ARCTIC LUNAR

09.03.2026.

Dear NavList members,

Lars Bergman recently challenged us to find longitude from lunar observation data that were performed on board “Vega” — the famous 1878-1880 expedition ship (expedition was the first who managed to navigate through the Northeast Passage and the first that circumnavigated Eurasia). Lars wanted us to do the calculations using only methods that were available in the end of 19th century. One peculiarity: the initial data does not contain altitudes. We have lunar distance, but no altitudes at all. So, what do we have? We know year, date and local mean time. We know exact latitude. We know temperature and preasure. And yes, we know that the longitude is 180° +/-10°.

To solve the problem here we can use the solution that was well know in 18th and 19th centuries: **clearing lunars using calculated altitudes.**

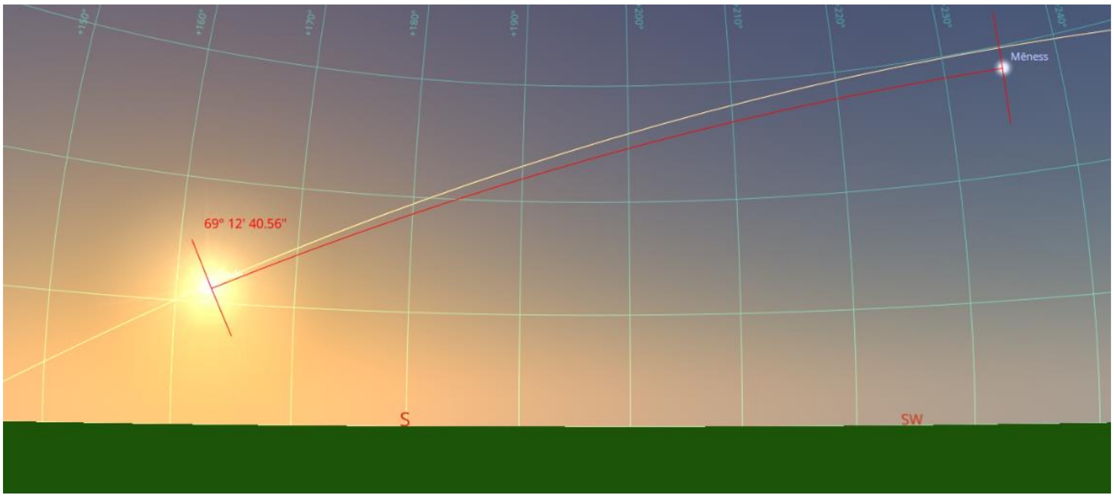
Nevil Maskelyne used it back in 1761 in his voyage to St. Helen even before Nautical Almanac era. Matthew Flinders used it when he charted the coasts of Australia in the early 19th century etc. We can now open the old logbooks and see calculations made by their hands.

The method of clearing lunars using calculated altitudes was published in many popular navigation manuals of that period (it was included in N.Maskelyne’s “British Mariners guide” in 1763, in Bowditch throughout the 19th century , Moor, Thomson’s tables etc.).

At first glance it may seem that even if the navigator knows excatly his latitude, he should have a very good estimated longitude to get reasonably good altitudes for clearing lunars. It seems so obvious — if my longitude is off, for example, by 15°, then the position of the celestial bodies would be so different, that calculated altitudes would be useless for clearing lunars. Right?

NO! Your calculated altitudes will be only off by maybe one arcminute or several arcminutes. And this is magical: we don’t know our longitude, but we can almost excactly calculate the altitudes! Of course there is no any magic here, because we know excact latitude and exact local time. This is a key factor here. The relatively small error in calculated altitudes will be caused only by celestial bodie’s position change during the time period corresponding to longitude error, not by Earth rotation’s effects on altitude, which changes the altitudes considerably. When we calculate altitudes at different assumed longitudes, we also calculate the corresponding UT for each assumed longitude. This provides that the altitudes are very similar. And in practise there will be rarely cases when uncertainty in longitude is more than 15° (1h error in time). What will be the angular distance the Moon makes in 1 hour? About 30’. But these 30’ will usually affect the altitude only partly (except if ecliptic is positioned perpendicular to horizon).

