circ 41'$
Parallax	$+45'$
Semidiameter	$-15'$
H_o	$34^\circ 11'$

Refraction Errors

Still another factor to be taken into consideration is refraction. If a fishing pole is partly submerged under water, it appears to bend at the surface. This appearance is caused by the bending of light rays as they pass from the water into the air. This bending of the light rays, as they pass from one medium into another, is called *refraction*. The refraction of light from a celestial body, as it passes through the atmosphere and through the astrodome, causes errors in sextant observation.

ATMOSPHERIC REFRACTION. As the light of a celestial body passes from the almost perfect vacuum of outer space into the atmosphere, it

is refracted as shown in figure 15-4 so that the body appears a little higher above the horizon than it really is. Therefore, the correction to the H_s for refraction is always minus. The higher the body above the horizon, the smaller the amount of refraction and, consequently, the smaller the refraction correction. Moreover, the greater the altitude of the aircraft, the less dense the layer of atmosphere between the body and the observer, hence, the less the refraction.

The appropriate correction table for atmospheric refraction is listed inside the back cover of all four books used for celestial computations, namely the *Air Almanac*, and each of the three volumes of H.O. 249. This table, shown in figure 15-5, lists the refraction for different observed altitudes of the body and for different heights of the observer above sea level. The values shown are subtracted from H_s or added to H_c .

DOMES REFRACTION. Light rays from a celestial body are usually refracted as they pass through the plastic astrodome. The sextant supporting arm, furnished with the instrument, holds the sextant in the correct position in the astrodome to reduce this dome refraction to a minimum.

The amount of dome refraction depends on the shape and construction of the particular dome. Domes are far from being optically perfect, and since they are somewhat flexible, their shape may be altered at the time of installation. Therefore, dome refraction is unpredictable.

The dome can be calibrated by taking observations through it from a known position on the earth. Dome refraction errors as large as $30'$ have been reported. Dome refraction is usually posted on a calibration card on or near the dome. The *Air Almanac* has a table of dome refractions for a "standard" dome (not applicable to the periscopic sextant). Since there are many types of domes in use in the Air Force, always use the calibration card if one is attached to the dome.

Instrument Errors

Certain errors of observation are a result of mechanical faults in the sextant. An aircraft sextant is a delicate instrument which can easily get out of alignment.

INDEX ERRORS. Hard knocks or jars can change the alignment of the index and field prism, thereby causing index error. Some sextants have index errors posted on the sextant or on the sextant case.

Acceleration Errors

Presently, the only practical and continuously available reference datum for the definition of the true vertical is the direction of the gravitational field of the earth. Definition of this vertical establishes the artificial horizon. It is also fundamental that the forces caused by gravity cannot be separated from those caused by accelerations by any self-contained means within the sextant. A level or centered bubble in the sextant indicates the true vertical only when the instrument is at rest or moving at a constant velocity in a straight line. Any outside force (changes in groundspeed or changes in track) will affect the liquid in the bubble chamber and, consequently, displace the bubble.

When the sextant is moved in a curved path (Coriolis, changes in heading, rhumb line) or with varying speed, the zenith indicated by the bubble is displaced from the true vertical. This presents a false artificial horizon above which the altitude of the celestial body is measured. Since the horizon used is false, the altitude measured from it is erroneous. Therefore, the accuracy of celestial observations is directly related to changes in track and speed of the aircraft. Acceleration errors have two principal causes: changes in groundspeed and curvature of the aircraft's path in space.

The displacement of the liquid and the bubble in the chamber may be divided into two vectors, and each vector may be considered separately. These vectors may be thought of as a lateral vector (along the wings) and a longitudinal vector (along the nose-tail axis of the aircraft). Any change in groundspeed can cause a longitudinal displacement. This change can be brought about by a change in the airspeed or the wind encountered, or the change in groundspeed brought about by a change in heading due to other factors (gyro precession, rhumb line error, etc.). A lateral displacement results from a number of causes, most of which will occur in spite of any efforts to hold them in check. These causes are termed Coriolis, rhumb line error, and wander errors.

