The Astrolabe

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Preface

This is not a book about astrolabes. Rather, it is about THE astrolabe ... and astrolabe-related instruments.

Most books and journal papers about astrolabes describe specific old instruments in great detail, often include biographical material about the maker and, possibly, source material used by the maker or a historical context. This book does not concentrate on specific instruments. Rather, it describes the principles on which all types of astrolabes are based, and includes concrete examples useable to design or make an accurate working instrument.

Clearly, there is a need for both types of treatments of scientific instruments in general and astrolabes and other pre-telescopic instruments in particular. Works describing the details of individual instruments and collections are invaluable to historians, curators and collectors. Such documentation stands at the pinnacle of historical scientific instrument research, and is invaluable for understanding the instruments themselves, their cultural environment and the history of scientific development. The corpus of available historical astrolabe references has been useful in tracing the development and spread of astronomical and mathematical knowledge, the development of instrument manufacturing and associated industries, student curricula in the Middle Ages and Renaissance and many other areas of historical interest.

Technical information is needed by historians to evaluate old instruments and is desired by those who want to understand the old sciences or make a reproduction of some type of old instrument. The material in this book is useful for students and scholars encountering astrolabes for the first time as well as interested readers of all kinds who are interested in the history of astronomy, pre-telescopic instruments or want to make or use an instrument reproduction.

The intention of this book is to fill a significant gap in technical astrolabe literature. There have been several attempts in the last half-century to document astrolabe principles and the technical details of astrolabe design. While these attempts have been extremely useful, they have not been completely satisfactory in availability, clarity, completeness or accuracy. To my knowledge, there is no single treatise or book in English providing the technical information required to understand all types of astrolabes completely, or to design and make one. I believe the detailed treatment of astronomical quadrants in modern terms is unique.

We will discuss the technical details of the common types of astrolabes and related instruments, and include information needed by the interested reader to design any type of astrolabe instrument. Sample computer programs and useful programming examples are also included with the idea that no technical subject can be considered to be completely covered today without considering computer implementations. Computer generated instruments allow accurate and inexpensive reproductions to be made, and offer the potential for new ways to experiment and exploit the concepts embodied in these instruments.

There is an additional motivation to update astrolabe perceptions. I feel strongly the instruments described in this book have relevance in the modern world as working instruments and teaching aids. Study of the astrolabe by students of astronomy is virtually unmatched for providing an intuitive feel for the motion of the Sun and stars and how celestial positions are measured.

An update to the mathematical foundations of the astrolabe is also merited. A rich literature of the mathematical foundations of the stereographic projection and its application to instruments survives, but it is almost impenetrable by modern readers. Mathematicians and astronomers who wrote about astrolabes in the 17^{th} century and earlier were well versed in Euclidean geometry, a skill largely replaced by analytic geometry in the modern world. The old astrolabists were accustomed in thinking about quantities in terms of ratios, which seems very foreign today.

Instructions in the old literature on how to make an astrolabe rely heavily on geometric constructions, which is understandable since trigonometric procedures were not well defined at the time nor widely understood, and the sheer labor involved in performing all of the required calculations was intimidating to even the most skilled mathematicians. Computers have freed us from these restrictions, and it seems timely to state the instrument design parameters in terms familiar to the modern reader that can be easily implemented on a computer. A few of the old constructions are included for completeness, but the clear thrust is to state the technical material in modern terms. Leaving the astrolabe mired in the past and burdened with obsolete technology would deny it its future.

It has often been said the only way to truly understand an instrument is to make one. There is little information available on how to make an astrolabe. The skills and tools required to make a working metal astrolabe are a science in itself and far beyond the scope of this work. In addition, new methods of working and marking metal instruments are constantly being developed, and it is not possible to cover a meaningful subset of the available techniques in any useful detail.

However, you can create a perfectly useable instrument at very little cost by printing the components on your computer and assembling them. A computer-generated representation of an astrolabe is within your reach if you are willing to invest the time and effort. It is fun and rewarding. Frankly, writing a computer program to draw an accurate and complete astrolabe is probably more work than the old makers exerted in making their instruments, but does not require the mechanical and artistic dexterity. The question of whether this is a virtue of computers or yet another example of computers compromising yet another old-world craft is not discussed. There are many figures in this book representing complete instruments or parts of instruments. All of these figures were created with computer programs I wrote. I am a mediocre programmer and far from accomplished mathematically, but I found good results are possible with diligence.

Many scholarly papers on astrolabes assume the reader is familiar with astrolabe principles and already understands the various scales, which may not be a good assumption, even for advanced readers. It is not possible to include every variation of every scale that has ever appeared on astrolabes in a book of reasonable size. However, the material covered should give a sound basis for interpreting old instruments and, for those scales not covered directly, may provide a starting point for figuring out the scale's intended use and how it was drawn.

The following material is provided for each type of astrolabe and scale on the instrument:

A description of the instrument or scale
A brief history.
How to use it.
How to design it.

I have seen and held many old astrolabes, but I have never made detailed measurements of an old instrument. They are too valuable and delicate for the curators of the collections to allow me to subject them to hands-on analysis. I have never even seen some of the instruments described in this book in person, and I have had to reconstruct them from pictures and the few available written descriptions. I believe the recreations shown in the figures are accurate within the qualifications mentioned in the text, but any errors are mine alone.

I am also not a historian, and I am not trained in the very specialized skill of historical research. Neither am I competent to read much of the source material, which is written in Latin and Arabic. I have an extensive astrolabe library and have tried to study every relevant reference available in English or French. I freely admit this book is useful only as an introductory reference on the history of the instruments discussed, but I am hopeful the historical material is sufficient for the interested reader to pick up the trail and follow it. Any errors in the historical discussions are not from lack of trying to be complete and accurate.

If you have scanned the contents of this book you will have noticed there are quite a few mathematical equations. Don't let this scare you off. You can skip the math without loss of continuity. However, you will find the math to be easy and useful if you decide to make an instrument. Geometrical and mathematical methods are both shown, and you do not need to be good at math to understand the astrolabe or to make one. The mathematical approach lends itself to computer generated astrolabes, but is not needed to understand the principles. On the other hand, the mathematically inclined should find the analytical treatment to be fun and insightful. The level of math required is mostly high school geometry and trigonometry, with a bit of basic analytic geometry.

The astrolabe is an astronomical instrument, and some knowledge of positional astronomy is needed to understand the astrolabe and its uses. A brief outline of the required astronomical background is included, but you may find it necessary to refer to more complete texts for full mastery of the necessary background.

Introductory material outlining the essentials of positional astronomy and computer implementation of astronomical calculations is included, as are references to more detailed and complete sources.

There was almost no useful technical information about astrolabes when I first became interested in them and I had to derive all of the math and figure out the details by myself. To me, having an equation providing some value is useless unless I understand where it came from. It is not practical to include every step of every derivation, but an outline of the derivation of every equation unique to the stereographic projection as applied to the instruments is included.

The following symbols are used throughout this book:

h,a	altitude of a celestial body above the horizon
A	azimuth of a celestial body
H	hour angle of the Sun or star
δ	Declination
α	Right ascension
φ	terrestrial latitude
β	celestial latitude
λ	celestial or terrestrial longitude
ε	obliquity of the ecliptic

A glossary containing definitions of specialized terms is included. Glossary words are shown in **boldface** the first time they are used in the text. Words defined in the text are shown in *italics* when they are defined.

I have tried to include everything I have learned about astrolabes in over 25 years of study. I hope this book will be a compact and useful technical reference for astrolabe studies. This book will have fulfilled my wishes if it makes your journey into the fascinating world of pre-telescopic astronomical instruments and the history of astronomy faster, easier and more fun.

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"But considre wel that I ne usurpe not to have founden this werk of my labour or of myn engyn. I n'am but a lewd compilator of the labour of olde astrologiens, and have it translatid in myn Englissh oonly for thy doctrine. And with this swerd shal I sleen envie." Geoffrey Chaucer, "The Astrolabe - Bread and Milk for Children", ca 1391.

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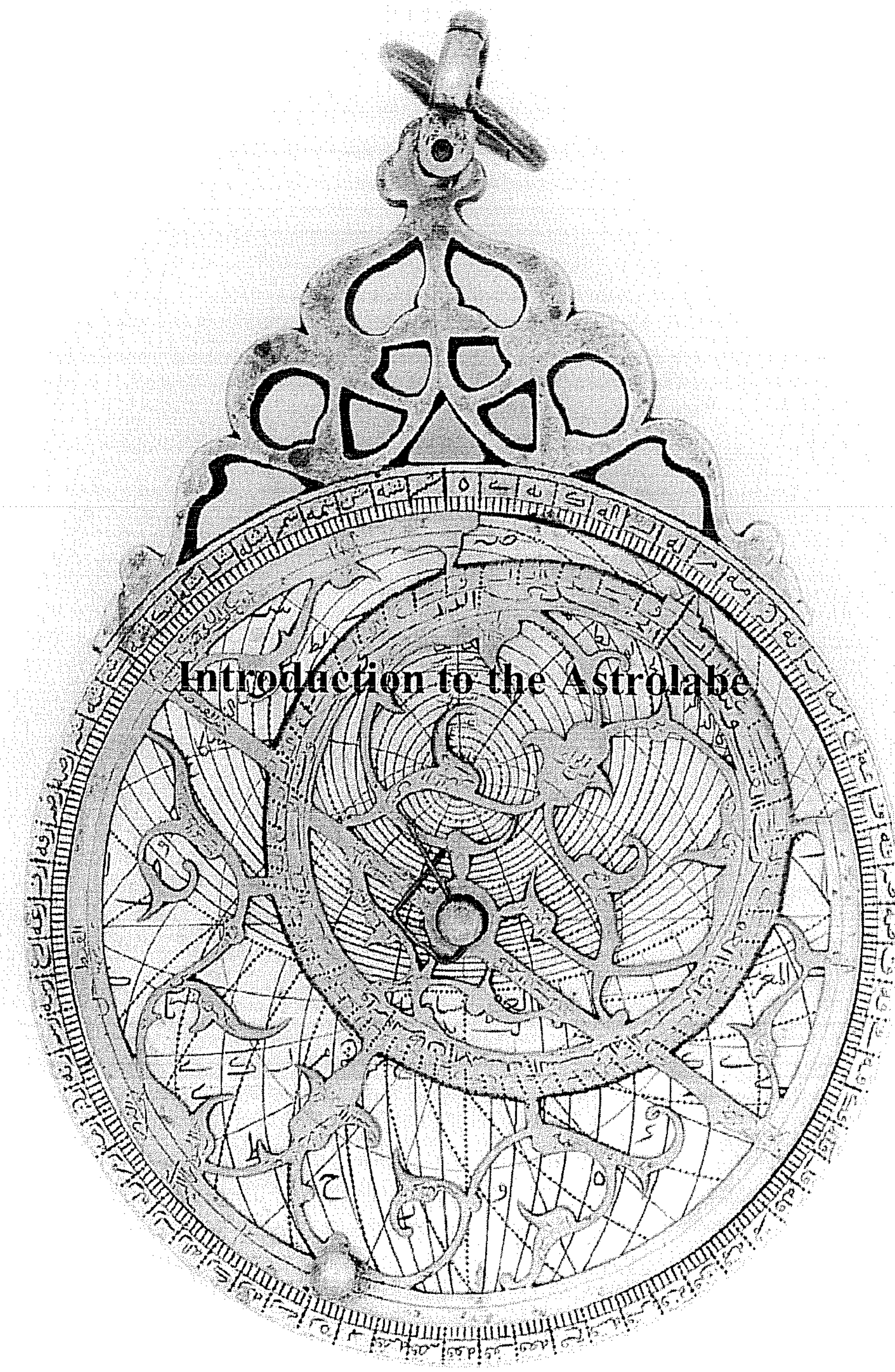
Introduction to the Astrolabe



Chapter 1 - The Astrolabe

"I know that I am mortal and the creature of a day; but when I search out the massed wheeling circles of the stars, my feet no longer touch the Earth but, side by side with Zeus himself, I take my fill of ambrosia.". Claudius Ptolemy

Awe at the majesty and beauty of the night sky is a feeling that unifies all cultures from all ages. The daily cycle of the Sun and the annual march of the stars inspire poets and scientists, mystics and saints, artists and barbarians. The celestial universe defines our cycles, warms our souls and excites our intellect.

Man has always wanted to grasp the universe and hold it in his hands to study and admire its regularity and mysteries. The closest this dream has come to reality is in the form of the astrolabe. No other heavenly model expresses the grace and symmetry of the heavens in such an intuitive way. Its graceful arcs and finely engraved components are both elegant and mysterious. It is a device of wonder to those who do not understand its simple elegance and a source of admiration to those who do. This very special instrument has excited people for nearly two thousand years.

What is an Astrolabe?

The astrolabe is an astronomical instrument used to solve problems related to time and the positions of the Sun and stars.

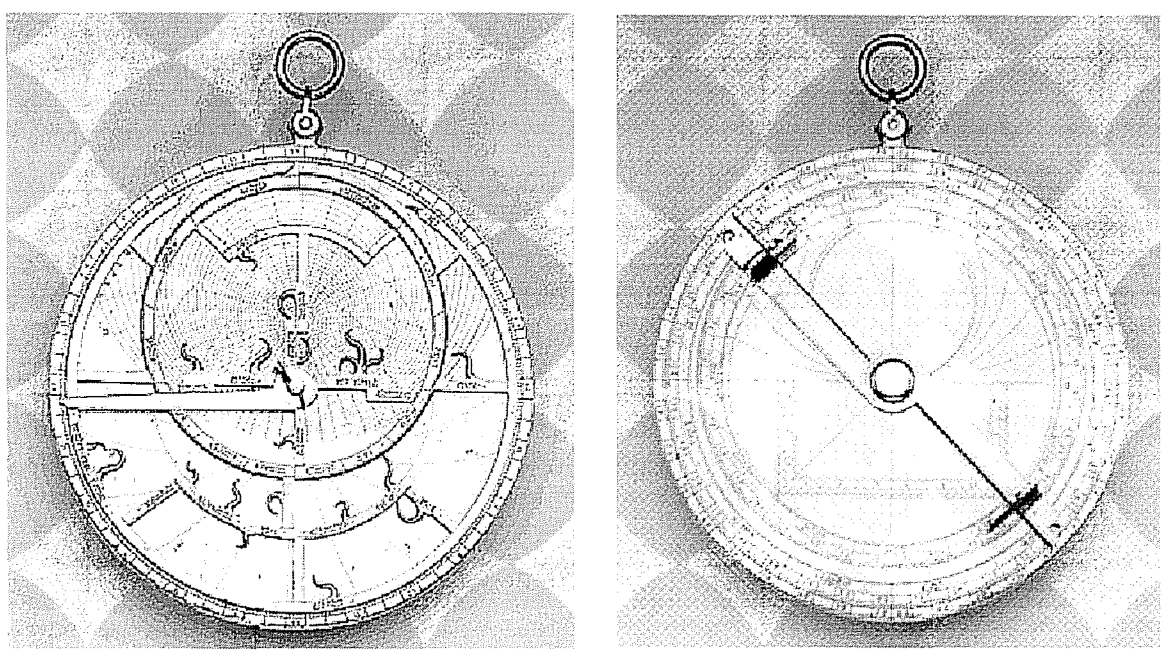


Figure 1-1. Astrolabe by Jean Fusoris, ca. 1400 (Adler Planetarium)

It can be used to solve a great many astronomy problems requiring considerable mathematical calculations if you didn't have such a marvelous device. The name comes from the Greek *aster*, meaning star, and *lambanein* (imperf. labein) meaning take, seize, catch, grasp and, derivatively, apprehend, comprehend, understand. Combined as the word astrolabe, the name astrolabe means an instrument for understanding the stars. It can be used as a star finder to find stars and other objects in the sky or to solve a problem such as finding the time by using the position of the Sun or a known star. The astrolabe is both a map of the heavens and a portable computer for solving astronomical problems. All this in an instrument you hold in your hands.

An example of a common astrolabe is shown in Figure 1-1. This instrument was made in France by Jean Fusoris in the late 14th century. It is made of brass, as are most surviving instruments, and is a little over six inches (16 cm.) in diameter. This type of astrolabe is known as a planispheric astrolabe. The word "planispheric" relates to the representation of a sphere on a plane and refers to the grid of curved lines on the front of the instrument.

The primary purpose of the astrolabe is to show the user the positions of the Sun and selected stars at a specific place at a given local time. This is done by drawing the sky on the face of the astrolabe and marking it so positions in the sky are easy to find.

Most astrolabe problems were solved using the front of the instrument. The front of an astrolabe has two types of parts: fixed and rotating. The fixed parts represent time scales and a view of the sky as seen from a specific latitude. The rotating parts simulate the daily rotation of the sky. To use an astrolabe, you adjust the moveable components to a specific time and date. Once set, much of the sky, both visible and invisible, is represented on the face of the instrument. This allows a great many astronomical problems to be solved in a visual way.

The astrolabe is intended to be used for both observation and computation. For observation, it is fitted with a ring so the instrument can be hung vertically while the position of the Sun or a star is measured using moveable sights and a scale on the back of the instrument.

Strictly speaking, the word "astrolabe" has always meant any instrument used to measure the **altitude** of celestial objects above the local horizon. Astrolabe instruments have taken many forms. The mariner's astrolabe was a graduated circle used by Renaissance navigators to determine the latitude at sea by measuring the Sun's noon altitude or the altitude of the north celestial pole (Figure 1-2). The *prismatic astrolabe* is a modern professional astronomical instrument used to measure the meridian altitude of a star very precisely.

In common use, however, "astrolabe" is usually intended to mean the type of medieval astronomical instrument that is the subject of our discussion — a portable instrument capable of measuring the altitude of the Sun or a star and used to solve astronomical problems.

The planispheric astrolabe should not be confused with the popular star finders found in book stores and museum book shops. Star finders, known as "planispheres", are a handy aid for finding stars in the sky and are based on a different projection. The only use of a planisphere star finder is to find stars. The planispheric astrolabe has many more uses.

The planispheric astrolabe was by far the most popular astronomical device from its introduction until it was rendered obsolete by more specialized and accurate instruments in the late 17th century. It was used in both the Islamic world and Medieval and Renaissance Europe, and many educated people learned the basics of its use. A well-made astrolabe was treasured in all cultures that used them, which accounts for the relatively large number of surviving instruments. Tragically, many instruments have been lost and many made of wood or paper have simply deteriorated.

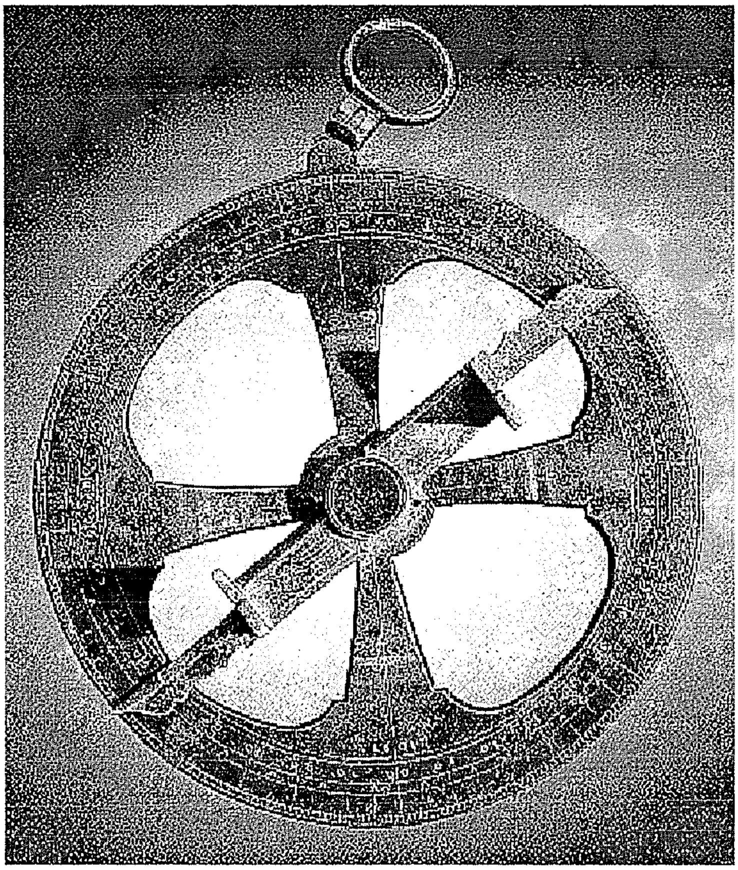


Figure 1-2 Mariner's Astrolabe

An Instrument with a Past and a Future

Surviving astrolabes serve many purposes. Examination of old astrolabes gives historians insight into the history of science and astronomy. Detailed examination of astrolabes by known makers charts the flow of technical information before printing was invented. The scales engraved on astrolabes and treatises explaining their uses give insight into cultures and the development of geometrical and analytical techniques. The metal provides a peek at historical metallurgy. Old astrolabes are an invaluable resource for historians.

But the astrolabe is more than a historical curiosity. Its use as a basic tool of astronomy education was recognized early, and astrolabe enthusiasts have continued to urge its use to introduce students to basic positional astronomy. One great virtue of the planispheric astrolabe is the *gestalt* learning experience it provides. The student learns basic astronomical concepts and vocabulary simply by learning to operate an astrolabe. It is fun for the student and efficient for the teacher. Creative educators all over the world use the astrolabe as a teaching tool. The astrolabe has been used in modern astronomy education for students from early primary school to university level.

For more advanced users, the astrolabe provides insight into timekeeping conventions and stimulates creative uses of the technology. Astrolabes are used by architects and photographers to determine Sun and shadow angles. Hikers use astrolabes to find how long it will be until sunset in order to plan stops. Astrolabes have been used to plan camping trips, sailing events, amateur astronomy star parties and a host of other activities. Craftsmen gain satisfaction making reproductions of old instruments. Teachers use astrolabe principles to cement mathematical principles.

The continuing interest in astrolabes confirms the durability of principles developed more than two thousand years ago and reaffirms the astrolabe as one of the most creative and useful devices ever devised. The following pictures of currency incorporating an astrolabe illustrate the astrolabe's continuing appeal.

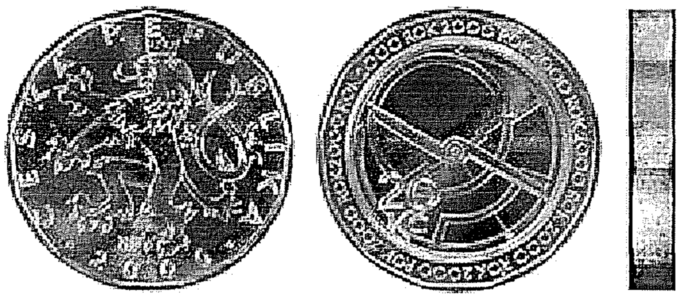


Figure 1-3. Czech Republic 20 koruna coin

Figure 1-4. Iraq 10,000 dinar note

A very basic introduction to the planispheric astrolabe follows. Much of the material in this section will be familiar to anyone with a background in astrolabes, but a scan of the material is encouraged to become familiar with our use of the terminology. All of the following material will be repeated in greater detail in the descriptions of the individual components.

The Principle of the Astrolabe

The flexibility and ease of use of the astrolabe derives directly from the method used to represent the sky on the face of the instrument.

The sky is hard to visualize. From time immemorial, people have looked at the sky trying to understand what is going on up there. We are confined to a place and, even under the best conditions, all we can see is part of the sky, and what we see is constantly changing. The Sun and stars rise and set over the course of a day, different stars are visible at different times of the year, and the Sun's position varies daily.

Conventions have evolved over millennia to describe the sky in an understandable, repeatable way, and many methods of visualizing the universe have been tried. Emperor Frederick III carried the solution of this problem to the extreme by having a special room constructed with a wire cage showing celestial positions as a roof. Frederick could sit in his room and see how the heavens behave through his dome. Even if we had Frederick's resources, this rather extreme solution does little to solve the problem of seeing the entire sky since it is not at all portable and works only for your castle in any case.

Globes showing the stars have been made, but we have to imagine we are in the center of the globe in order to relate the figures to what we see in the sky. Globes are expensive and hard to make accurately, and are poor at illustrating the movement of the sky over the course of a year. Globes are not a very effective astronomical tool.

A less expensive variation of globe including the essential information about how the sky works is an *armillary sphere* (Figure 1-5). The Latin word for "ring" is *armilla*; thus, an armillary sphere is a sphere made out of rings. Some very elaborate armillary spheres have been made since antiquity and some have been equipped with accessories allowing observations to be made[1]. Using an armillary sphere is not a bad way to start learning about the sky, and they were popular in the Renaissance and are still made, but new ones tend to be more decorative than useful. The main problem with an armillary sphere is it is not very portable; you can't put it in your pocket, and they are easily damaged. Also, the ability of an armillary sphere to provide accurate celestial positions is limited, and you can't use it for many practical problems.

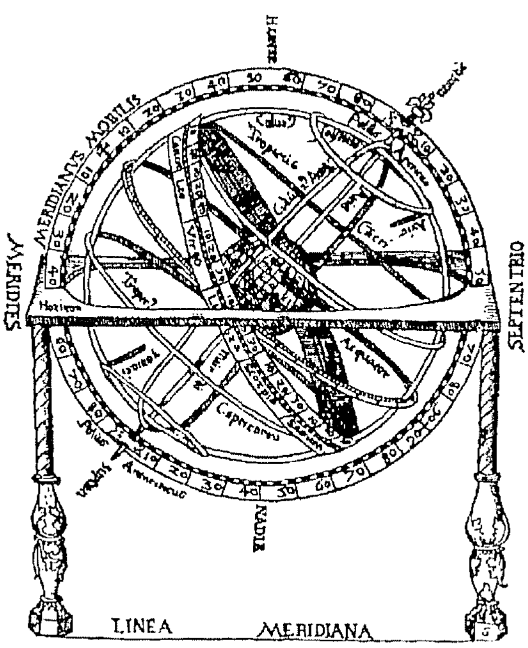


Figure 1-5. Armillary Sphere from the *Cosmographia* of Gemma Frisius

Sundials offer a means for finding the time from the position of the Sun. Many beautiful and ingenious sundial designs have developed, and the natural simplicity of seeing the time from the Sun's shadow is still enchanting. The problem with sundials is they only work on sunny days for a single location. The time can be found with an astrolabe day or night, and many time related problems can be solved without even going outdoors.

[1] Ptolemy describes an armillary sphere for conducting observations which he called *astrolabon organon*.

The popularity of the astrolabe derives from the fact that it is intuitive and both portable and flexible. You can carry it around with you, and you can use it for a number of practical applications. But, why does it work?

The planispheric astrolabe and other pre-telescopic astronomical devices use a method to represent the celestial sphere on a plane called the **stereographic projection**. The stereographic projection, like other projection methods, is a technique for representing a three-dimensional space on a two-dimensional plane.

Many types of projection are used for different tasks. The most familiar projection we encounter is the perspective used by artists to make a three-dimensional world look realistic on a flat painting.

The planispheric astrolabe and related instruments are unthinkable without the stereographic projection, and some knowledge of its characteristics is required to understand them. There is considerable discussion later on the details of how the projection is applied to instruments.

The Greeks studied many types of projection and evaluated their characteristics. They called the process of representing a sphere on a plane "unfolding the sphere." Among the methods studied was the method shown in Figure 1-6.

The sphere in the figure is the celestial sphere of arbitrary but very large diameter, with the Earth at the center. The equator and tropics are shown on the celestial sphere as they would be seen from the Earth. In the stereographic projection the eye is assumed to be at one of the poles of the sphere being projected (the south pole for astrolabes designed to be used in the northern hemisphere). A line is drawn from the pole to the point on the sphere to be projected. The projected point is where the line crosses the projection plane. The projection plane for a planispheric astrolabe is the plane of the equator.

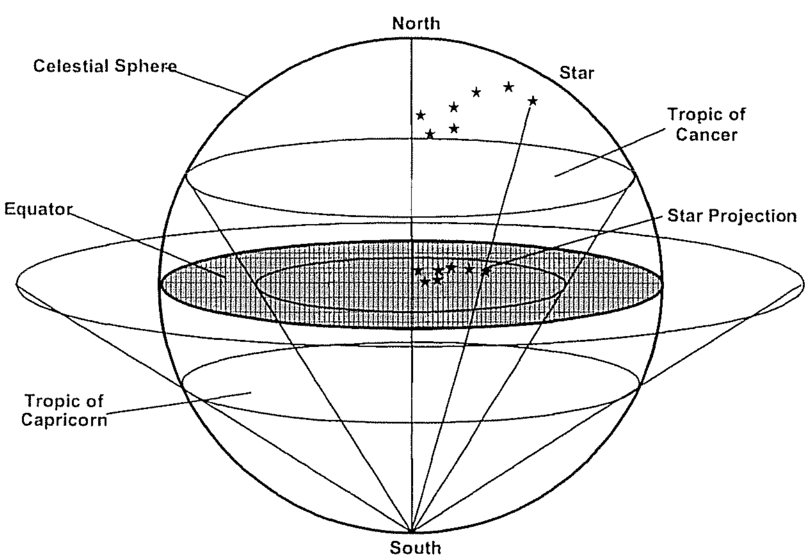


Figure 1-6. The Principle of the Stereographic Projection

The projection plane for the stereographic projection does not have to be on the equator. Modern high precision terrestrial maps and stars charts use the stereographic projection with a projection plane tangent to the sphere and centered at the place of interest, usually the zenith for a particular latitude. The projection principles are identical. The stereographic projection is also used in sciences other than astronomy, such as crystallography and cartography.

The stereographic projection has two properties making it ideal for solving astronomical problems:

- In the stereographic projection, circles on the celestial sphere are projected as circles on the projection plane (the equator for the planispheric astrolabe). This means the circles such as the tropics and the ecliptic are projected as circles and altitude and azimuth arcs are preserved. This property is particularly easy to demonstrate for the tropics, which, since they share a common center with the axis of the Earth, are clearly projected as circles.

- Angles between objects on the projection are the same as the angles on the sphere (conformal mapping). This allows direct measurements of angles on the projected plane.

These properties are ideal for astronomical purposes since most celestial positions are measured as angles along circles. Note, however, the stereographic projection distorts the sizes of objects with greater distortion for greater distances from the projection axis.

In theory it is possible to project almost the entire celestial sphere, but astrolabes constructed for use in the northern hemisphere show the celestial sphere only as far south as the Tropic of Capricorn[2]. This range includes the Sun's annual motion and represents almost all of the sky visible from northern temperate latitudes. Limiting the plate representation avoids distortion resulting from including far southern constellations. Such distortion as remains is acceptable since our interest is in solving problems rather than celestial cartography.

Using the stereographic projection, it would be possible to project the positions of the stars and planets at any time onto a piece of paper and then take measurements of relative positions. It is likely Hipparchus did this exact thing in the second century BC. It would, however, be impractical to attempt to project the sky at every instant for every location onto paper. The real genius of the astrolabe is how it makes use of the projection to allow the instrument to be used at any time and to be flexible enough to be used at many locations.

Because all components on the front of an astrolabe are made using the same projection of the sky, they can be lined up and the positions shown are true representations of the sky at that time. In addition, angles measured on the projection are equal to the same angles measured in the sky.

All projection techniques have properties that change what is on the sphere to what is seen on the projection. Some of these changes are desirable and some less so. For example, the familiar Mercator projection maps that are a staple of classrooms have the desirable characteristic of showing lines of latitude and longitude as straight lines. They have the undesirable characteristic of grossly distorting areas close to the pole, thus making Greenland appear to be as large as North America. The stereographic projection distorts the sizes of shapes in the sky far from the pole.

The application of the stereographic projection to the astrolabe is presented in detail in the following chapters.

[2] To project the entire celestial sphere would require an infinitely large astrolabe.

The Parts of the Astrolabe

Figure 1-7, is a depiction of the parts of a European Renaissance astrolabe in the Louvain tradition. These parts can be crude and simple or rather ornate and complicated. The astrolabe in the figure is quite sophisticated. Astrolabe components have different names depending on the language. Both Arabic and Latin names are shown and the most common modern terminology is used in the text.

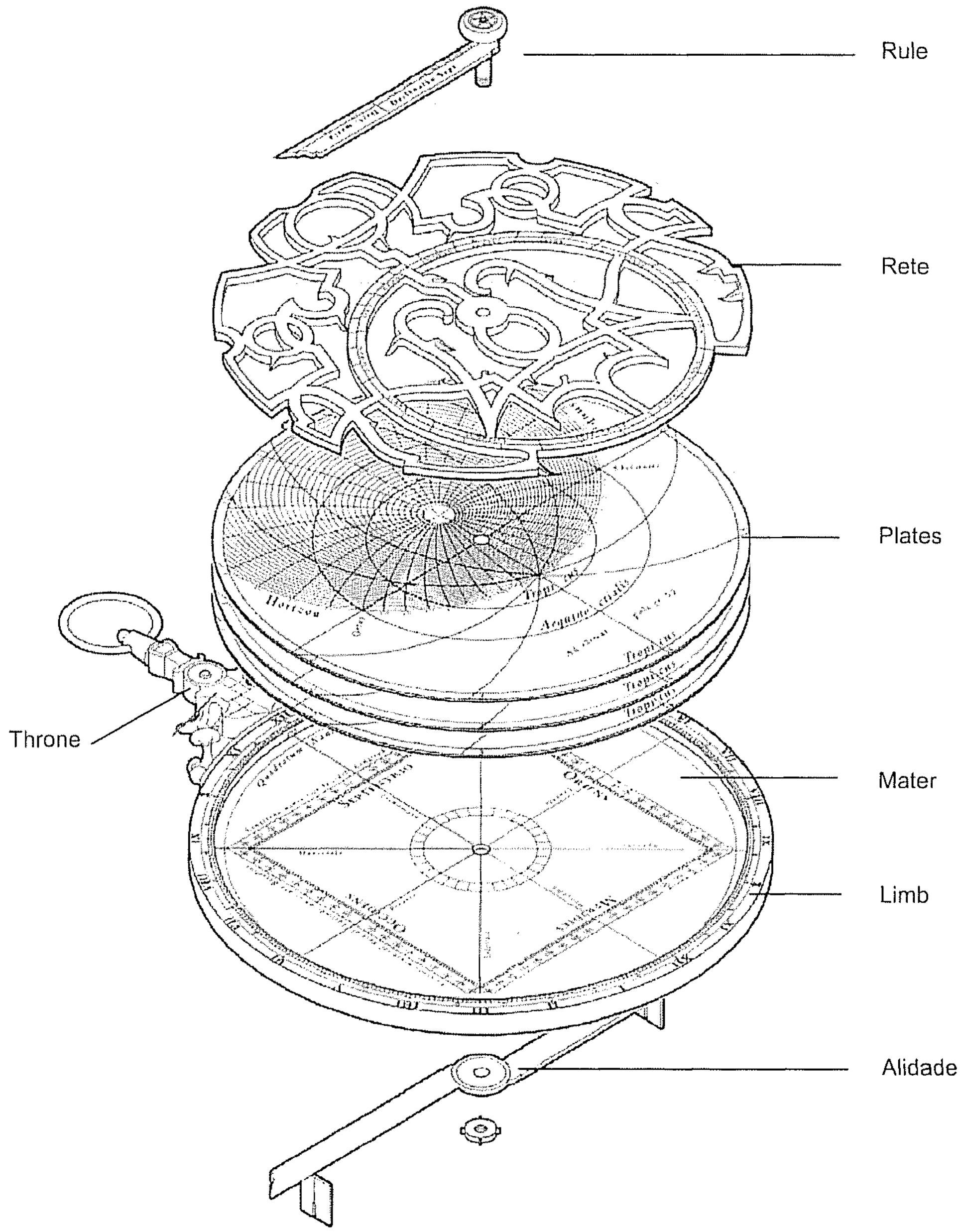


Figure 1-7. The Parts of the Astrolabe

Following is a very general overview of the parts of the astrolabe. Each element will be discussed in detail in subsequent chapters.