Now let’s return to Arctic Lunar example. When I start analysing historic lunars usually I open Stellarium to see the configuration of celestial bodies. This is a very good way of reconstructing what the historical navigator saw when he did his observations. The key points I look at here are: the aprproximate altitudes of the bodies and the direction of lunar distance arc at the both ends (is it close to vertical, close to horizontal, slanted, if slanted, how large is the angle with horizon. These angles correspond to sextant inclination angles while taking lunar respectively from one and other end of lunar arc). This gives a very good sense of how much the effects of refraction and paralax will affect the lunar distance and if the altitude will be calculated, it shows how the error in assumed longitude will affect the lunar distance.



We can see that Sun's altitude is about 10°, Moon's altitude about 30°.

In this case the lunar arc at the Sun's end forms about 25° angle with horizon, but the angle at the Moon's end of lunar arc is almost horizontal. Here we can conclude that:

a) there will be no effect of Moons SD compression here, because lunar arc is almost horizontal;

b) Moon's altitude is close to 30°, therefore the efects of refraction because of altitude uncertanty will be minimal. Besides it will be of little effect on lunar arc;

c) Computed Moon's altitude will not be considerably affected by even several hours of time uncertainty, because Moons motion during this period of time would be more or less horizontal;

d) Sun's SD compression will have some effect, but very small;

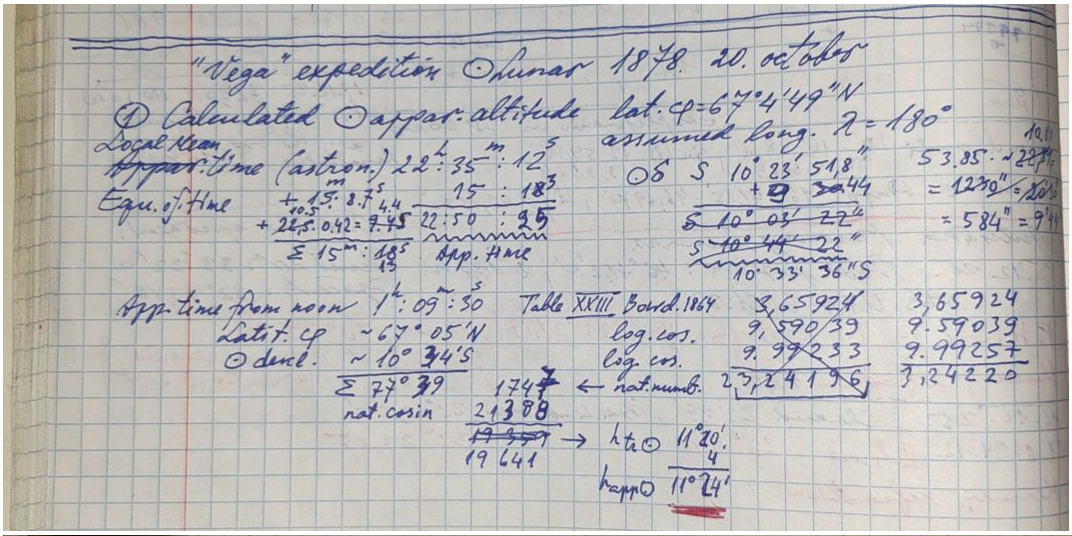
e) Effects of refraction at Suns's altitude at about 10° will be more noticable, but as the lunar arc is so slanted, they will not very noticeably affect along the lunar arc. Errors in assumed longitude will not be of noticable effect. If the lowest object was Moon, the situation would be more unfovarable.