CHANGES IN GROUND SPEED. This error is caused primarily by changes in airspeed or changes in wind velocity. Errors due to changes in groundspeed will normally be small. The computation of the amount of error is based on the assumption that a one-knot error in true airspeed will produce

a one-knot error in groundspeed. If this occurs during a one-minute shooting period, it will cause up to three nautical miles of error in the LOP, irrespective of the speed of the aircraft. The maximum error would occur on a speed line shot.

Wind velocity also causes changes in groundspeed but the amount cannot be determined. This change in groundspeed (acceleration-deceleration) will cause the liquid to be displaced, with the subsequent shifting of the bubble creating a false horizon. Notice in figure 15-6 how the horizon is automatically displaced by keeping the bubble in the center while these changes are taking place.

CORIOLIS FORCE. Any freely moving body traveling at a constant speed above the earth is subject to an apparent force which deflects its path to the right in the Northern Hemisphere, and to the left in the Southern Hemisphere. This apparent force and the resulting acceleration were first discovered shortly before the middle of the nineteenth century by Gaspard Gustave de Coriolis (1792-1843) and given quantitative formulation by Ferrel. The acceleration is known as Coriolis acceleration (or force) or simply Coriolis, and is expressed in Ferrel's law.

The navigator must realize that the bubble sextant indicates the true vertical only when the instrument is at rest or moving at a constant speed in a straight line as perceived in space. If the earth were motionless, this straight path in space would also be a straight path over the surface of the earth; conversely, a straight path over the motionless earth would also be a straight path in space.

When the aircraft is flying a path curved in space to the left, the fluid in the bubble chamber is deflected to the right and the bubble is deflected to the left of the aircraft's path over the earth. When the aircraft is flying a curved path in space to the right, the reverse is true.

In figure 15-7, the aircraft is represented as flying on a curved path to the left. Note that in the inset representing the bubble chamber, the heavy black bubble is indicated in its approximate position representing the true vertical.

However, the observer always seeks to center the bubble, and on this beam shot, facing to the right side of the aircraft to observe the body, he would tip the sextant up. This would tilt the bubble horizon from its true position, producing a smaller sextant reading than the true