The main body of the instrument is usually called the *mater* (Latin for *mother*, English *may'ter* Latin *mahter*). It is called the *umm* (*mother*) on Arabic astrolabes. The mater of old instruments was usually about six inches (15 cm) in diameter, although much smaller and larger instruments were made. The mater is usually made of brass and consists of two parts: a solid disk defining the back of the instrument and a ring of the same diameter (the *limb)* cast as a unit or soldered to the back and engraved with time and degree scales.

The thickness of the limb ring creates a hollow in the mater used to store *plates* (also called *tympans*) displaying the stereographic projection of the sky for a specific latitude. Most astrolabes included several plates, each engraved on both sides to extend the range of locations where the astrolabe could be used. The astrolabe was adapted to a latitude by taking the instrument apart and inserting the desired plate on top of the stack and reassembling the astrolabe. Many astrolabes were made for a single latitude, particularly those made of paper or cardboard, where the stereographic projection was drawn directly on the mater.

Fitted over the plate is a pierced disk, the *rete* (*reetee*, Latin for *net*, Arabic *'ankabūt*), showing the locations of a number of stars and the stereographic projection of the ecliptic. As much of the rete disk as possible is cut away so the user can see the plate under the rete. The rete has pointers showing the positions of bright stars and the ecliptic circle engraved with a scale of the Sun's longitude, almost always in the form of the signs of the zodiac.

Many European astrolabes and a very few Islamic instruments had a rotating bar (the *rule)* over the rete serving the same purpose as the hand of a clock for finding the time and having many other uses.

The *throne* (Arabic *kursī*) is attached to the top of the mater. The throne of European astrolabes was generally fairly small and functional. It was quite elaborate on many Islamic astrolabes, particularly those from Persia. The suspensory apparatus (Latin *armilla*, Arabic *al-'urwa*) was attached to the top of the throne and normally consisted of a ring and swivel to allow the astrolabe to hang vertically from the thumb or a peg.

The back of the astrolabe (not shown in the figure) had many scales depending on where and when the astrolabe was made. All astrolabes had a scale of degrees for making observations using the *alidade* (Latin *regula*, Arabic *al-'iḍāda*). The alidade was free to rotate around the center of the back and had sights used to find the altitude of the Sun or a star. The alidade may have other uses with the scales included on the back.

The Front of the Astrolabe

The front of the astrolabe (Latin *facies*, Arabic *wajh*) contains scales for time and degrees and a stereographic representation of the celestial sphere.

The principal element of the front of the astrolabe is the *plate* or *tympan* (Latin *tabula*, Arabic *ṣafīḥa*), occasionally *climate*, which is the stereographic projection of the local coordinate system. The plate contains arcs representing the stereographic projections of the local horizon and circles of altitude and azimuth. A different plate is needed for each latitude at which the astrolabe will be used and most astrolabes included several plates covering a reasonable range of latitudes. Each plate was normally cut to include a lug or other retaining device to lock it in place.

The arcs on the plate will be discussed in detail later. For now, it is sufficient to understand the purpose of the main elements.

Visualize the astrolabe laying flat with the top pointing south.

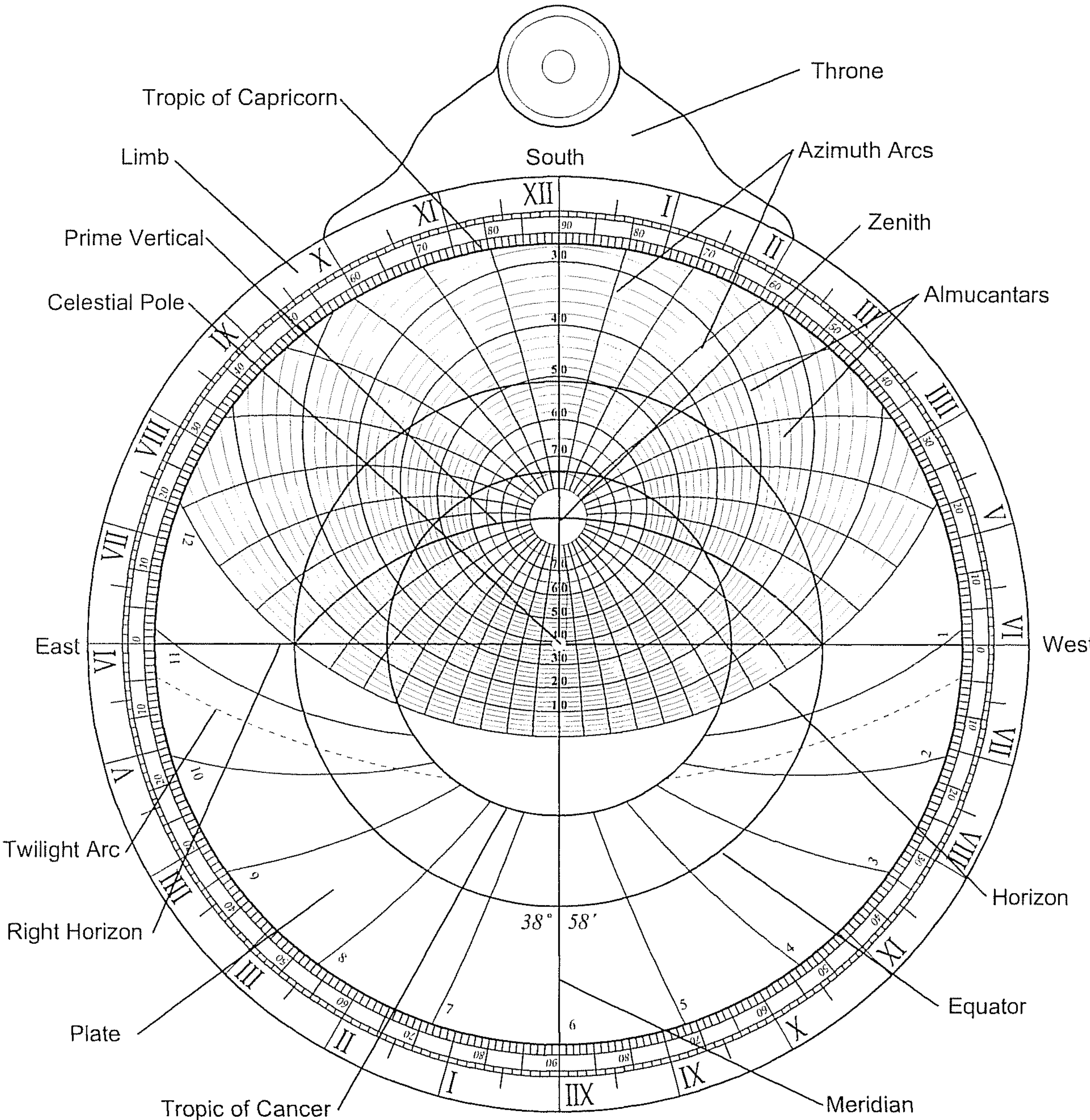


Figure 1-8. European Astrolabe Front

Referring to Figure 1-8, the vertical plate diameter represents the local meridian — the north–south line passing through your location. South is at the top. The astrolabe is oriented with the top pointing south and held flat, like a compass. The horizontal plate diameter connects east on the left and west on the right. It can simply be called the "east-west line" or the **right horizon** (i.e. the horizon at the equator). The center of the plate is the projection of the north celestial pole. The outer circumference of the plate is the Tropic of Capricorn, and the projection of the equator and the Tropic of Cancer are concentric circles.

It is most natural to describe the location of something in the sky by its angle above the horizon (*altitude*) and its angle from the meridian (*azimuth*). The web of arcs show the stereographic projection of the circles of equal altitude and azimuth. The lower arc is the local horizon, which is also called the **oblique horizon** to differentiate it from the right horizon. A celestial object is visible if it is above the horizon. The arcs above the horizon show the altitude above the horizon. If the altitude of the Sun or a star is, say, 30° above the horizon, the stereographic projection of the object's position will lie somewhere on the 30° altitude arc. The altitude arcs are often called *almucantars*[3] in astrolabe literature. The zenith is the point directly overhead. The arcs seeming to radiate from the zenith show angles of azimuth, the angle of something from the meridian. The azimuth arc from the eastern intersection of the horizon, through the zenith and to the west point on the horizon, is the **prime vertical**. The other plate elements will be discussed in due time.

The rim of the mater, the *limb* [Latin *limbus*, Arabic *ḥujra* ('side/enclosure') or *ṭawq* ('ring')] almost always contained a scale of degrees representing hour angles or right ascensions. It also contained a scale of time dividing the day into two twelve-hour parts on most European astrolabes and some Islamic instruments. Noon is at the top and midnight at the bottom.

The top of the mater, the *throne* (Arabic *kursī*), was more or less elaborate depending on the style of the time and place of manufacture. The throne on European astrolabes tended to be rather simple and functional. The thrones on some Islamic instruments, particularly those from Persia, could be very elaborate. A ring was attached to the throne through a hole that is exactly in line with the meridian. The astrolabe is suspended from the ring for making observations.

The Rete and Rule

A pierced disk representing the rotating sky is inserted into the mater on top of the plate.

It is called the *rete* (Latin for "net" or *al-shabaka* also "net" in Arabic, also called the *aranea* [Latin] or *al-'ankabūt* [Arabic], both of which mean *spider*). The word "rete" is Latin and should be pronounced "reetee", but almost everyone says "reet". The rete has two essential elements. The pointers on the rete indicate the positions of fixed stars. The stars can be seen to rise, culminate and set by rotating the rete.

The offset circle on the rete represents the path of the Sun over the course of the year: the ecliptic. The ecliptic was divided by the Sun's longitude, which was always shown by the sections of the zodiac.

The rete in Figure 1-9 is from the Fusoris astrolabe in the previous chapter.

Most European astrolabes had a clock type hand called the *rule* (Latin *ostensor*), used to show the position of the Sun in the ecliptic or to indicate the time. The rule might be divided to allow the determination of the declination of any celestial object represented on the front of the astrolabe. The rule might cover only half the front or it may extend across the entire face of the instrument.

The plates, rete and rule are held in the mater by a pin. On old instruments the pin had a slot for a wedge to lock the components in place. This wedge was usually made in the shape of a horse and was often aptly yclept the *horse*. Later instruments, and any astrolabe made today, would use a screw and nut.

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[3] From the Arabic *al-muqanṭara*.

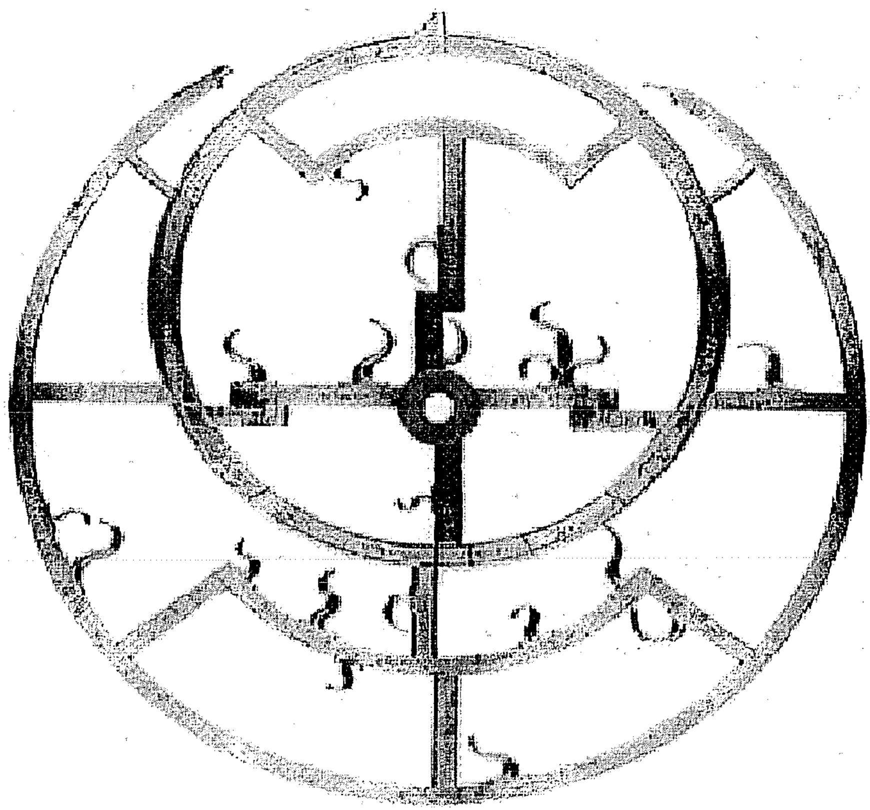

Figure 1-9. Rete

The Back of the Astrolabe

The back of the astrolabe contains scales for measuring the altitude of the Sun or a star and, on European instruments, scales for finding the Sun's longitude. Other scales were included depending on where and when an astrolabe was made.

The back of a representative European astrolabe is shown in Figure 1-10. The essential scale which must be included on all astrolabes is the scale of degrees in the outer margin. This scale was used to measure the altitude of the Sun or a star using the alidade.

The alidade was free to rotate around the center and had a fiducial edge indicating the current setting. The alidade was equipped with sights, normally small holes through which a star could be sighted or the rays of the Sun could shine through for making altitude measurements. The alidade is also be used in conjunction with other scales on the back of the instrument.

Alidade examples are shown in Figure 1-11 and Figure 1-12. The counterchanged alidade was common on European astrolabes, while the straight-bar alidade was usually found on Islamic instruments.

The scales on the back of the astrolabe varied considerably depending on whether the instrument was of Islamic or European origin. The only absolutely required scale is a scale of degrees in the margin for measuring celestial altitudes.

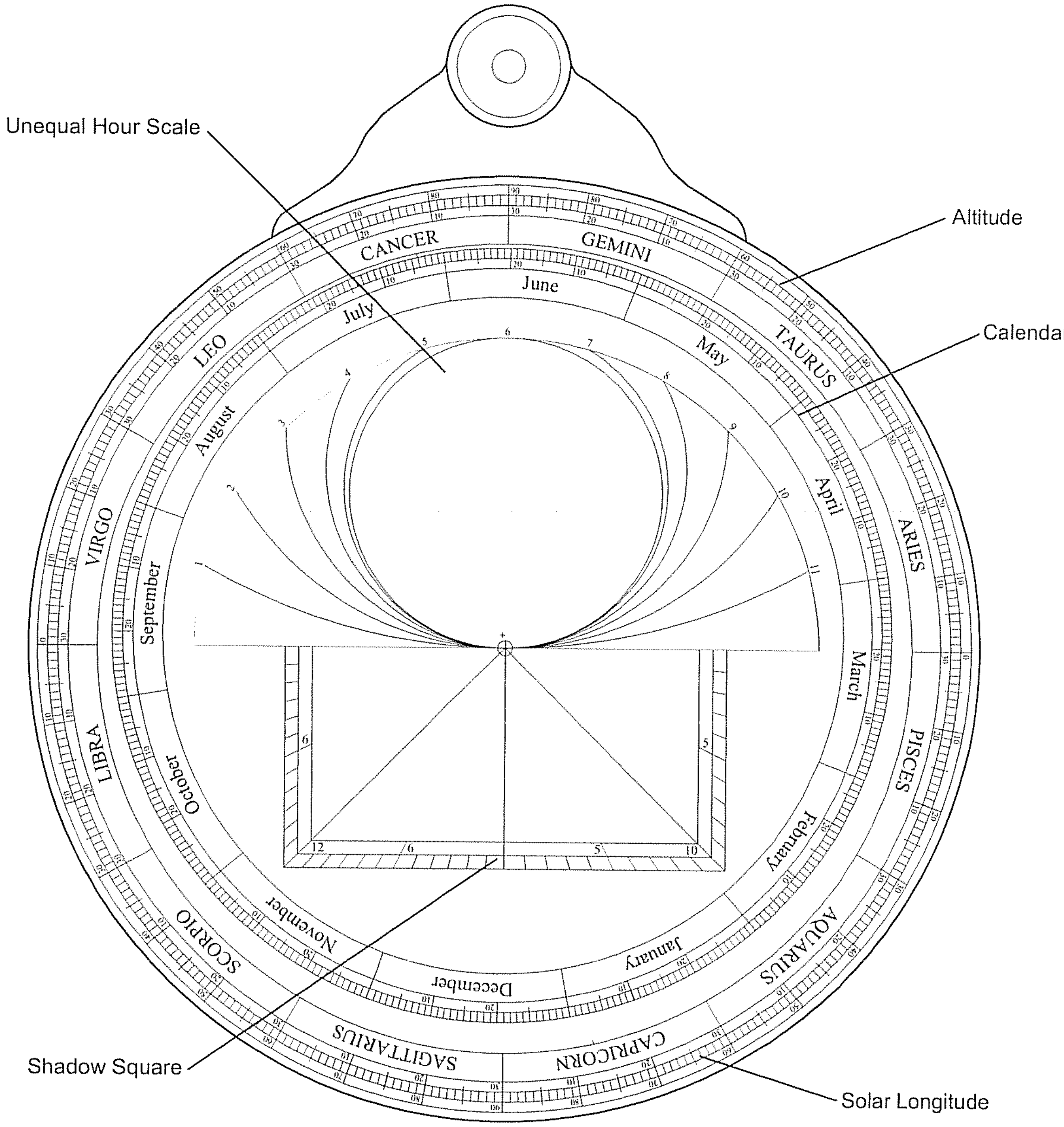

Figure 1-10. European Astrolabe Back

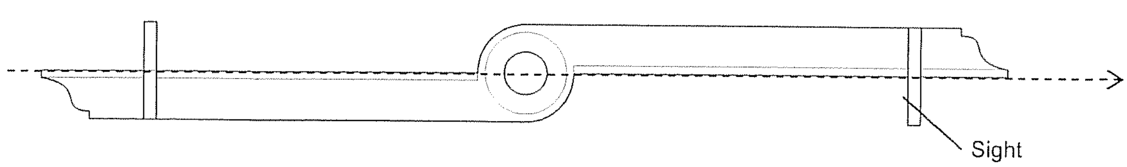

Figure 1-11. Counterchanged Alidade

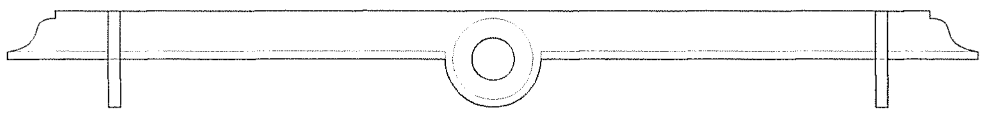

Figure 1-12. Straight-bar Alidade

In use, the astrolabe is suspended vertically by the ring in the throne, usually from the thumb or a peg, for measuring celestial altitudes. This establishes an artificial horizon for making altitude measurements. At night, a star is located by looking through the sights on the alidade and its altitude noted from the engraved scale. In the day, the Sun's altitude is found by allowing sunlight to shine through the holes in the sights.

The Sun's longitude for a day must be known in order to set the front to the correct day and time. All European astrolabes included a calendar and a scale of solar longitudes represented by the zodiac for finding the Sun's longitude for any day of the year. The edge of the alidade was aligned with a date and the corresponding solar longitude was read from the zodiac scale.

The calendar scale was usually drawn as an eccentric in keeping with the Ptolemaic astronomical roots of the instrument. The calendar on some astrolabes was drawn concentric with the center of the back. The eccentric calendar required more astronomical knowledge to design correctly but was easier to make. Concentric calendars were much more difficult to make accurately.

The unequal hour scale on the back of the astrolabe in Figure 1-10 is a universal scale and was fairly common on European astrolabes and was also found on Islamic astrolabes and quadrants. It is used to estimate the seasonal hour from the Sun's altitude and meridian altitude. A wide variety of other scales appeared on European astrolabes.

The earliest astrolabe treatises called the alidade the "diopter".

The diagram labeled as the *shadow square* in Figure 1-10 was used to solve simple trigonometry problems described by shadows cast by a vertical or horizontal gnomon.

The back of Islamic astrolabes was quite different. There was tremendous variety in Islamic astrolabes and the backs of specific instruments were characteristic of regional traditions. An example that can be considered representative of a late Western Islamic instrument is shown in Figure 1-13. Islamic instruments often contained scales related to the five daily Muslim prayers, which are astronomically defined.

The scale of degrees around the outer margin of the back is the same as on European instruments and is used in the same way. The arcs in the upper right quadrant of the example are used to find the direction of Mecca for prayers (the *qibla*). The scale in the upper left quadrant is used to find the values of sines and cosines and was used to solve a wide range of trigonometric problems. The shadow square is the same as on European instruments, but was to the added function of determining the time of Muslim prayers defined by shadows. The scale around the lower margin is a scale of cotangents that extends the range of the shadow square. All of these scales will be discussed in detail later.

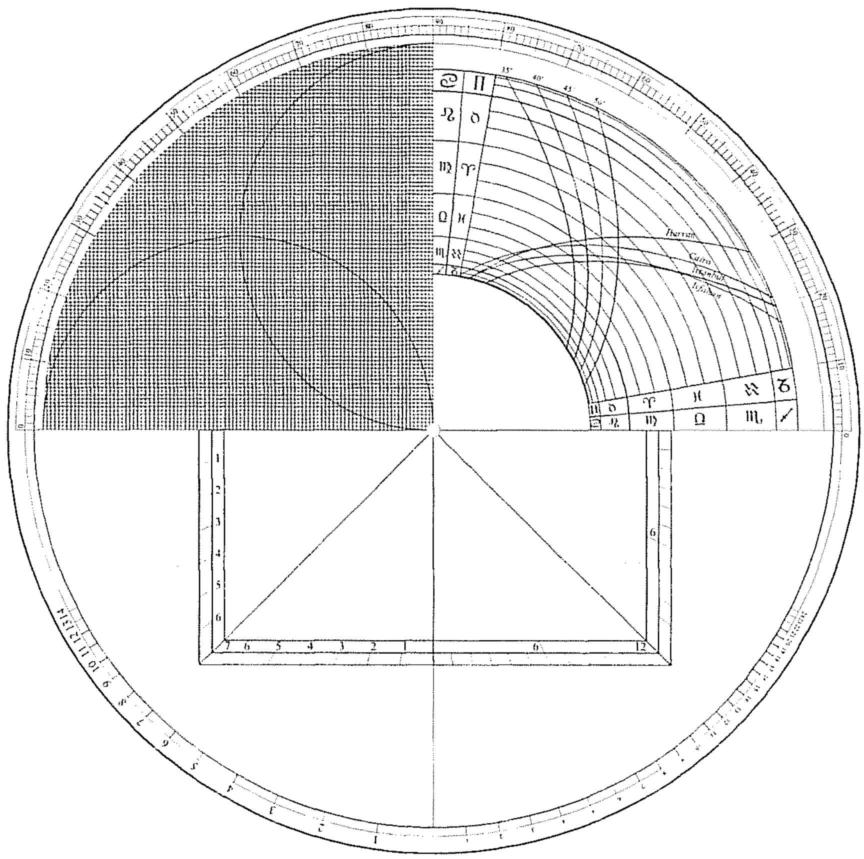

Figure 1-13. Islamic Astrolabe Back

Later chapters will cover details of each scale on each type of astrolabe and provide the technical details needed to make a working instrument.

An Illustrated Example

Following is an illustrated example of how an astrolabe is used to find the time at night by reading the altitude of a known bright star. The objective is to read the altitude of a star and use the reading to find the apparent solar time. Each step is illustrated in Figures 1-14 through 1-16.

The example uses an astrolabe made for latitude 38° 58′ North latitude and 77° West longitude, which is near Washington, DC. The principles are identical for any location.

It is November 9 in the late evening and the night is clear. You look up at the sky and see hundreds of stars. You can locate some of the familiar constellations, and the "Summer Triangle" is easy to see in the western sky. You find Altair and choose it to be the star you measure, because it is bright and not too high in the sky. Four steps are needed to find the current apparent solar time.

1. Suspend the astrolabe by your thumb and raise it above eye level. Hold the astrolabe steady and move the alidade on the back until you can see Altair through the alidade sights. Hold the alidade in position and read Altair's altitude as 40° (Figure 1-14). Remember the 40° altitude.

2. Line up the alidade so its edge is on November 9 on the calendar scale (Figure 1-15). It is late in the day, so move the alidade about ¾ of the way across the division for the day. Read the Sun's longitude from the zodiac scale as about Scorpio 17.5° (you can read the scales to about the nearest .25 degree or day).
3. Turn the astrolabe over. Rotate the rete until Altair is exactly on the 40° altitude circle to the west of the meridian (Figure 1-16).
4. Holding the rete in position, move the rule to Scorpio 17°.5 on the ecliptic circle. The rule points to 7:50 PM on the limb (Figure 1-16). This is the apparent solar time.

Astrolabes and sundials show apparent solar time, which is the way time was told until time zones were established in the 19th century. Apparent solar time is the time according to the Sun's position relative to your meridian. Noon apparent solar time is when the Sun is due south of your location. The angle of the Sun from the meridian measured on the equator (the **hour angle**) determines the time. The Sun appears to go around the Earth in 24 hours, or 15° per hour. If the Sun's hour angle is 15° west of the meridian, it is one hour after noon or 1 PM, apparent solar time.

Modern timekeeping uses time zones which are centered on each 15° of longitude from Greenwich. The Sun moves 15° in an hour, or four minutes for each degree of longitude you are from the center of your time zone.

We are at 77° west longitude in our example, which is 2° west of the time zone center at 75°. Therefore, the Sun will reach our meridian eight minutes later than the center of the time zone. We have to correct the apparent solar time read from the astrolabe for this eight minutes to get our zone time. This is called the *longitude correction* and is subtracted from apparent solar time to get zone time. Our longitude correction is –8 minutes since we are west of the time zone center.

5. The approximate civil time, therefore, is 7:50 - (-8 minutes) = 7:50 + 8 minutes = 7:58 PM. To be completely accurate, this time should also be corrected for the **equation of time** for the date. Classical astrolabes did not have a provision for correcting for the equation of time.

An experienced user could perform this operation in less than a minute and get a result accurate to within a few minutes, depending somewhat on the quality of the astrolabe.

What have we done to find the current time? We have aligned the stereographically projected ecliptic with the stereographically projected local coordinate system. When a star is placed at its correct altitude, the ecliptic is also at its correct position. When the rule is placed on the Sun's longitude on the ecliptic, it is on a radius from the center of the plate. Hour angles are projected as diameters of the plate, so the radius through the Sun is the Sun's hour angle. The time scale on the limb is divided into hours at 15° per hour. So, the rule, which represents the hour angle in this application, points to the time.

Note also on Figure 1-16 the point where the rule intersects the ecliptic is the Sun's position at that time, which is well below the western horizon. The Sun's altitude can be used to find the time during the day. In this case, the Sun's longitude as marked by the rule is set to the Sun's measured altitude and the time is read from the rule.

The calculations required to get the same result are outlined in the chapter on astronomical calculations (Page 366).

We have not yet discussed all of these results, but they will become clear as the components are discussed in detail. Refer to the Astronomical Background chapter if any of the terms are unfamiliar.

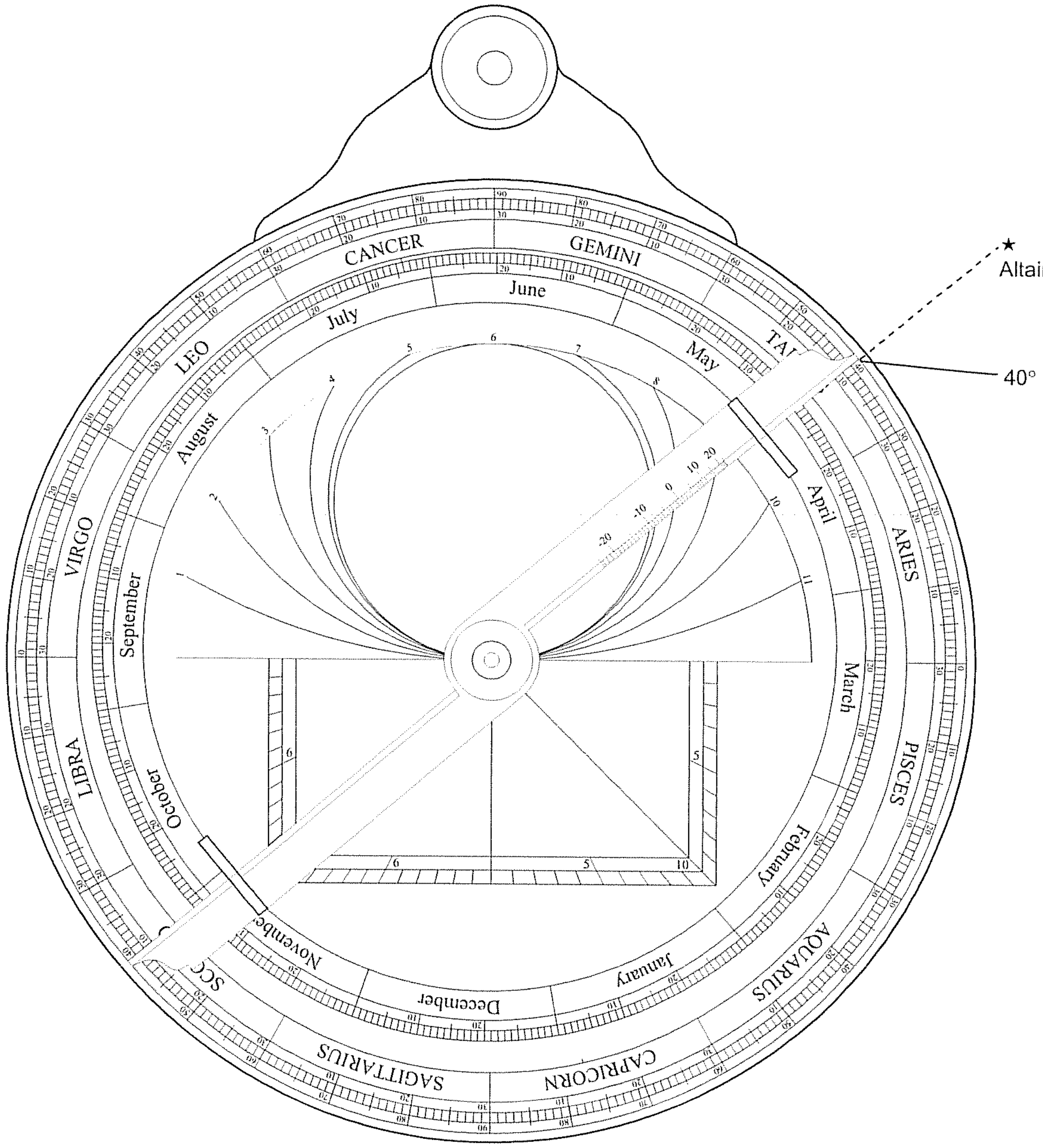

Figure 1-14. Reading altitude of Altair as 40°

You read the altitude of a celestial object by looking through the sights on the alidade. The angle of the object above the horizon is read from the outermost scale.

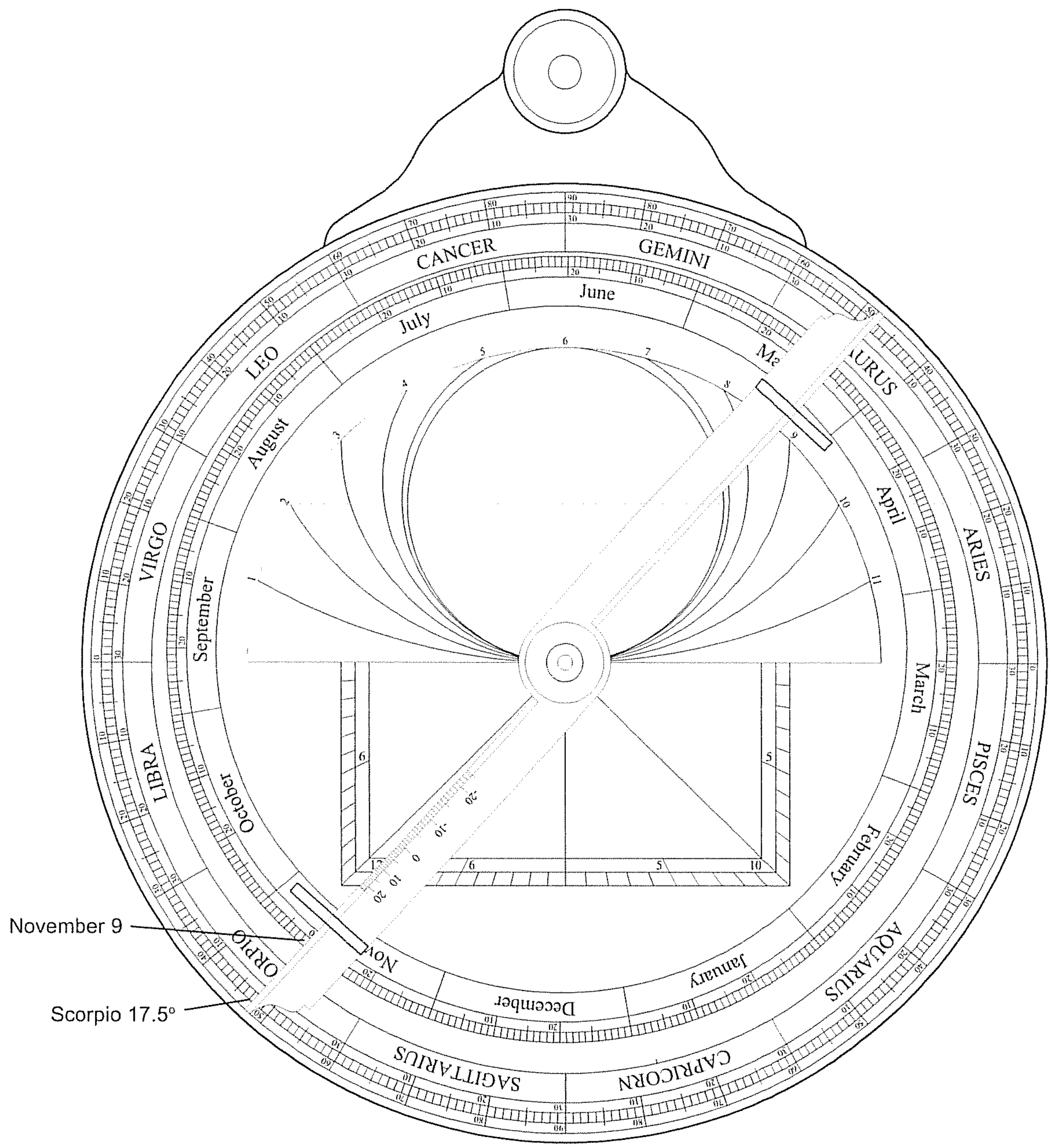

Figure 1-15. Setting alidade to November 9, 10 PM

When you set the rule on a date on the calendar scale, the edge of the alidade shows the Sun's longitude in the zodiac. In the example, each day tic is midnight for the date. We set the alidade to a little before the November 9 date tic for 10 PM.

