

An Arctic lunar

The first thing to do is to find out at which approximate GMT the observation took place. In the following, all times will be astronomical, i.e. civil time minus 12 hours.

The observation time is stated as 20.4^d which we interpretate to be $\pm 0.05^d$. Thus October 20th, 9.6^h \pm 1.2^h, or in the interval 8^h24^m ... 10^h48^m.

The longitude is given as $180^\circ \pm 10^\circ$ which corresponds to $12^h \pm 40^m$. As LMT = GMT + long (east longitude positive), together with the given LMT of 22^h35^m this gives a GMT interval of 9^h55^m ... 11^h15^m.

The common part of these two intervals is 9^h55^m ... 10^h48^m, and we can expect that the GMT found from the lunar should be somewhere within these limits.

In this interval, less than one hour in length, the equation of time, EqT, will be pretty constant. According to NA, EqT varies $9.7^s/24^h \approx 9.6^s/24^h = 0.4^s$ per hour. We can thus be quite confident that we know the local apparent time, LAT, with sufficient accuracy. LAT is the same as local hour angle, LHA, of the Sun.

In order to avoid as much interpolation as possible we should choose an integer hour value of GMT, as the Moon's data is tabulated hourly in the NA. In this case the choice is obvious: GMT = 10^h.

Then, EqT = 15^m8.7^s + 10 · 0.4^s = 15^m12.7^s
 add LMT 22^h35 11.7
 and you get LAT 22 50 24

To convert to LHA in degrees: 22 · 15° = 330°
 50 · 15' = 750' = 12°30'
 24 · 15" = 360" = 6'
 LHA_{sun} 342°36' = -17°24'

The Sun's declination is found by linear interpolation between mean Noons on 20th and 21st respectively. The change in 24^h is 21'28" = 20'88" = 1288". For 10^h the change is 10/24 · 1288" = 5/12 · 1288" = 5/6 · 644" \approx 5 · 107" = 535" = 8'55".

Add the Noon value for 20th S 10°24' 5"
 S 10°33' 0"

Together with the known latitude of N 67°4' 49" we could calculate the Sun's true altitude, h.

A formula suited for logarithmic calculation is

$$\sin h = \sin \text{dec} \cdot \csc \text{aux} \cdot \cos(\text{lat}-\text{aux}), \text{ where } \cot \text{aux} = \cot \text{dec} \cdot \cos \text{LHA}$$

aux is just an auxiliary angle.

dec	-10°33'	0.72 9923 n	9.26 2673 n
LHA	-17°24'	<u>9.97 9658</u>	
aux	-11° 2' 37"	0.70 9581 n	0.71 7705 n
lat	67° 4' 49"		
lat-aux	78° 7' 26"		<u>9.31 3437</u>
h	11°20' 40"		9.29 3815

To clear the lunar, we also need the apparent altitude of the Sun, h'.

$$h = h' - \text{ref}(h') + \text{par}, \text{ thus } h' = h + \text{ref}(h') - \text{par}$$

To start we replace h' by h and see if the result thus obtained equals h , if not we have to iterate. The mean refraction for h is $4' 40'' = 280''$ which should be multiplied by a factor of 1.096 to cater for temperature and pressure. $280'' \cdot 1.1 = 308'' = 5' 8''$. The parallax is $8''$ giving a first estimate of h' as $11^\circ 25' 40''$. Now the refraction for $11^\circ 25' 40''$ is $278'' \cdot 1.1 = 306''$, i.e. the calculated h will be $2''$ too high, from which $h' = 11^\circ 25' 38''$. The refraction and parallax values have been taken from af Klint's "Nautiska och logarithmiska tabeller". My oldest af Klint is the fifth edition from 1895, but I guess that earlier editions had identical tables in this respect.

To find the Moon's true altitude, H , we take the declination directly from NA. To find the hour angle, consider the following:

$$\text{LHA}_{\text{moon}} = \text{GHA Aries} + \text{SHA}_{\text{moon}} + \text{long}$$

$$\text{LHA}_{\text{sun}} = \text{GHA Aries} + \text{SHA}_{\text{sun}} + \text{long}$$

Subtracting the second equation from the first and rearranging, gives

$$\text{LHA}_{\text{moon}} = \text{LHA}_{\text{sun}} + \text{SHA}_{\text{moon}} - \text{SHA}_{\text{sun}} = \text{LHA}_{\text{sun}} + \alpha_{\text{sun}} - \alpha_{\text{moon}}, \text{ where } \alpha_{\text{sun}} \text{ and } \alpha_{\text{moon}} \text{ are the right ascension for Sun and Moon, respectively.}$$

α_{moon} is found directly in NA. For the Sun, we have to interpolate between the Noon values. The change between 20th and 21st is 224^s and $10/24 \cdot 224^s = 5/12 \cdot 224^s = 5/3 \cdot 56^s = 280^s/3 = 93^s = 1^m 33^s$. Adding the 20th mean Noon value you get $13^h 41^m 39^s$ and after subtracting the Moon's right ascension we get a difference of $4^h 21^m 46^s$. To convert to degrees

$$\begin{array}{r} 4 \cdot 15^\circ = \\ 21 \cdot 15' = 315' = \\ 46 \cdot 15'' = 460'' + 230'' = 690'' = \\ \alpha_{\text{sun}} - \alpha_{\text{moon}} \\ \text{LHA}_{\text{sun}} \\ \text{LHA}_{\text{moon}} \end{array} \qquad \begin{array}{r} 60^\circ \\ 5^\circ 15' \\ \underline{11' 30''} \\ 65^\circ 26' 30'' \\ \underline{-17^\circ 24'} \\ 48^\circ 2' 30'' \end{array}$$