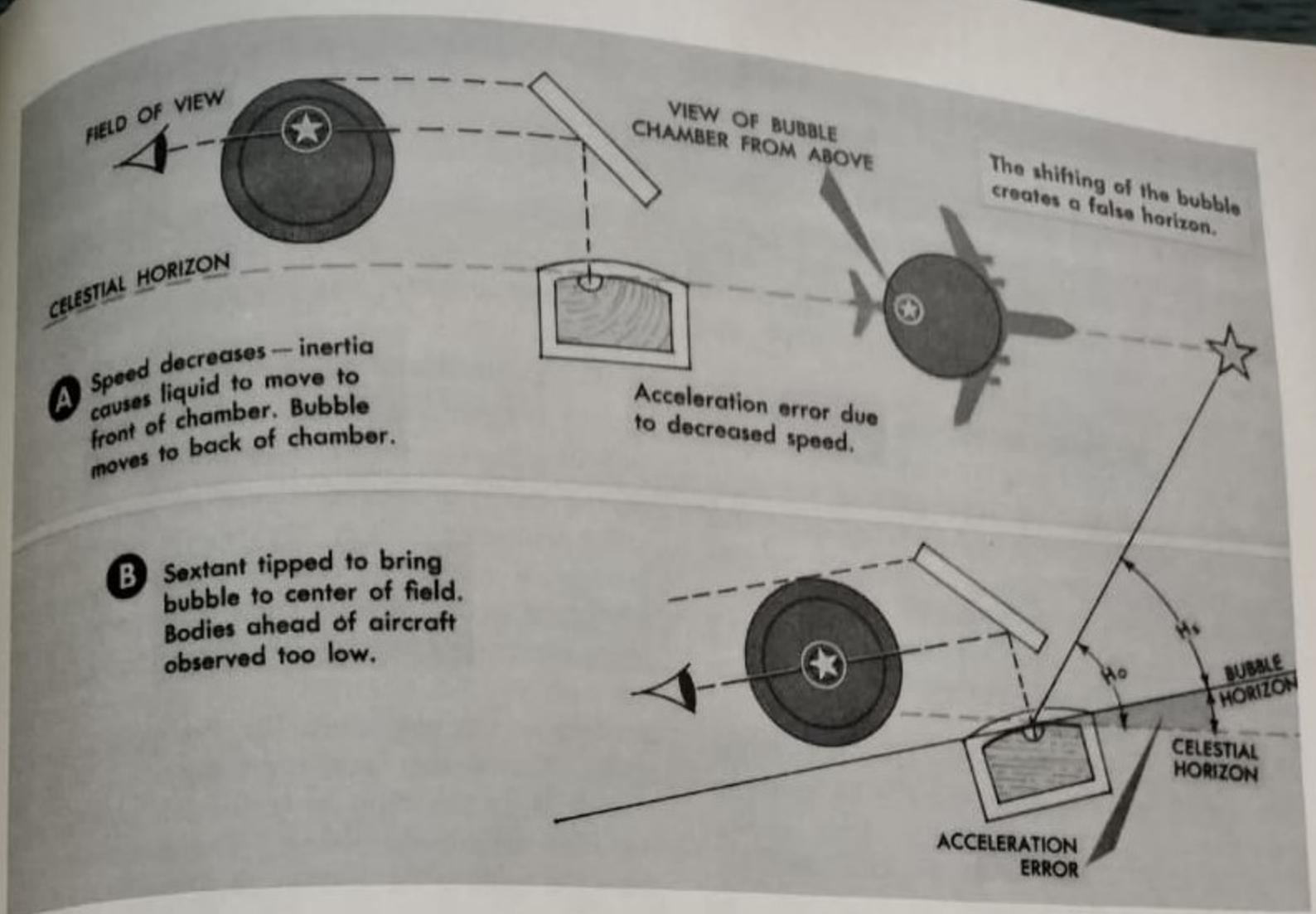


Figure 15-6. Acceleration/Deceleration Errors

value. Following the rule — the smaller the H_o , the greater the radius of the circle of equal altitude—the LOP will fall farther from the subpoint than the true LOP. Obviously, if the erroneous LOP falls farther from the subpoint, it will fall to the left of the true LOP and the correction to the right is valid. Corrections for Coriolis error are shown on the inside back cover of the almanac as well as in all volumes of H.O. 249.

Coriolis acceleration is:

- Directly proportional to the straight line velocity.
- Directly proportional to the angular velocity of the earth.
- Directly proportional to the sine of the latitude.
- At right angles to the direction of motion of the particle, and therefore it can only influence its direction, never its speed.

RHUMB LINE ERROR. As long as a constant true heading is flown, the path of the aircraft will be a rhumb line. Because a rhumb line on the earth's surface is a loxodromic curve, it is also a curved line in space. If the aircraft is headed in a general

easterly direction in the northern hemisphere, the apparent curve is to the left and becomes an addition to the Coriolis error. By the same token, if headed in a westerly direction in the northern