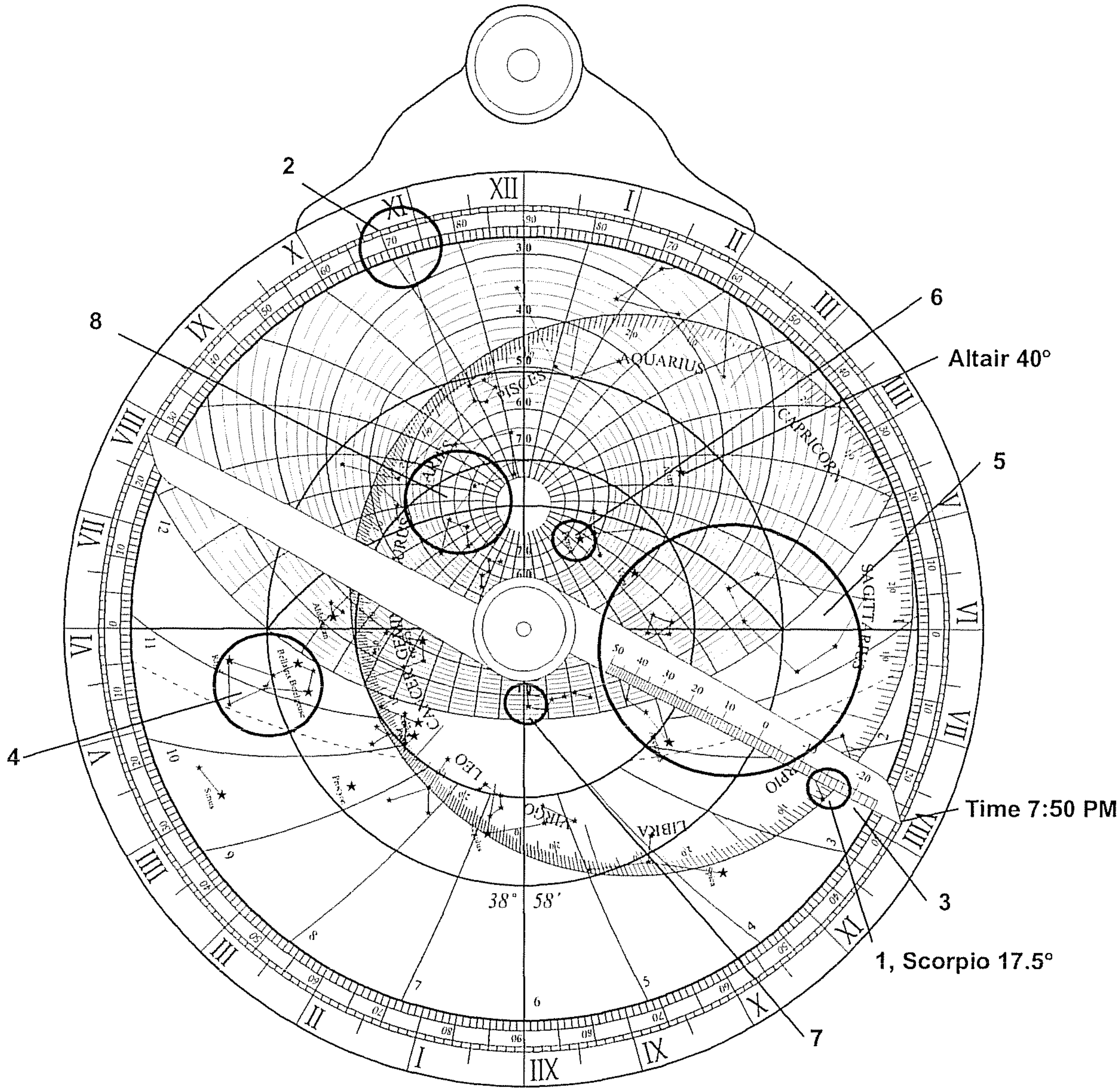

Figure 1-16. Astrolabe set for November 9, Altair at 40°

While we have the astrolabe set, let's see what else the astrolabe shows:

1. The declination of the Sun is about -17° as shown by the divisions on the rule.
2. The sidereal time is 22:50 shown on the limb by the pointer extending from Aries 0°.
3. The Sun's position on the ecliptic shows it is about halfway through the third unequal hour of the night.
4. Orion and Gemini are about to rise.
5. Ophiuchus and Boötes are setting.
6. Deneb in Cygnus is high in the sky at an altitude of about 65° with an azimuth 30° north of west
7. The "pointer" stars in the big dipper are near lower culmination.
8. Andromeda and Pegasus are nearing the zenith (this would be a good time to get out your binoculars and look for M31, the great galaxy in Andromeda).

Types of Astrolabes

The planispheric astrolabe was by far the most widely used astronomical instrument of its time, but variations were developed in an effort to provide more general or less expensive instruments. This section presents a very general overview of the types of astrolabe that have been made and includes pictures of representative instruments so you can relate the elements of the astrolabe discussed so far with actual instruments.

The purpose of an astrolabe is to allow the user to solve astronomical problems with adequate accuracy without having to do any math, and to visualize the movements of the Sun and stars. Several types of astrolabe have been developed.

The Spherical Astrolabe

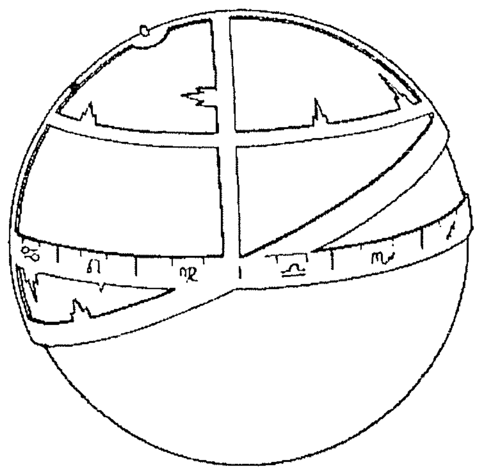

Figure 1-17. Spherical Astrolabe

The spherical astrolabe was apparently an attempt to make it easier to visualize the sky and still solve basic problems. It consisted of a pierced cap free to move around on a sphere marked with latitude circles (Figure 1-17). The cap included a representation of the equator and ecliptic and included pointers for a few stars. It was difficult to use, and the cap is very fragile. Only two examples survive, and this form will not be discussed further.

The Planispheric Astrolabe

The planispheric astrolabe is the principal focus of our discussion and will be discussed in great detail in subsequent chapters.

Planispheric astrolabes were made in sizes ranging from a few inches to several feet in diameter. They were used in cultures ranging from India through the entire Medieval Islamic world and all over Europe. Instruments varied widely in detail depending on where and when they were made and the skill and technical proficiency of the designer. It is known that many later instruments were made by teams with specialists providing the layout, casting, engraving and finishing.

Most early astrolabes were apparently single examples stimulated by individuals and executed by a single person or a small team. Some surviving astrolabes, particularly the most complex Islamic examples, were so elaborate they appear to be presentation pieces rather than working instruments. This also appears to be true for some of the finest European instruments which were made for wealthy patrons or clients and were probably never used for any practical purpose.

Does this mean astrolabes were simply playthings for the rich? Not at all. Having a really nice astrolabe must have been considered a status symbol reflecting the intellectual achievement of the owner. Lesser mortals had to make do with cruder instruments made of paper and wood, most of which have been lost.

The following pages include examples of two astrolabes representative of their type and source.

The astrolabe shown in Figure 1-18 (front) and Figure 1-19 (back) was made in the Nuremberg workshop of Georg Hartmann (1489-1564) in 1532. It is catalog number W-272 in the Adler Planetarium and Astronomy Museum[4] and is 13.7 cm (5.4 inches) in diameter.

This and the other surviving Hartmann astrolabes are particularly interesting because there is indisputable evidence Hartmann's workshop was a very early example of mass production on a modest scale[5]. It is clear the components of a series of instruments were made in batches and then finished and used in the final product. This is an excellent example of the predominate European astrolabe style from the 15th century.

It is difficult to define a "typical" astrolabe from the world of Islam. The Medieval Islamic world stretched from Persia in the east to Andalusia (Spain) in the west, and a wide variety of styles and scales were developed, some of them quite elaborate and complex. The astrolabe shown in Figure 1-20 and Figure 1-21 is catalog number A-70 in the collection of the Adler Planetarium and Astronomy Museum. It was made in 1647/8 [1057 AH] in Lahore by Ḍiyā' al-Dīn Muḥammad (flourished: 1635-1680) and is brass and copper with a diameter of 12.3 cm (4.85 inches).

[4] Webster, Roderick and Marjorie, *Western Astrolabes*, Adler Planetarium and Astronomy Museum, Chicago, 1998, pp. 53-55.
[5] Lamprey, John P., "An Examination of Two Groups of Georg Hartmann Sixteenth-century Astrolabes and the Tables Used in their Manufacture," Annals of Science, 54 (1997), 111-142.

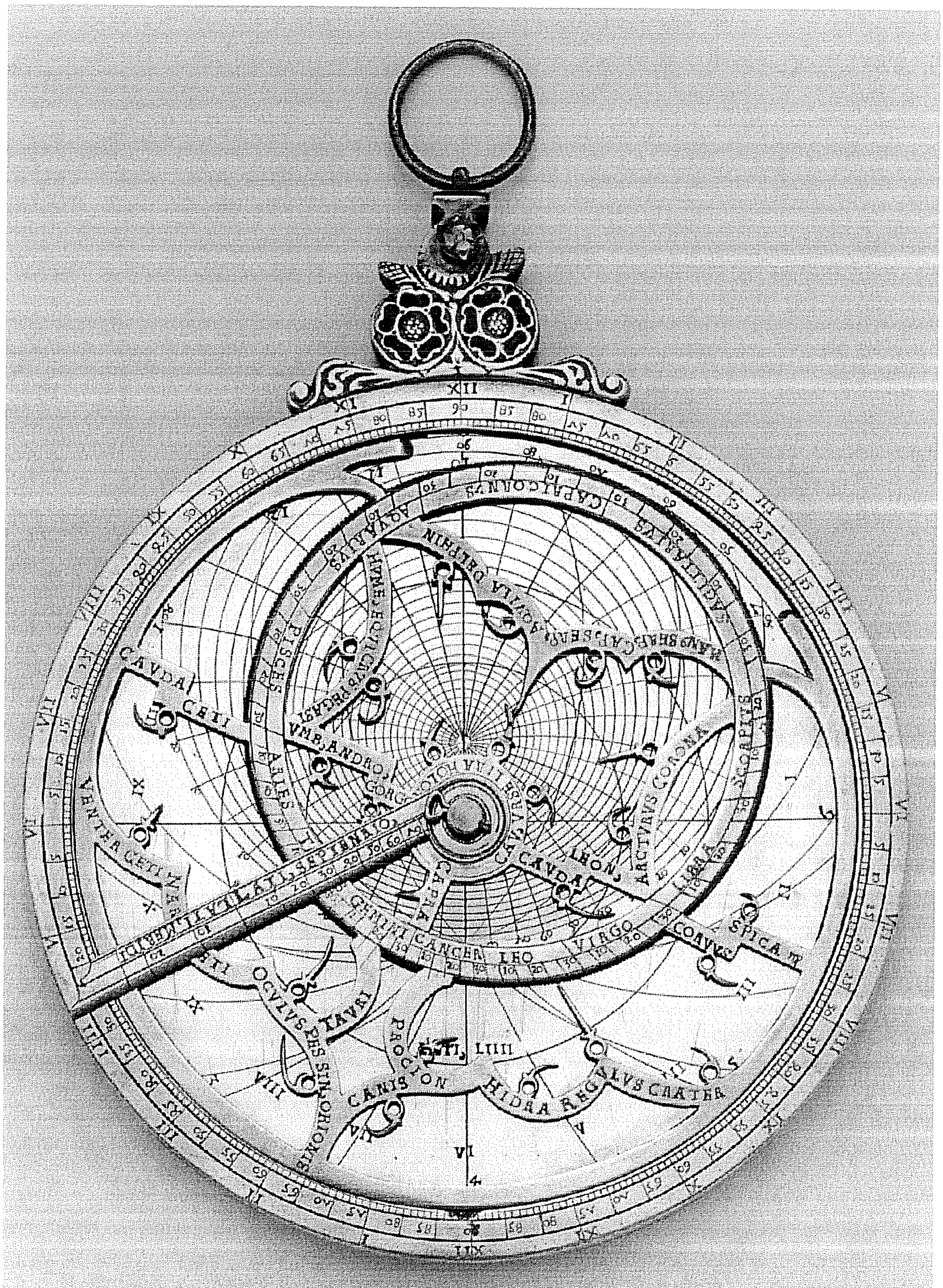

Figure 1-18. Astrolabe by Georg Hartmann

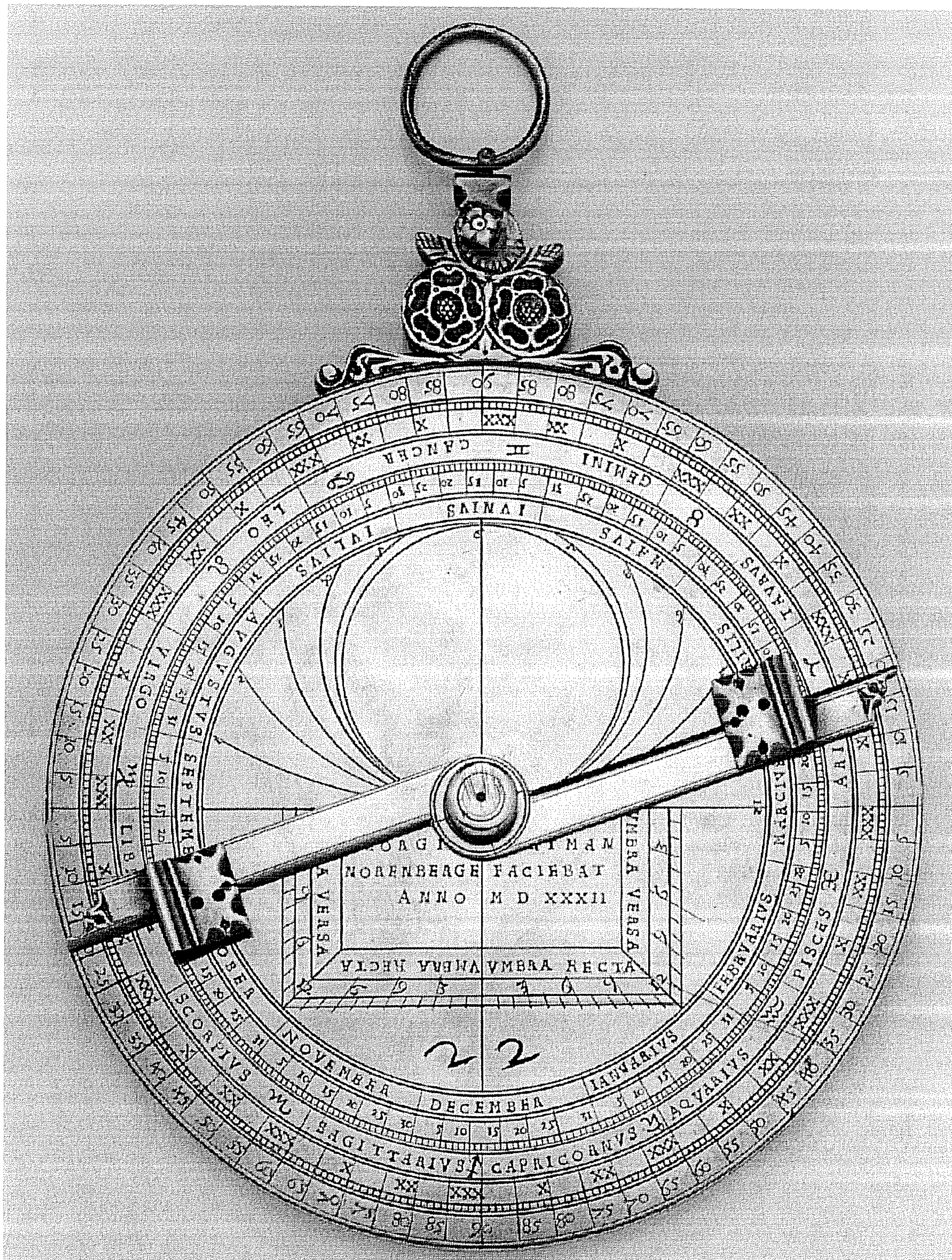

Figure 1-19. Hartmann Astrolabe Back

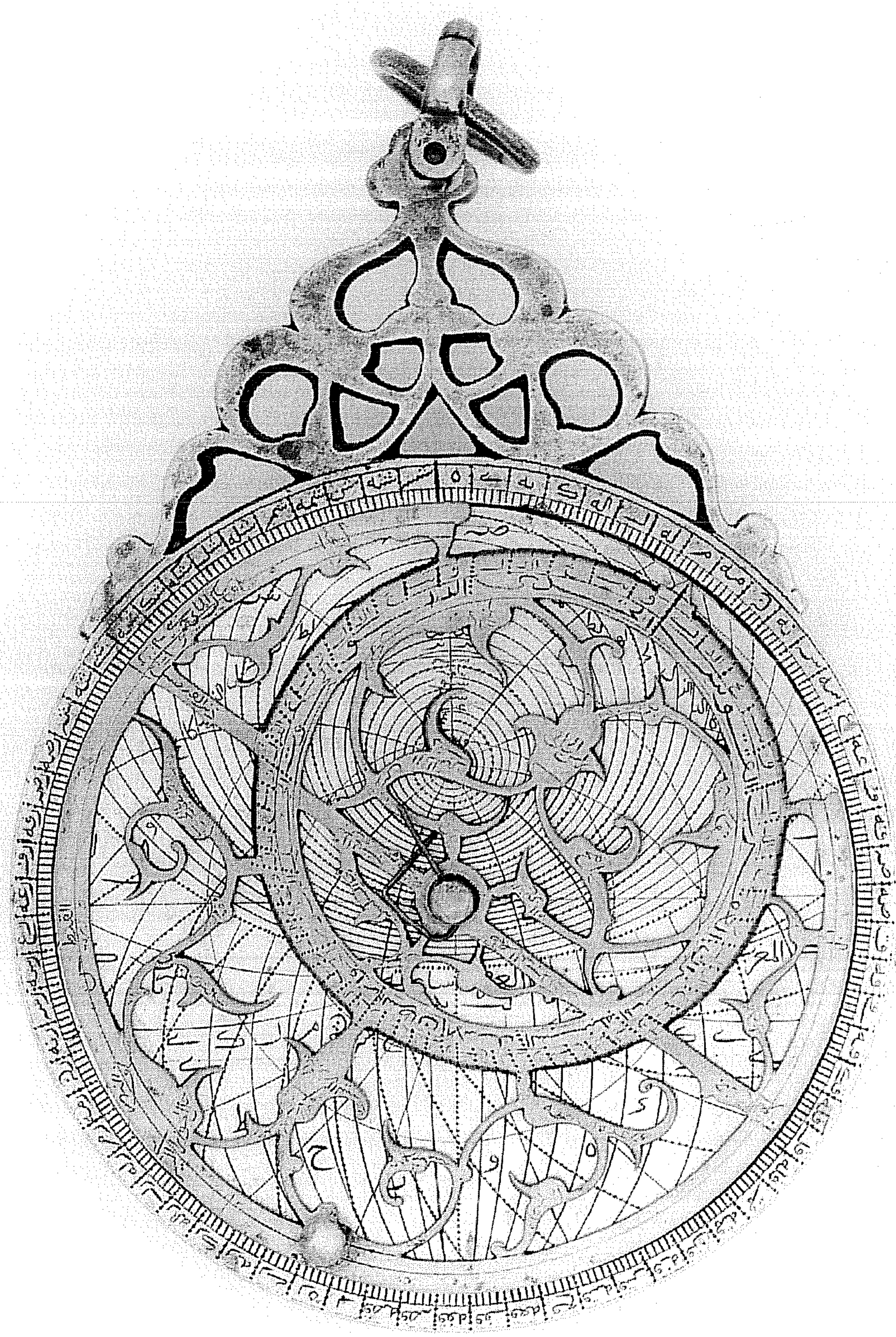

Figure 1-20. Islamic Astrolabe Front

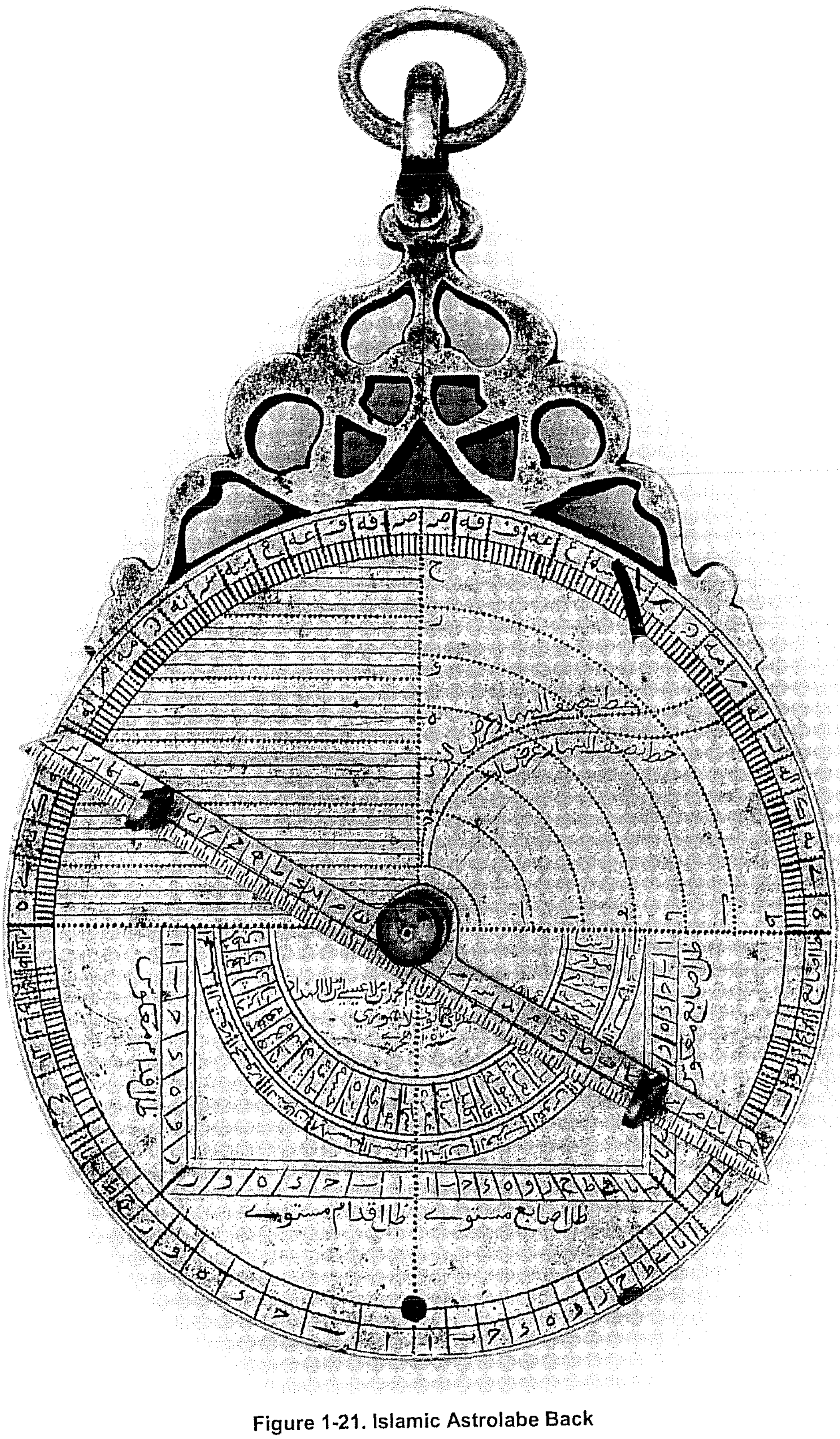

Figure 1-21. Islamic Astrolabe Back

Universal Astrolabes

Astrolabe variations intended to make the instrument usable at any latitude were investigated in the Islamic world by the 11th century and were described in European texts, but did not gain much European popularity until the 16th century. Two versions that had been studied centuries earlier were described and published around 1550. Gemma Frisius of Louvain in Belgium stimulated the production of an astrolabe based on the stereographic projection onto the **solsticial colure** originally described in 11th century Toledo (Figure 1-22). This type of instrument is known as the *saphea arzechelis* or simply the *saphea*. Juan de Rojas, a student of Gemma's, wrote on a similar instrument using the orthographic projection (Figure 1-23). Both types of instrument are described in detail in later chapters.

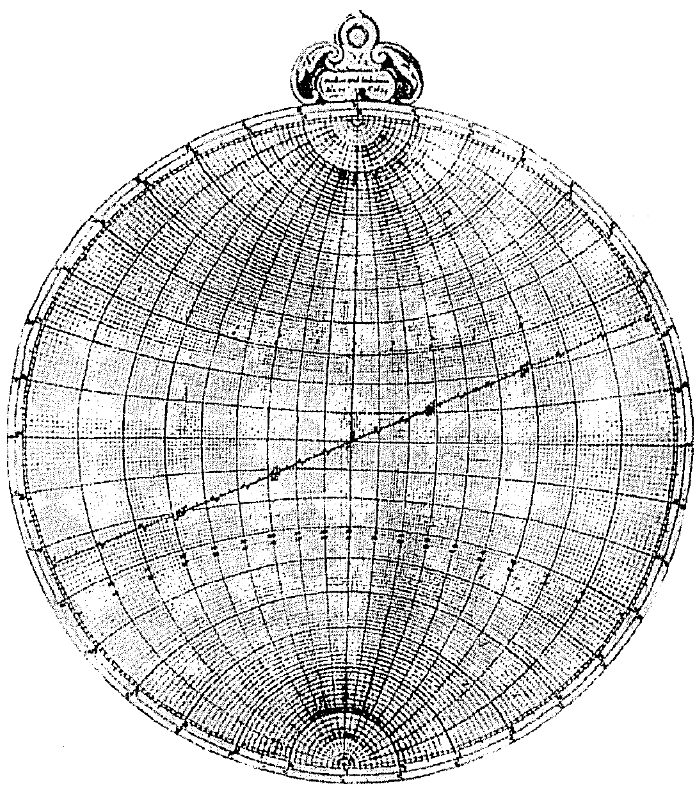

Figure 1-22. Saphea Universal Astrolabe

Astrolabe Quadrants

Astrolabes were always expensive and, in general, only the very rich could afford a good brass instrument. Instruments in the form of the less expensive quadrant providing some of the functions of a full astrolabe were developed. A quadrant containing the principal arcs of the astrolabe plate was popular in the Islamic world. Another somewhat more universal quadrant was described in the 13th century and became rather widely published in Europe, but few examples survive. This type of astrolabe quadrant became known as the *quadrans novus* (new quadrant) to differentiate it from the horary (time-telling) quadrant available for several hundred years known as the *quadrans vetus* (old quadrant). Both types of quadrants will be described in detail.

Other easier to use quadrants for finding the time were developed in the 17th century and gained a measure of popularity. Some of these quadrants are described in detail later.

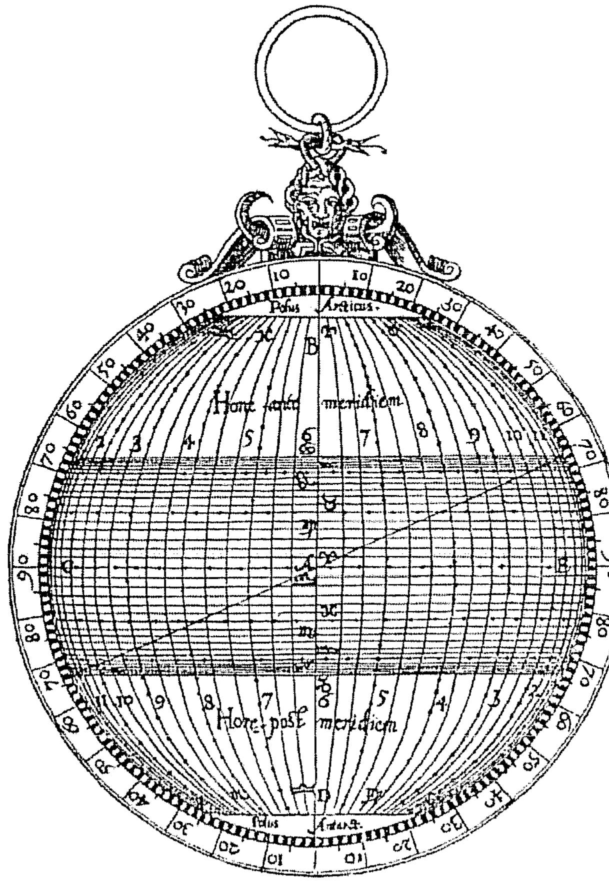

Figure 1-23. Rojas Astrolabe

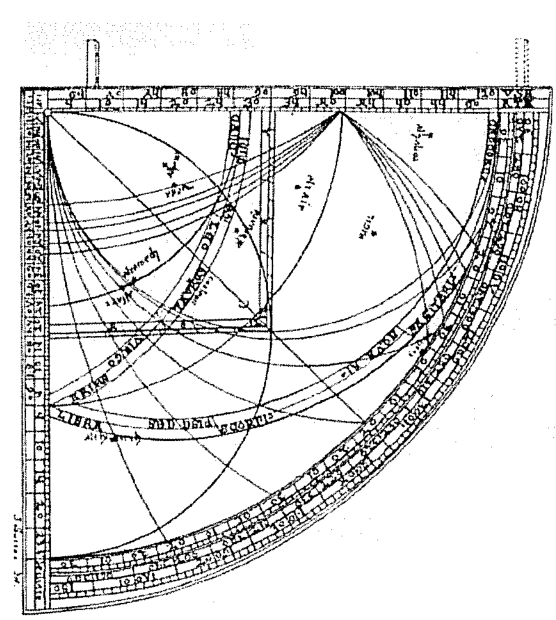

Figure 1-24. Astrolabe Quadrant (*quadrans novus*)

Horizontal Instruments

The stereographic projection was also applied to astronomical devices using the horizon as the projection plane. Several types of horizontal instrument were defined and implemented by Islamic makers, but only one gained much European popularity, a horizontal stereographic projection incorporated into a sundial by William Oughtred in the early 17[th] century. The applications of these instruments are described in a later chapter.

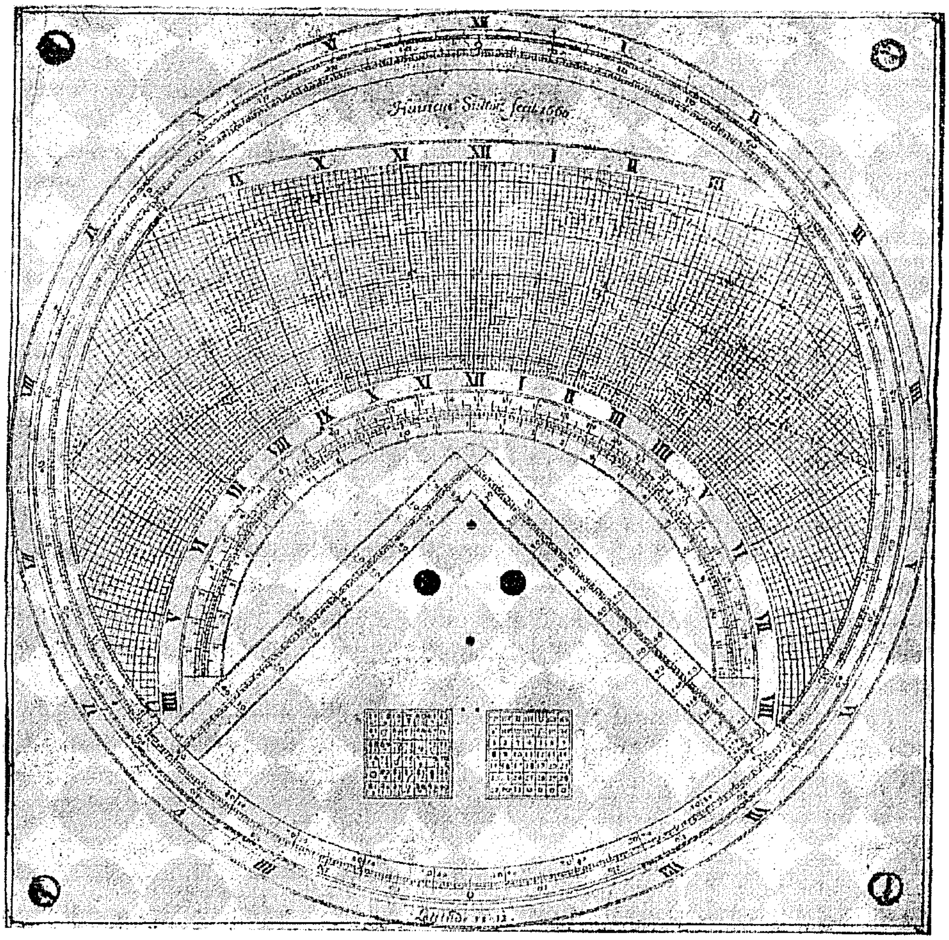

Figure 1-25. Double Sundial with Horizontal Stereographic Diagram

Astrolabe Clocks

One of the great appeals of the astrolabe is the dynamic view of the heavens provided. This view is enhanced when the astrolabe components are connected to a gear train that continuously keeps the correct alignment. A number of monumental astrolabe clocks were built in Europe, and astrolabe table clocks were popular masterpieces of guild artisans. The basics of these mechanisms are presented in a later chapter.

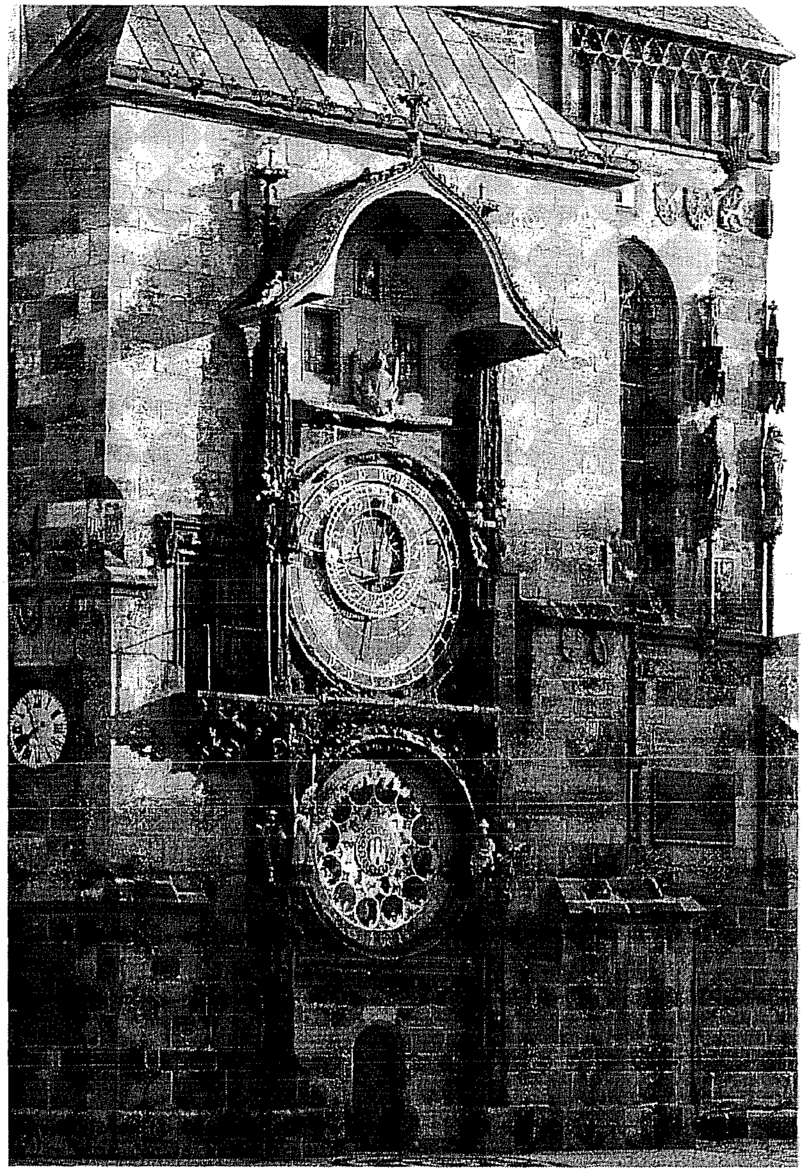

Figure 1-26. Astrolabe Clock on Prague City Hall

Computer Astrolabes

Computers present opportunities unimaginable to the old astrolabists. Computer-based tools are used to design and lay out instruments, and computer controlled processes are used to fabricate astrolabe components accurately. The results make it much faster and cheaper to create an astrolabe or astrolabe reproduction. Such technological advances serve to extend the life of the astrolabe even further.

