

latitude and declination are of contrary names, or subtracting if of the same name, to get the value  $K \sim d$ .

as the calculated altitude. The difference between this altitude and the true altitude found with the sextant gives the altitude intercept 14 miles. Since the true altitude is greater than the calculated, the direction is toward the observed body.

**Azimuth.**—The azimuth is taken from the diagram shown in Fig. 217c. On the left-hand side of the page enter with  $18^\circ$ , cross on this horizontal line until it intersects the curved line of declination  $21^\circ$ , pass up this line in a vertical direction until the altitude curve  $33^\circ$  is intersected, then pass horizontally to the right side of diagram, and thus read the azimuth  $20^\circ$ . Since the sun bore southeast, the azimuth is S.  $20^\circ$ E.

Table with columns: H.A., 56° 30', 57° 00', 57° 30', 59° 00', 59° 30', H.A. Rows contain numerical data for various celestial observations.

Table with columns: 160, 165, 170, 175, 180. Rows contain numerical data for various celestial observations.

FIG. 221.—Page from "H.O. 214."

With the value  $K \sim d$ , turn to Table B (Fig. 217b) and with the value to the nearest minute,  $55^\circ 43'$ , find the number 24927, and add together and  $B$  to get 26437. Now, entering Table B (Fig. 217b) at the bottom and looking in the column for this same number, the altitude is found to be

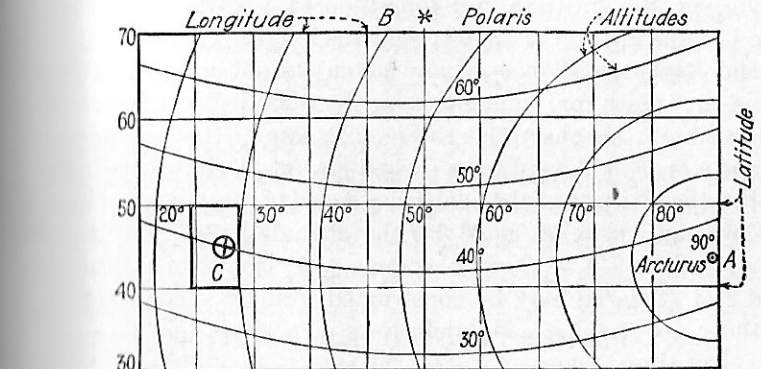


FIG. 222.—Substellar points.

**For Meridian Altitude.**—If the LHA is 0, the sun is on the meridian, and  $A = 0$ , and

$K = \text{latitude}$

Since  $0 + B = B$ , it is only necessary to enter Table B with  $K \sim d$  and pick out from  $B$  the  $H_c$  direct. It is not necessary in this case to write down  $A$ ,  $B$ , or  $\log H_c$ . Also since the body is on the meridian, the azimuth is not required, and the  $a$  is applied to the assumed latitude to get the latitude direct.

**Form Used.**—The column form of work sheet as shown will be found convenient in the air, since several sights can be worked on one page and the form need not be written each time. Also since similar terms in all sights appear on the same line, a ready check on the work is afforded. It is customary to make the small correction for refraction mentally.

**"Star Altitude Curves."**—From any given position on the earth, at any given instant of sidereal time, there is only one possible altitude for each fixed star. The "Star Altitude Curves" take advantage of this fact to do in advance a great part of the work otherwise required of the navigator.

The simultaneous altitudes of two stars, together with the Greenwich sidereal time of observation, definitely determine a point on the earth's surface. This may be put in graphic form by plotting the altitudes against the latitude and local sidereal time. The simultaneous altitudes thus determine by the curves a latitude and a corresponding local sidereal time. The local sidereal time found from the curves combined with the Greenwich sidereal time gives the longitude of the observer. When both latitude and longitude are determined without reference to the dead-reckoning position, right ascension, declination, hour angle, or azimuth. No plotting whatever is required to obtain a fix. The same computation for the latitude and longitude of a definite position is reduced to one subtraction of time to find the longitude.