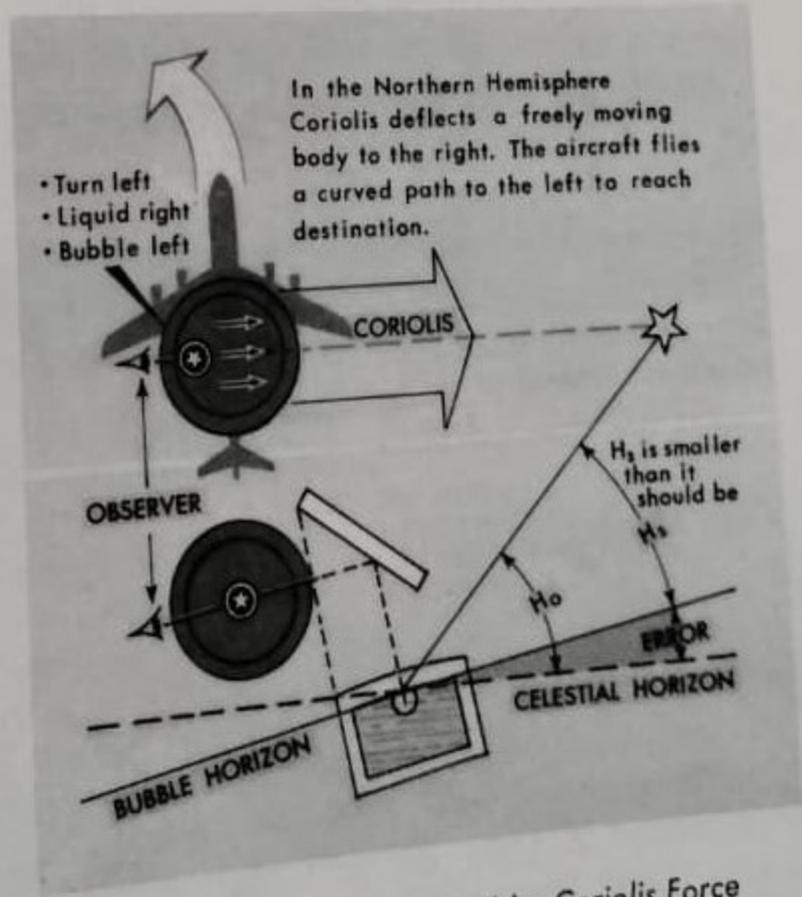


Figure 15-7. Error Caused by Coriolis Force

hemisphere, the apparent curve is to the right, or opposite that of Coriolis force as shown in figure 15-8.

There are notable exceptions to this. When flying north or south, the aircraft is flying a great circle and there is no rhumb line error. Also, when steering by a free running compensated gyro, the track approximates a great circle and eliminates rhumb line error.

Prior to the wide use of high speed aircraft, the rhumb line error was not considered, because at speeds under 300 knots the error is negligible. However, at high speeds or high latitudes, rhumb line error is appreciable. For example, at 60° north latitude with a track of 100° and a groundspeed of 650 knots, the Coriolis correction is 15 nautical miles right and the rhumb line correction is 10 nautical miles right.

The correction for rhumb line error is included with the Coriolis correction and can be found in the combined Coriolis/rhumb line table shown in figure 15-9. These tables are computed for each 50 knots of groundspeed.

WANDER ERROR. A change in track can be produced by changes in the wind velocity, by heading changes between limits of the autopilot, by heading changes produced from changing magnetic variation, and by heading changes derived from pilot manual steering errors. As with the Coriolis force and rhumb line errors, correction tables have been developed for wander error. Values extracted from the wander correction table, shown in figure 15-10, are to be applied to the Ho.

Use the following information as entering argu-

ments for the determination of the correction taken from the table.

The heading at the beginning of the observation was 079.3°.

The heading at the end of the observation was 081.3°.

The observation was taken over a two-minute period.

The groundspeed was 450 knots.

The true azimuth of the body was 130°.

Following the instructions shown at the bottom of the table, enter the numerical portion of the change of track per two minutes. In this case, the groundspeed is 450 knots and the change in track per two minutes is 2°. Since the heading at the end of the observation is greater than the heading at the beginning, the change is 2° to the right. Notice that you must know whether the change is to the right or to the left to determine the sign of the correction. The factor obtained from the table is $12 \times 2 = 24$.

