LONGHAND SIGHT REDUCTION USING A COMPACT HAVERSINE TABLE AND AZIMUTH GRAPH

Last July Hanno Ix presented a challenge to the NavList Forum (http://www.fer3.com/arc/) for the developement of a manageable means of sight reduction without the use of logs, slide rules, or calculators and if possible to eliminate special rules and ambiguities. We began with the classic Law of Cosines navigational triangle formula.

$\text{Sin } h = \text{Sin } L \text{ Sin } d \sim \text{Cos } L \text{ Cos } d \text{ Cos } t$

This formula requires three time consuming error prone multiplication steps. Next to try was the Davis haversine cosine formula.

$\text{hv } ZD = \text{hv } (L \sim d) + \text{hv } (t) \text{ Cos } L \text{ Cos } d$

This formula was almost manageable with two multiplication steps but is still classified as time consuming and error prone.

The breakthrough came with the discovery of the haversine cosine Doniol formula in the 1977 volume 1 Bowditch. The beauty of this formula caught my eye as I was leafing through the pages. Not perfect but very close to perfect with just a single multiplication step and a few specal case rules.

$\text{Sin } h = n - (n+m)a$

$n = \text{Cos } (L - d)$ $m = \text{Cos } (L + d)$ same name
$n = \text{Cos } (L + d)$ $m = \text{Cos } (L - d)$ contrary name
$a = \text{hv } (t)$

The haversine cosine Doniol formula was rewritten by Hanno Ix to all haversines which has only one multiplication step and NO SPECIAL RULES. Beauty was looking me square in the eye.

$\text{hv } ZD = n + [1 - (n + m)] (a)$

$n = \text{hv } (L - d)$ $m = \text{hv } (L + d)$ same name
$n = \text{hv } (L + d)$ $m = \text{hv } (L - d)$ contrary name
$a = \text{hv } (t)$

ZD = Zenith Distance
Hc = 90 - ZD
L = Latitude
d = Declination
t = Meridian Angle
hv = Haversine

For practical use a format was created to fit on a 3x5 index card :

L same name		
d	A	B
L - d	hv	hv
L+ d	+hv	+P
	1-	Inv hv
		ZD
t	hv	Hc
	P	

L contrary name		
d	A	B
L + d	hv	hv
L - d	+hv	+P
	1-	Inv hv
		ZD
t	hv	Hc
	P	

L = Latitude
d = Declination
hv = Haversine
Inv hv = Inverse Haversine
t = Meridian angle
P = Product
ZD = Zenith Distance
Hc = Calculated Altitude (Hc = 90 - ZD)

Azimuth is solved with Hanno Ix's fantastic azimuth graph which includes instructions directly on the graph.

There is an all haversine Doniol formula for azimuth arranged by Lars Bergman :

$(a - n) / [1 - (m + n)] = hv\ Z$

a = hv (90 + d) contrary name
a = hv (90 - d) same name
m = hv (L + Hc)
n = hv (L - Hc)

N.Lat.	Zn = Z	LHA > 180
N.Lat.	Zn = 360 - Z	LHA < 180
S.Lat.	Zn = 180 - Z	LHA > 180
S.Lat	Zn = 180 + Z	LHA < 180

d = Declination
L = Latitude
Hc = Calculated Altitude
hv = Haversine
Zn = Azimuth

The 3x5 index card format for practical azimuth reduction :

```
L
Hc
L - Hc          hv
L + Hc         +hv
                 1-
                    ...............  C   {  C = 1-sum
PD               hv
L - Hc          -hv
                    ................ D   { D = sum
D/C
Inv hv
Z                Zn
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The draw back to this azimuth formula is a division step. Rounding to 2 places before dividing (D/C) makes for a manageable longhand division with the resulting azimuth good to the nearest degree.

Summary of benefits for the all haversine Doniol sight reduction longhand method :

1. Full coverage of latitudes and declinations.
2. Uses DR as the assumed position .
3. No special rules.
4. Ultra compact.
5. Quick solution time.
6. 1' precision.
7. Not reliant on electronics or mechanical devices.