One innovation unthinkable only a decade ago is a totally computerized astrolabe. An astrolabe drawn on a computer screen offers capabilities that capitalize on and extend the capabilities of a static instrument. For example, a computer astrolabe can be programmed to draw plates for any latitude in an instant. Far more stars can be included on the rete and the positions of the Sun, moon and planets can be shown for any date and time. The display can be animated to illustrate astronomical principles and the basics of orbital mechanics. Figure 1-27 shows a computerized astrolabe written by the author set for the instant when Neil Armstrong first set foot on the moon as seen from the Manned Spaceflight Center in Houston.

The computer astrolabe can even be portable when run on a laptop computer.

Some of the elements of a computerized astrolabe are discussed in a subsequent chapter.

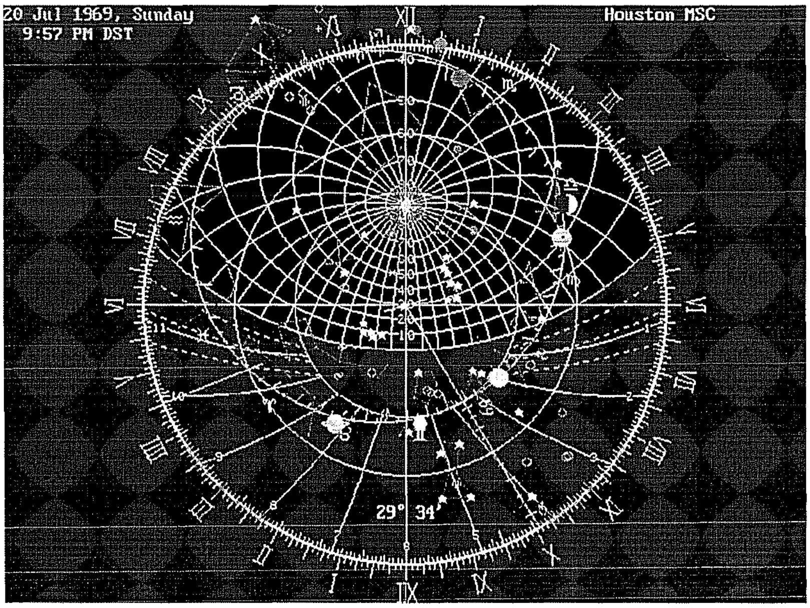

Figure 1-27. Computer Astrolabe Screen

Appreciation of the astrolabe and its derivatives is enhanced by understanding its origins and development. The history of the astrolabe is outlined in the next chapter.

" ... truste wel that alle the conclusions that han be founde, or ellys possibly might be founde in so noble an instrument as is an astrelabie, ben unknowe parfitly to eny mortal man in this regioun, as I suppose." Geoffrey Chaucer, "The Astrolabe - Bread and Milk for Children", ca. 1391.

Chapter 2 - A Concise History of the Astrolabe

The origins of the astrolabe are clouded in obscurity. True astrolabes existed by the late 4th century AD, but the steps leading to the application of the stereographic projection to instruments and combining the components to make the first astrolabes are not documented. Few astrolabe-related sources from the ancient world survive, and these are incomplete and confusing. Much high quality historical research using sparse and widely separated sources has allowed a picture of early astrolabe history to be pieced together, but the subject is far from clear. For the earliest origins of the astrolabe we can only speculate. The later history of the astrolabe is well documented by original and translated treatises on its theoretical foundations, design, construction and use, along with a good number of surviving instruments. Historians have supplemented astrolabe-specific sources with more general astronomical works to arrive at conclusions on the likely evolution of the instrument.

Following is a very brief historical summary of the development of the astrolabe and how it spread throughout the western world in the Middle Ages. While the history of the astrolabe is interesting in and of itself and is an excellent barometer for measuring the spread of scientific knowledge, it is also a meaningful element of the larger historical canvas painted over many centuries in many cultures. The interested reader is encouraged to explore the broader historical foundations of Greek science and culture, the birth and spread of Islam, medieval culture and historical scientific thinking. The real value of studying the history of the astrolabe is the insight it gives into to the minds of thinkers from the past, and one cannot help but be impressed by their intelligence and dedication.

The history of the astrolabe can reasonably be divided into five periods:

> **Foundation**. In this period the stereographic projection was developed and refined. True astrolabes did not exist during this period, but early devices based on the projection were made. The foundation period of the astrolabe can be placed from 150 BC or earlier to about the fourth century AD.
>
> **Development.** During the development period, the construction and scales on the front of the astrolabe instrument were defined, and its theory, construction and use were documented. Astrolabe development was entirely in the Middle East. The instrument we know today as the planispheric astrolabe was virtually complete in concept by the year 900.
>
> **Diffusion.** The use and manufacture of astrolabes spread rapidly once the instrument was fully developed and its many uses were appreciated. Diffusion through the Islamic world was very fast, and the instrument passed into India from Persia and to Europe from Muslim Spain. The astrolabe was widely accepted as a basic astronomical tool in Europe by the end of the 13th century.
>
> **Refinement**. Various improvements in scales on the front and back of the instrument were applied regionally to improve the utility of astrolabes in various cultures. New types of astrolabes were designed for special purposes, and astrolabes usable in any location were developed. Astrolabe theory reached its pinnacle and was well documented.. The refinement process, which overlaps the astrolabe's diffusion somewhat in time, continued through the 17th century.

Decline. Use of the astrolabe declined in the west as more accurate and specialized astronomical devices became available and as interest in astrology declined. The astrolabe continued to be used in the Islamic world for a much longer time, but after about 1300 was replaced by cheaper, more accurate quadrants for finding the time. Astrolabe production in the west declined dramatically in the 18th century and few astrolabes have been made since except to reproduce classic instrument or for education. Use in parts of the Islamic world continued through the 19th century.

Interest in the astrolabe has been high since the second half of the 20th century for its value in historical research into medieval astronomy and in studies of the diffusion of knowledge.

Practical uses for the astrolabe continue and recognition of the value of the astrolabe as a tool for basic astronomy education has resurfaced.

Foundations of the Astrolabe: Origins of the stereographic projection

The astrolabe is inconceivable without the stereographic projection[6], but it is not known exactly when or by whom the projection was first defined and studied. The origins of the stereographic projection were in classic Greece. Given their interest and sophistication in geometry, the Greeks were as comfortable with various types of projection as we are with columns of figures.

Agartharchus, an Athenian artist ca. 470 BC, applied the concept of projection onto a plane surface to the theory of perspective[7]. Eudoxus of Cnidus (408-355 BC), a student of Plato, is credited with a new form of sundial called "the spider's web (arachne)" that some sources wrongly say may have been a crude form of astrolabe. It is easy to accept that a projection as simple as the stereographic projection would have been investigated.

It is likely Apollonius (ca. 225 BC), the great codifier of conic sections, studied the stereographic projection based on the fact he included several theorems having direct application to the proof of preservation of circles[8].

The most influential individual on the theory of the stereographic projection was Hipparchus who was born in Nicaea in Asia Minor (now Iznik in Turkey) about 180 BC, but studied and worked on the island of Rhodes. Hipparchus, who also discovered the precession of the equinoxes and was influential in the development of trigonometry, is often given credit for discovering the stereographic projection in ancient writing. Since none of Hipparchus' writing survives we cannot be sure of this point, but numerous references by later writers credit him. It is widely recognized that Hipparchus was the most accomplished ancient astronomer, and it is likely his reputation was so great he was incorrectly credited with undeserved accomplishments by ancient commentators. Although Hipparchus' knowledge of the stereographic projection is a matter of speculation, it is likely Hipparchus redefined and formalized the stereographic projection, may have been aware of its main characteristics and applied the projection to celestial maps. If the projection were known before Hipparchus, his main innovation might well have been to move the origin of the projection to the south celestial pole to improve the utility in the northern hemisphere. Hipparchus was most likely studying one of the most difficult problems of ancient astronomy, determining the length of the day as a function of latitude (the so-called "rising time" problem). The solution clearly depends on the latitude of the observer, and it was a problem of some magnitude to find a solution. Hipparchus did not have spherical

[6] The stereographic projection was named by François d'Aguilon (Franciscus Aguilonius) in *Opticorum libri sex* (Antwerp, 1613) from στερεος (solid). Hence, stereographic is "drawing solids (on a plane)"
[7] Smith, D. E., *History of Mathematics*, Dover, NY, 1951. vol. 1, p. 79.
[8] Neugebauer, Otto, *A History of Ancient Mathematical Astronomy*, Springer-Verlag, New York (1978). V B 3, 1 pp. 858-60.

trigonometry to solve this type of problem and the stereographic projection worked nicely. He did not invent the astrolabe, but at the very least, he refined the projection theory.

The oldest surviving writing on the stereographic projection was from the famous Claudius Ptolemy (ca. 150 AD) who wrote on it in his work known as the *Planisphaerium*. There are tantalizing hints in Ptolemy's writing he may have had an instrument that could justifiably be called a precursor of the astrolabe. Any instrument of Ptolemy's was definitely not a true astrolabe, as it did not have a suspension or alidade for making observations. At best, his instrument would have been for a single latitude (Lower Egypt) with a rotating star field, and it probably covered a larger portion of the sky than later astrolabes. Ptolemy was deeply interested in "the greatest always invisible circle" so any instrument of his would have covered the celestial sphere to the Antarctic Circle. Ptolemy does not claim invention of the stereographic projection but credits Hipparchus. Ptolemy also refined the fundamental geometry of the Earth-Sun system used to design astrolabes. Ptolemy is also credited with removing the hour arcs above the horizon to reduce congestion of the plate.

Note the stereographic projection was nameless in its early days and was usually described as "unfolding the sphere" by the Greeks. It should also be mentioned that writings in Greek referred to instruments used to observe celestial objects as astrolabes (*astrolabos*), but these references should not be confused with the instrument later commonly called the astrolabe. Any instrument for measuring celestial positions relative to the horizon is still called an astrolabe.

The earliest evidence of an astronomical machine is in the writing of the Roman author and architect, Marcus Vitruvius Pollio (ca. 88 - ca. 26 BC), who in *De architectura* [Book IX, Chap. 8, 8-15] describes a clock (a clepsydra or water clock) made by Ctesibius (ca. 285-222 BC) in Alexandria, which predates both Apollonius and Hipparchus. Vitruvius does not mention the type of projection used by Ctesibius, but does say the hour arcs were constructed with the use of the analemma. There are two reasons I do not believe the network was stereographic: the theory was not developed when the clock was made and Vitruvius says the water flow changed to account for the change in length of the day, which would not be necessary for a stereographically projected network. In any case, this type of machine had significant influence on the history of machines in general and clocks in particular.

A later form of clock known as an *anaphoric clock*[9] had a rotating disk with a field of stars and the ecliptic with holes for a peg representing the Sun behind a wire frame indicating the hours of the day (the position of the Sun would have to be set manually by moving the peg each day). Similar constructions dated from the first to third century have been found in Salzburg and northeastern France, so such mechanisms were apparently fairly widespread among Romans. It is almost certain the famous Tower of the Winds in Athens contained a similar mechanism using the stereographic projection.

The wire framework (the spider) has been represented as an early use of the stereographic projection. Figure 2-1 is the form suggested by Drachmann. It contains arcs for the unequal hours for the entire day and declination circles for the beginning of each zodiac sign. The mesh in the figure is for Alexandria at 31° 13′ N in 100 AD. A plate with stars and the Sun's daily position rotated in (approximately) a day behind this wire mesh. The dot on the ecliptic is the peg for March 7. The clock shows it is near the end of the 8th hour of the day. The plate is conjectural and the stars included are just the brightest ones and may not be the ones used on the original which had drawings of the constellation asterisms. Sirius is the bright star above the eastern horizon and the outline of Orion can be discerned to the northwest of Sirius. Altair has set just inside the Tropic of Cancer. Assuming the clock was accurate, which is highly unlikely, it would have had a very informative display.

[9] The term "anaphoric" clock does not have an agreed definition. Some say it is any clock with a projection of the sky, such as an astrolabe clock. Others say it is a clock with a rotating star field. The name derives from *anaphora*, meaning 'rising', and specifically the rising of stars.

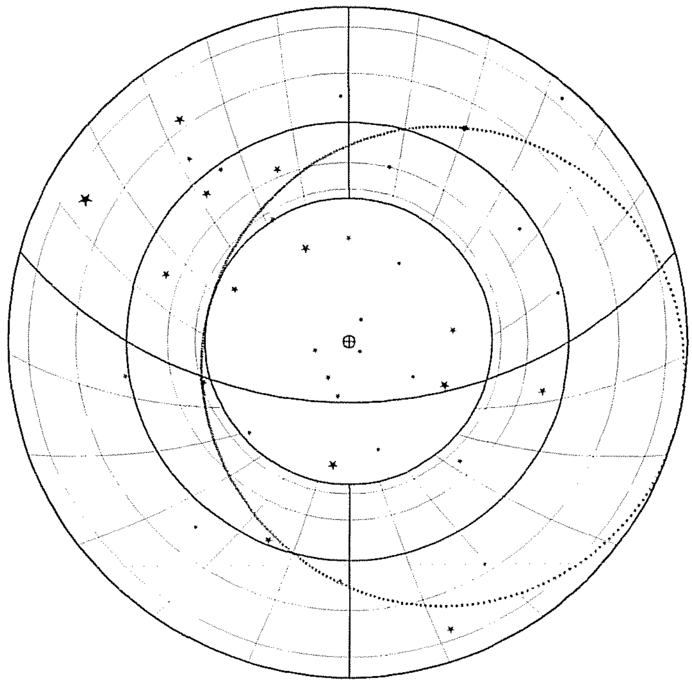

Figure 2-1. Anaphoric Clock Framework

A small portable sundial which is reliably dated to 143-44 AD, and is thus contemporaneous with Ptolemy, clearly uses the stereographic projection and its use of interchangeable plates for different locations anticipates the astrolabe form[10].

Development of the Astrolabe

No one knows exactly when the stereographic projection was actually turned into the instrument we know today as the astrolabe. Theon of Alexandria (ca. 375) wrote a treatise on "the little astrolabe" (apparently to differentiate it from Ptolemy's *astrolabon organon* which was a form of armillary sphere) that does not survive except for the table of contents, which was preserved by Ya'qūbī in his *History of the World* (ca. 880)[11]. This treatise was evidently the basis for most writings on the subject in the early Middle Ages. Synesius of Cyrene, Bishop of Cyrenaica (370-413) may have had a form of astrolabe constructed. This is plausible since Synesius was a student of Hypatia, Theon's daughter. It could, however, have been a variant of the astrolabe of Synesius' own design. The evidence of Synesius' instrument is documented in a letter to his friend Paeonius, in which he dedicates the gift of an instrument. The letter is very hard to understand, and there is significant disagreement among historians as to whether the instrument in question can be an early form of astrolabe.

The earliest surviving descriptions of actual instruments were written by John Philoponos of Alexandria (Johannes Grammaticus) in the sixth century (ca. 530), and a century later by Severus Sebokht (ca. 660) of Nisibis, Bishop of Kennesrin, Syria, although it is likely Sebokht's work was derivative of Theon[12]. It is certain true astrolabes existed by the seventh century.

[10] Turner, Anthony J., *The Time Museum: Time Measuring Instruments. Part 1. Astrolabes/Astrolabe Related Instrument"*, The Time Museum, Rockford, Il, 1985. pp. 10-11.
[11] Neugebauer, O., *Astronomy and History: Selected Essays*, Springer-Verlag (1983)
[12] Neugebauer, Otto, "The Early History of the Astrolabe", Isis 40 (1949), pp, 240-256.

The utility of the astrolabe as an aid in determining astronomically defined Islamic prayer times was recognized as soon as the theory became available through translation of Greek texts in the 8th and 9th centuries. Prior to this time, historical evidence on the level and sophistication of astrolabe knowledge is sparse and widely separated. The historical trail is much clearer and unbroken once Islamic scientists and mathematicians began writing astrolabe treatises. Astrolabe history from the ninth century on is documented by many treatises, copies of treatises and actual signed instruments, supplemented by a wide variety of other sources.

The Islamic capital was well established in Baghdad by the end of the 8th century after the turbulent years of expansion, and the empire was relatively peaceful in comparison to earlier years. Border skirmishes with the Byzantine Empire to the north continued, to the irritation of both sides, as they needed to deal with internal and theological rifts of higher priority. Several peace treaties between the Caliphate and the Byzantine emperors provided for interchange of documents from which the Arabs got a tremendous amount of Greek source material. The translation of Greek manuscripts was at a peak under the caliphates of Hārūn al-Rashīd (r. 786-809) and his son, al-Ma'mūn (r. 813-833) (al-Rashīd was the caliph of the *1001 Nights*). Among the works translated were Ptolemy's *Mathematical Syntaxis,* which had been renamed *Almagest* ("The Greatest") in Byzantium, and other works on astronomy, including material on astrolabes. Much of the translation from Greek and Syriac into Arabic was done in Baghdad by Jews and Nestorian Christians. Harran became an early center of astrolabe production[13].

Muslims first encountered the astrolabe in Harran in the mid-8th century. The first Muslim to actually make an astrolabe was al-Fazāri, one of the first Muslim astronomers known to us. The oldest known astrolabe is from the late 8th century. The location of this this small instrument (8.5 cm), which was in the Baghdad museum, is not currently known. It includes 14 stars, plates for the seven classic climates with no meridian or azimuth arcs and only the altitude scale on the back[14].

There was no analytic approach to the stereographic projection in the Greek foundation. al-Khwārizmī was the first to apply analytic methods to astrolabe design in the mid-9th century. However, most medieval Islamic astrolabes were designed using tables prepared for that purpose rather than from first principles. For example, al-Khwārizmī supplied tables of almucantar radii and centers for a range of latitudes[15].

Knowledge of astrolabes was widely available by the 9th century. The eminent scholar, Abū al-Abbās Aḥmad ibn Muḥammad ibn Kathīr al-Farghānī[16] (fl. ca.820 - after 861) wrote his *Book on the Construction of the Astrolabe* (*Kitāb fī ṣan'at al-asṭurlāb*) in part because he felt astrolabes were being made and used by rote without true understanding of the underlying principles. He was the first to publish a proof of the circle preservation property of the projection and provides complete instructions for design.

The oldest surviving dated instrument was made by Nasṭūlus, an Egyptian, in 927/8 (315 AH)[17]. Some undated examples are even older. There are a total of eleven astrolabes in collections dateable from before 1000 (only three have actual dates). This is an amazing number of instruments to survive for such a long time and reflects the esteem in which the instruments have always been held.

[13] Many astrolabists of the 9th and 10th centuries bore the *nisba al-Harrani* indicating an origin in Harran but living elsewhere, probably Baghdad. [Charette, private communication]
[14] King [2005], p. 411.
[15] King [1983], pp. 25-26 and Charette [2004].
[16] Arabic names often have many elements. *ibn* means "son of" and is often used in a genealogical sense. *al* (*at, ar, ad*) is the definite article, like the English "the". The origin of a person is given by the *nisba*, as al-Ṭūsī is "the Tusian", meaning he was born in Tus.
[17] King, D. A., "A Note on the Astrolabist Nasṭūlus / Basṭūlus ", Archives Internationales d'Histoire des Sciences 28 (1978), pp. 115-118.

Later, such notable Persian scholars as al-Bīrūnī (973 - 1048 [363 - 440 AH]) and Naṣīr al-Dīn al-Ṭūsī (1201 - 74 [598 - 673 AH]) wrote detailed treatises on the astrolabe[18]. 'Abd al-Raḥmān ibn 'Umar al-Ṣūfī (986-7 [376 AH]) wrote an amazing treatise of 386 chapters presenting 1000 uses for the astrolabe[19].

The earliest astrolabes were apparently greatly influenced by Philoponos and were supplied with plates showing almucantars for the seven classic climates (page 58). The back of the earliest astrolabes contained only the altitude scale. The astrolabe was mature by the year 900 and a 10th century astrolabe is instantly recognizable as an astrolabe. The basic form of a brass instrument body (the *mater* or *umm*) with a divided rim (the *limb*) holding several latitude plates, and equipped with a rotating rete with the ecliptic circle and star pointers and a suspension and alidade was universal. Astrolabe makers had access to detailed instructions for design including lists of appropriate stars to include on the rete and instructions for users were available.

Figure 2-2 shows a possible reconstruction of some of the earliest astrolabes based on comments from the old treatises[20]. It does not include the right horizon or azimuth arcs, and the equator and Tropic of Cancer are shown only under the horizon. The limb is divided by degrees with no time scale. It is instantly recognizable as an astrolabe plate nonetheless. The plate latitude is the climate of Rhodes (p. 58). It is not surprising that even the oldest astrolabes included unequal hour arcs since timekeeping in that era was almost universally based on unequal hours. In fact, it is likely the inclusion of unequal hours derives directly from the anaphoric clock. Some, and possibly most, of the earliest astrolabe plates did include the right horizon.

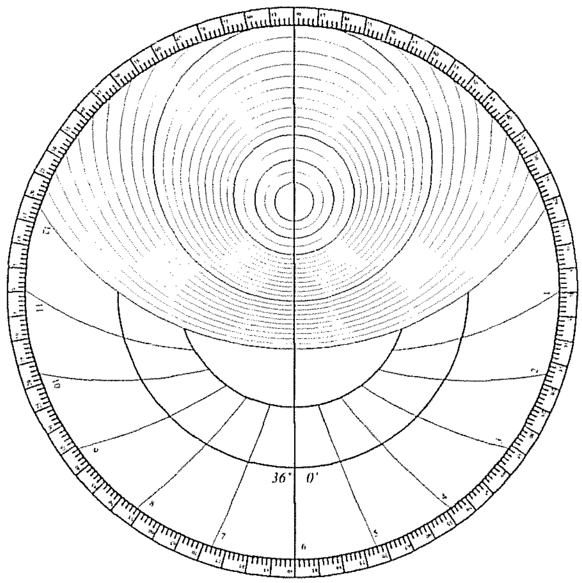

Figure 2-2. Possible very early astrolabe plate

The development of additional astrolabe scales by Islmaic astronomers documented in surviving treatises before the end of the 10th century included[21]:

[18] Hartner, Willy, "The Principle and Use of the Astrolabe", *Oriens Occidens: Ausgewählte Schriften zur Wissensschafts und Kulturgeschichte. Festschrift zum 60. Geburtstag* (Collectanea III), Hildesheim, 1968.
[19] Kennedy, E. S. and Destombes, Marcel, "Introduction to Kitāb al 'Amal Bil Asturlāb", Osmania Oriental Publications Bureau, Osmania University, Hyderabad, India, 1968.
[20] See also, Stautz [1994].
[21] King, D. A., "A Supplement to the Standard Literature on the Astrolabe", unpublished, 2002.

- Azimuth curves (early 9th century).
- Twilight arcs and daylight prayer plates (10th century)
- Plate of horizons (mid 9th century).
- Ecliptic plate
- Shadow scale (9th century).
- Sine quadrant (9th century).
- Prayer time scales.
- Seasonal hours
- Azimuth of the qibla
- Calendar/Longitude scales

Development of trigonometry also aided in the design and layout of astrolabes.

Dispersion of Astrolabe Knowledge

The astrolabe moved quickly from eastern Islam (the Mashriq) to India and through North Africa (the Maghrib) to Muslim Spain (Andalusia). Instruments dating from the eleventh century were made in Toledo. Knowledgeable treatises from Spain date from around 1025, but clearly astrolabe knowledge was available in Western Islam earlier. A treatise on the use of the astrolabe by ibn al-Ṣaffār (417 AH / 1026) became very influential in Europe as a Latin translation (usually falsely attributed to Māshā'allāh or, occassionally, Maslama ibn Aḥmad al-Majrīṭī, al-Ṣaffār's teacher). The translation of this treatise by John of Seville in the middle of the 12th century incorporated both construction and use instructions and was re-edited, copied and expanded many times to become the most widely used astrolabe reference and standard text. All of the early astrolabe treatises were based in some way on earlier Western Islamic treatises and surely contributed to the adoption of Arabic names for stars and other astronomical elements.

It is possible there was another path for astrolabe knowledge into Europe by way of Venetian traders but this route is not well documented.

An early champion of the value of Islamic science to Europe was Gerbert of Aurillac (ca. 945-. 1003) who spent some years in Spain and studied in Barcelona and may have been the first to transmit the new knowledge into Christian Europe. Gerbert mentions astrolabes in surviving correspondence. He rose rapidly in the church hierarchy, eventually becoming Pope Sylvester II (999-1003), the first French Pope. It is interesting that Gerbert commented on the utility of astrolabes for determining the canonical hours for prayer, but he felt finding the time was unnecessary for the general public. No commentary could underscore the lack of concern about timekeeping in medieval Europe more powerfully.

Transmission of Islamic scientific knowledge in general, and astrolabe principles in particular to Christian Europe was aided by Christian monasteries on the border with Andalusia. Notable was Santa Maria de Ripoll, a Benedictine monastery near the Pyrenees whose monks translated many Arabic documents for their own use in the 10th and 11th centuries. One manuscript includes at least eleven sections concerning astrolabes. The rapid movement of this knowledge is demonstrated by the fact that Count Hermann Contractus (Hermann the Lame) (1012-1054), a student at the Reichenau monestary school in Germany[22], wrote a treatise on the astrolabe based on a Latin translation of the Ripoll manuscripts by Llobet of Barcelona. The path of dissemination of Islamic science was apparently from Catalonia to Lorraine, Liège, Gorz, Cologne and to the rest of Europe[23]. The astrolabe reached England by the late 11th century.

[22] Hartner Ibid, p.290.
[23] Turner, Anthony J., *The Time Museum: Time Measuring Instruments. Part 1. Astrolabes/Astrolabe Related Instruments*. The Time Museum, Rockford, Il, 1985, p.16..

Introduction of Islamic astronomy into Europe was revolutionary. European astronomical knowledge prior to 1000 was virtually restricted to computus, the calculation of the date of Easter and related ecclesiastical days. The computus rules had been argued and refined for several hundred years and were commonly taught to clerics. However, the methods were approximate and little more than arithmetic procedures for calculating the date of the pascal full moon with no generally understood underlying theory. In fact, there was virtually no European appreciation of geometric astronomical constructions or analytic procedures for predicting future astronomical events. Observations of celestial phenomena were crude and descriptive. European astronomical knowledge in this era was more literary and philosophical than scientific, and such astronomical discussion as there was celebrated the glory of God's creations[24].

Islamic astronomy was built on geometrical models justified by observation. The astrolabe introduced both of these scientific approaches to Europe. The astrolabe was not the only element that began the process of establishing astronomy as a science in the Latin West, but it was arguably the most significant. The fact that the astrolabe brought a hitherto impossible level of precision to monastic timekeeping was a very positive factor in the astrolabe's rapid adoption.

The earliest Latin astrolabe treatises were based on Arabic translation, were not very well organized, and often contained meaningful errors (such as incorrect instructions on how to divide the ecliptic). Adelard of Bath (ca. 1080 – ca. 1160) traveled extensively in the Middle East, where he learned Arabic and the basics of Islamic science and astronomy. He dedicated an astrolabe treatise to Henry Plantagenet (Henry II) in 1147. New and better treatises evolved in the 13th century as more experience was gained. The most widely used treatise was compiled from several texts, mainly the translation of ibn al-Ṣaffār mentioned above (falsely attributed to Māshā'allāh), which became the standard text for astrolabe construction and use. A notable entirely European contribution was *De plana spera* in the early 13th century by Jordanus de Nemore, which presented a complete theoretical foundation for the stereographic projection[25].

A variation of the common astrolabe to make it usable at any latitude with a single plate was developed by Abū Isḥāq Ibrāhīm ibn Yaḥyā al-Naqqāsh, known as ibn al-Zarqālluh (1028-1087)[26] in Toledo in the 11th century, based on a modification of the *lamina universal* of 'Alī ibn Khalaf, another Toledan astronomer of the same century. This form of astrolabe became known as the *saphea arzachelis* in Europe. It is described in detail in a later chapter.

The astrolabe was firmly established in Europe by the end of the 13th century.

Prayer Times

The potential for the astrolabe to determine prayer times was an early attraction for its adoption in both Islam and Europe. There is no question astrolabes were used for this purpose, but it is also clear that older traditions continued after the astrolabe was available. Christian monasteries continued to use descriptive astronomy to estimate the times of night prayers.

A mosque has always had two key positions: the religious teacher or *imam* and the *muezzin*, who was responsible for announcing the call to prayer. A third official, the *muwaqqit*, emerged after about 1250, particularly in the Mamluk Sultanate in Egypt and Syria and was responsible for determining the qibla and regulating the times of of prayers, which are to be called by a *muezzin*.

[24] McCluskey, Stephen C., *Astronomy and Cultures in Early Medieval Europe*, Cambridge University Press, 1998, pp 131-164.
[25] Thomson, Ron B., *Jordanus de Nemore and the Mathematics of Astrolabes: De plana spera*, Pontifical Institute of Mediaeval Studies, Toronto (1978)
[26] ibn al-Zarqālluh, Latinized as Arzachel, was also the author of the Toledan Tables of planetary positions that was a mainstay of European astronomy until publication of the Alphonsine Tables in the early 14th century.

Five daily prayers are required of all devout Muslims[27]. The times of the prayers are all astronomically defined. The *ṣubḥ* (morning) prayer time, also called the *fajr* (dawn) prayer time, begins at the beginning of morning twilight and continues to sunrise. The *ẓuhr* prayer time begins after local noon, when the Sun has passed the local meridian. Various values were used to determine this time but one that prevailed in many areas was defined by the time when the shadow of a vertical gnomon had increased to one-fourth the length of the gnomon over the noon shadow. The mid-afternoon *aṣr* prayer has two possible starting times, one early and one late. The choice of which definition to use depends on local preferences. The *Shafi'i* legal school (*madhab*) specifies the early *aṣr* prayer time to start when the shadow cast by a vertical gnomon is equal to the shadow length at noon plus the length of the gnomon. The prayer time ends when the shadow increase has reached twice the length of the gnomon. The *Hanafi madhab* specifies the late start when the shadow length is the noon length plus twice the length of the stick and ends at sunset. The *maghrib* prayer time starts at sunset when the Sun has completely disappeared below the horizon. The *'ishā'* prayer time starts at the end of evening twilight. and ends at dawn. Several definitions of twilight were used, mostly in terms of the angle of the Sun below the horizon. The literature on this subject is vast.

Islamic astrolabes often included scales or indicators specifically intended for the determination of prayer times. Many astrolabes had a curve indicating twilight at 18° below the horizon. Other values, such as 16°, 17°, 19° and 20° have also been used. Current usage varies by local custom. All Muslim prayers are performed while facing Mecca and some astrolabes had scales for finding the direction of Mecca (*qibla*) from several cities. Astrolabes from western Islam frequently include arcs for the prayer times. Specific applications of prayer times to instruments are discussed in more detail beginning on page 85.

The Islamic science of determining the times of required prayers was called *'ilm al-mīqāt* in medieval Arabic. *mīqāt* (plural *mawāqit*) means "appointed time" and was also used elliptically to refer to the time of prayer. After 1250 the term *'ilm al-mīqāt* assumed a broader meaning that encompassed the whole area of practical astronomy including spherical astronomy, astronomical instruments, qibla determination, timekeeping and lunar crescent visibility[28]. A new profession developed for experts in *mīqāt* which led to a dramatic increase in texts devoted to all *mīqāt* topics. These professionals may not have been directly associated with a mosque, although they may have provided services such as producing tables of prayer times or instruments. They may also have tutored apprentice muezzins and interested students.

Texts and tables produced by the *mīqāti* covered the entire range of practical Islamic astronomy and led to a rich literature in instrumentation including astrolabes, sundials and quadrants. For example, a 14th century Mamluk treatise by Najm al-Dīn al-Miṣrī includes detailed descriptions of over one hundred astrolabe, sundial and quadrant variations. Not all of the instruments described were particularly practical or ever became popular, but the treatise is a dramatic example of the virtuosity of a medieval *mīqāti*. The 13th-century comprehensive manual on practical astronomy and instrumentation by al-Marrakushī became the standard reference for Mamluk, Yemeni and Ottoman instrument specialists[29].

Christian monastery life in the Middle Ages was strictly regulated and most of each day was dedicated to meditating, reading, and prayer. The ritual of reciting psalms at specific times dates back to the earliest days of Christianity. The term "Canonical Hours" was first used by St. Benedict in *The Rule*, written about 530 for use by the monks at Monte Cassino, and later spread throughout Europe. The prayers to be said each day and at each prayer time were later collected into books called *The Book of Hours*. Some of the most beautiful of all medieval manuscripts had this form.

[27] For a complete discussion of medieval Islamic timekeeping and prayer times see King [2004].
[28] Charette [2003] p. 6
[29] Charette[2003] p. 9

The canonical hours were originally defined by the unequal hour when the prayer started. The day began with "Matins" at the ninth (unequal) hour of the night, about 3 AM. "Lauds" were said at about the beginning of morning twilight, after reading and meditation, followed by breakfast and the "Prime" prayer at sunrise. Work time followed until "Terce" was said at the third hour of the day. "Sext" was said at the sixth hour (noon), followed by reading, meditation, and rest. "None" was said in mid-afternoon at the ninth hour of the day, followed by dinner and work. "Vespers" were said at sunset, followed by more reading and meditation. The final prayer of the day was "Compline", said at bedtime, ending at the beginning of the third hour of the night.

All of the canonical hours could be found with the astrolabe. Matins presented a particularly difficult problem and the abbot was directly responsible for ensuring the monks were awakened at the correct time, although this task was normally delegated to pairs of monks who took turns staying up and would wake the abbot at the appropriate time. A tradition of descriptive astronomy based on the rising and setting of stars evolved to estimate the correct time, and a variety of techniques, such as timed recitation of psalms, calibrated candles, and crude water clocks were used on cloudy nights. The early development of clocks was clearly motivated by the need to sound the canonical hours. There is no question the astrolabe was also used, but traditions persisted in many monasteries.

Astrology

It must be noted the astrolabe would never have attained the level of popularity it achieved if its uses were only astronomical. There are simply not that many common citizens who care about astronomy as a science. Astrology, on the other hand, was a deeply embedded element of culture at the time of the earliest astrolabe development and continued in both Islam and Christianity.

Astrology has had a major influence on the history and development of astronomy. The ancient astronomers were motivated to measure the positions of the stars and planets and to keep track of eclipses for astrological reasons (the word "horoscope" is from the Latin *horoscopium* which, in turn, is from the Greek ωροσκοποζ which combines the words for "hour" and "observer"). Their terminology, measurements and techniques were the foundation for the astronomical knowledge that eventually evolved. It is interesting that scientific heroes, such as Tycho Brahe and Johannes Kepler were employed as court astrologers.