Next, enter the graph portion of the table with the value of the factor (24) and the value of the azimuth of the body minus the value of track. The graph is so constructed that it must be entered with $Z_n - Tr$. In this case, the azimuth is 130° minus a track of 080°; the value thus determined is 050°. Following the rules set down in steps two and three at the bottom of the table, the correction found is 19'. Since the change in track is to the right, the correction is to be subtracted from the Ho. This is determined by referring to the signs shown at the ends of the arc

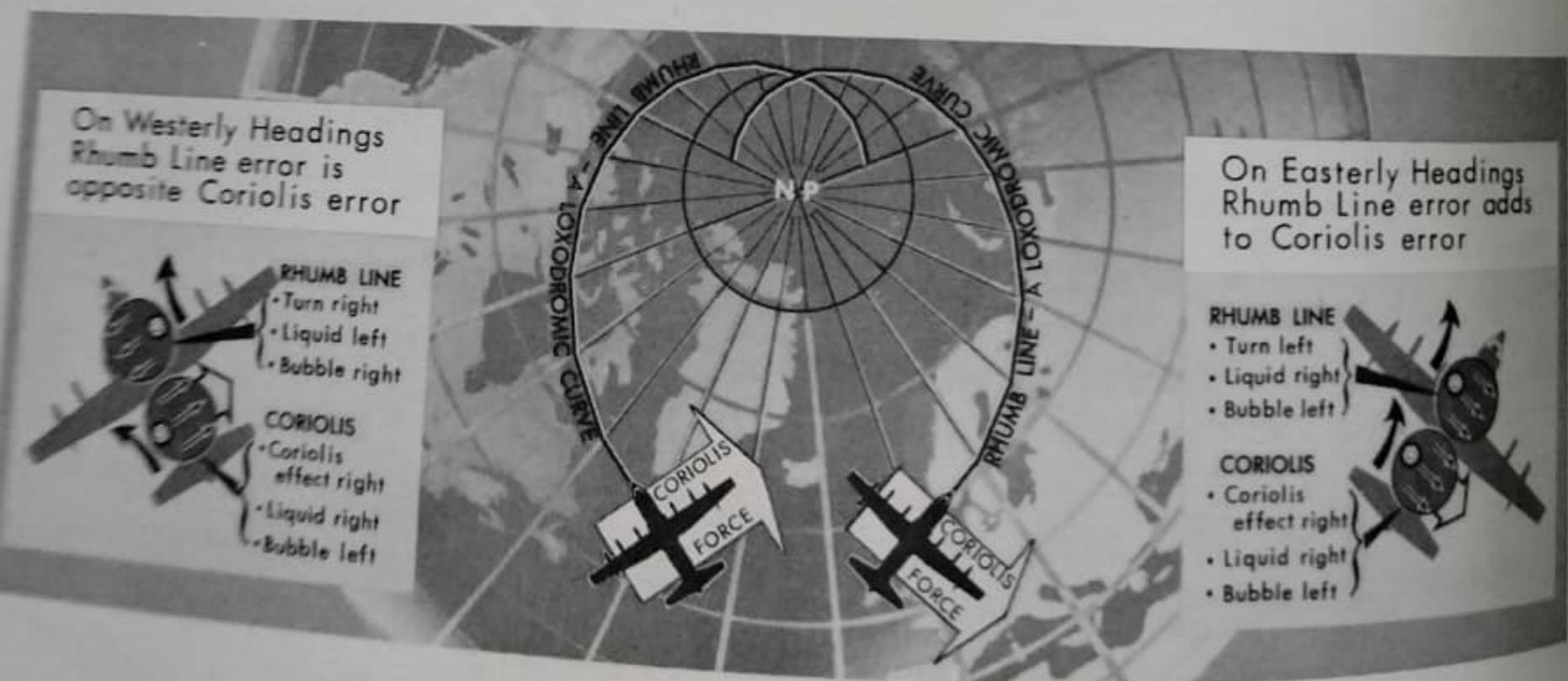


Figure 15-8. Coriolis/Rhumb Line Errors in Northern Hemisphere

GROUND SPEED 300 KNOTS

TR. →	GROUND SPEED 300 KNOTS																		
	270	260	250	240	230	220	210	200	190	180	170	160	150	140	130	120	110	100	90
LAT. ↓	270	280	290	300	310	320	330	340	350	0	10	20	30	40	50	60	70	80	90
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	1.1	1.1	1.1	1.2	1.2	1.2	1.2	1.3	1.3	1.4	1.4	1.4	1.5	1.5	1.6	1.6	1.6	1.6	1.6
20	2.2	2.2	2.2	2.3	2.3	2.4	2.4	2.5	2.6	2.7	2.8	2.8	2.9	3.0	3.1	3.1	3.1	3.2	3.2
30	3.2	3.2	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.1	4.2	4.3	4.4	4.5	4.6	4.6	4.7	4.7
40	4.0	4.0	4.0	4.1	4.2	4.4	4.5	4.7	4.9	5.1	5.2	5.4	5.6	5.8	5.9	6.0	6.1	6.1	6.2
50	4.5	4.5	4.5	4.7	4.8	5.0	5.2	5.5	5.7	6.0	6.3	6.6	6.8	7.0	7.2	7.4	7.5	7.6	7.6
60	4.6	4.6	4.7	4.9	5.1	5.4	5.7	6.0	6.4	6.8	7.2	7.6	7.9	8.2	8.5	8.8	8.9	9.0	9.0