**Graphical Representation.**—At any given instant a star is directly over (*i.e.*, in the zenith for) some point on the earth called the substar point. On a small-scale chart such as Fig. 222, assume that the substar point for one star is at *A*, and for a second star at *B*, shown just off the chart. At *A* the altitude of the zenithal star is  $90^\circ$ , and at a distance of 600 miles (60 nautical miles equal  $1^\circ$ ) the altitude is  $80^\circ$ , and at 1200 miles the altitude is  $70^\circ$ , etc. In the same way, curves of altitudes for the second star (Polaris) may be constructed from *B*. These circles of equal altitudes are nothing more than lines of position laid down for a given instant of time.

Referring again to Fig. 222, suppose the altitude of star *A* is observed to be  $28^\circ$ , and of star *B*,  $40^\circ$ ; then the intersection of these two altitude circles at *C* is the observer's position. There are two positions possible which are on the same two circles of altitude, but only the one for which the curves are constructed will give the proper value of the local sidereal time.

If the star *A* of  $0^\circ$  declination is on the prime vertical with the observer on the equator, its altitude at point *C* will increase at the rate of  $15^\circ$  every 4 min. of elapsed sidereal time. The star *A* changes altitude from  $0^\circ$  to  $90^\circ$  in 6 hr., therefore, 1 hr. equals  $15^\circ$ , or 4 min. equals  $1^\circ$ . Four minutes after the altitude of star *A*, as observed from *C*, is  $28^\circ$ , it will have increased to  $29^\circ$ , and 4 min. later, to  $30^\circ$ , etc. This may be graphically represented by having the time scale increase toward the right as shown in Fig. 222. Instead of considering that the altitude increases for the passing of time, picture the time increasing, as represented by the local-sidereal-time scale, for greater altitudes. For other latitudes and declinations, the change of altitude would be less than  $1^\circ$  for 4 min. of time, but the figures given illustrate the principle. The "Δt" in "H.O. 214," shows the change of altitude for change of time.

Since the azimuth is at right angles to the line of position and the altitude increases when the body is approached, the "Star Altitude

Curves" give the approximate azimuth at a glance. In Fig. 223 the azimuth of Vega will be seen at once to be rising because the altitude increases with time, and to be nearly east because the altitude curves for Vega run nearly north and south. Given the approximate local sidereal time and latitude, the curves give the name, azimuth, and approximate altitude of the star to be observed. The curves may be used conveniently for star finding.

Provision is also made for the accurate simple use of any edition of the curves for a date earlier or later than the date of publication. This is accomplished by applying to the sextant altitude a correction for the desired date. The figure below each star's name in Fig. 223 is the correction to be applied for the annual change in altitude, the sign showing whether it is applied for a date *later* than the epoch for which the curves are computed and positioned. Figure 223 shows a sample page of the new curves reduced one-half.

**Sidereal Time and Longitude.**—*Local sidereal time* (LST) is found from the "Star Altitude Curves" by projecting the altitude intersection to the top or bottom scale. Longitude is the difference between *Greenwich sidereal time* (GST) and LST. GST may be determined by any of several different methods:

1. By GST watch showing GST in time units.
2. By GST watch showing GST in arc units.
3. By converting Greenwich civil time (GCT) to GST in arc by means of the *Air Almanac*, or by means of a mechanical time converter.

When using GST in time units, LST is taken from the top scale of the "Star Altitude Curves," and the difference is longitude in time units which should be converted to arc units. When GST in arc units is used, LST is taken from the bottom scale of the "Star Altitude Curves."

The *Air Almanac* gives GST in arc (GHA of  $\Upsilon$ ) for 10-min. intervals with a convenient interpolation table for minutes and seconds from 0 to 10 min. This is perhaps the most satisfactory way of finding longitude when the *Air Almanac* is available. Remember that the hour angle of the sun,  $\Upsilon$ , is sidereal time.

**Example.**—At any time, any place, observed with an adjusted bubble sextant the altitude of Vega to be  $39^\circ 35'$  and the Greenwich sidereal time of observation to be  $10^h 15^m 29^s$ . Immediately thereafter observed the altitude of Polaris to be  $28^\circ$ . The star Vega is observed to be in the east and rising. Required, a fix.