Many old astrolabes had astrological features allowing the user to determine the astronomical elements of horoscopes. The aspects of a horoscope involve the positions of the planets and the ecliptic for a certain date and time. The astrologer interprets the aspects to advise his client. The astrolabe was a convenient way to determine the astronomical data for a horoscope, because much astrological stress was placed on the position of the ecliptic. Of particular interest were the point of the ecliptic intersecting the eastern horizon (the ascendant), the ecliptic degree on the western horizon (the descendent) and the ecliptic degree on the meridian (the degree of mid-heaven). In use, the astrolabe is set to the time and date of interest (birth, death, coronation, etc.), and the ecliptic degrees are read directly. The astrolabe could also be used to determine the limits of the "houses" which are very important in astrology.

Refinement of the Astrolabe

Except for variations attempting to make an astrolabe with a single plate usable in any latitude, the modest changes in astrolabe construction over time have not affected the basic principles of the instrument. This is convincing proof of the elegance and utility of the fundamental design. That is not to say, however, the astrolabe did not evolve. Scales were added, changed, and discarded on both Islamic and European instruments to accommodate local customs and changes in astronomical conventions. The refinement process was more than changes to the instruments, but also included improvements in the level of sophistication and confidence in the literature, extensions of uses, and better manufacturing techniques.

Virtually all astrolabe variations and enhancements were developed in the world of Islam and later imported to Europe by translated texts. Even those innovations which were not documented in works available to Europeans were generally anticipated by Islamic makers with few exceptions, and it took hundreds of years for some of them to be developed independently. Examples are the use of both north and south projections on quadrants, which were well documented in Islam by the 13th century and did not appear on European instruments until the 17th century. Gunter's quadrant and de la Hire's astrolabe and scales related to equal/unequal hour conversion and astrological houses were, however, uniquely European.

Some of the earliest astrolabes did not have the right horizon on the plate and may not have had the equator drawn. Both elements became universal over time. In addition, the convention of including equal hours on the limb was established in Europe by the late-14th century as clocks established this timekeeping standard. Several different conventions for the azimuth arcs were used depending on the origin of the instrument. Some astrolabes used azimuths in even degrees, others used the nautical winds in several varieties.

The back of the instrument offered the most fertile ground for experimentation. The scales for conversion between calendar dates and the Sun's longitude are required but do not take up much space so there is plenty of room available for any element the maker might wish to include.

European instruments clearly had no need for the Islamic scales related to prayer times, and it is easy to get the impression that makers were at somewhat of a loss as to what to use to fill the back. Many timekeeping conventions have been used throughout history. Some of them divide the day and the night into 12 equal periods (hours). Since the length of the day and the night change over the course of the year, more so the farther one is from the equator, the length of a daylight and nighttime hour also changed. This is actually not as awkward as it appears at first glance. People in the Middle Ages were accustomed to thinking of quantities as ratios: A is twice as big as B or B is one-half of A. It is actually quite easy to think of time in terms of "the night is three-fourths over" or "one sixth of the day remains until sunset". Many astrolabes had arcs to determine the unequal hour from the position of the Sun.

Astronomers prefer for time to be continuous in order to compare observations taken at different times of the year. It is awkward to have to compare an observation taken during the 5th hour of the night in October to one made in the 9th hour of the night in May so astronomers have always preferred to count hours as equal, each being 1/24 of the day. Astrolabes began to include equal hours and unequal hours, and scales to convert between to the two conventions in the 14th century as clocks using equal hours became popular. Jean Fusoris, the French scholar and craftsman, created an astrolabe design in the late 14th century employing all the relevant time-keeping systems in an astrolabe of elegant and simple design that became the most widely used form from that time forward.

A variation of the astrolabe with folded scales in the form of a quadrant appeared in the early-14th century. The astrolabe quadrant (or *quadrans novus*) was somewhat harder to use than a traditional astrolabe, but was simpler to make, and therefore less expensive, and apparently enjoyed some popularity. An easier to use quadrant variation closer to an astrolabe was popular, particularly in the Ottoman Empire. These astrolabe quadrants are described in later chapters.

European education in medieval times was virtually confined to the church The university system began to develop, and learning became available to a much wider audience in the late Middle Ages. The classic curriculum consisted of the seven liberal arts and was taught as the "trivium" (grammar, logic and rhetoric) and the "quadrivium" (astronomy, arithmetic, geometry and music), all in Latin. Astronomy was treated as a philosophical subject in the early medieval period, but gradually become more scientific. The astrolabe was adopted as a basic tool for teaching astronomy soon after its introduction into Europe.

Astronomy education was quite different in the Middle Ages than today. Students were taught some of the basics of Ptolemaic astronomy, introduced to the cosmological system of the time (the planets were on invisible rotating spheres with an outermost sphere (*primum mobile*) moved by angels to impart motion to all the rest), calendar basics and how to compute moveable Christian feasts such as Easter (the computus) and enough positional astronomy to understand how horoscopes were derived. Most people who attended university went for two years, starting at about 16 years of age. It took about 18 years to become a Doctor of Theology.

The first monumental clocks were built in the 14th and 15th centuries, and several included astrolabe dials which caused the rete to rotate once in a sidereal day. Clock makers, particularly in Augsburg, made many table clocks with astrolabe dials. It is also clear one use of astrolabes in this period was to set clocks, which were notoriously inaccurate. Astrolabe dials on monumental clocks sometimes used a stereographic projection with the projection origin at the north pole, which causes the horizon to arc in the opposite direction. This form gives a view with the Sun farther from the horizon when the Sun's declination is greater, as in the summer, and was apparently felt to be easier for the man in the street to understand. Astrolabe clocks are discussed in a later chapter.

The first European treatise on the use of the astrolabe in the vernacular was written in French by Pèlerin de Prusse in 1362 at the request of the Dauphin Charles, later Charles V (reigned 1363-1380). In about 1390, no less of a literary figure than Geoffrey Chaucer wrote a treatise on the astrolabe in vernacular English for his 10-year-old son, Lewis, which a later scribe with a sense of humor apparently subtitled "Bread and Milk for Children." This work, which is hard going for an informed adult, much less a child, demonstrates a high level of astronomical knowledge and, as a vernacular work, received fairly wide circulation. A notable astrolabe treatise, *Elucidatio fabricae ususque astrolabii*, by Johannes Stöffler, who was professor of mathematics at the University of Tübingen[30], published in 1512, established something of a standard for astrolabe design. It was common to refer to the normal planispheric astrolabe as *Stöffler's astrolabe* in Renaissance discussions.

Many 16th century European astrolabes included arcs defining the astrological "Houses of Heaven", using a system popularized by Regiomontanus, who is often incorrectly credited with its definition which was probably formulated by his teacher, Georg Peurbach (1423-1461). It is important to note that many systems of astrological houses were used and the one normally included on astrolabes was not necessarily the most popular.

The growth of university education and the permeation of astrology through European society led to increased demand for astrolabes in the 15th century. Most instruments were made by the owner or purchased from a skilled worker as a special order prior to this time. The increased demand led to the formation of a few workshops specializing in astrolabe production. Most such *ateliers* were dominated by a single individual whose knowledge and skill defined the finished product and generally closed when the master retired or died. Clock makers might also provide astrolabes. An important workshop was founded by Jean Fusoris in Paris in the late 14th century. Fusoris' instruments set a design standard that had a major influence on astrolabe production throughout Europe.

Instrument making was concentrated in Nuremberg and Augsburg in Germany in the 15th and 16th centuries. Many elaborate and beautiful clocks with astrolabe dials were constructed by the instrument makers' guild in Augsburg. A particularly interesting workshop was founded about 1525 by Georg Hartmann (1489-1564) in Nuremberg. Analysis of surviving Hartmann

[30] Johannes Kepler also studied at Tübingen some 80 years later.

instruments shows his shop used an early form of mass production, where the components were made in batches and assembled into instruments of a common design. Hartmann also published astrolabes printed on paper that could be mounted on wood and assembled into a functional, yet inexpensive instrument.

Instrument production began to develop in the Low Countries about 1530. Louvain in Belgium became a center of astrolabe production around the shop started through the influence of Gemma Frisius (1508-55) but actually run by his nephew, Gualterus (Walter) Arsenius. Arsenius' astrolabes set a standard for craftsmanship and beauty never surpassed. Some of Gemma's astrolabes had a standard planispheric astrolabe on the front and a universal astrolabe based on the saphea arzachalis on the back (which Gemma called "astrolabum [sic] catholicum" or "universal astrolabe"). Thomas Gemini, an engraver in Arsenius' shop, migrated to England, and his engraving style had a large influence on English instrument-making. Another maker of note was the German Erasmus Habermel.

Based on the number of surviving instruments, astrolabe production peaked in the 16th century. It is possible, however, many astrolabes made of paper have not survived due to their fragility. Astrolabes were made all over Europe by the 17th century, and it is known many were made for export where the maker would sell a mostly finished instrument to be completed and signed by the retailer. Clearly, there were supporting industries surrounding the instrument makers for services such as gilding and casting interchangeable parts.

In the East, Islamic influence spread through Persia, Afghanistan and into India, all of which adopted the astrolabe. Persian instruments are often quite ornate and beautifully executed with elaborate zöomorphic rete designs and complex suspensions. Sir Jean Chardin, a French jeweler who lived in Isfahan in the early 17th century, commented that astronomers made their own instruments and had to be skilled in the art of instrument construction to be accepted as *savants*[31]. Chardin also comments that well-made astrolabes were highly prized and valued as jewels. It must have been something of a status symbol to have a fine astrolabe in the Islamic world as some instruments are so elaborately decorated they appear to be presentation pieces rather than working instruments. An excellent example is an astrolabe from 1698 presented to Mahmud Agha, chief of the arsenal under Shah Husain I (1694-1722) of the Safavid dynasty[32]. As chief of the arsenal, Mahmud Agha was also in charge of education. The astrolabe is still in mint condition and shows no sign of use.

The oldest surviving Indian instruments are from the 16th century, although the astrolabe was introduced to India much earlier, and reflect a much simpler, almost stark, design.

Juan de Rojas Sarmiento of Monzón, Spain, published a commentary on a universal astrolabe based on the orthographic projection of the celestial sphere onto the solsticial colure in 1550. Rojas had studied under Gemma Frisius in Louvain where he met Hugo Helt, who assisted in the commentary and wrote the section on the instrument's construction. Astrolabes based on the orthographic construction had been discussed earlier, and Rojas did not claim to invent the method or assert he was the first to apply it to the astrolabe, but this form did not gain much attention until Rojas' publication.

Phillipe de la Hire (1640-1718) developed a third form of universal astrolabe in late 17th century that attempted to resolve some difficulties of both the saphea and Rojas versions. This method was described by Nicholas Bion (1652-1733) in 1702, but apparently no la Hire astrolabes were ever made of metal as interest in the astrolabe had declined by this time.

[31] Michel, Henri, "Méthodes de tracé et d'exéction des Astrolabes persans", Ciel et Terre, 12, 1941, p. 4.
[32] Saliba, George A., "The Buffalo Astrolabe of Muhammad Khalil", Al-Abhath, XXVI, 1973-1977, pp.11-18.

Decline of astrolabe popularity

The astrolabe declined in popularity as instrument-making grew from a trade into an industry. Instrument makers and their clients demanded more specialized and accurate instruments, and replacements for the functions of the astrolabe appeared rapidly in the 17th century. Professionals adopted the telescope and other more accurate devices for astronomical observations. The invention of the pendulum clock by Christian Huygens in 1652 allowed clocks to be made that did not need frequent setting, and the timekeeping uses of the astrolabe were no longer needed in a society with ready access to the time from clocks and watches. Wider education, acceptance of Kepler's system of astronomy, growth in understanding of the Newtonian world, and other factors contributed to a rapid decline in reliance on astrology. These factors combined to make the astrolabe irrelevant in European society, and production in Europe had virtually ceased by the end of the 17th century.

Such forces were not at work in the East, and astrolabe use continued at a high level, until the end of the 19th century in some places. The value of an astrolabe for determining astronomically defined prayer times was not reduced by newer technology, and astrology continued to be practiced.

Interest in the astrolabe widened in the last half of the 20th century. The inherent beauty of classical astrolabes has been an attraction for collectors, historians view the astrolabe as a mirror of intellectual achievement, and its value as a teaching aid for basic positional astronomy principles has once again been recognized. Inexpensive astrolabe reproductions are available to any interested student.

It should be noted that astrolabe concepts are still in wide use. The famous Swiss watch maker Ulysse Nardin sells a exquisite astrolabe wristwatch described in a later chapter. There are a number of navigational aids based on the same principles.

The stereographic projection is introduced in the next chapter before proceding to the detailed examination of the astrolabe components.

Chapter 3 - The Stereographic Projection

This chapter presents the basic theoretical foundation of the stereographic projection, including its mathematical characteristics and proofs of the conservation of circles and conformality of projected arcs.

Knowledge of the stereographic projection is not needed to admire or use an astrolabe, but some background on the projection is needed to understand why it works and to design or make one. The mathematically inclined also insist that assertions about the properties be proven from first principles.

The level of mathematics used is high school plane geometry and trigonometry, and all readers are encouraged to at least scan the chapter contents, but the mathematical details can be skipped without loss of continuity or returned to when the content is needed.

Additional details of the stereographic projection as applied to each specific astrolabe component are presented in the sections concerning the components.

Why the Astrolabe Works

The planispheric astrolabe works because all of the components on the front of the instrument are identical projections of the celestial sphere. The rotation of the rete to a specific date and time represents the sky accurately because of the circle preservation property of the projection. The rule points to the time on the limb because of the angle preservation property. Similarly, the rule divisions show declinations on the sphere because of the preservation of angles.

Principles of the Stereographic Projection

The problem addressed by the stereographic projection is how to represent a spherical surface on a plane. Many projections have been devised as solutions to this problem. All of them introduce some form of distortion and have advantages for certain applications and disadvantages for others. For example, the popular classroom Mercator projection maps of the world, which use a cylindrical projection, have the advantage of representing lines of latitude and longitude as parallel straight lines, which makes it easy for students to locate a place by its geographical coordinates. The disadvantage of this projection is the severe distortion of areas near the poles, which makes Greenland, for example, appear ridiculously large.

In the general form of the stereographic projection, a ray originates at a point on the surface of the sphere to be projected and is directed to another point on the sphere's surface. The projected point is located at the point where the ray passes through a plane perpendicular to the diameter originating at the projection origin and intersecting the sphere.

The projection plane can be anywhere within or tangent to the sphere as long as it is perpendicular to the axis of the sphere defined by the projection origin. The projection plane is defined by either a great circle or tangent to the sphere in most applications of the stereographic projection, although in some mapping applications the projection plane is defined by a small circle near the periphery of the sphere.

The origin of the projection is called the *aspect*. The aspect may be at a defined point of a physical sphere, such as one of the poles or the equator of the Earth, or a defined point on the surface of an abstract construction such as the celestical sphere. The mathematical treatment is slightly different depending on the location of the projection plane, but the basic properties of the projection are the same.

Regardless of the projection plane, the stereographic projection has two characteristics that are particularly valuable for astronomical applications:

- Circles on the sphere are projected as circles on the projection (circle preservation).

- Angles on the sphere are preserved on the projection (conformality).

These characteristics are ideal for astronomy since most celestial positions are described as angles measured on arcs of circles.

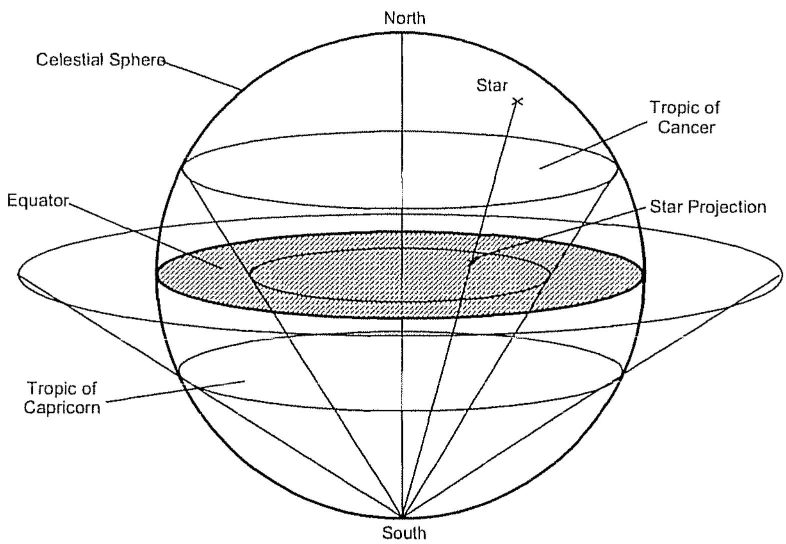

Figure 3-1. Principle of the stereographic projection

As applied to the planispheric astrolabe (Figure 3-1), the origin of the projection is one of the poles of the celestial sphere and the projection plane is defined by the celestial equator, which is assumed to be coplanar with the Earth's equator (the rigorous definition of the celestial equator is rather complex, but this assumption is valid for all but the most exacting situations).

The stereographic projection is ideal for certain applications including maps that cover small areas, crystallography, astronomy applications including star maps and, of course, the astrolabe. Familiar examples of the use of the stereographic projection are U.S. Coast and Geodetic Survey topographic maps and high precision star charts.

The disadvantage of the stereographic projection is distortion in size of objects very far from the pole of the projection.

Cartographers call the variation of the stereographic projection used on astrolabes a polar secant perspective stereographic projection. Polar means the projection origin (aspect) is a pole. Secant means the projection plane passes through the sphere, and perspective means the projection rays pass through the sphere. A more common version of the stereographic projection used by cartographers is azimuthal, which means the projection plane is tangent to the sphere.

There is no surviving documentation describing why the equator was chosen as the projection plane. Use of the equator turns out to be convenient, but it is not strictly required.

Application of the stereographic projection to the planispheric astrolabe

How the stereographic projection is used to define the points and circles projected on the plate of a common astrolabe is introduced in this chapter. Modern methods are used. Readers interested in the mathematical approach used in the Middle Ages are referred to Thomson for a thorough treatment.

The application of the stereographic projection to the astrolabe is to project the celestial sphere onto the plane of the equator. The celestial sphere is viewed as a real sphere with all of the celestial objects on its surface and the Earth at its center. Clearly, this view does not match the physical realities of the universe, but this view of the celestial sphere is perfectly valid for this use. The radius of the celestial sphere is taken as an arbitrary value, R.

The simplest demonstration of the use of the stereographic projection is to consider the projection of a point on the sphere at a known declination angle, δ, from the projection plane (the equator). Figure 3-2 shows the projection of such a point.

The objective is to find the distance, r, of the projected point from the center of the projection plane, O.

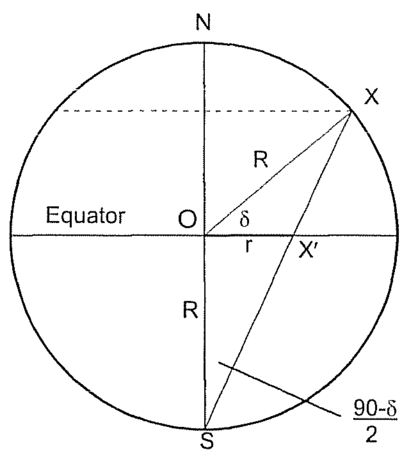

Figure 3-2. Stereographic projection of point with declination δ

A line is drawn from the center to the point X with an angle of δ from the equator (Figure 3-2). Then a projection ray from the south celestial pole, S, is drawn to X. The stereographic projection of X is where this line crosses the equator at X'. r is the distance of X' from the center, O. R is the radius of the celestial sphere and is totally arbitrary.

$\angle SOX = 90° + \delta$.

ΔOSX is an isosceles triangle with the equal sides defined by R, the radius of the sphere.
The sum of the interior angles of any triangle is 180° and the base angles of an isosceles triangle are equal.

$$\therefore \angle OXS = \angle OSX = \frac{180^\circ - (90^\circ + \delta)}{2} = \frac{(90^\circ - \delta)}{2}$$

ΔSOX′ is a right triangle. Therefore, $\mathbf{r = R\tan\frac{(90-\delta)}{2} = R\left(\frac{\cos\delta}{1+\sin\delta}\right)}$

We will call this relationship ***The Fundamental Equation of the Astrolabe.***

If X is a point on a small circle parallel to the equator (shown by the dotted line in Figure 3-2), all points on the small circle have the same declination, and all points on the circle will project at the same distance, r, from the center. Therefore, all circles on the sphere parallel to the projection plane will project as circles.

Using identical logic, it is easy to show that circles can be drawn in the same way using the difference in declination from a given circle.

This method of locating circles of known declination difference is directly applicable to drawing the equator and tropics, each of which has a difference in declination equal to the obliquity of the ecliptic (currently about 23° 26′). In this construction, the outer circle is the Tropic of Capricorn and the other circles are drawn based in their difference in declination.

The Half Angle Theorem

Graphical constructions of the arcs on an astrolabe plate use "The Half Angle Theorem", which provides an easy way to find the radii of stereographically projected declination circles. Referring to this simple construction as a theorem overstates its complexity, but the term is used here to be consistent with the literature.

The Half Angle Theorem: A chord of a circle drawn to meet a radius drawn at an angle α from a diameter subtends an angle of $\beta = (90 - \alpha)/2$ with an orthogonal diameter.

See Figure 3-3.

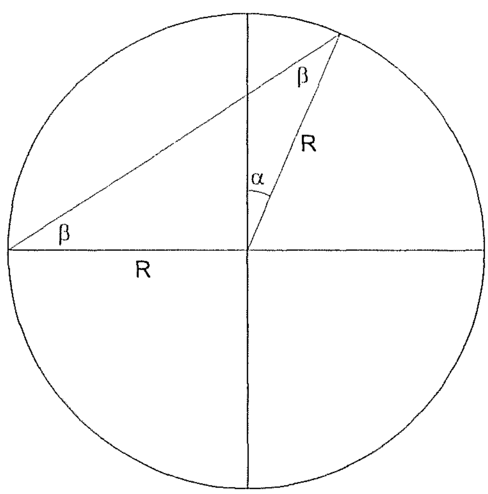

Figure 3-3. The Half Angle Theorem

Simply note that the sum of the interior angles of a triangle is 180°. The triangle created by the chord is isosceles since two legs of the triangle are radii of the circle. Therefore, the base angles are both β. $\beta + \beta + (90° + \alpha) = 180°$ and $\beta = (90 - \alpha)/2$.

This construction is the basis for many of the graphical methods for astrolabe plate layout. Note the chord intersects the vertical diameter at a distance **R tan (90 - α)/2** as above.

Stereographic Projection Proofs

In this section proofs from several sources of the properties of the stereographic projection are presented. This section can be skipped without loss of continuity.

Preservation of circles based on theorem from Apollonius[33]

Of all the proofs for preservation of circles in the stereographic projection, perhaps the most elegant is based on a theorem from Apollonius of Perga (ca. 225 BC). This theorem (Proposition 1:5) is so applicable to this problem that its existence has raised speculation Apollonius knew the stereographic projection.

Theorem. Two families of circular sections exist on an oblique circular cone. The first family is produced by planes parallel to the base cutting the cone. The second family is produced by planes inclined to the base at an angle equal to the oblique base angle. (Note the two families coincide on a right circular cone.)

Proof. Refer to Figure 3-4, which is a cross section of an oblique circular cone.

The angles at C, F and K are equal. The angles at D, E and H are equal. FE is a plane parallel to the base which generate circles by definition of the cone. It is to be proved that a plane through HK and perpendicular to the plane BCD generates a circle.

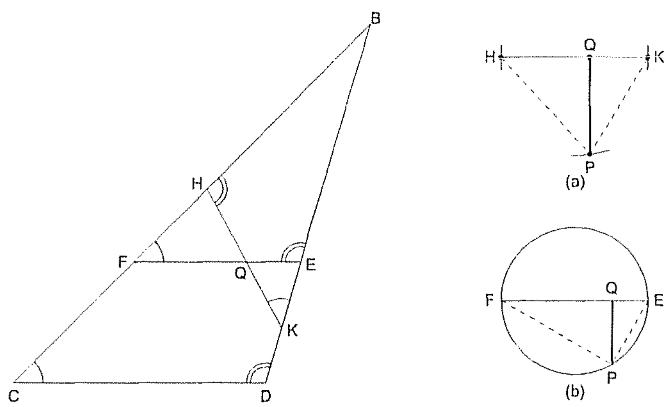

Figure 3-4. Circles generated in an oblique circular cone.

Let P be a point on the section generated by the plane (a). $PQ \perp HK$. Line FE is a diameter of the circle parallel to the base and also contains Q (b).

$PQ^2 = EQ \bullet FQ$ (this is easily demonstrated by the fact that ΔEFP is a right triangle. $FE^2 = FQ^2 + EQ^2 + 2QP^2$. $QP^2 = EQ \bullet FQ$ (by completion of the square).

ΔEKQ and ΔHFQ have identical interior angles are, therefore, similar. Hence, $EQ/KQ = HQ/FQ$ and $EQ \cdot FQ = KQ \bullet HQ = PQ^2$.

Therefore, $PH \perp PK$ and P lies on a circle of diameter KH. Q.E.D.

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[33] Neugebauer, Otto, "A History of Ancient Mathematical Astronomy", Springer-Verlag, New York (1978). V B 3, 1 pp. 858-59.

Circle preservation in the stereographic projection.

The above theorem is used to prove circles are preserved in the stereographic projection. SPQ is an oblique circular cone. It is necessary and sufficient to prove that $\angle a = \angle a'$ and $\angle b = \angle b'$ (Figure 3-5) to meet the requirements for the intersecting plane to generate a circle.

The angles at N and Q are equal because they have the chord PS in common. $\angle NSP = 90\text{-}a$. $SN \perp OP'$. Therefore, $\Delta SOP'$ is a right triangle and $a = a'$.

ΔSPQ and $\Delta SP'Q'$ share the same base angle, c. Therefore, $b = b'$ and the construction meets the requirements of the theorem above. Q.E.D.

There are several other ways to prove this property. A particularly simple one is from analytic geometry, where it is shown that the angle required to produce a circle when an elliptic cone is cut is the angle of the equator.

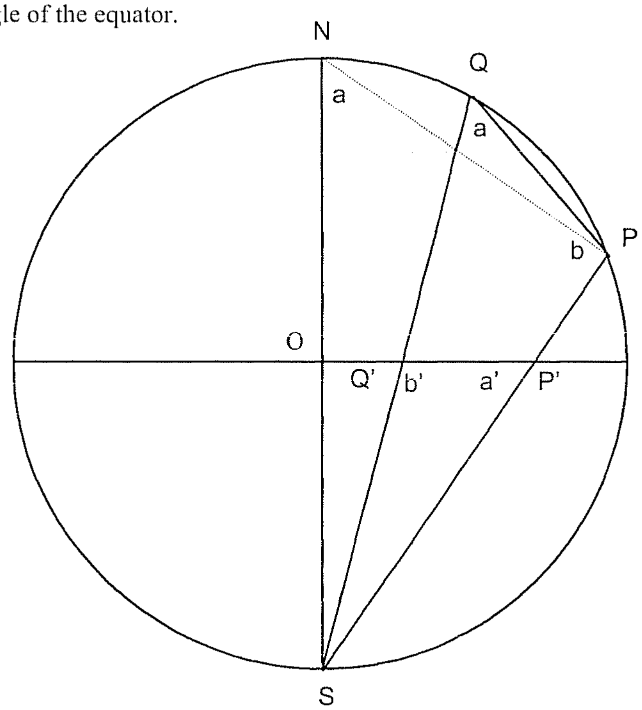


Figure 3-5. Circle preservation in the stereographic projection.

Conformality

Conformality is the property of the stereographic projection that angles on the sphere are preserved in the projection. There are several proofs of this property[34]. The following proof is a particularly simple one[35].

This proof is based on the property of the stereographic projection that any circle on the sphere passing through the projection origin is projected as a straight line and vice-versa — straight lines on the projection are mapped to circles on the sphere passing through the projection origin.

See Figure 3-6. This proof uses a variation of the stereographic projection with the projection plane tangent to the north celestial pole. The projection principles and conclusions are just as

[34] Preservation of angles under the stereographic projection was first proved by Thomas Harriot (1560-1621).
[35] Neugebauer, *ibid*, pp. 859-60.

valid for this projection plane as for the equator, but the mechanics of projecting a point are slightly different. In the figure, N is the north celestial pole, and S is the projection origin at the south celestial pole. O is the center of the sphere. P is the point on the sphere to be projected and P′ is the projected point.

Let t_1 and t_2 be tangents to the sphere intersecting at P and defining the angle α at their intersection. t'_1 and t'_2 are their projections defining the angle α' on the projection plane. Conformality means $\alpha'=\alpha$.

The plane containing t_1 intersects the sphere as a circle passing through S, so t_1 projects as a straight line t'_1 passing through P′. The tangent τ_1 to this circle at S is parallel to t'_1 because all tangents at S will be parallel to the projection plane. Similarly, the plane defined by t_2 and S has the tangent τ_2 parallel to t'_2. Therefore, the angle between t_1 and $t_2 = \alpha'$. But, the angle between τ_1 and τ_2 at S is the same as the angle α between the tangents t_1 and t_2 and $\alpha'=\alpha$. QED

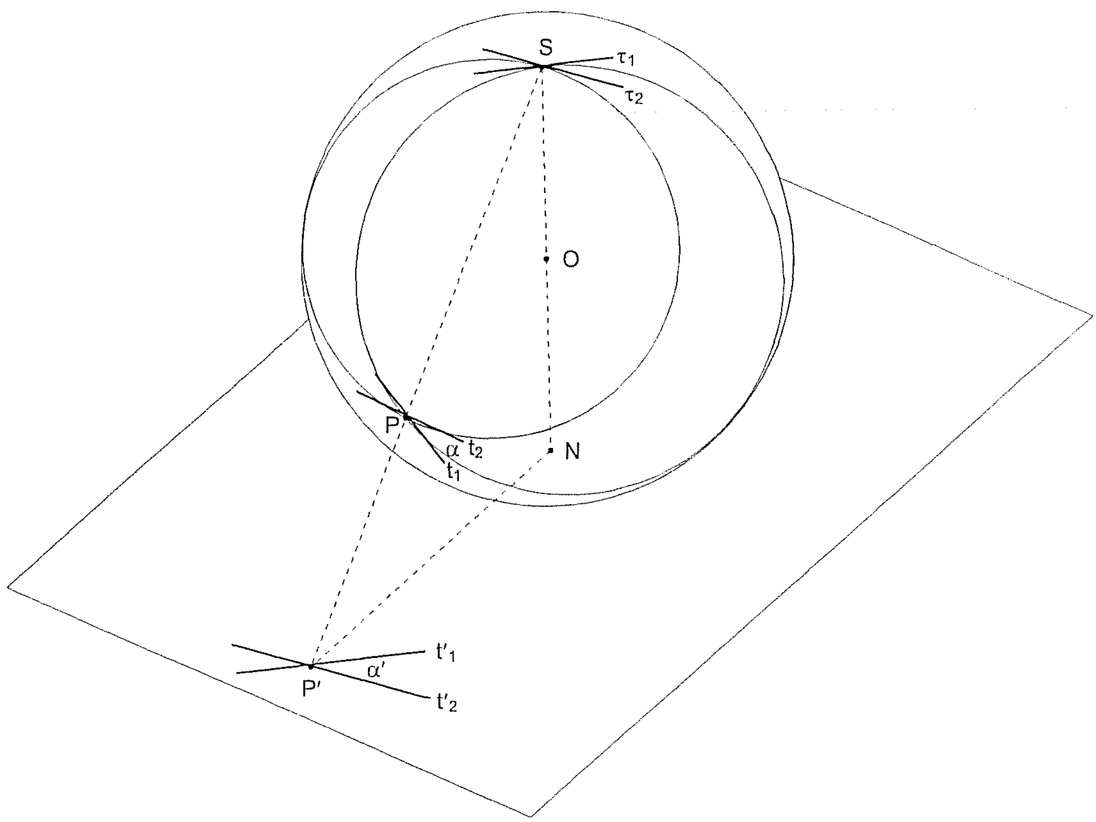


Figure 3-6. Conformality of Stereographic Projection

Michel and Thomson provide slightly different proofs.

Note that angles between radii of the sphere are not necessarily preserved by the projection. This property is why the methods for dividing the ecliptic and determining azimuth arcs are rather more complicated than other circles. Apparently, lack of understanding of this point led several medieval writers to give incorrect methods for dividing the ecliptic.

Almucantar Centers

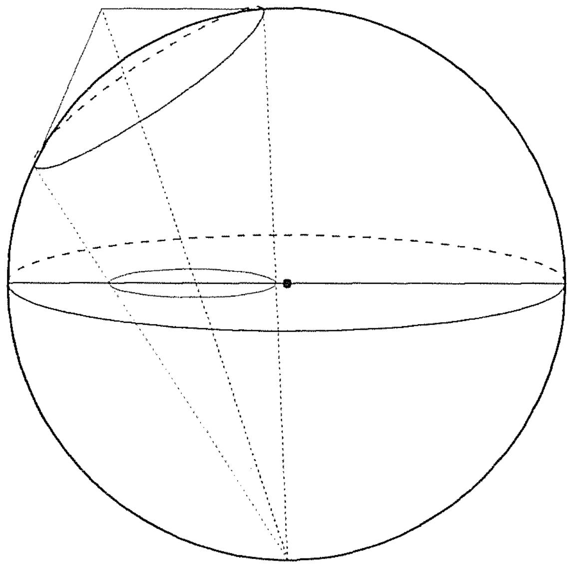


Figure 3-7. Alcumantar Center Projection

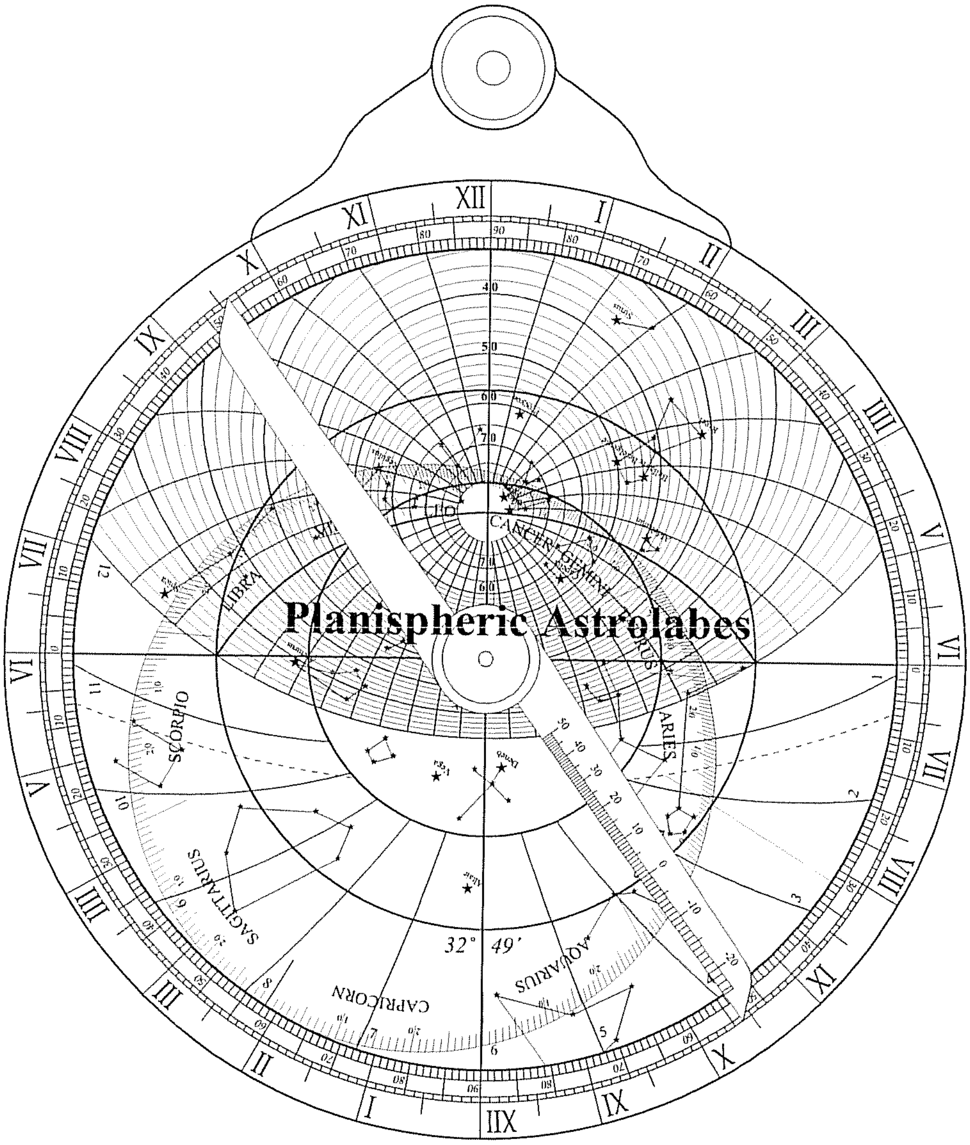
Following is a theorem attributed to Chasles that offers a method for locating the center of projected almucantars.