70	3.8	3.8	4.0	4.3	4.6	5.1	5.6	6.1	6.8	7.4	8.0	8.6	9.2	9.7	10.2	10.5	10.8	10.9	11.0
80	0.3	0.4	0.8	1.3	2.0	3.0	4.0	5.2	6.4	7.7	9.0	10.3	11.5	12.5	13.5	14.2	14.7	15.0	15.1
89	67.2	66.1	62.6	57.4	50.1	40.2	30.0	17.9	5.3	7.9	21.0	33.6	45.7	55.9	65.8	73.1	78.3	81.8	82.9

GROUND SPEED 650 KNOTS

TR. →	GROUND SPEED 650 KNOTS																		
	270	260	250	240	230	220	210	200	190	180	170	160	150	140	130	120	110	100	90
LAT. ↓	270	280	290	300	310	320	330	340	350	0	10	20	30	40	50	60	70	80	90
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	1.9	1.9	1.9	2.0	2.1	2.3	2.4	2.6	2.8	3.0	3.1	3.3	3.5	3.7	3.8	3.9	4.0	4.0	4.0
20	3.6	3.6	3.7	3.9	4.1	4.4	4.7	5.1	5.4	5.8	6.2	6.6	7.0	7.3	7.6	7.7	7.9	8.0	8.1
30	5.0	5.0	5.2	5.4	5.8	6.3	6.7	7.3	7.9	8.5	9.1	9.7	10.3	10.8	11.2	11.6	11.8	12.0	12.1
40	5.8	5.9	6.1	6.5	7.0	7.7	8.4	9.2	10.1	11.0	11.8	12.7	13.5	14.2	14.9	15.5	15.8	16.0	16.1
50	5.7	5.8	6.1	6.6	7.4	8.3	9.3	10.5	11.8	13.0	14.3	15.6	16.7	17.7	18.6	19.4	19.9	20.2	20.3
60	4.2	4.3	4.8	5.6	6.6	8.0	9.4	11.1	12.9	14.7	16.5	18.3	20.0	21.5	22.8	23.9	24.6	25.1	25.2
70	0.9	0.6	0.2	1.3	3.0	5.2	7.5	10.2	13.1	16.0	19.0	21.8	24.5	26.8	29.0	30.7	31.8	32.6	32.9
80	18.0	17.5	15.9	13.6	10.1	5.6	0.8	4.8	10.7	16.8	22.8	28.7	34.3	39.1	43.6	47.1	49.4	51.0	51.5
89	334.8	329.2	313.0	288.9	254.5	208.2	160.1	103.6	44.6	17.0	78.7	137.6	194.1	242.2	288.5	322.9	347.0	363.2	368.8

Figures in **BOLD FACE** type are plotted in a direction opposite to that of coriolis force.