**Solution** (Using GST watch).—(1) The altitude of curve of Polaris indicates the latitude ( $30^\circ$  to  $40^\circ$  N.) in which the observer is located. (2) Follow through the curves until the altitude of the star Vega is approximately  $40^\circ$  and rising, or take the difference between the approximate longitude in time and the watch (GST) to find the approximate LST and turn to that page of the curves (Fig. 223). (3) Find the intersection of the curves for the two altitudes observed. This point projected vertically to the time scale at the top or bottom gives the local sidereal time ( $14^h 09^m 38^s$ ).

of the place. The difference between the local sidereal time from the scale and the observed Greenwich sidereal time gives a longitude of  $5^{\text{h}}05^{\text{m}}51^{\text{s}}$ , this being in units of time, and when converted into arc gives a longitude of  $76^{\circ}28'W$ . (4) The point of intersection projected *horizontally* to either of the latitude scales gives a latitude of  $38^{\circ}57'.5N$ . Note that the Polaris altitude curves are not horizontal and should not be followed to pick latitude from the scale.

Figure 224 shows the solution of four examples, using the *Almanac* to find GHA  $\uparrow$  (GST).

**Practical Use of Adjusted Altitude Line.**—Regardless of the method used, the difficulty of taking simultaneous altitudes of two or of three stars complicates celestial air navigation. This difficulty may be reduced by using the "Star Curves" in a manner similar to "Precomputed Altitudes" described later.

Suppose a plane making 300 m.p.h. on course  $240^{\circ}$  true is at  $A$  Lat.  $70^{\circ}N$ ., Long.  $56^{\circ}W$ . at 1800 GST, or 1416 LST (1800 less  $3^{\text{h}}44^{\text{m}}$  for  $56^{\circ}W$ . longitude). In 10 min. the plane would travel 50 miles on  $240^{\circ}$  and, with a page of the curves used as a Mercator chart, as shown in Fig. 225, would arrive at point  $B$ , in Lat.  $68^{\circ}36'N$ . Because of the distance traveled in 10 min., the altitude of Vega would change from  $44^{\circ}36'$  at  $A$  to  $43^{\circ}55'$  at  $B$ . In the elapsed 10 min. Vega's altitude increases at point  $B$  from  $43^{\circ}55'$  to  $44^{\circ}48'$  at  $C$ . The combined effect of change of position and 10 min. of elapsed time would change Vega's altitude from  $44^{\circ}36'$  to  $44^{\circ}48'$ . If Vega's altitude at 1800 GST is  $44^{\circ}36'$  and at 1810 GST is  $44^{\circ}48'$ , then for 1805 GST Vega's altitude is shown at once to be  $44^{\circ}42'$ . In other words, the line  $AE$  is the locus of simultaneous altitudes of Vega, Capella, and Polaris, provided the plane remains on its schedule. If at 1830 GST Vega's altitude is  $45^{\circ}00'$ , the plane is at some point  $D$  about 12 miles off course to right. If an observation of Polaris gives an altitude of  $67^{\circ}40'$  at 1830, the intersection with Vega gives  $D$  as the definite fix, and the plane is shown to be 13 miles  $247^{\circ}$  true from the scheduled 1830 position.

The adjusted altitude line may be laid down for any course and speed. By its use the problem of advancing lines may be greatly simplified.

**Other Methods of Using Curves.**—A transparent template may be used over the curves in the book to find a position without any calculation. The latitude and longitude are etched on the transparent template, making of it a Mercator chart to the same scale as that of the curves. Positions may be plotted on the template; also, courses and bearings. With this arrangement, the latitude and longitude may be determined without writing a single figure—simply by orienting the LST scale with a point on the transparent cover, and then by marking the intersection of the altitude curves. The intersection of the altitude curves shows the latitude and longitude on the transparent cover.