Theorem: The center of the stereographic projection of a small circle not passing through the projection origin is the projection of the apex of a the cone circumscribed on the sphere, tangent to the small circle.

Figure 3-7 shows the construction. The generators of the cone are tangent to the sphere around the circumference of the small circle. Tangents to the sphere project as lines and the angle of the generators meeting at the apex of the cone are preserved in the projection. Therefore, the cone generators project as radii of the projected circle, locating the center.

This theorem can be used to derive relationships for calculating the center of projected circles.

Planispheric Astrolabes



Chapter 4 -The Planispheric Astrolabe Front

The components of the planispheric astrolabe will now be discussed in detail. Each component, its uses and construction will be described.

The Mater and Throne

The mater (or *umm*) is the main body of the instrument. It provides for storage of plates, and the border (the *limb*) contains degree scales on all astrolabes, and time scales on European instruments.

The mater may be cast and the recess turned or, as is usually the case, made from a flat sheet of brass with a torus for the limb riveted or soldered to the base. The recess in the mater could be turned from thick brass sheet, but this was never done on old astrolabes due to the cost of brass and the technical difficulty considering the machining tools available at the time.

The throne and suspension provide the means by which the astrolabe is held for making observations. The suspension should be as free and frictionless as possible so the astrolabe is self-leveling and creates an artificial horizon and vertical plane when suspended for a measurement. In normal use, the throne points south when the astrolabe is held horizontally.

The throne on Islamic instruments was usually triangular in shape of with more or less decoration depending on the age and origin of the astrolabe. Some instruments, particularly those from Persia, had wildly ornate thrones with finely engraved patterns.

The throne on European instruments ranged from a very simple, rudimentary attachment point for the suspension ring on early astrolabes, to relatively elaborate gimbaled mechanism with several degrees of freedom on later high quality instruments. Considerable artistic and technical variation is seen on the thrones of old astrolabes.

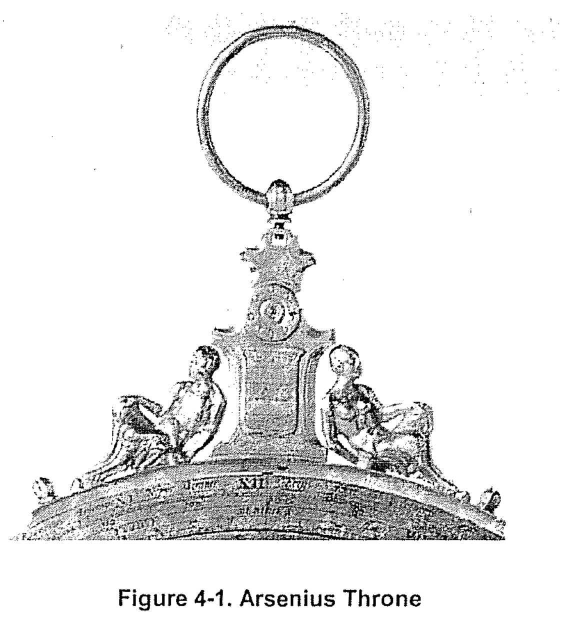


Figure 4-1. Arsenius Throne

The throne on European astrolabes was often cast separately and riveted or soldered to the mater. It often included rosettes or reclining figures to add an artistic touch on later instruments. The suspension on European astrolabes usually consisted of a single ring attached to a swivel free to rotate in the plane of the mater.

The throne in Figure 4-1 is from a spectacular astrolabe made by Gualterus Arsenius in the mid-16th century. It contains all of the elements of the finest Renaissance thrones.

The throne on Islamic instruments was often cast with the mater or cut and fretted from the mater plate. Islamic throne designs varied widely and were often quite ornate. The suspension was usually two interlocking rings.

The Limb

The limb of the mater contains a degree scale on all instruments from all sources and almost always has a time scale on European astrolabes. Islamic astrolabes did not normally have a time scale. As with all astrolabe scales, there is no universal method used for dividing the limb.

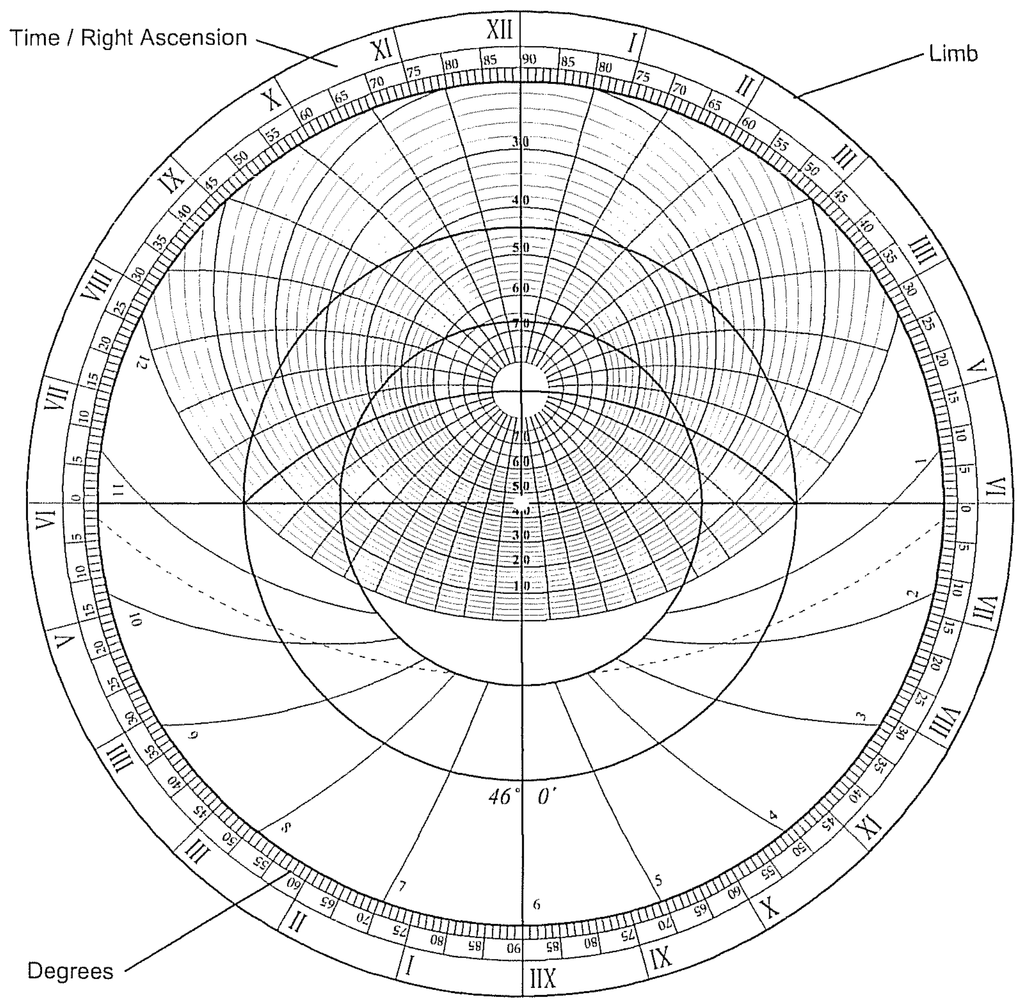


Figure 4-2. Stöffler Style Limb

Almost all astrolabes have a scale of degrees around the limb with every degree shown. The degree scale is usually divided with a tic for each degree and longer tics for each five degrees. The degree tics are labeled in various ways on old instruments including 0° at the top with 90° quadrants or continuing to 360°.

Earlier instruments did not have a separate time scale in minutes but merely indicated the hours with a long division each 15 degrees, thus each degree tic represents four minutes. There may be old instruments with a time scale showing each minute, but such an instrument would be rare and the scale would be very dense and hard to read. A time scale divided for each five minutes would be sufficiently accurate and much easier to read. The time scale is also used for right ascensions.

Note the degree and time scale on the limb are the stereographic projection of angles on the equator. Time is specified by the Sun's hour angle with 15° per hour. The degree scale can be viewed as either the stereographic projection of hour angles or right ascension, both of which are measured on the equator.

Hours on old instruments were always in Roman numerals. It was normal, but not universal, to show the fourth hour as IIII instead of IV on both astrolabes and clocks. There is still debate about whether this was done as an artistic element to balance the mass of the VIII for the eighth hour or was a convention for certain uses.

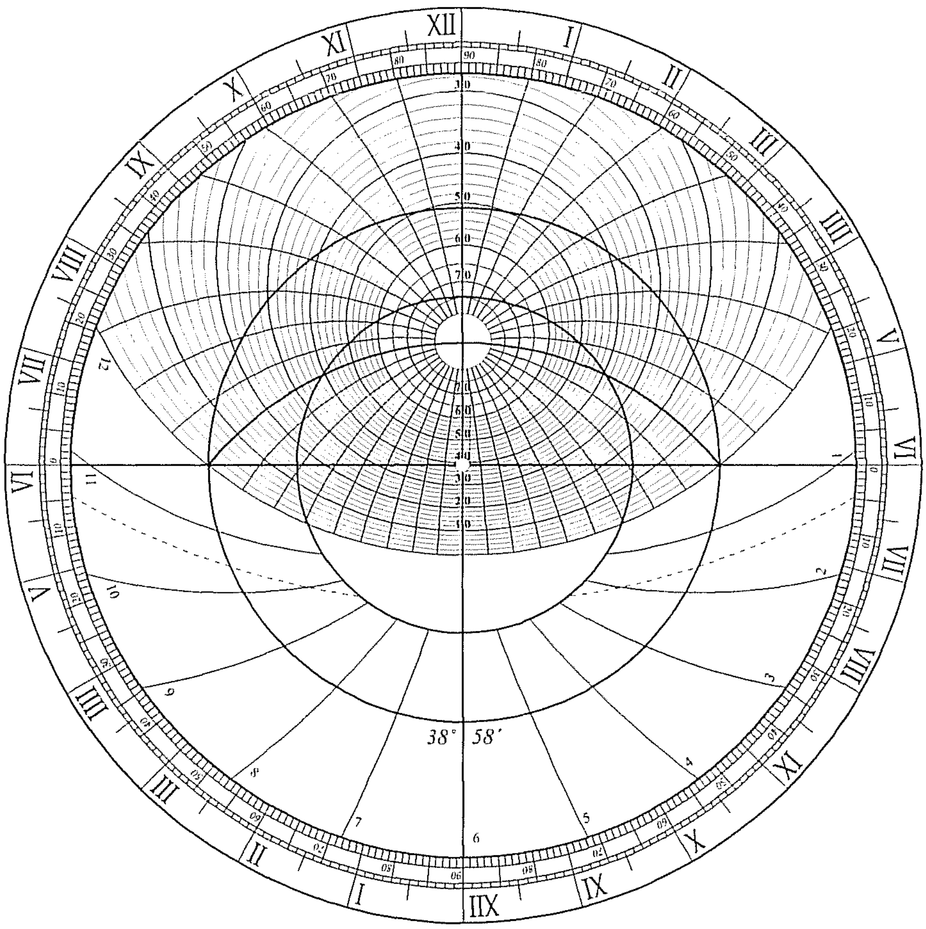


Figure 4-3. Late Renaissance Limb

The limb in Figure 4-2 shows a limb style common on early Renaissance astrolabes. It is similar to the style recommended by Stöffler and is the style used by Hartmann on the astrolabe shown in Chapter 1. There is considerable variation in artistic execution of this basic style on old astrolabes. It is a workable form, but lacks grace. Determination of the minute is left to the user to estimate from the angle at four minutes per degree. This limb style clearly demonstrates the casual attitude about time precision in those days, since the user would have to exert some effort to get the time more accurately than about a quarter hour.

Figure 4-3 is an example of a limb style that became more popular in the 16th century. This style includes an hour scale with each five minutes indicated. The hour scale dominates this style and may indicate the increasing importance of timekeeping accuracy as clocks became more widely used. This style is generally more appealing to the modern eye.

As with all astrolabe elements, it is necessary to study the old instruments to appreciate the variety of artistic approaches and the maker's view of utility used in the wide range of limb designs.

The Plate

The plate is the heart of the astrolabe, and the accuracy of the instrument is defined by the accuracy of the plate. The plate's accuracy has several elements. The almucantars must be drawn accurately, the center of the plate must be accurately drilled and there must be an adequate number of almucantars to meet the owner's needs. The plate resolution may be an almucantar for each degree or for each 2, 3, 5 or 6 degrees. Realistically, an astrolabe with a resolution less than 3° per almucantar cannot be considered to be a working instrument. Most old instruments had 3° resolution. Some astrolabes have been made with 1° resolution, but they are either rather large and heavy or the almucantars are very close together. The plates in most of our figures have 2° almucantar resolution.

Astrolabes almost always included several plate disks. Each disk was engraved with a plate for a different latitude on each side, thus doubling the number of latitudes covered for a given number of plates. An astrolabe with a single plate integrated into the mater for use in a single location would be adequate for many users. There were surely a great many astrolabes made of wood and paper for a single latitude, but few survive. An astrolabe with a single plate integrated into the mater for use in a single location would be adequate for many users.

The earliest astrolabes, which were deeply influenced by Greek tradition, included plates for the latitudes of the *climates*. The climates of the world were defined by Ptolemy to be the latitudes where the length of the longest day of the year varied by one-half hour. Ptolemy calculated the latitude corresponding to a 15-minute difference in the length of the longest day (using a value of 23° 51′ 20″ for the obliquity of the ecliptic) for 39 latitudes, which covered the Earth from the equator to the North Pole[36]. The ones called the classic *climata* were for half-hour differences in the longest day covering the then populated world. The values given by Ptolemy are:

		Latitude	Longest Day
I	Meroe	16° 27′	13 h
II	Soene[37]	23° 51′	13 ½ h
III	Lower Egypt	30° 22′	14 h
IV	Rhodes	36° 0′	14 ½ h
V	Hellespont	40° 56′	15 h
VI	Mid-Pontus	45° 1′	15 ½ h
VII	Mouth of Borysthinia	48° 32′	16 h

[36] Neugebauer, O, *A History of Ancient Mathematical Astronomy*, Springer-Verlag, New York, 1975, pp. 43-44.
[37] Climate II was chosen to match the Tropic of Cancer. The latitude would be 23° 49′ by the definition.

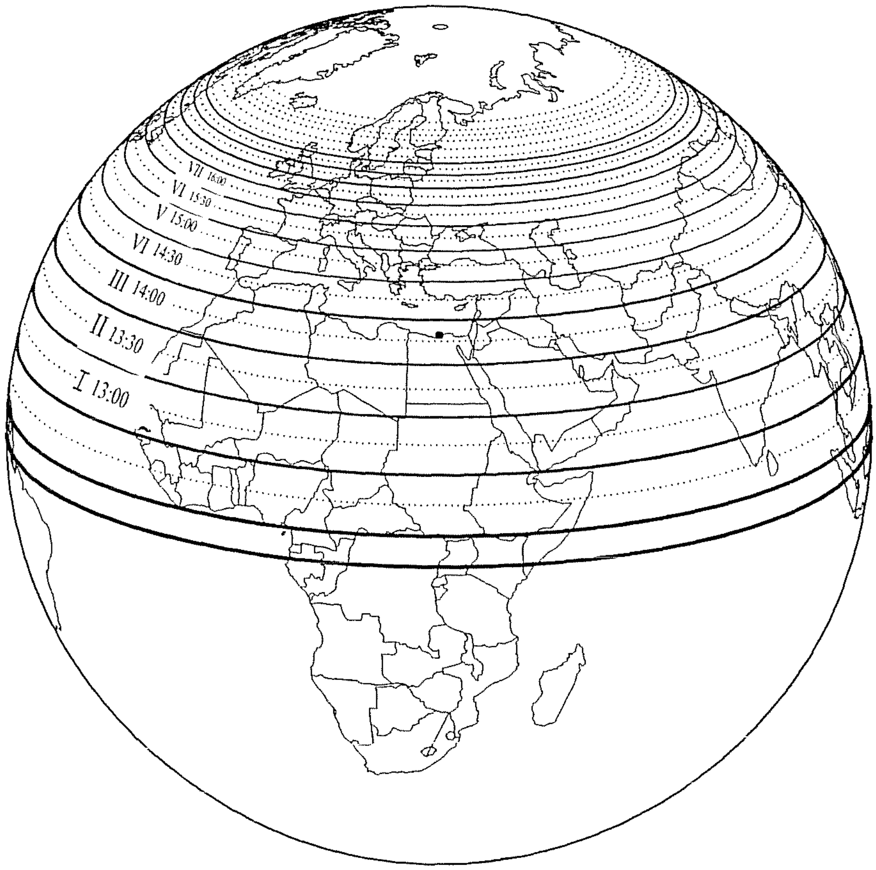


Figure 4-4. Classic Climata

Figure 4-4 shows the areas defined by the classic climates and illustrates how the entire populated world familiar to the Romans is included in the numbered climate zones. The complete set of zones covers virtually all of the northern populated world even today. An astrolabe equipped with plates of the climates would be usable anywhere in the entire populated ancient and medieval world. Later instruments included plates for specific cities or nearby regions, although plates for the climates continued to be used to make the instrument more universal.

Figure 4-5 shows a representative plate for a European astrolabe. Islamic astrolabe plates are almost identical, and the differences will be described as we proceed.

Plate Interior

The interior of the plate can be thought of as a special kind of graph paper for finding the location of celestial objects in the sky at your location. The main difference between normal graph paper and the graph on the plate is the lines on graph paper are normally straight while the astrolabe lines are curved. All of the curves on the astrolabe plate are arcs of circles.

The larger circles centered on the plate represent the Earth's tropics. The largest circle, which defines the outside of the plate, is the Tropic of Capricorn, which is the farthest south the Sun ever reaches. The middle circle is the equator and the smaller circle is the Sun's northern limit, the Tropic of Cancer. The circles defining the tropics are the same for all latitudes.

The straight lines drawn as diameters of the largest circle show direction. The vertical diameter goes north and south through your location, representing your **meridian**. South is at the top and east to the left. The horizontal diameter connects east and west and is the projection of the great circle perpendicular to the meridian. It is normally called the **right horizon**, the horizon at the equator. It is perhaps easiest to visualize the plate as lying flat on a table with the top pointing south. A star chart is held overhead. You look down on an astrolabe, like a compass.

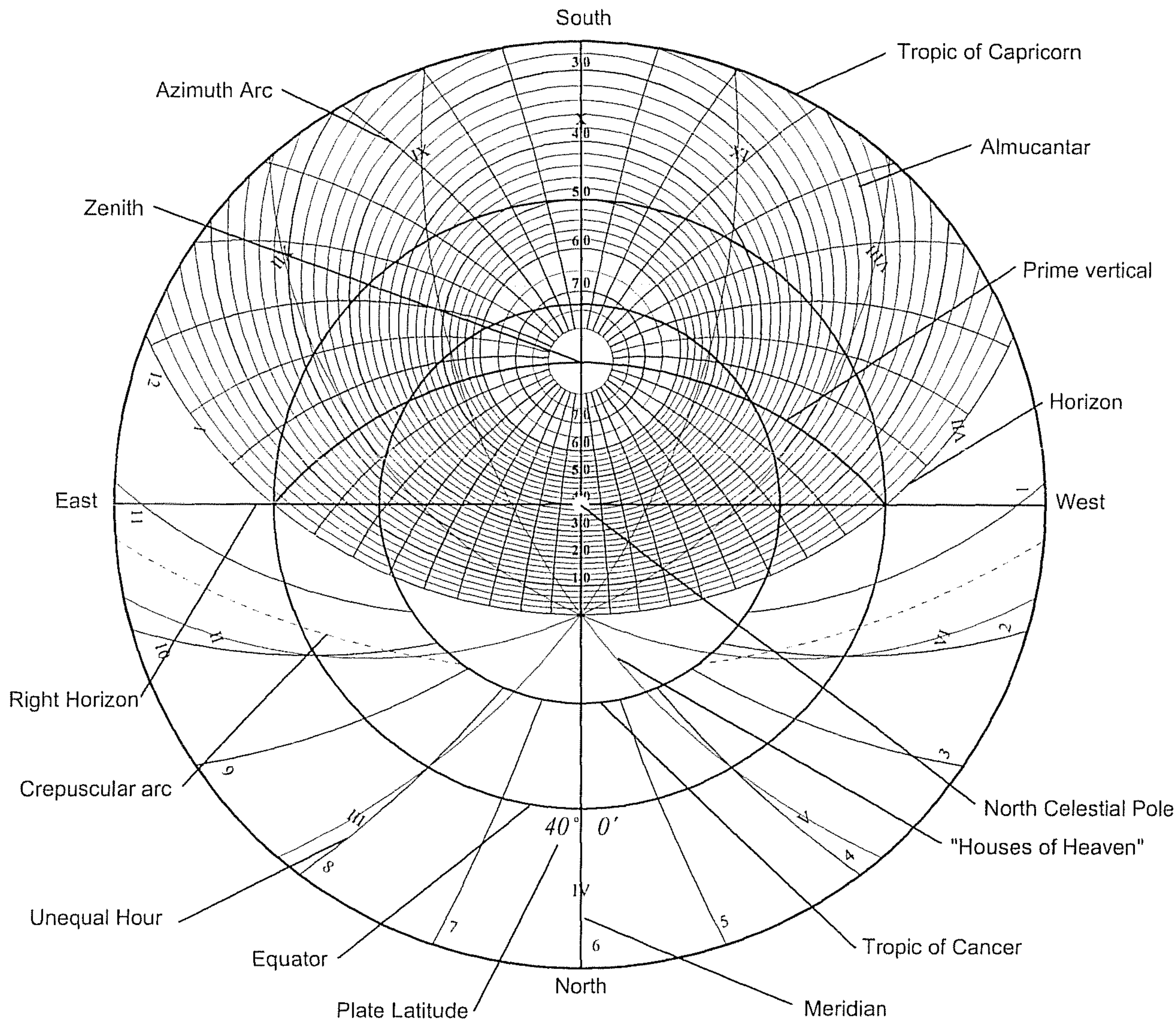


Figure 4-5. Astrolabe Plate

Apparently, some of the earliest astrolabes did not have the right horizon shown, and the equator and Tropic of Cancer were shown only below the horizon[38].

The plate is used to find the positions of celestial objects in the sky as seen by an observer at a specific location. The interior of the plate is the stereographic projection of the tropics and the

[38] Neugebauer, Otto, "The Early History of the Astrolabe", *Astronomy and History: Selected Essays*, Springer-Verlag, New York (1983), pp.278-294.

local horizontal coordinate system. The arcs on the plate represent positions in the sky. You can find anything in the sky if you know its angle above the horizon and the direction to look. The angle of something in the sky above its horizon is its **altitude** and its direction is its **azimuth.**

Look at the web of arcs inside the instrument in Figure 4-5. The lower solid arc represents your **horizon** (also called the **oblique horizon**), the line where the sky meets the Earth. Any celestial object above the horizon is visible to you (or would be if trees, mountains, buildings and clouds were not in the way). Any object below the horizon is not visible.

The circles above the horizon, the almucantars, represent lines of equal altitude above the horizon. All objects on the same circle are the same altitude above the horizon. Any object anywhere on, for example, the 50° circle has an altitude of 50°.

The numbers on the meridian show the altitude for each ten degrees. The point inside the smallest circle where the lines cross is your **zenith**[39], or the point directly overhead. The plate in the figure has an altitude circle for each two degrees up to 60° and each 5° from 60° to 80°. Notice the equator crosses the horizon in the east and west, as it should.

The arcs radiating from the zenith represent lines of equal azimuth. Astronomers measure azimuth angles as increasing as you go east from north, so an azimuth angle of 30° is 30°east of north and an azimuth of 270° is 270° east of north or due west. An older convention was to specify azimuth angles as east or west from south, a convention still used in celestial navigation.. Circles of equal azimuth are shown for every ten degrees in the figure. The azimuth arcs are also used to determine the **amplitude** of the Sun or a star.

The **prime vertical** is the arc in the sky from the eastern to the western horizon, passing through the zenith. Notice the azimuth circles for 90° and 270° intersect the horizon at east and west and define the prime vertical.

Any object visible in the sky from our location can be located with the circles of equal azimuth and altitude. For example, if a star's altitude is 40° and its azimuth is 200° you can find it in the sky by pointing south, raising your arm 40° and then turning 20° to the west.

Twilight Arcs

The dashed altitude circle below the horizon is 18° below the horizon and is called the *crepuscular* (twilight) *arc* (*linea crepusculi*). Sunlight can still be seen even when the Sun is below the horizon because the Earth's atmosphere bends the Sun's light rays and because sunlight is reflected by particles in the air,. Civil Twilight ends when the center of the Sun is 6° below the horizon. At this time artificial illumination must be used to see clearly. Nautical Twilight ends when the Sun is 12° below the horizon and the horizon cannot be seen at sea. Astronomical Twilight ends when the Sun is 18° below the horizon and you are in the full shadow of the Earth; there is no sunlight at all. Since the crepuscular arcs are only used with the Sun, they extend only from the Tropic of Cancer to the Tropic of Capricorn.

Most old astrolabes included only the 18° crepuscular arc as this arc was relevant for prayer times on Islamic instruments and represented the end of the day on European astrolabes. Some Western Islamic astrolabes mark the +18° arc instead of -18° and use the point opposite the Sun on the ecliptic to find the time. Astrolabes from Western Islam might include arcs showing the times of the *ẓuhr* and *aṣr* prayer times.

[39] The words 'zenith' and 'azimuth' both derive from the same Arabic word *samt* (direction). Azimuth comes from the plural; *al-sumūt*. Zenith comes from *samt al-ra* (the direction of the head), which in Latin becomes *cenith capitus*.

Unequal Hours

The solid arcs below the horizon connecting the Tropic of Cancer and the Tropic of Capricorn are used to determine the unequal hour of the day. Throughout recorded history, until reliable clocks became widely available in the 17th century, time was described as the fraction of the time from sunrise to sunset that had passed. In most cultures, the time from sunrise to sunset (or sunset to sunrise for hours at night) was divided into twelve equal parts and the time would be described as being, for example, in the 6th hour of the day for the portion of the day just before noon. The time duration of an unequal hour clearly depends on when sunset occurs, so the length of an hour varied during the year. In northern latitudes where the length of daylight varies widely over the course of the year, the duration of an unequal hour will also vary, the more so the farther north you are.

Telling time this way was not all inconvenient even though the length of an hour was different for different times of the year. If you know the current time is in the third hour of the day you know immediately the day is about one-fourth over. It is a very easy convention to get used to and can still be a more convenient way to tell time if your interest is how long it is until dark. For far northern latitudes, the length of an hour in the winter might be half the length of an hour in the summer, but the unequal hours are the same ratio year round. Several methods of counting the unequal hours have been used. The most common use was to count the hours of the day beginning at sunrise and the hours of the night beginning at sunset, and this is method shown in the figure. Care must be taken when referring to historical texts, because other methods of counting hours were used, such as numbering the hour as the number of hours until sunrise or sunset. This timekeeping convention was also called the *planetary hours*.

Daylight unequal hours are counted from the eastern horizon starting with 1, the first hour of the day, and continuing clockwise through the end of the 12th hour at the western horizon. The unequal hours of the night are the same, but starting at the western horizon at sunset.

The unequal hour of the night is found by setting the Sun and ecliptic to the appropriate almucantar and noting where this point falls in the unequal hour divisions. In the day, the point on the ecliptic opposite the Sun (the nadir) is used. Partial unequal hours can be found by noting the position of a fixed ecliptic point such as Capricorn 0° at the current time and then rotating the Sun to fall on the previous hour line. The part of the hour is the angular difference / 15.

Other Hour Arcs

Some astrolabes were made with hour arcs to show the time in other timekeeping conventions. For example, Italian hours (*horae ab occasu*) are equal hours (1-24) beginning at sunset and Babylonian hours (*horae ab ortu*) are equal hours (1-24) beginning at sunrise. These arcs are discussed later (Page 87).

Arcs for the times of Muslim prayers were common on astrolabes from western Islam. The midday and mid-afternoon prayers are defined by the Sun's altitude as determined by the length of a shadow and are marked as arcs in the unequal hours. They are drawn by finding the appropriate times at the equinoxes and solstices and drawing an arc through the three points[40]. See Page 85 for a more detailed discussion of prayer time arcs.

"Houses of Heaven"

The arcs crossing at the north point on the horizon are the, so called, astrological "Houses of Heaven". Astrology was a deeply embedded element of culture until the 17th or 18th century and the astrolabe was a convenient astrological computer. One component of horoscopes was, and is, the "House" in which the Sun or a planet is in at a given time. Many systems of defining

[40] King, David A., "An Astrolabe from 14th-century Christian Spain", Suhayl 3, 2002-03, p. 85.

houses have been developed. The one usually shown on astrolabes was the method popularized by the noted astronomer and mathematician, Regiomontanus [Johannes Müller] (1436-1476) which divides the equator into 12, 30° sections and projects the result onto the local horizon. This system was first described in an essay published in 1467[41]. The house arc construction is on Page 89.

Regiomontanus recommended the heavens be divided by great circles passing through the north and south points of the horizon and cut the equator into 12 equal parts.

The houses are numbered from I to XII counterclockwise from the eastern horizon. The house system shown on astrolabes is not the same as the one used in modern astrology, which is based on rising times. Note, however, houses mentioned in historical texts may or may not use the Regiomontanus method. For example, Chaucer's astrolabe treatise devotes considerable space to finding the house of the Sun and planets but in a different system. The use of the houses is discussed in some detail by Chaucer.

The houses are counted counterclockwise from the eastern horizon. For example, the Sun or a planet is in the 7th (VII) house just before sunset. Regiomontanus associated the influences of the houses as: I physique and appearance, II money, III relations and kin, IV lineage, V progeny, VI health, VII spouse, VIII death, IX religion, X employer, XI relations with friends, XII enemies[42].

The houses are never included on Islamic astrolabes although there is a construction related to "casting the rays" that appears similar, but with a different astrological meaning..

Plate Latitude

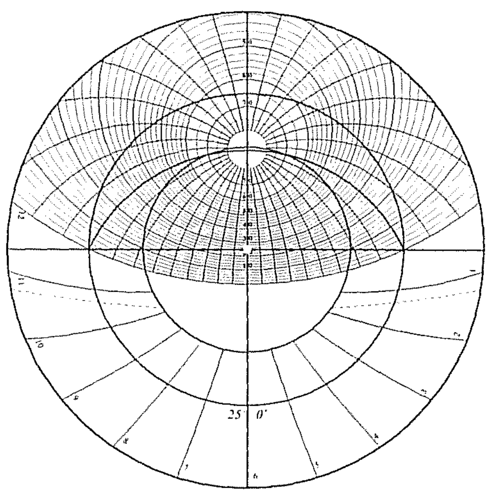


Figure 4-6. Plate for 25°

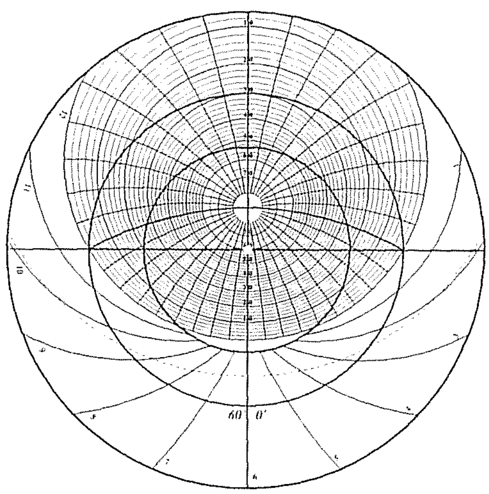


Figure 4-7. Plate for 60°

Your horizon changes if you go north or south. Since the particular piece of the sky you can see depends on your latitude, a different set of altitude and azimuth circles is needed for each latitude. The latitude for the plate is shown by the larger numbers near the bottom of the plate. The **Error! Reference source not found.** and 4-6 show astrolabe plates for latitudes of 25°

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[41] Gibbs, Sharon, "Astrolabe Clock Faces", *The Clockwork Universe. German Clocks and Automata 1550-1650*, Neale Watson Academic Publications, Inc., New York, 1980. p. 52.
[42] Gibbs [1980], pg. 52.

north, which is close to the latitude of Miami, and 60° north which is close to the latitude of Stockholm. The horizon for the 25° plate is very flat, because 25° north is fairly close to the equator and the length of the day is very constant, but the horizon for 60° north is nearly a complete circle, because there is a very large difference in the length of the day for a location so far north. Plates for additional latitudes are included at the end of this book. It is instructive to study the additional plates to understand how the latitude of a place affects the seasons.

Some Special Plates

The Plate of Horizons

Some astrolabes were equipped with a plate showing only the horizon for a number of latitudes. This plate allows the solution of problems relating to sunrise/sunset, the length of the day and the rising times of stars without the need for a complete plate for the latitude.

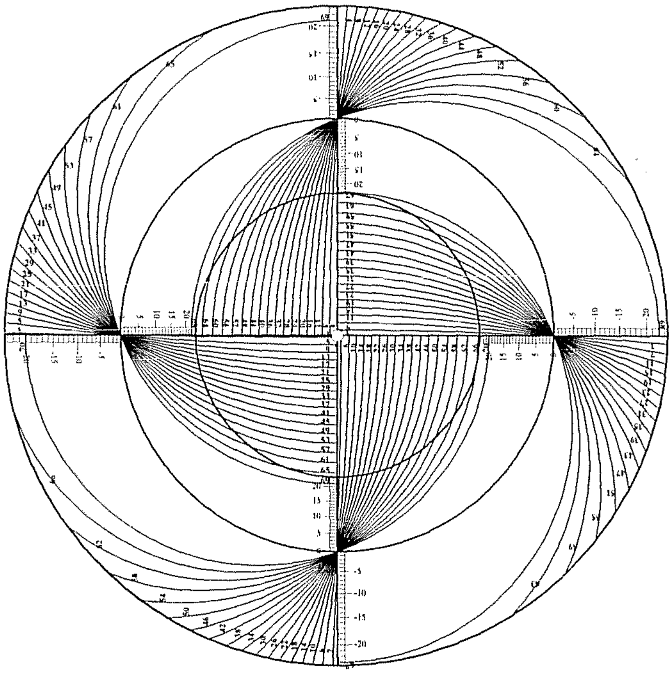


Figure 4-8. Plate of Horizons

Two forms of this plate were used. Figure 4-8 shows a style common on Islamic astrolabes, but was also used on European instruments. This horizon plate includes only half of each horizon, since all horizon problems can be solved using only one side of the arc. For example, to find the length of the day, the time of sunrise before noon is found and then doubled to find the day's length.