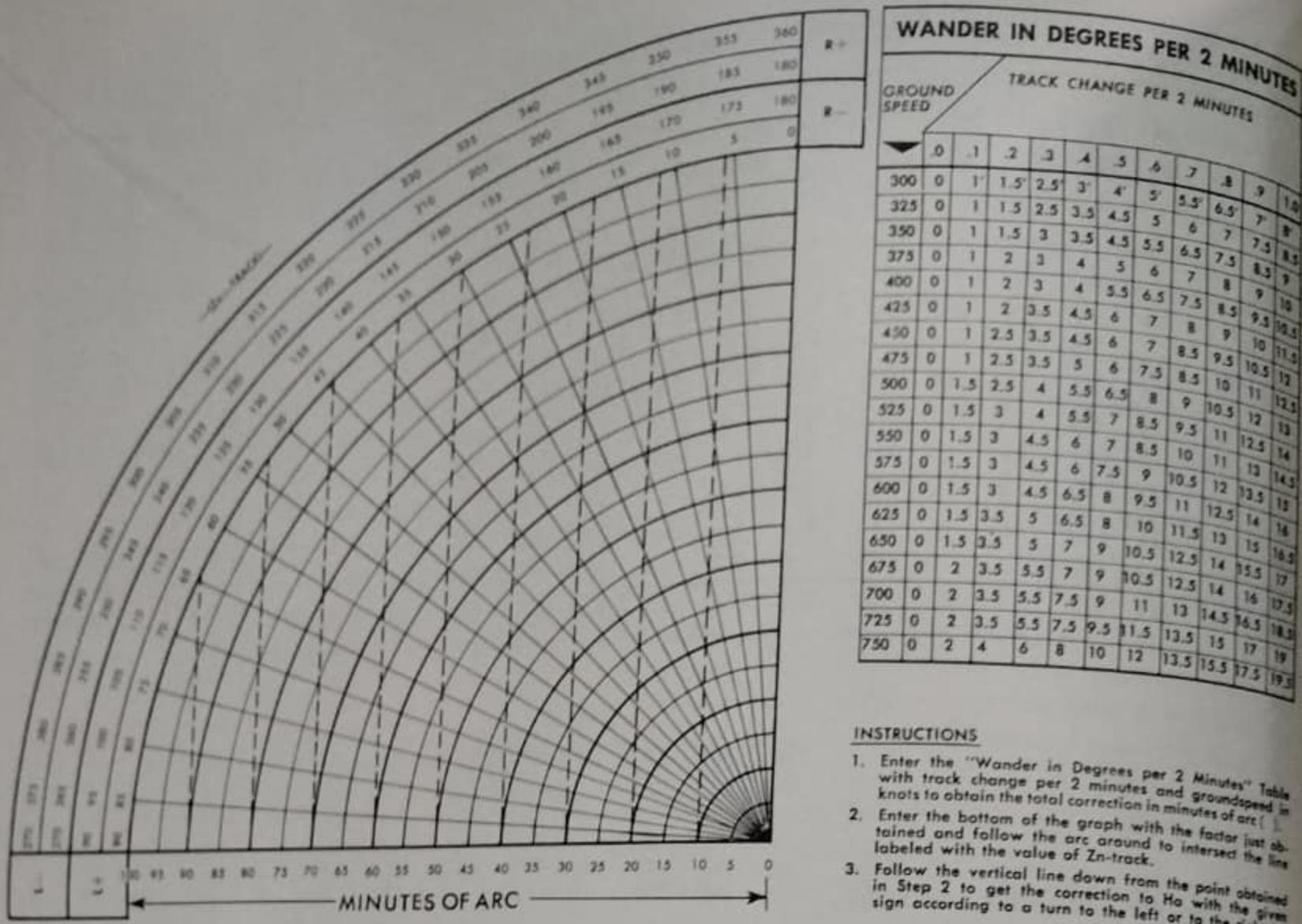
*Coriolis corrections alone are the figures in the 0° or 180° column.