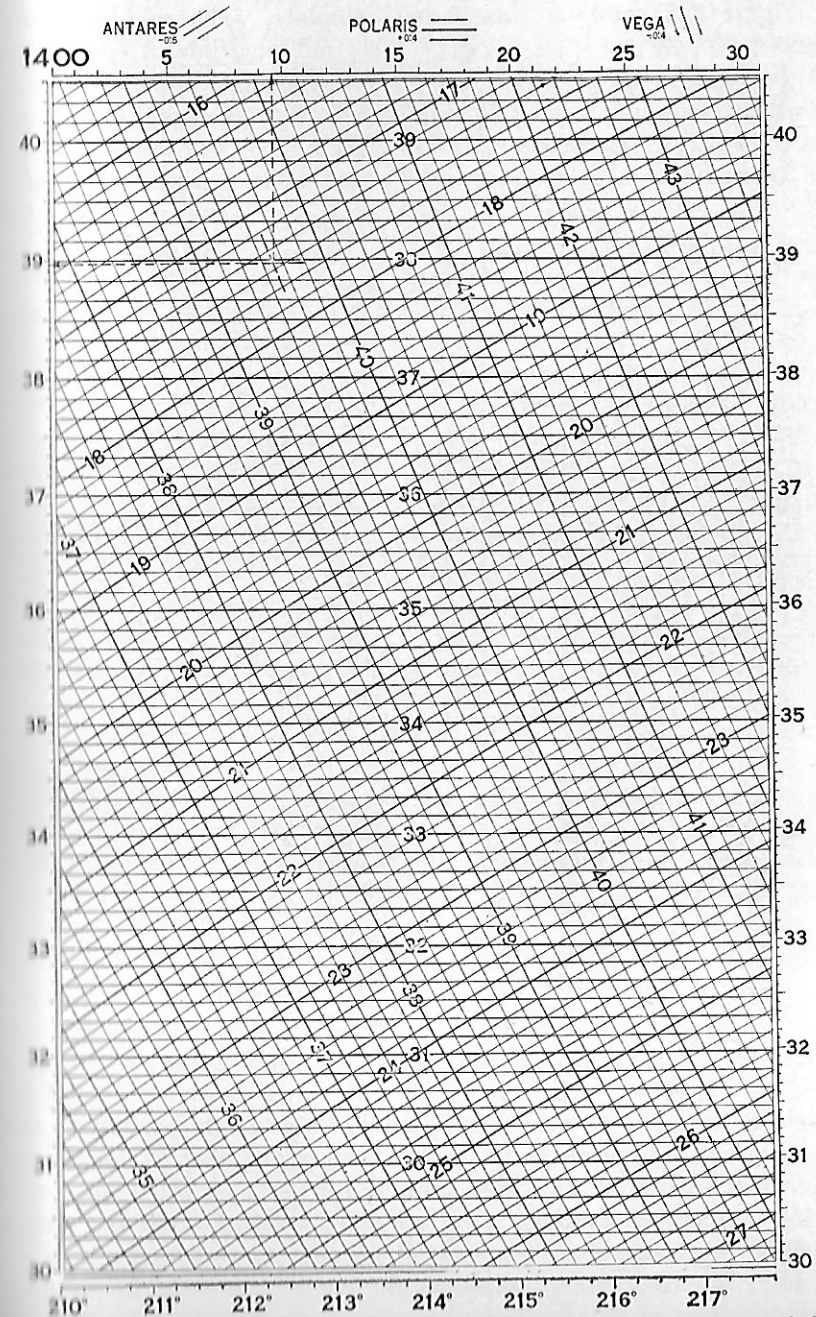


FIG. 223.—Sample page of 1938 edition of "Star Altitude Curves" reduced one-half. These curves are printed in three colors, black, red, and green.

The simultaneous altitudes of two stars, together with the Greenwich sidereal time of observation, definitely determine a point on the earth's surface. This may be put in graphic form by plotting the altitudes against the latitude and local sidereal time. The simultaneous altitudes thus determine by the curves a latitude and a corresponding local sidereal time. The local sidereal time found from the curves combined with the Greenwich sidereal time gives the longitude of the observer. Thus both latitude and longitude are determined without reference to the dead-reckoning position, right ascension, declination, hour angle, or azimuth. No plotting whatever is required to obtain a fix. The entire computation for the latitude and longitude of a definite position is reduced to one subtraction of time to find the longitude.

**Graphical Representation.**—At any given instant a star is directly over (*i.e.*, in the zenith for) some point on the earth called the substellar point. On a small-scale chart such as Fig. 222, assume that the substellar point for one star is at *A*, and for a second star at *B*, shown just off the chart. At *A* the altitude of the zenithal star is  $90^\circ$ , and at a distance of 600 miles (60 nautical miles equal  $1^\circ$ ) the altitude is  $80^\circ$ , and at 1,200 miles the altitude is  $70^\circ$ , etc. In the same way, curves of altitudes for the second star (Polaris) may be constructed from *B*. These circles of equal altitudes are nothing more than lines of position laid down for a given instant of time.