The example includes every horizon up to 70° arranged in quadrants. Each quadrant has one-fourth of the horizons and the horizons in a quadrant are separated by four degrees to keep the arcs from getting too close together. The horizon for 1° is on the eastern edge of the right horizon. The horizon for 2° is in the next quadrant going counterclockwise and is near the meridian. The horizon for 3° is in the next quadrant and so on.

In use, the horizon plate is rotated to the horizon of interest, aligned with the east and inserted like any other plate. This plate normally includes a declination scale in each quadrant.

Another form of horizon plate occasionally included was somewhat simpler and contained the complete stereographically projected horizons for a large range of latitudes. The plate in Figure 4-9 shows a representative sample.

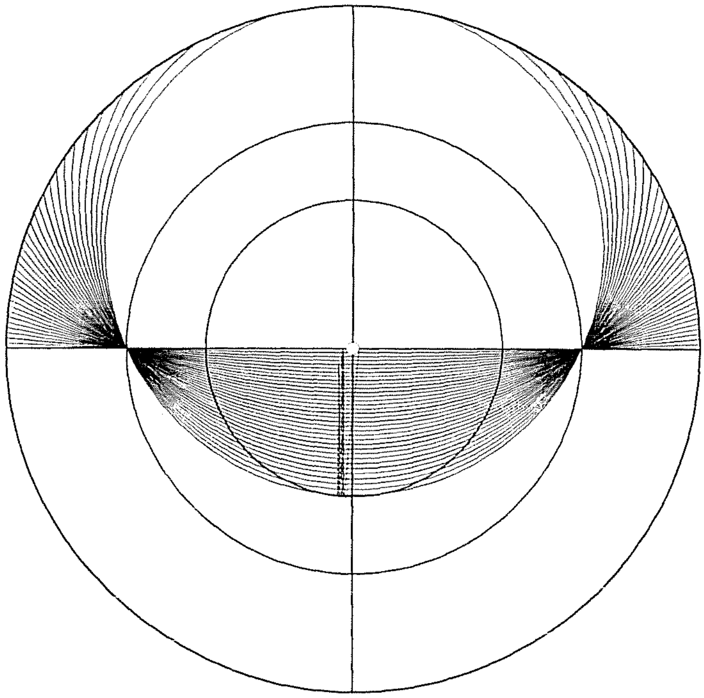


Figure 4-9. Horizon Plate Variation

Plate for 0° and 90°

A special plate divided for the equator (0° latitude) and the North Pole (90° latitude) is sometimes included. This plate can be rotated so the half of interest is at the top of the astrolabe. The plate for the equator is useful for solving problems common in the study of the history of astronomy involving rising times (i.e. the elapsed time for a specified number of ecliptic degrees to rise). The horizon at the equator is called "sphaera recta". The horizon at other latitudes is called "sphaera obliqua". The "rising time at sphaera recta" is often shortened to "ascensio recta" or "right ascension". The astrolabe was commonly used to solve these problems before the development of spherical trigonometry.

The plate for the North Pole shows declination directly. At the North Pole the horizon is the equator and declinations can be read directly from the plate. Negative declinations cannot actually be seen from the North Pole but the plate divisions extend to the Tropic of Capricorn. The azimuth arcs correspond to the hour angle of the Sun or a star. If the rete is set to 0 hr sidereal time, the azimuth arcs correspond to right ascension. Another use of the plate for the North Pole is to locate planets or other objects not included on the rete.

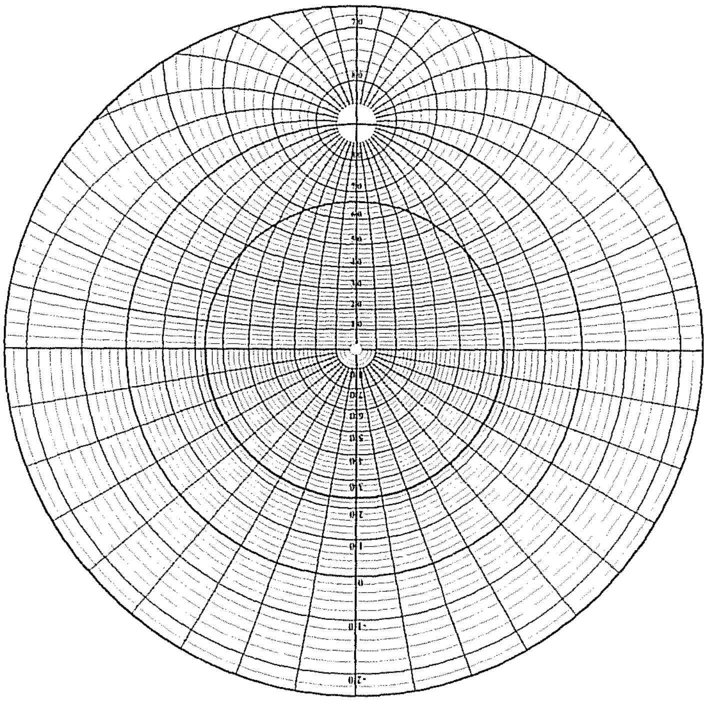


Figure 4-10. Plate for 0° and 90°

Plate of Ecliptic Coordinates

If the plate latitude is (90° - the obliquity of the ecliptic) ≈ 66° 33′, the horizon corresponds to the ecliptic and the zenith is the ecliptic pole. When this plate (Figure 4-11) is inserted in the astrolabe, and the ecliptic circle on the rete is aligned with the horizon, ecliptic latitude and longitude can be read directly from the plate. It can also be used to convert between ecliptic and equatorial coordinates.

The construction of the plate of ecliptic coordinates is the same as any other plate except the altitude (latitude) and azimuth (longitude) arcs are drawn for the entire plate to enable you to read the latitude and longitude of any object on the rete or to find the declination and right ascension corresponding to a latitude and longitude..

Other plates, some of them quite bizarre, have been described in the literature. See the chapter on astrolabe variations for examples.

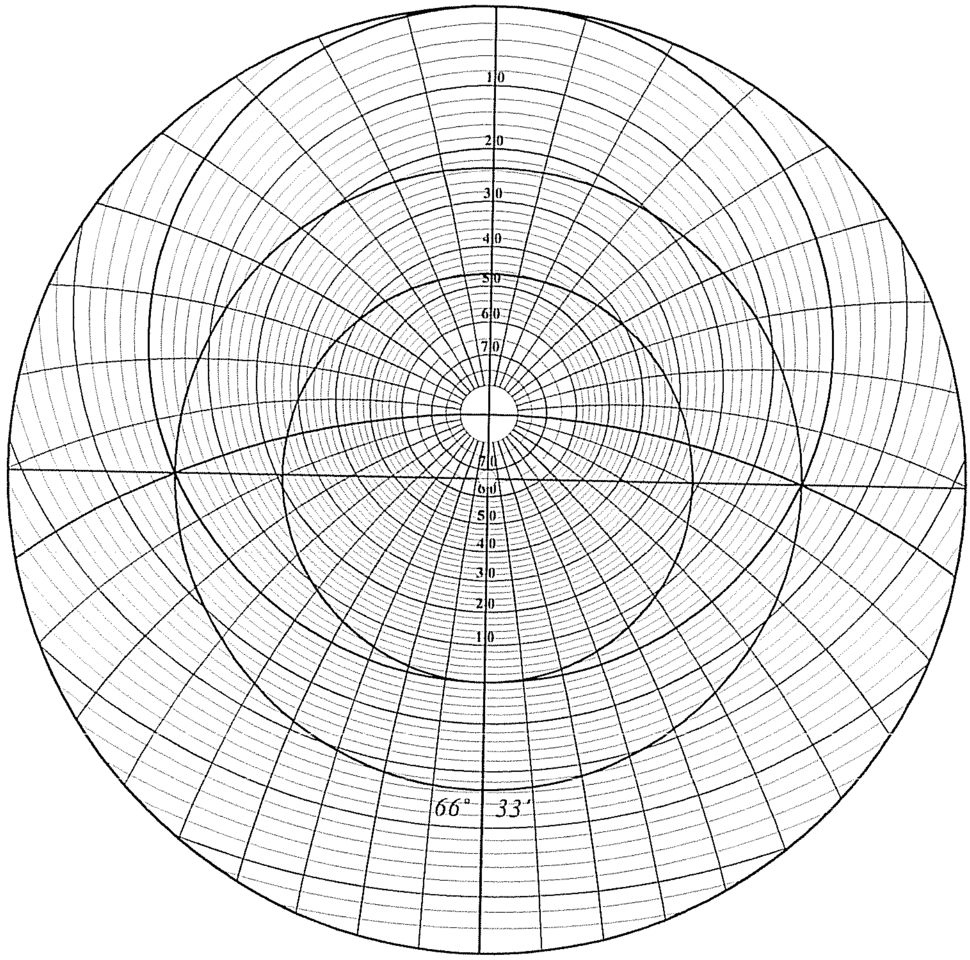


Figure 4-11. Plate of Ecliptic Coordinates

Chapter 5 - Drawing the Astrolabe Plate

The specific application of the stereographic projection to the astrolabe plate will now be discussed. The plate is sufficiently complicated to deserve a separate chapter to treat it completely.

Constructions for the other components are covered in the chapter describing the component. Both analytic and graphical methods are shown. This subject is rather technical and involves some basic math. This material can be skipped without loss of continuity and returned to when needed, however the results of this chapter will be used extensively in later chapters and should be studied by those interested in understanding how an instrument is designed.

Analytic methods are used to calculate the location of the center and the length of the radius of the arcs on the astrolabe plate. The calculated dimensions are used to draw the arcs. This method is required to draw an astrolabe component with a computer and is also useful for checking values drawn using graphical methods. It is very difficult to transfer calculated values to the plate accurately by hand using a compass and rule but a great many, possibly most, old astrolabes were made this way with the values taken from tables. The analytic methods presented are equivalent to those used to create the tables.

Graphical methods are used to draw the arcs directly onto the plate using a compass, straight edge, and protractor. The arcs will be accurate with minimal measuring with this technique. Many construction lines will be drawn which must be erased from the final plate, which is not as hard as it sounds if a light touch and sharp tool are used. Not all steps in the graphical constructions will be covered in the following discussion, as it is assumed basic plane geometry constructions, such as bisecting a line or erecting a perpendicular, are known.

Less common constructions are covered in the text. Excellent drafting skills are needed, and the effort required to draw a plate using classical methods will inspire great admiration for the skills and tenacity of the old astrolabe makers. Several graphical plate layout methods have been described in the old literature. Alternative methods are shown as appropriate. One point must be made concerning graphical constructions; there is no "right" or "wrong" method. In the final analysis, any graphical method producing the right result is fine. All graphical methods use the half-angle theorem to construct triangles with the desired base angles and intersect the meridian at the correct point. The only meaningful differences in the various graphical layout methods is how many calculations have to be made and how many lines are drawn.

Both methods require the basic dimensions of the instrument to be defined before beginning. The starting point for the plate is its circumference, the Tropic of Capricorn. The radius of the Tropic of Capricorn is R_{cap} in the following discussion.

The plate disk must be prepared before beginning either method of layout. The plate circle must be cut and turned to the exact size including provision, such as a lug, for locking the plate in place. The center must be accurately located and the meridian drawn exactly on the vertical. The right horizon must be drawn at a right angle to the meridian, intersecting exactly in the center.

The Equator and Tropic of Cancer

The simplest of all astrolabe plate elements are the equator and Tropic of Cancer. These are both circles of fixed declination. If the radius of the outer circle defining the tropic of Capricorn is

R_{Cap}, and the obliquity of the ecliptic is ε (currently about 23° 26′), the radius of the equator (R_{eq}) and the of the Tropic of Cancer (R_{can}) are calculated using the fundamental relationship as:

$$R_{eq} = R_{cap}\tan\left(\frac{90-\varepsilon}{2}\right) \qquad R_{can} = R_{eq}\tan\left(\frac{90-\varepsilon}{2}\right)$$

If the radius of the equator is given then:

$$R_{cap} = R_{eq}\tan\left(\frac{90+\varepsilon}{2}\right)$$

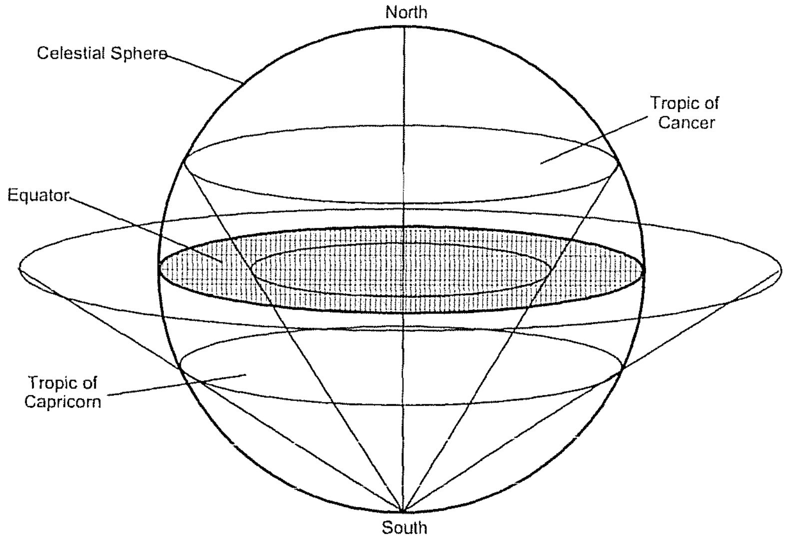
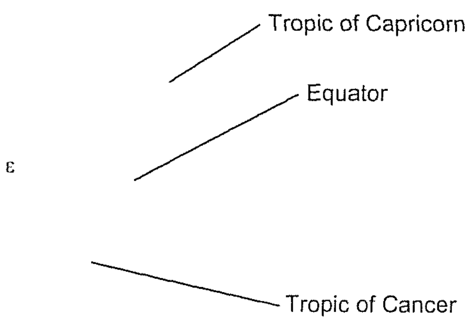


Figure 5-1. Projection of Tropics

Circles of R_{eq} and R_{can} are drawn with their centers at the center of the plate.

Grapical Construction of the Equator and Tropic of Cancer

Drawing the equator and Tropic of Cancer graphically is a straightforward application of the half-angle theorem. Figure 5-2 shows the plate disk with the outer diameter equal to the Tropic of Capricorn.



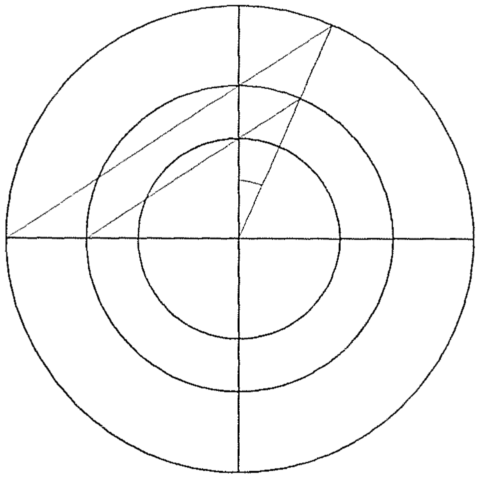


Figure 5-2. Equator and Tropic of Cancer Construction

Locate the point on the circumference of the circle at angle ε from the meridian. Draw a line from this point to the point where the right horizon meets the circumference. The point where the line crosses the meridian is on the circumference of the circle desired, the equator if the outer circle is the Tropic of Capricorn and the Tropic of Cancer if the outer circle is the equator. This is simply a direct application of the half-angle theorem.

Horizon and Almucantar Projection

Horizon

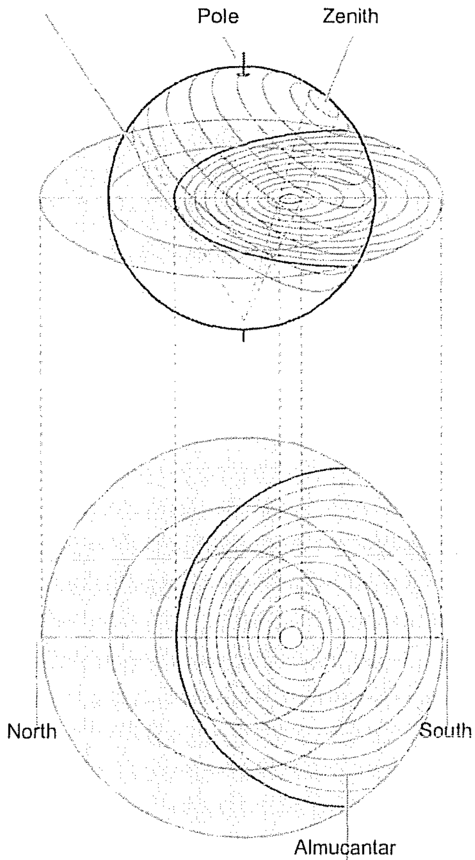


Figure 5-3. Horizon and Almucantar Projection

The projection of the horizon and almucantars forms the heart of the astrolabe and must be done with great care and foresight.

Conceptually, the projections are simple. We know the circles defining the horizon and altitudes above the horizon are circles and will be projected as circles by the stereographic projection. Any circle is completely defined if the location of its center and the radius are known. Both of these parameters can be found if we can locate the ends of a diameter.

Figure 5-3, illustrates the concept. The meridian is a common diameter of the horizon and almucantars. A method is needed to find the points where the circles intersect the meridian. The circle centers and radii can be found from these coordinates.

Only the part of the circle inside the Tropic of Capricorn is actually drawn, but as much area outside Capricorn as necessary is required to define the limits of the circles.

Horizon Projection

Figure 5-4 shows the projection of the local horizon for latitude φ. The position of the horizon is determined by the fact that the angle from the horizon to the north pole is equal to the latitude. Z is the zenith.

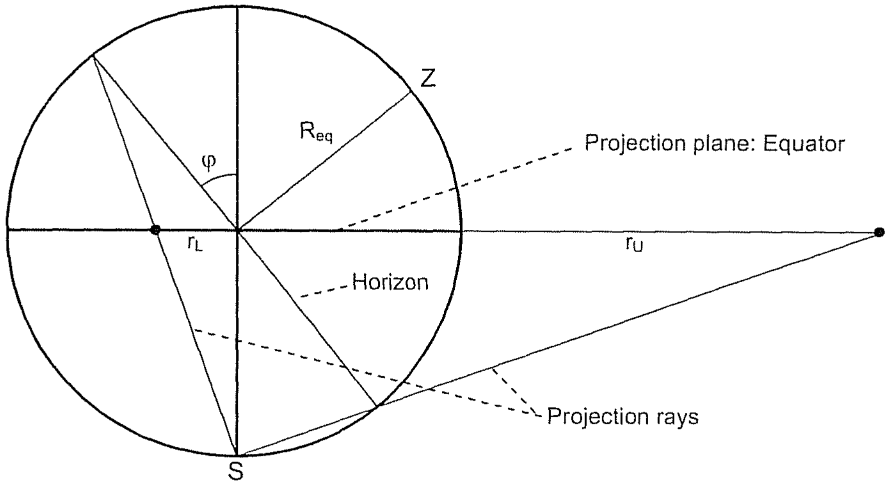


Figure 5-4. Horizon projection

In this construction, the outer circle is the equator. Since a circle is completely determined by locating any two points on the ends of a diameter, all that is needed is to find the extreme limits of the horizon on the meridian. The circle center will be midway between these points.

The fundamental equation is used to determine the projection of the horizon. The declination of the lower limb of the horizon is (90-φ) so the distance of the projected lower limb from the center,

$$r_L = -R_{eq} \tan\frac{(90-(90-\varphi))}{2} = -R_{eq} \tan\frac{\varphi}{2}.$$

Similarly, the upper limb of the horizon is projected as:

$$r_U = R_{eq} \tan(90-\frac{\varphi}{2}) = R_{eq} \cot\frac{\varphi}{2}.$$

Noting the distance of the center of the horizon circle above the center of the plate is halfway between the extremes: $y_C = (r_U + r_L)/2$. Using the identity $\tan 2A = 2 \tan A / (1 - \tan^2 A)$ the center coordinate of the horizon circle is:

$$y_C = \frac{R_{eq}}{\tan\varphi} = R_{eq} \cot\varphi = R_{eq} \tan(90-\varphi)$$

Similarly, the radius of the horizon circle, $r_H = r_U - r_C$ and using the identity $\tan A/2 = \sin A/(1+\cos A)$, we find:

$$r_H = \frac{R_{eq}}{\sin\varphi}$$

These two relations tell us the horizon center forms a right triangle with hypotenuse r_h and sides r_{eq} and y_c with a base angle at the center of φ. This triangle is useful in graphical constructions.

The declination of the zenith is φ, therefore the projection of the zenith, y_Z, is:

$$y_Z = \tan(\frac{90-\varphi}{2})$$

We can also calculate where the horizon intersects the Tropic of Capricorn quite easily. Sunset at the summer solstice will be where the horizon intersects the Tropic of Capricorn. We note the hour angle of sunset H, is given by **$\cos H = -\tan \varphi \tan \delta$** where δ is the Sun's declination. At the summer solstice the Sun's declination is equal to the obliquity of the ecliptic, ε. Therefore, the hour angle of the intersection of the horizon and Capricorn is **$\cos H = -\tan \varphi \tan \varepsilon$**. Similarly, the angle up from the right horizon is (90 - H) so the angle **h** from the E-W line is found from **$\sin h = \tan \varphi \tan \varepsilon$**. This relationship is useful when drawing a computer generated astrolabe or in the design of astrolabe quadrants.

Graphical Horizon Construction

Figure 5-5 shows the graphical construction for determining the location of the horizon on a astrolabe plate.

The equator must be drawn before the horizon. To determine the position of the horizon circle for latitude φ, an arc equal to the latitude is measured from the meridian on the equator. A line connecting the eastern edge of the equator is connected to the end of the arc. The point where the line crosses the meridian is the zenith (Z).

Similarly, an arc equal to the latitude is measured on the equator down from the right horizon, and the end of the arc is connected to the eastern end of the equator. The point where the line crosses the meridian is the lower limb of the horizon arc, H_L.

To locate the upper end of the horizon circle, a line is either drawn through the center from the arc just used or an angle equal to the latitude is measured up from the right horizon, and a line is drawn from the eastern edge of the equator until it intersects with the meridian at H_U.

The center of the horizon circle, y_C, is midway between these points.

The radius of the horizon arc is the distance from either H_U or H_L to y_C.

The complete horizon circle is shown in the figure, but only arc inside the Tropic of Capricorn is drawn on the astrolabe plate.

The validity of this construction can be deduced easily from the logic used to locate circles of equal declination and noting the declination of the lower horizon intersection is (90-φ) and the declination of the upper intersection is -(90-φ).

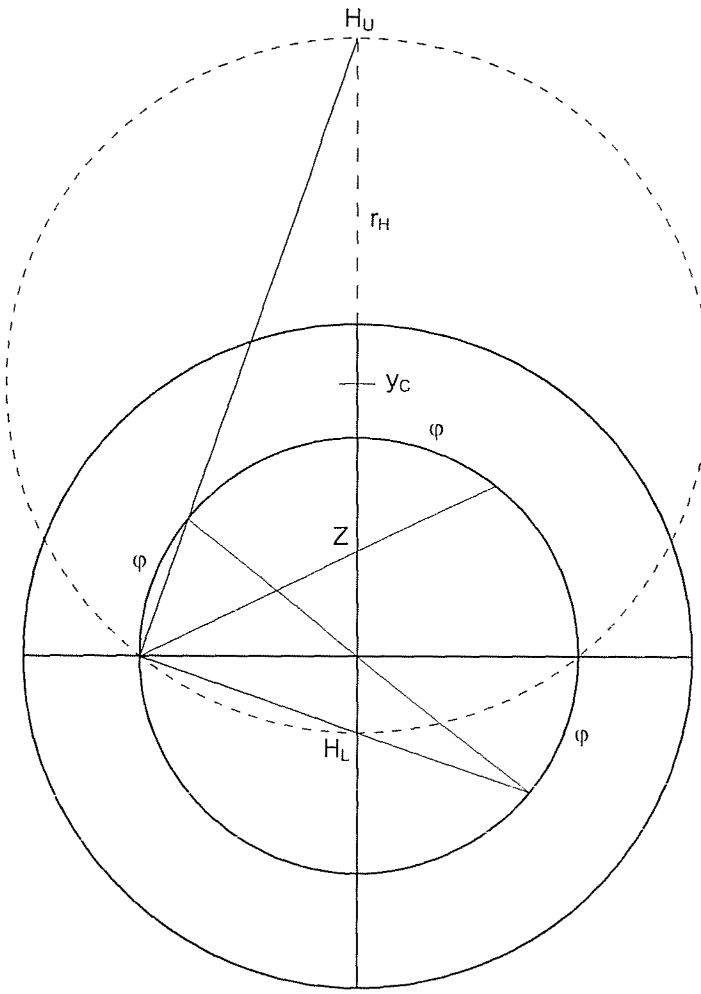


Figure 5-5. Graphical horizon construction.

Altitude circle (almucantar) projections

The construction of altitude circles closely follows the horizon construction. Figure 5-6 shows the basic layout. This figure illustrates the 20° almucantar for 40° latitude.

Using logic identical for defining the horizon, the lower intersection of the altitude circle is: $r_L = R_{eq} \tan\frac{(\varphi - a)}{2}$ and the upper intersection is $r_U = R_{eq} \cot\frac{(\varphi + a)}{2}$. The center of the altitude circle is midway between these two points.

It can be shown the radius of the altitude circle can be calculated directly as:

$$r_A = R_{eq}\frac{\cos a}{\sin\varphi + \sin a}$$

and the distance of the center of an altitude circle from the center of the instrument is:

$$y_A = R_{eq}\frac{\cos\varphi}{\sin\varphi + \sin a}$$

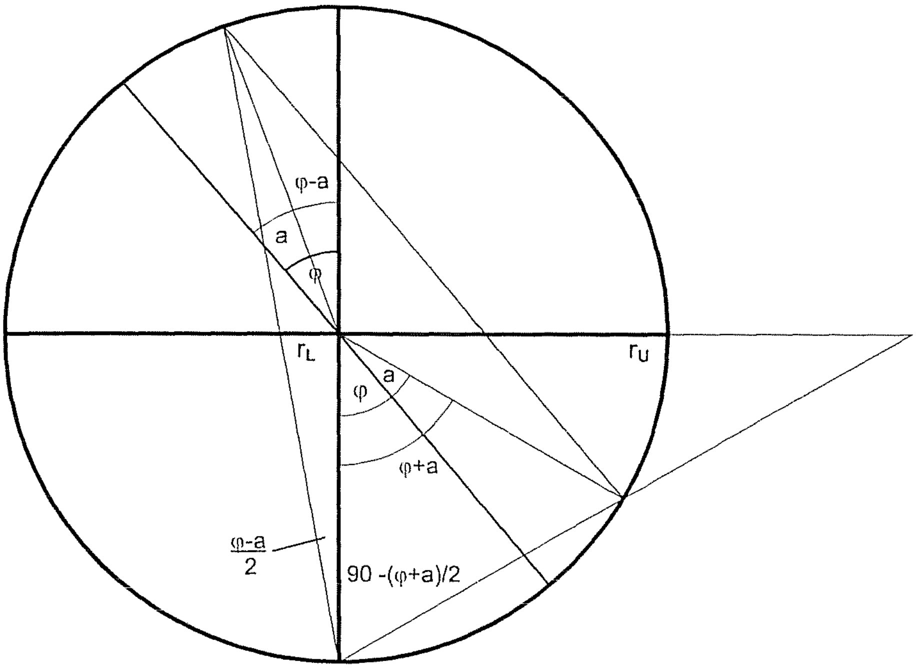


Figure 5-6. Altitude circles (almucantars)

To derive these relations note the radius of the altitude circle $\mathbf{r = (r_U - r_L)/2}$ and the identities:

$$\tan a = \frac{\sin a}{\cos a}$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

If $\mathbf{A = (\varphi + a) / 2}$ and $\mathbf{B = (\varphi - a) / 2}$, then $\mathbf{r_A = R_{eq}/2((\cos A \cos B + \sin A \sin B)/\sin A \cos B)}$. Considering the denominator of this expression, a straightforward application of the second identity to the numerator and the third identity to the denominator of the equation for the radius, substituting back and collecting terms gives the desired result. The procedure for the center is identical starting with $\mathbf{r_C = (r_U + r_L)/2}$.

Altitude circles are normally drawn at 1, 2, 3 or 5 degree intervals on the plate of an astrolabe. The choice of resolution depends mainly on the size of the instrument; the larger the plate, the better the potential resolution. The altitude arcs are drawn only to the Tropic of Capricorn. This restricts the number of problems involving stars that can be solved, but covers the entire annual motion of the Sun. Many astrolabes included segments of an altitude arc for -18° to show the end of sunset or the beginning of dawn. These arcs, called a *crepuscular* (twilight) arcs are drawn only from the Tropic of Cancer to the Tropic of Capricorn since they are used only for the Sun.

Calculating the radius and center of the altitude arcs can be done with a very simple computer program. However, computer drawn arcs are more difficult since the arcs must be "clipped" by the Tropic of Capricorn. See page 378 for details.

Almucantar Center Projection

The equation above for calculating the location of the center of an almucantar circle can be derived directly from the theorem that projects the almucantar center from the apex of a cone circumscribed around the almucantar.

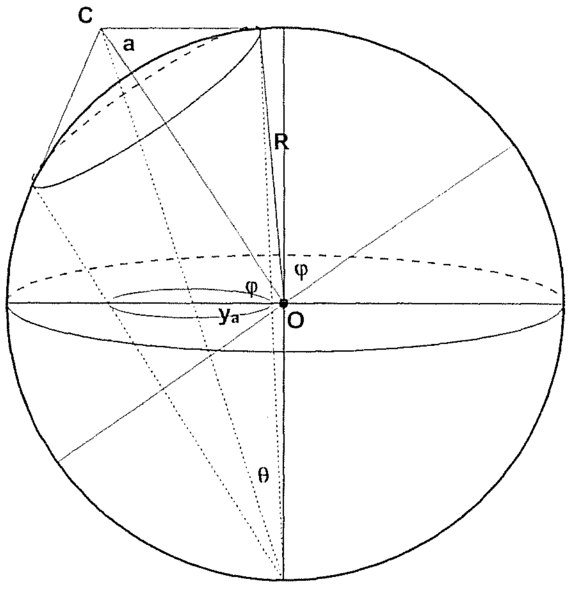


Figure 5-7. Almucantar Center from Cone Theorem

The objective is to find the the distance from the center of the sphere to the projected almucantar center, y_a. Note that $y_a = R \tan \theta$, where θ is the angle from the vertical axis to the projection ray of the cone apex. We, therefore, need to find a relationship for θ.

The angles shown in Figure 5-7 can be easily confirmed with reference to the previous proofs.

OC = R / sin a. The x,y coordinates of **C** relative to **O** are $C_x = OC \cos \varphi$, $C_y = OC \sin \varphi$. The coordinates of **C** relative to **S** are $C_{Sx} = OC \cos \varphi$, $C_{Sy} = OC \sin \varphi + R$.

$\tan \theta = OC \cos \varphi / (OC \sin \varphi + R) = (R/\sin a) \cos \varphi / [(R/\sin a) (\sin \varphi + R)]$.

Simplifying gives $\tan \theta = \cos \varphi / (\sin \varphi + \sin a)$.

Graphical Altitude circle (almucantar) projections

The standard graphical method of laying out the almucantars is almost identical to the horizon construction. See Figure 5-8, which shows the construction for the 20° almucantar for latitude 40°. The construction can be made on either side of the meridian.

To find the lower almucantar intersection with the meridian, measure an angle of (φ - a) down from the right horizon. Note the resulting point will be above the right horizon if a > φ. Draw a line from this point on the equator to the intersection of the right horizon and the equator on the opposite side of the meridian. The lower intersection of the almucantar circle is where this line intersects the meridian.

To find the upper almucantar intersection with the meridian, measure (φ + a) up from the right horizon. Draw a line from the intersection of the equator and right horizon intersecting the angle just drawn and extend the line to the meridian, which may have to be extended. The upper intersection is the point where this line intersects the meridian.. Both the horizon and almucantar circles are shown in the figure.

The almucantar center is halfway between the two points just located.

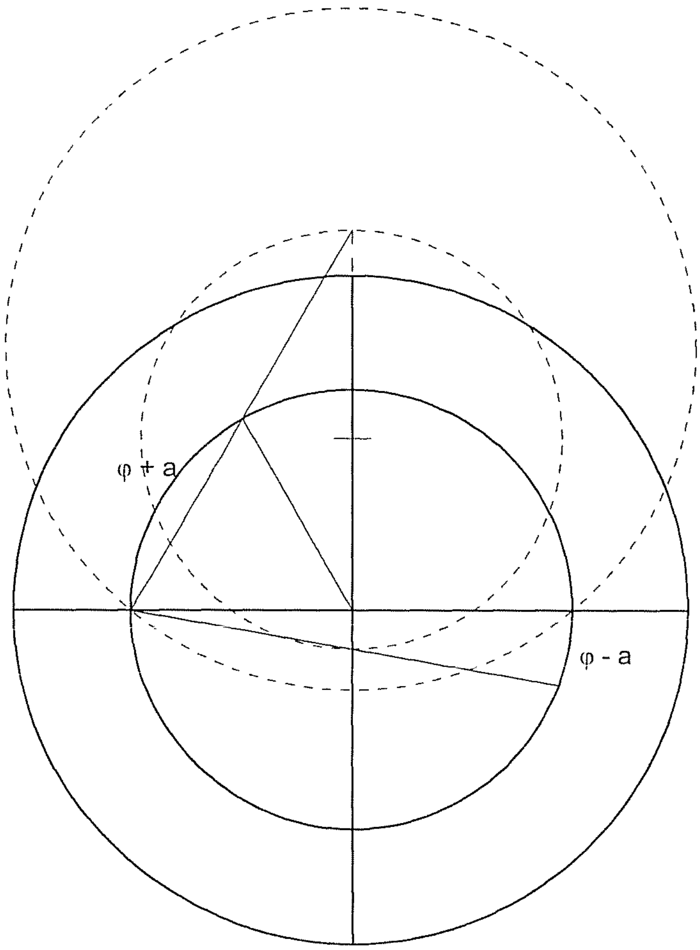


Figure 5-8. Graphical Almucantar Construction

Almucantar alignment with meridian

It is also interesting to note the almucantar intersections with the meridian are distributed exactly by declination. This characteristic is true for all plates for all latitudes, but will involve different almucantars depending on the plate latitude.

The almucantar for an altitude of (90 - φ) will be tangent to the equator. Recall the upper almucantar intersection with the meridian is: $r_U = r_{eq} \cot (\varphi + a)/2$. If $a = (90 - \varphi)$, then $r_U = r_{eq} \cot (45) = r_{eq}$.

For any altitude difference n with (90-φ) the almucantar intersection will be the declination difference. For, $a = (90 - \varphi + n)$. $r_U = r_{eq} \cot\{[\varphi + (90 - \varphi + n)] / 2\} = r_{eq} \tan (90- n) / 2$, which is the equation for the distance on the plate for declination n. This characteristic is immediately seen if a rule graduated by declination is aligned with the meridian, and is rather dramatically demonstrated on a Prophatius astrolabe quadrant (page 247).