Now we can calculate H :

dec	14° 52' 22"	0.57 5820	9.40 9381
LHA	48° 2' 30"	<u>9.82 5160</u>	
aux	21° 40'	0.40 0980	0.43 2795
lat	67° 4' 49"		
lat-aux	45° 25' 1"		<u>9.84 6302</u>
H	29° 12' 48"		9.68 8477

Moon's HP is given for Noon and Midnight mean time; on the 20th the variation is $26.2''/12^h$ and interpolation gives the excess value for 10^h as $10/12 \cdot 26.2'' = 131''/6 = 21.8''$. Adding the Noon value gives $\text{HP} = 58' 52.4'' = 3480'' + 52'' = 3532''$. To cater for the oblateness of the earth, $10''$ should be subtracted, giving $\text{HP}' = 3522''$, the log of which is 3.5468.

We also need an approximate value of the Moon's azimuth in order to correct the altitude used to calculate the parallax.

dec	15°	log cos	9.985
LHA	48°	log sin	9.871
alt	29°	log sec	<u>0.058</u>
azim	S55°W	log sin	9.914

That the angle lies between south and west is easily seen if you make a sketch of the triangle. The correction, to be added to the altitude, is 4.5' with this azimuth and latitude.

In analogy with the Sun, we have $H' = H + \text{ref}(H') - \text{par}$ and we need to iterate a few times. The refraction value changes quite slowly. If we guess that H' is around 50' less than H we have as a start

$H' + \text{corr}$	28° 27'	log cos	9.9441
HP'		log	<u>3.5468</u>
par 3097" =	51.6'	log	3.4909
H'	28° 23'		
ref	<u>2.0'</u>		
H_{est}	29° 12.6'	This is close to our calculated H. A new try:	

H'	28°23' 12"		
$H' + \text{corr}$	28°27' 42"	log cos	9.9441
HP'		log	<u>3.5468</u>
par 3097" =	51'37"	log	3.4908
ref 1'47" · 1.1 =	<u>1'58"</u>		
H_{est}	29°12'51"	Only 3" off =>	
H'	28°23' 9"		

Remains pre-clearing the measured lunar distance.

D_{meas}	69° 1' 50"
SD_{moon}	16' 4"
augm	7"
SD_{sun}	16' 7"
contr _{sun}	<u>2"</u>
D'	69°34' 6"

To summarize, so far:

h	11°20' 40"	true altitude Sun's centre
h'	11°25' 38"	apparent altitude Sun's centre
H	29°12' 48"	true altitude Moon's centre
H'	28°23' 9"	apparent altitude Moon's centre
D'	69°34' 6"	apparent distance between centres

To clear the distance, a multitude of different methods exist. Here a method by Bremiker will be used. In short,

$$C = \cos H' \cdot \cos h' \cdot \sec H \cdot \sec h$$

$$d = H - h$$

$$d' = H' - h'$$

$$\cos d'' = \cos d' / C$$

$$\cos D'' = \cos D' / C$$

$\sin(z/2) = \sin((d - d'')/2) \cdot \sin((d + d'')/2) \cdot \csc((D'' + z/2)$; initially let z in the last factor be zero; for most cases this is sufficiently accurate; otherwise iterate once.

As $z/2$ and $(d - d'')/2$ are small angles, a simplified formula is

$$z = (d - d'') \cdot \sin((d + d'')/2) \cdot \csc D''$$

$$D = D'' + z ;$$

To the thus found D a correction for the side parallax of the Moon is to be added.

h	11°20' 40"	log sec	0.00 8569
h'	11°25' 38"	log cos	9.99 1305
H	29°12' 48"	log sec	0.05 9081
H'	28°23' 9"	log cos	<u>9.44 4367</u>
C		log	0.00 3322
D'	69°34' 6"	log cos	<u>9.54 2937</u>
D''	69°43'51"	log cos	9.53 9615
d	17°52' 8"		
d'	16°57'31"	log cos	<u>9.98 0692</u>
d''	18°20' 0"	log cos	9.97 7370
d - d''	- 28' 2" = - 1682"	log	3.22 5826 n
(d + d'')/2	18° 6' 9"	log sin	9.49 2366
D''	69°43'51"	log csc	<u>0.02 7762</u>
z	-557" = <u>- 9'17"</u>	log	2.74 5954 n
D	69°34'34"		
side par	- <u>6"</u>		
D final	69°34'28"		

Now remains to find out GMT:

IX ^h	70°11' 6"	2690	
Midnight	<u>68°56'56"</u>	2672	-18
	36'38"	6914	
Preceding	9 ^h		
approx time	1 ^h 8 ^m 3 ^s	4224	
corr 2 nd diff	<u>5^s</u>		
Obs'd GMT	10 ^h 8 ^m 8 ^s		
LMT	<u>22^h 35^m 12^s</u>		
longitude	12 ^h 27 ^m 4 ^s E = 186°46' 0" E = 173°14' 0" W		

The result published by the Vega expedition was 12^h27^m 6^s, i.e. 186°46'30" E or 173°13'30" W.