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The *Air Almanac* gives GST in arc (GHA of  $\Upsilon$ ) for 10-min. intervals with a convenient interpolation table for minutes and seconds from 0 to 10 min. This is perhaps the most satisfactory way of finding longitude when the *Air Almanac* is available. Remember that the hour angle of Aries,  $\Upsilon$ , is sidereal time.

**Example.**—At any time, any place, observed with an adjusted bubble sextant the altitude of Vega to be  $39^\circ 35'$  and the Greenwich sidereal time of observation to be  $19^h 15^m 29^s$ . Immediately thereafter observed the altitude of Polaris to be  $37^\circ 58'$ . The star Vega is observed to be in the east and rising. Required, a fix.

**Solution** (Using GST watch).—(1) The altitude of curve of Polaris indicates the band of latitude ( $30^\circ$  to  $40^\circ$  N.) in which the observer is located. (2) Follow through the curves until the altitude of the star Vega is approximately  $40^\circ$  and rising, or take the difference between the approximate longitude in time and the watch (GST) to get the approximate LST and turn to that page of the curves (Fig. 223). (3) Find the exact intersection of the curves for the two altitudes observed. This point projected *vertically* to the time scale at the top or bottom gives the local sidereal time ( $14^h 09^m 38^s$ ).

Figure 226 shows a transparent template. The mid-longitude is marked 70°, 80°, etc., as desired. The mid-longitude of the template is then aligned with the last digit of degrees and exact minutes of GHA  $\Upsilon$ , in which position the star altitude curve may be correctly traced on the template. Referring to Fig. 224, example 1, the mid-longitude of the template would be marked 80°, and aligned with (28) 3°38', in

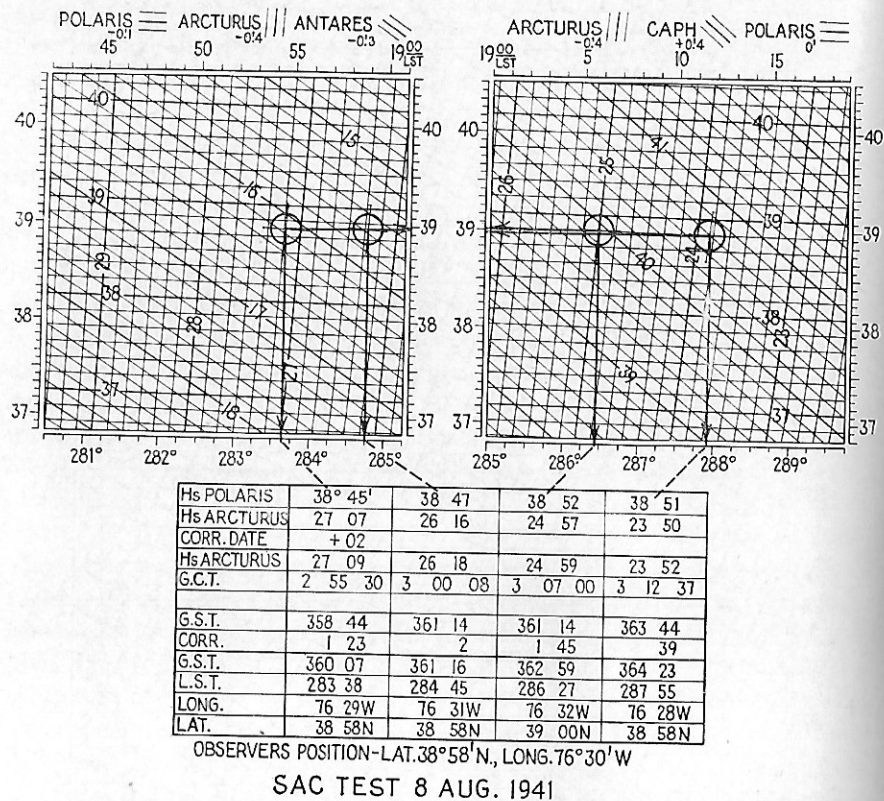


FIG. 224.—Solution of four examples using the Air Almanac to find GHA  $\Upsilon$  (GST).

which position the Arcturus curve would be correctly positioned under the template. In addition to the book form, the Star Altitude Curves may be lithographed on strips suitable for use in a roller map holder with a transparent celluloid cover on which are etched the latitude and longitude to the same scale as that of the curves.