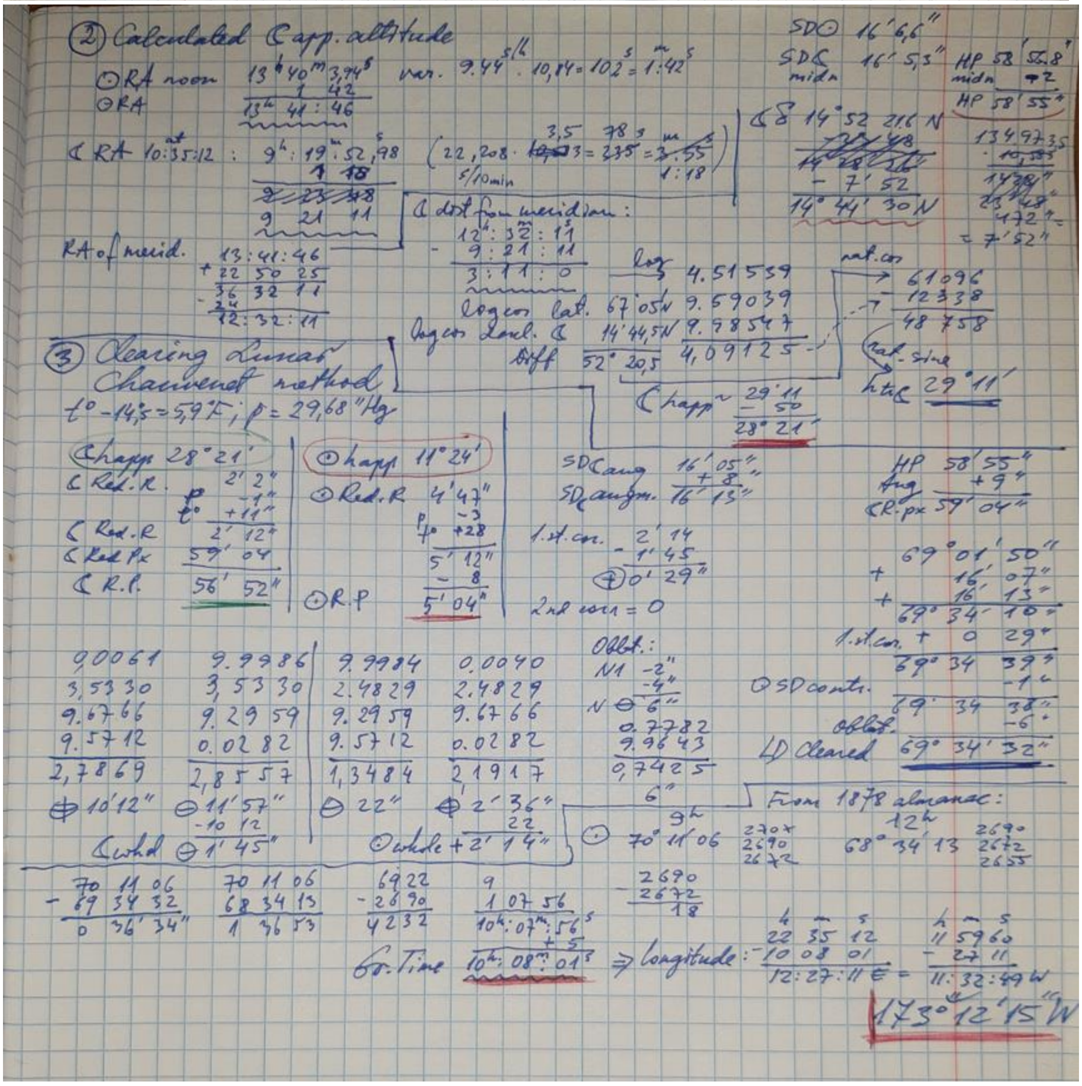
f) The Sun is close to South. If the latitude was uncertain, Sun's altitude would be affected by this latitude error (not our case). The Moon is close to SW, if there is an error in local time, it would affect the Moon's altitude only partly (the full effect would be in E/W direction).

g) the non standart temperature will minimal effect on cleared lunar distance (effects of refraction works verticaly, but in this case lunar arc is very slanted, thus diminishing impact on lunar arc) , therefore clearing this lunar using historicaly so called "approximate" clearing methods (ignoring temperature, preasure, earth oblateness etc.) will provide quite good results. Besides the high latitude provides that any uncertainties in longitude will not produce large errors in distance on the surface of the Earth.

CALCULATIONS

I performed calculations using oldschool pencil/paper method. To get astronomical data I used 1878 Nautical Almanac from here:
https://babel.hathitrust.org/cgi/pt?id=njp.32101050586617&seq=197





As you can see there are 3 basic steps here:

1) finding Sun's apparent altitude at assumed longitude (in this case assumed longitude was 180°). Calculated Sun's apparent altitude: 11° 24';

2) finding Moon's apparent altitude at assumed longitude. Calculated Moon's apparent altitude: 28° 21';

3) clearing lunars and finding GMT and longitude. Cleared lunar distance: 69° 34'32"