Figure 15-9. Combined Coriolis and Rhumb Line Correction

in the table. Figure 15-11 shows the effect of this correction.

If the heading and airspeed are the same at the beginning and the end of a shooting period, there will be no wander error. This is assuming

that a change in heading produces an equal change in track, and a change in airspeed produces an equal change in groundspeed. Further, this only applies if the body is continuously collimated during the observation time.



INSTRUCTIONS

1. Enter the "Wander in Degrees per 2 Minutes" Table with track change per 2 minutes and groundspeed in knots to obtain the total correction in minutes of arc.
2. Enter the bottom of the graph with the factor just obtained and follow the arc around to intersect the line labeled with the value of Zn-track.
3. Follow the vertical line down from the point obtained in Step 2 to get the correction to Ho with the given sign according to a turn to the left or to the right.

Figure 15-10. Wander Correction Tables

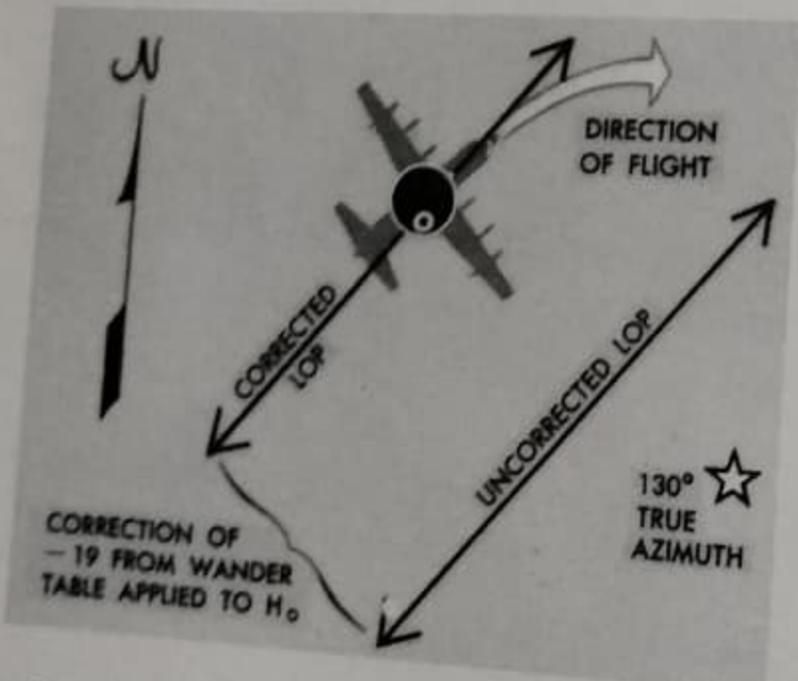


Figure 15-11. Wander Correction Applied to Ho

The amount of force applied over a given time to displace the liquid, and subsequently the bubble, in one direction will be equalized

by an amount of force over a given time in the opposite direction to bring the aircraft back to its original heading. Therefore, if the heading (track) and airspeed (groundspeed) are the same at the beginning and end of a shooting, there will be no acceleration error caused by heading or airspeed changes, and no correction is necessary for wander or groundspeed change (a constant wind is assumed over the shooting period).

MINIMIZING ACCELERATION ERRORS. Errors due to acceleration are minimized by the following procedures:

1. Maintain aircraft heading, or desired change of heading, and airspeed as constant as possible (maintenance of track and groundspeed during a shooting period).
2. Average the altitude over a period of time.
3. Apply corrections for known acceleration errors to all observations.