The slight advantage gained in the use of the template by saving the subtraction of time for longitude when working direct from the curves is offset by a slight loss in accuracy, and the necessity of carefully placing the longitude scale at a definite point on the LST scale.

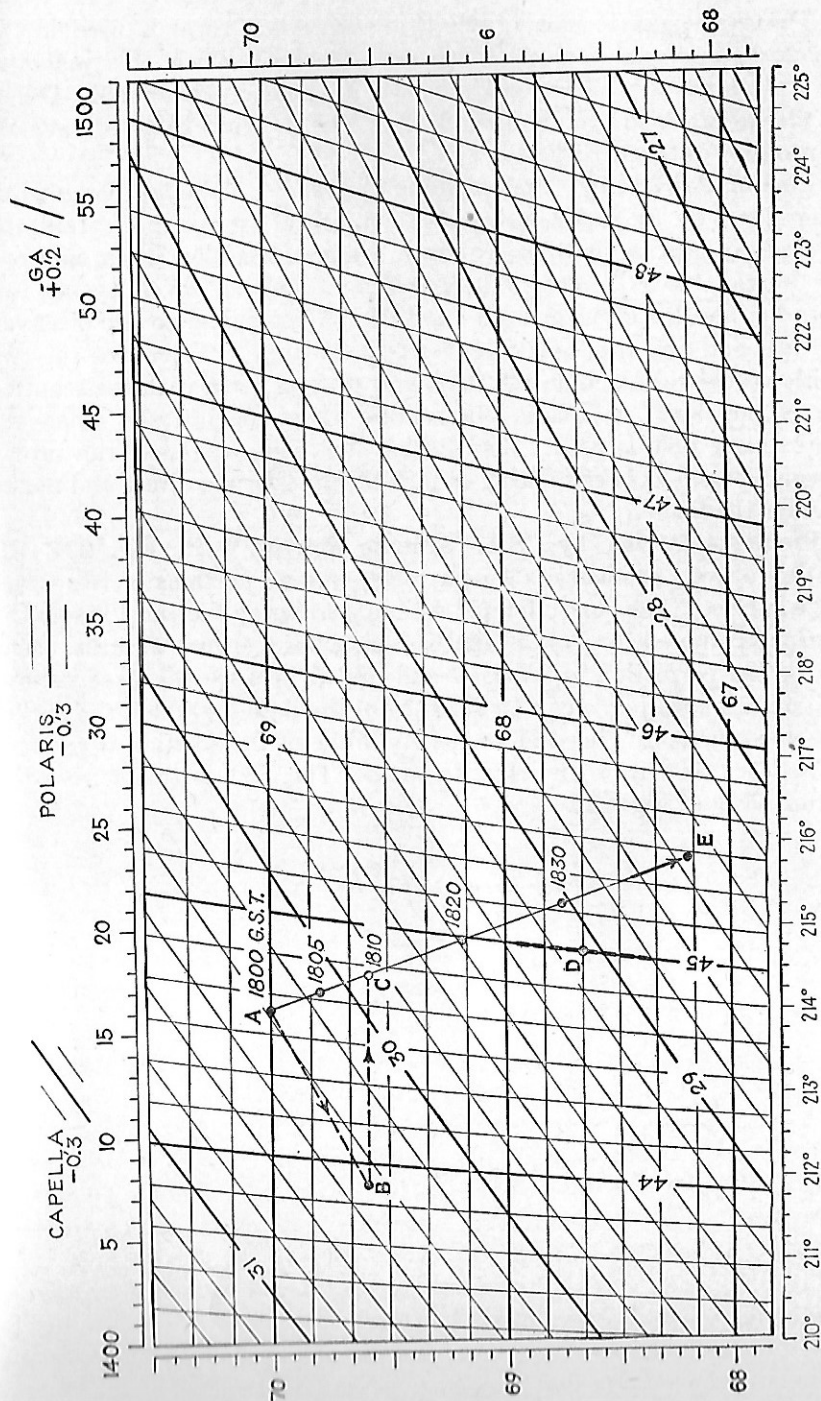


FIG. 225.—Practical use of adjusted altitude line. The curves in this figure are constructed for epoch 1940.

The same general idea of the Star Altitude Curves is used in the construction of the Baker navigation machine. With this machine, the altitude curves on a transparent sheet are passed over a map. Different sheets are used for different declinations in order to give a general solution for different bodies.