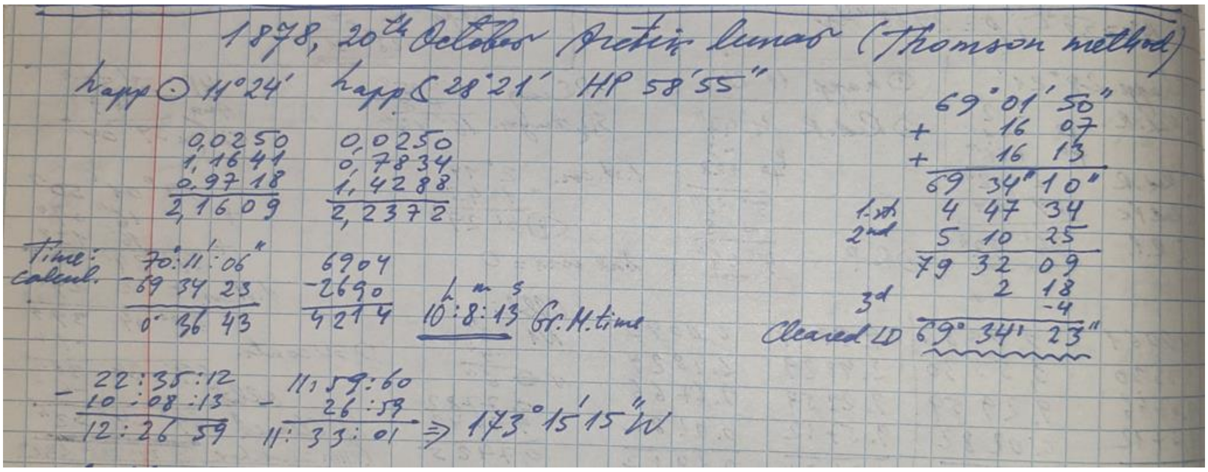
For both altitude calculations I used method from Bowdich (mid 19th century). For clearing lunars I used 19th century Chauvenet method, which is very accurate.

All calculations are performed using log tables from 19th century Bowditch manuals.

My results are:
GMT (astron.): 10h:08m:01s (it would correspond to 22h:08m:01s UT1)
Longitude: 173° 12' 15"W

(These results are calculated using the Almanac from 1878, not modern ephemeris data!)

We can test how correct was above mentioned statement that this lunar is quite insensitive to inaccuracities of faster simplified lunar clearing methods. Let's clear this Arctic Lunar using Thomson's method (similar to Bowditch 2nd method):



We get difference in cleared distance only 9" comparing with Chauvenet's result! In Chauvenet solution I used second differencies for GMT calculation. If these extra 5 seconds would be taken into consideration the final longitude for Thomson's solution would be:

173° 16' 30"W.

Difference between these two points 173° 12' 15" and 173° 16' 30"W is 4'15". This is only 1,66 nautical miles on 67° latitude!

EFFECTS OF ALTITUDE ERRORS ON CLEARED DISTANCE IN THIS ARCTIC LUNAR EXAMPLE

How much this Arctic Lunar example is sensitive to errors in assumed longitude?

Let's calculate the altitudes for Arctic Lunar example using modern tools and modern ephemeris (just to get results faster). For clearing I will use Paul Hirose's Chauvenet method calculator. Below is a table with results:

Assumed longitude	UT1 corresponding to assumed longitude	Calculated apparent altitude of Sun	Calculated apparent altitude of Moon	Cleared lunar distance with calculated altitudes
170°E	23:15:12	11°24'27"	28°19'10"	69°34'22,7"
180°	22:35:12	11°25'02"	28°21'13"	69°34'22,9"
170°W	21:55:12	11°25'37"	28°23'10"	69°34'22,0"
150°W	20:35:12	11°26'47"	28°26'50"	69°34'22,1"
0°	10:35:12	11°35'30"	28°43'38"	69°34'12,3"

In the last row of the table I included even totaly absurd case — longitude that is about 180° in error, assuming the navigator has no idea where he is (he assumed that the ship is on Greenwich meridian – on the other side of the planet). Of course this is just for fun. But how large is error in cleared distance in this totaly extreme case? Lunar distance is in error of only 10" (less then 0,2')!!!

It means that this lunar example can be solved without knowing longitude at all!

In the table cleared distance is shown with accuracy of 1/10" just to show how incredibly small are the effects caused by errors in altitudes in this particular case. Sun's altitude can be off by about 6' and Moon's altitude can be off by about 15' to cause 0,1' error in cleared lunar distance in this case.

Modris Fersters