**Line of Position by "Star Altitude Curves."**—Although designed to determine a fix by simultaneous observations of two or more stars, the curves may also be used to lay down a line of position when only one star is available. To plot a line of position from the curves, assume two latitudes, and for each, pick off the LST corresponding to the observed altitude, and find the longitudes for each latitude. Then plot the two positions so determined on the chart and connect them with the required line of position. A Polaris observation gives the latitude when the approximate LST is used. The Polaris line will, of course, run nearly east and west. It is customary to pick off the latitude direct and not to consider the line.

**Problems Solved by "Star Altitude Curves."**—In Fig. 224, the solution of four problems is shown together with portions of two pages of the curves. The correction for date appearing on the third line of the solution is ignored for Polaris, and is four times 0.4' for Arcturus. The sign of the correction is reversed and added because the observations were made about four years in advance of the date for which the curves were constructed. The GST in the sixth line of the solution is given in arc and is taken from the *Air Almanac*. The LST is taken from the bottom scale of Fig. 224.

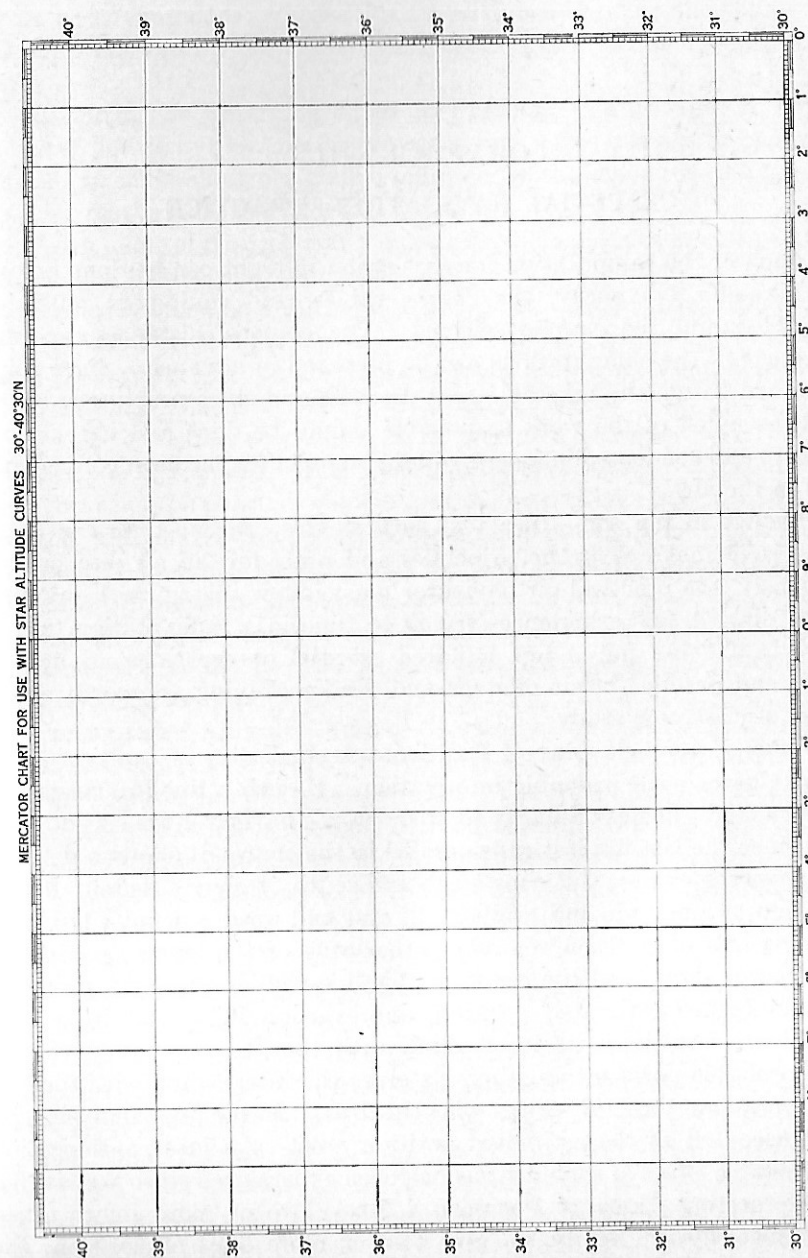


FIG. 226.

# Air Navigation

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