The figure below illustrates this characteristic for the almucantars south of the zenith. In this case, the plate is for 40° and is divided with an almucantar for each degree of altitude. The 50° almucantar is tangent to the equator and the almucantars in both directions from the equator align with the declination tics on the rule.

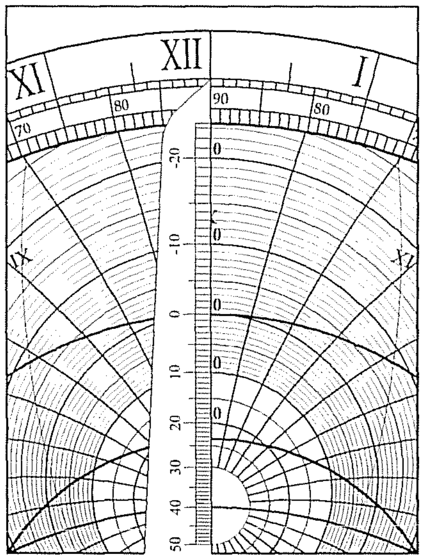


Figure 5-9. Almucantar Alignment with Declination

Another way of looking at it is to note the upper limb of each almucantars is the Sun's maximum altitude for the corresponding declination: $h = 90 - \varphi + \delta$. This approach is conventient when the plate latitude is not a whole degree. For example, for a plate latitude of 36.5°, the almucantar corresponding to a declination of 10° is 63.5° or the declination corresponding to the 44° almucantars is -9.5°.

This characteristic of the projection has been known since ancient times, and was used to determine the declination distance of celestial objects on instruments not including a divided rule. In use, the first object of interest was aligned with the meridian and note was made of its position. The second object was then brought to the meridian and the difference in declination was shown directly by the almucantars. A declination scale was actually marked on a plate of one old instrument (see King [2001], p. 412).

Almucantar separation

The lower intersections of the almucantars with the meridian between the zenith and the horizon may appear to be evenly spaced, but they are not. The graph in Figure 5-10, shows the spacing between lower meridian intersections of adjacent almucantars for three latitudes. The graph was calculated for a large radius plate to expand the abscissa, but the shape of the curves is the same for all plate sizes. Notice the separation between almucantars is minimized close to the pole where the altitude is equal to the plate latitude. Measurement of the spacing between almucantars is one technique for validating the accuracy of a plate[43]. Fake astrolabes may not show this subtle characteristic.

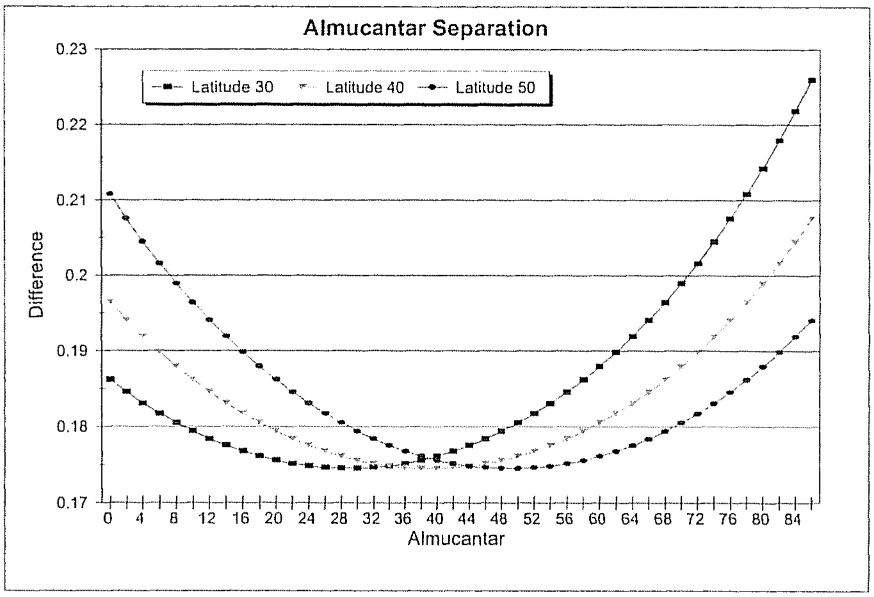


Figure 5-10. Almucantar Separation

Azimuth arc projections

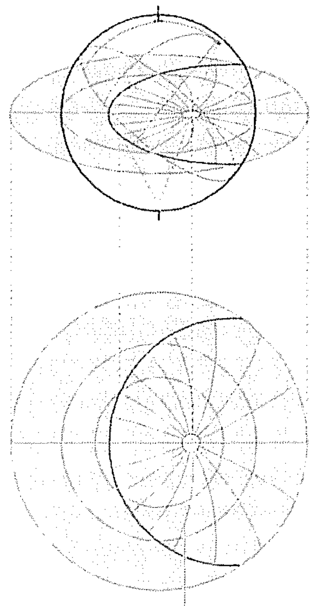


Figure 5-11. Azimuth Projection

The projection of the azimuth arcs is shown in Figure 5-11. The figure is a good illustration of what we are trying to achieve, but it is not very informative about how to go about it.

[43] Lamprey, John P., "An Examination of Two Groups of Georg Hartmann Sixteenth-century Astrolabes and the Tables Used in their Manufacture," Annals of Science, 54 (1997), 111-142.)

The characteristics of the projected azimuth arcs are easiest to visualize from a discussion of the characteristics of the azimuth arcs themselves. By definition, all azimuth arcs pass through the zenith and the nadir (the point on the celestial sphere opposite the zenith) on both the celestial sphere and on the projection of the sphere. Since circles are preserved by the stereographic projection, the centers of all the azimuth arcs must lie on a line midway between the zenith and nadir. The angle between arcs is also preserved by the projection. Therefore, each arc makes an angle with the meridian equal to the azimuth angle, A.

Refer to Figure 5-12. Z is the projection of the zenith. N is the projected nadir. O is the center of the plate. The dashed line below the east-west line shows the line of azimuth centers halfway between the zenith and nadir.

y_Z is the distance of the zenith above O, the center of the instrument, $y_Z = R_{eq} \tan[(90-\varphi)/2]$. y_N is the distance of the nadir below the center, $y_N = -R_{eq} \tan[\,(90+\varphi)/2]$. Let y_C be the distance of the intersection of the line of azimuth centers with the meridian below the center:

$y_C = (y_Z + y_N)/\,2$ y_C will be a negative value.

The distance from the line of azimuth centers to the zenith is y_{Az}:

$y_{Az} = y_Z - y_C = (y_Z - y_N)/2$

A relatively simple reduction of this equation using the identities: $\tan(\frac{90 \pm a}{2}) = \frac{1 \pm \tan(a/2)}{1 \mp \tan(a/2)}$

and $\cos 2a = \frac{1-\tan^2 a}{1+\tan^2 a}$ yields: $y_{Az} = \frac{R_{eq}}{\cos\varphi}$

Computationally, any of the above relationships for y_{Az} can be used.

Considering the triangle defined by the center and radius of the azimuth arc, let x_A be the distance from the meridian to the azimuth center.

$x_A = y_{Az} \tan A.$

The radius of the azimuth arc is:

$R_A = y_{Az} / \cos A.$

Note only one-fourth of the azimuths must be calculated. Each calculation draws one arc representing two azimuths. The arc on the other side of the meridian can be drawn by symmetry by simply making the center coordinate negative.

A clear distinction must be drawn between how the azimuth arcs are labeled and how they are drawn. The angle A in the drawing is the angle of the azimuth circle radius with the meridian line, 30° in the figure. This angle corresponds to azimuth arcs for (90°-A)/(270°-A) and. (90°+A)/(270°+A). The arcs are labeled for the azimuth angle from north or the meridian.

The azimuth arc is drawn only from the Tropic of Capricorn to the horizon. Notice the azimuth arcs intersect the horizon at a right angle, as they should.

There have been several customs defining which azimuth arcs to include on a plate. The modern definition of azimuth leads us to use arcs at so many degrees from north (0-360°), or the angle from the southern meridian (0-180°, E or W),usually each five or ten degrees. Other methods have been to use the mariner's 32 compass points (N, N by E, NNE, NE by E, NE, etc.) or the 24 winds

The simplest azimuth arc is the prime vertical. The prime vertical is the azimuth arc passing through the east and west points on the horizon. The prime vertical's center is on the meridian and the radius of the prime vertical circle is y_{Az}. The prime vertical also intersects the horizon on the right horizon at the equator at a right angle from the preservation of angles in the projection.

Graphical Azimuth Arc Construction

The azimuth arcs are produced graphically using the same technique. The distance on the meridian between the zenith and nadir is bisected, and the line of azimuth centers is located perpendicular to the meridian. The azimuth angle is measured from the meridian and a line drawn to the line of centers. A circle is drawn from the intersection point on the line of centers intersecting the zenith. Only the part of the circle above the horizon is drawn.

Actually drawing the azimuth arcs can be quite difficult. The arcs close to the meridian have a very large radius, and drawing circles with such a large radius is tricky, even on a computer. Some computer circle drawing algorithms break down for such large radii, and it may be necessary to modify or rewrite circle drawing functions. Drawing the large radii arcs by hand will involve using a trammel bar compass or calculating a few points on the circumference and connecting them with a smooth curve. Old instruments usually included azimuth arcs only for each 10°, which simplifies the problem somewhat.

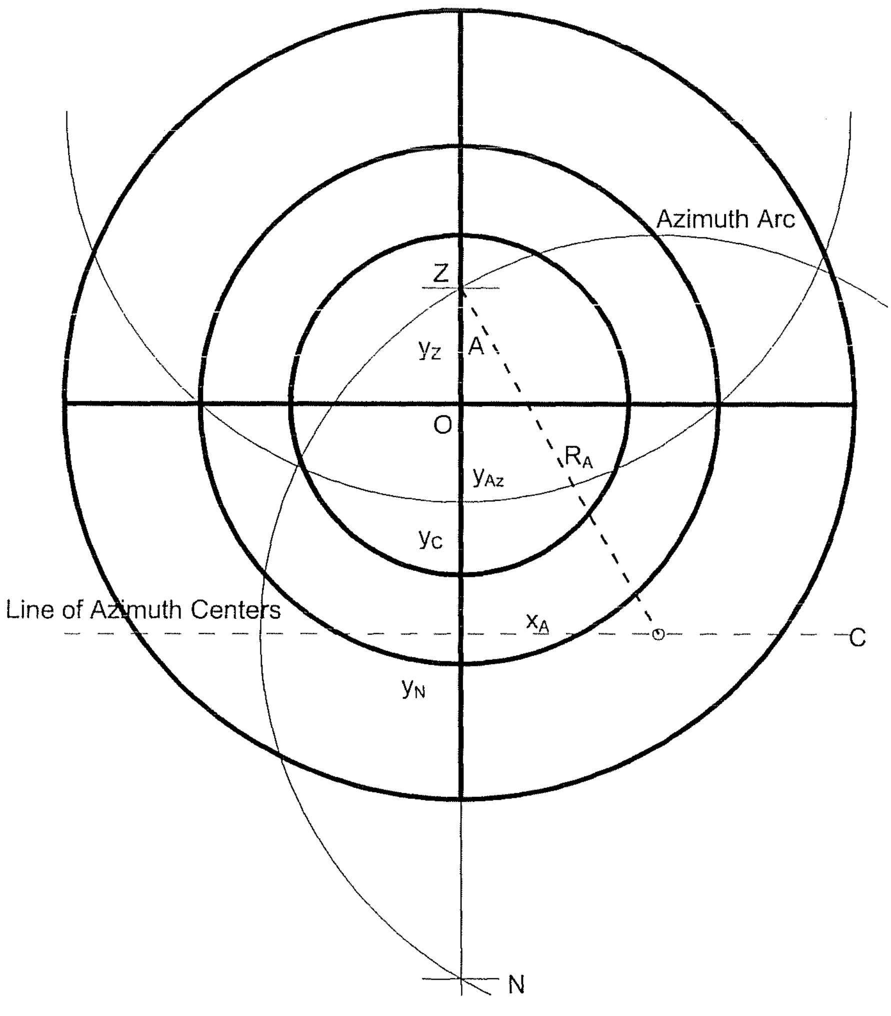


Figure 5-12. Azimuth arc construction.

Unequal hour arcs

The unequal hour arcs are relatively easy to draw. Recall the purpose of the unequal hour arcs is to determine what fraction of the day or night has passed. The time from sunrise to sunset is divided into 12 equal periods, as is the time from sunset to sunrise. Therefore, all we have to do is divide the plate below the horizon into 12 equal sections.

To draw the unequal hour arcs, divide the section of the equator, Tropic of Cancer and Tropic of Capricorn below the horizon into 12 equal sections each. This defines three points for each arc. The unequal hours are drawn as a circle with these three points on the circumference.

Strictly speaking, the unequal hour arcs are not arcs of circles, but the difference with a circle is so slight it it hard to see, even at large magnification and they are always drawn as arcs of circles. To be precise, the individual points derived by dividing the area below the horzon into 12 equal parts are stereographic projections of the unequal hours for a specific day. The unequal hour curves are arcs connecting the stereographically projected points for all days between the limits of the Sun's motion..

Figure 5-13, shows the unequal hour arcs drawn two ways. The solid lines are drawn as arcs of circles. The dashed lines superimposed on the solid lines are drawn using connected line segment for each degree of declination. The difference between the arcs can be seen at high magnification. The maximum deviation from a circular arc is in the area about halfway between the autumnal equinox and the winter solstice, and is no more than the width of the dashed line. Clearly, there is no reason to use anything but a circle to approximate the unequal hour arcs. You can construct the curves from connected line segments as in the figure, or define a cubic spline to fit the curve if you want to go to the trouble, but such a refinement seems excessive.

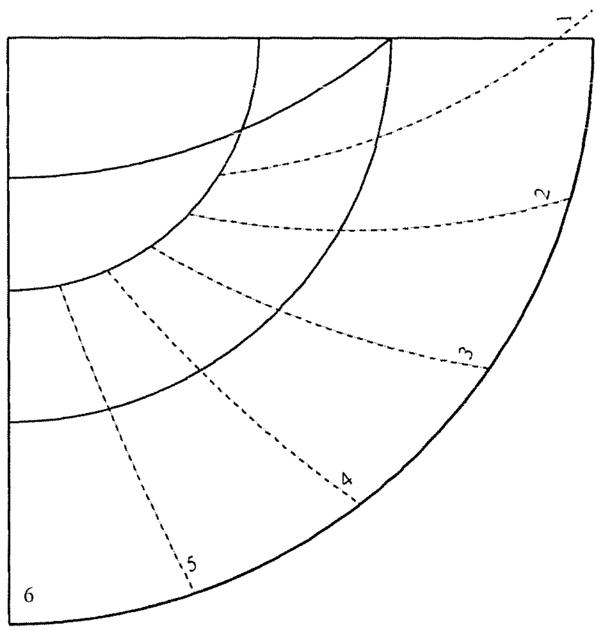


Figure 5-13. Unequal Hour Arc Circles

Purists note the unequal hour arcs do not reflect the Sun's change in declination over the course of a single day, which makes it theoretically invalid to use the Sun's nadir to determine the unequal hour. This is true, but this refinement seems excessive considering the accuracy of the instrument as a whole, and the fact that the unequal hours arcs show only the whole hour divisions in any case.

There are at least two ways to find the points for the unequal hours. The points on the equator are trivial; the unequal hour curve intersects the equator each 15° of the equator semi-circle below the right horizon. The positions of the unequal hour arc intersections with the tropics can be found by calculating the hour angle of sunset and then dividing by 6, the number of unequal hours from noon to sunset. The hour angle of sunset is **cos H = tan φ tan δ**. For the tropics, $\delta = \pm \varepsilon$ depending on the tropic. There are six unequal hours from noon to sunset, so the angle on the tropic for each unequal hour is $\alpha_H = H / 6$. For unequal hour i, the coordinates of the intersection point is $x = R_T \sin i\, \alpha_H$, $y = R_T \cos i\, \alpha_H$, where R_T is the radius of the tropic circle on the plate. The intersection of the tropics and horizon can also be found using the circle intersection routine described on page 378.

For a computer generated astrolabe, a sample routine to draw a circle from three points is on page 377 in the chapter on Computers and Astrolabes.

A closed solution to finding the center coordinates and radius of the unequal hour arcs can be derived from Figure 5-14[44]. The figure shows the unequal hour centers for the third hour before or after noon. For hour **i** from noon, the angle of the hour arc intersection with the equator is **15i**. The same hour arc will intersect the Tropic of Cancer at **ci**, where **c** is the angle per hour on Cancer. **x** is distance from the plate center to the unequal hour arc center. In the figure, the third hour after noon is used as an example, which is the 9th unequal hour of the day or night, so I = 3.

$$x = \frac{R_{Eq}^{\ 2} - R_{Can}^{\ 2}}{2 R_{Can} \sin(15i - ci)}, \quad R^2 = R_{Eq}^{\ 2} + x^2$$

The coordinates of the unequal hour arc center are at: $x_c = x \cos 15i$, $y_c = x \sin 15i$ with the appropriate sign applied for each side.

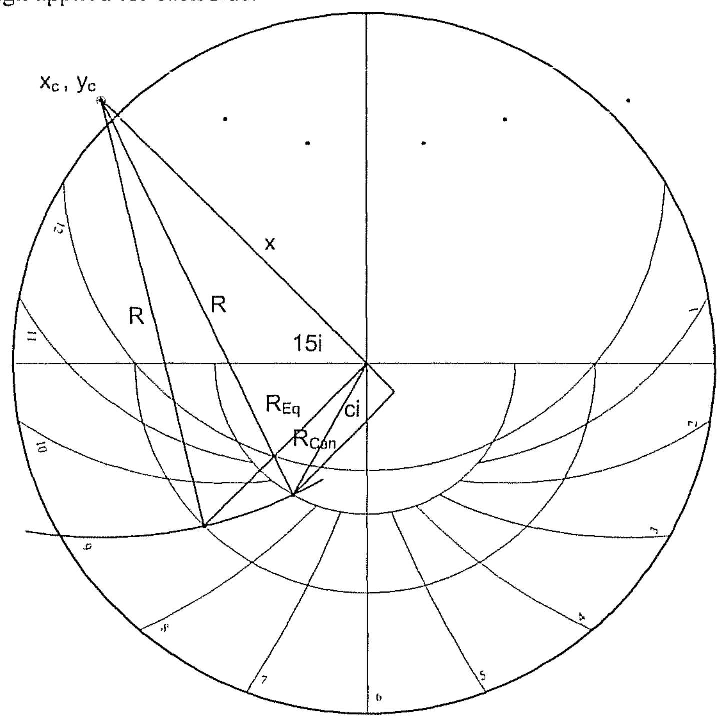


Figure 5-14. Unequal Hour Solution

The unequal hour arcs are drawn from the Tropic of Capricorn to the Tropic of Cancer. The arc for the 9th hour in the figure has been extended a bit to show its location. The intersection points with the tropics can be found using the circle intersection routine mentioned above.

[44] See also, Tardy [1999].

The unequal hours are labeled beginning with the first unequal hour at the western horizon.

To draw the unequal hour arcs graphically, you need to divide the segment of the tropics below the horizon into 12 equal sections. This is easy for the equator since the horizon divides it in half, so each section is 15°. The arcs of Cancer and Capricorn below the horizon can be measured with a protractor or read from the degree scale on the limb. Divide the angle found by 12 to get the length of each arc, and then mark the plate carefully for each division. The center and radius of the circle defined by the three points can be found with a bit of geometrical construction (Figure 5-15).

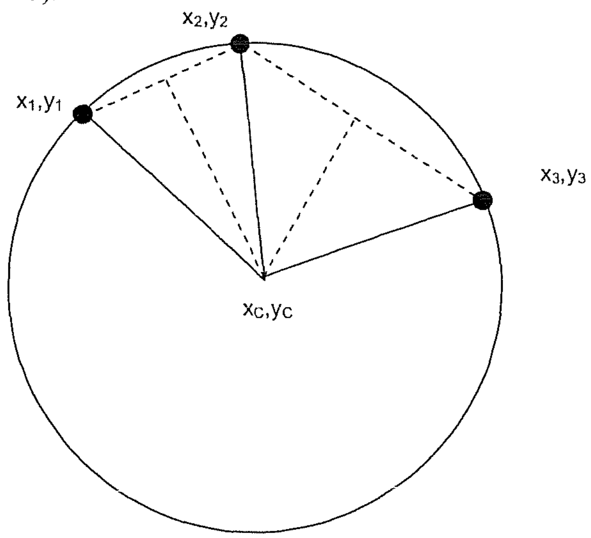


Figure 5-15. Constructing a Circle from Three Points

Draw lines connecting pairs of points. Bisect each line and construct a perpendicular. The point where the perpendiculars intersect is the center of the circle.

Islamic Prayer Times

The times of Islamic prayers are all astronomically deteremined and all can be found with an astrolabe[45]. The first prayer time of the day (*maghrib*) begins at sunset: when the Sun is on the western horizon. The *'ishā'* prayer time begins at nightfall, usually taken as when the Sun is 18° below the western horizon. The *fajr* prayer time begins at morning twilight, again when the Sun is 18° below the eastern horizon (other values are used depending on local practice) and ends at sunrise when the Sun is on the eastern horizon. The *ẓuhr* prayer time begins after the Sun has passed the meridian and the *aṣr* prayer time begins in mid-afternoon.

The prayer times related to twilight and sunset are easily defined and shown on the astrolabe, subject to variations in local customs. The *ẓuhr* and *aṣr* prayers are more complicated.

Astrolabes from Western Islam sometimes included arcs showing the times of the *ẓuhr* and *aṣr* prayers and this discussion is restricted to Maghribi and Andalusi practice. Such curves were very rare on astrolabes from eastern Islam.

The definition of when the *ẓuhr* and *aṣr* prayer times is based on the increase in length of the shadow of a vertical gnomon over the shadow length at local noon. The *ẓuhr* prayer time begins

[45] For a complete discussion of the history and practice of Islamic prayer times see King [2004].

when the increase in the length of the shadow from the noon shadow is ¼ the length of the gnomon. The *aṣr* prayer time begins when the shadow increase equals the length of the gnomon and ends when the shadow increase is twice the length of the gnomon. This convention is not universal in Islam and other definitions have been used, but all are based on the shadow increase.

Various methods have been used to determine when these conditions occur. Strangely, there is little evidence actual gnomons were used in practice and only one example of a gnomon in a mosque survives, at the 7th century Mosque of Janad in the Yemen[46].

An approximate formula of Indian origin for determining the unequal hour from the shadow increase was frequently used. In the following formula, **T** is the unequal hour from sunrise or sunset. The result must be subtracted from 12 for afternoon hours.

$$T = \frac{6n}{\Delta z + n}$$, **n** = gnomon length in feet and **Δz** = increase in shadow length from noon

This formula gives the same unequal hour for every day of the year and for every latitude. It is, however, surprisingly accurate for middle latitudes. For the beginning of the *ẓuhr* prayer time ($\Delta z = n/4$) the formula gives $T = 4.8$. Since the *ẓuhr* prayer time is after noon, the hour must be adjusted by substracting from 12 giving 7.2, or a little over one unequal hour after noon. For the start of the *aṣr* prayer time, $\Delta z = n$ and giving 9, the end of the ninth unequal hour or the beginning of the tenth unequal hour. Mid-afternoon in other words. The *aṣr* prayer time ends when $\Delta z = 2n$, at the end of the tenth hour.

Another somewhat more rigorous formula, also of Indian origin, was also used.

$$\sin(15T) = \frac{\sin h}{\sin H}$$, where **h** = Sun's current altitude, **H** = Sun's noon altitude

Once again, this formula gives surprisingly good results for middle latitudes. This is also the formula on which the universal horary quadrant is based. Arcs showing the unequal hour of the *ẓuhr* and *aṣr* prayers was sometimes shown on astrolabe plates from the Maghrib and Western Islam. An example of how these arcs would appear is shown in Figure 5-16.

Note also, the crepuscular arc at -18° is used to indicate the morning and evening prayer times.

The curves in the figure are drawn three ways. The solid arc results from the use of the approximate sin formula above. The dashed arc is calculated using exact values. There is a dotted arc close to the solid arc showing the result if it is drawn as the arc of a circle using three points defined at the solstices and the equinox. As can be seen from the figure, all three methods give good results. It should be noted that these are not the only methods used for this problem.

The exact values are calculated by finding the Sun's required altitude ($\cot h = \cot H + \Delta$) and the hour angle is calculated with the standard equation and converted to unequal hours.

[46] King [2004] p. 469.

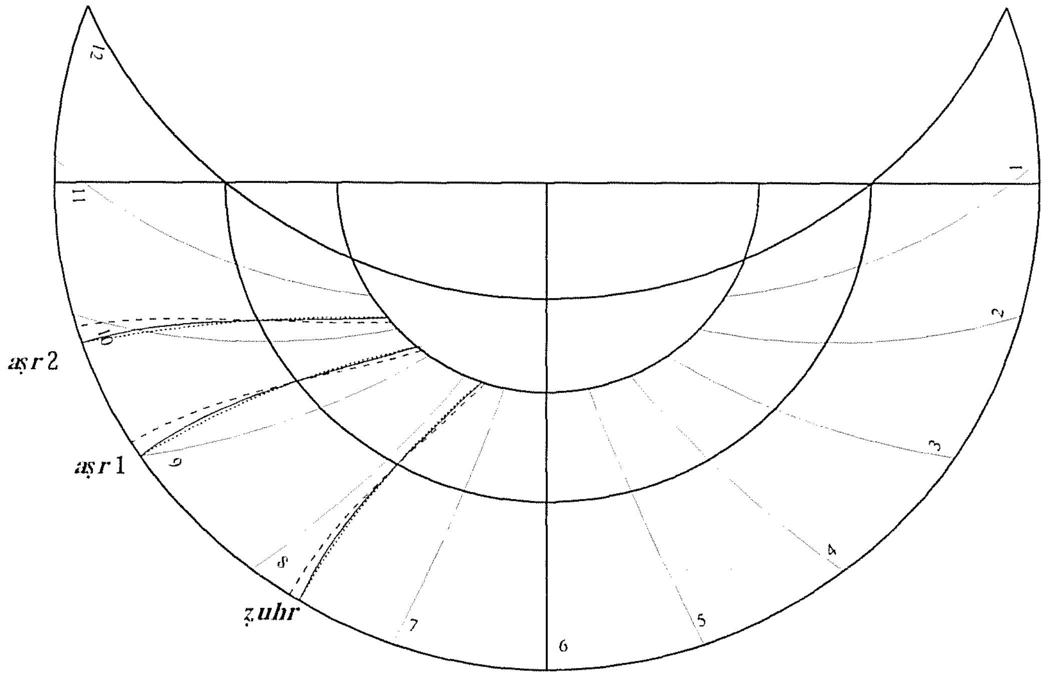


Figure 5-16. Islamic Prayer Time Arcs

Equal Hour Arcs

As mentioned earlier, some locales used equal hours beginning at sunrise or sunset. Arcs representing these timekeeping conventions were sometimes included on astrolabe plates.

Italian hours (*horae ab occasu*) are 24 hours of equal length beginning at sunset (Figure 5-17). Arcs defining the equal hours are drawn below the horizon. The daytime hour is found using the Sun's nadir as with the unequal hour. The hour arcs which can be drawn below the horizon limit the times these arcs can be used. The figure is drawn for latitude 49°, which is in the area of southern Bohemia.

All of the arcs have the same radius as the horizon. Italian hours can be drawn in two ways. The tropics can be divided into 24 equal segment beginning at sunset and the arcs constructed from three points as segments of circles connecting the hour divisions.

Alternatively, the circle defined by the center of the horizon can be divided into 24 equal sections beginning at the southern meridian. The center of each arc is located by each 15° point on this curve. The dots in the figure are the centers of the arc circles. Each arc is drawn from this circle of centers. The equal hour arcs are drawn from Capricorn to either Cancer or the horizon since the arcs are concerned only with the Sun.

It is possible, and has been done, to include both Italian and Babylonian hours on the same plate to allow the coverage of the entire day by the hour arcs. The web of arcs becomes very congested, but the functionality may be slightly improved. On some Islamic plates (rare), equal hours arcs are extended only to the meridian, which makes a rather pleasant display when both types are included.

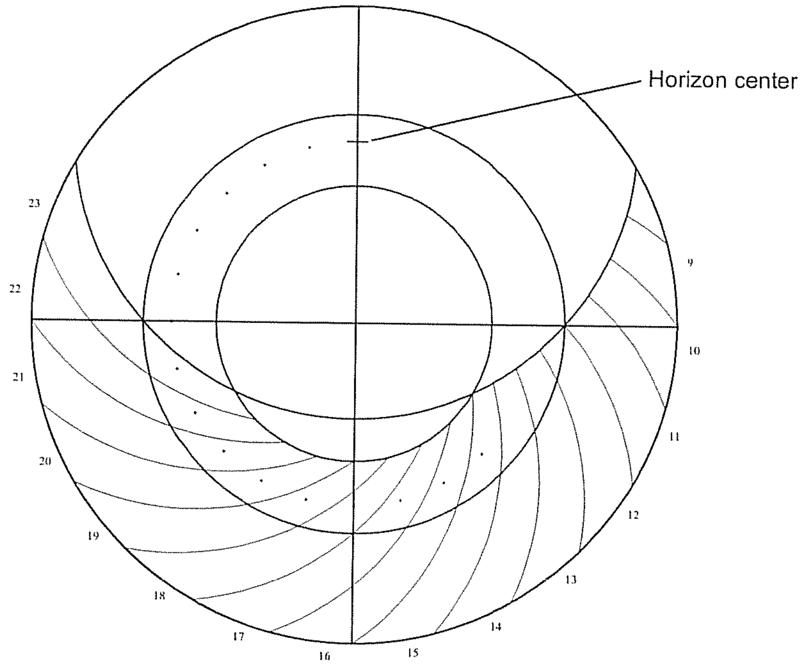


Figure 5-17. Italian Hours

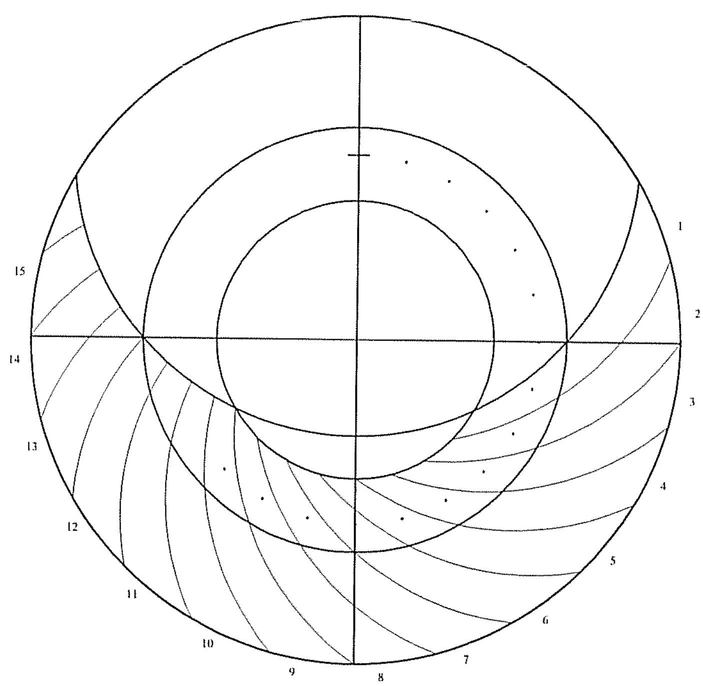


Figure 5-18. Babylonian Hours

Babylonian hours (*horae ab ortu*) are identical to Italian hours except the counting begins at sunrise (Figure 5-18).

Equal hour arcs are sometimes drawn with unequal hour arcs. The result is attractive, but rather congested.

Houses of Heaven

The "Houses of Heaven" as shown on old astrolabes follow a construction advocated by Regiomontanus in the 15th century. This system is based on the local horizon. The sky is divided into 12 houses, six above the horizon and six below. Each house is an arc in the sky from the north point on the horizon to the south point. Thus, the meridian is a house boundary. The other house boundaries are at 30° intervals from the meridian or horizon.

The arcs defining the houses are projected on the astrolabe plate and are defined by three points: the northern point on the horizon, the southern point on the horizon and the intersection of the house arc with the equator.

To draw the house arcs (Figure 5-19), divide the equator into 30° sections above the right horizon. Locate the north and south points on the horizon (this will require drawing the entire horizon circle for graphical constructions). Draw the arc of the circle thus defined within the Tropic of Capricorn. Only two circles define the houses with the other two circles drawn from symmetry. The houses are numbered consecutively from I to XII starting at the eastern horizon and continuing counterclockwise.

The location of the northern limit of the horizon circle is $\mathbf{R_{eq} \tan \varphi/2}$, and the southern intersection of the horizon circle with the meridian is $\mathbf{R_{eq} \cot \varphi/2}$. The houses can be drawn using the same techniques used for the unequal hour arcs for drawing a circle from three points. Note the centers of the house circles are all on a line passing through the center of the horizon circle and is parallel to the right horizon.

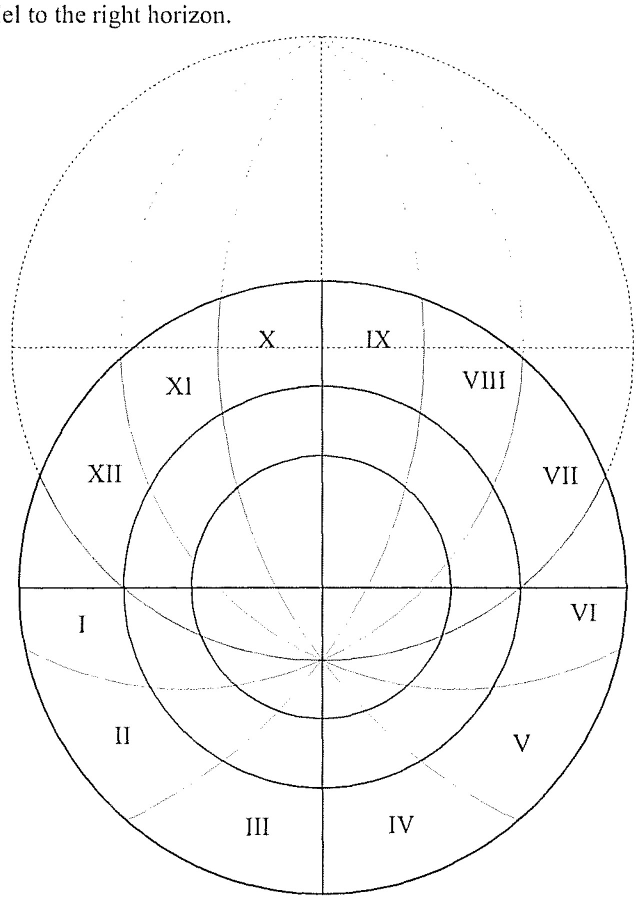


Figure 5-19. Houses of Heaven Construction

The astrolabe can be used with other house systems than the one described above. All house systems use the position of the ecliptic at the time of some meaningful event. The astrolabe rete is set to the time of the event and the location of the ecliptic is used to locate the houses.

Chaucer describes a system in which the arc from the point where the ecliptic intersects the eastern horizon (*ascendant*) and the southern meridian (*medium caelum*) is divided into three equal sections with the angles measured on the limb (which is equivalent to measuring them on the equator). Each of the three sections is a house for the instant. Lines drawn across the face of the astrolabe from these points define three more houses below the horizon and west of the meridian. Similarly, the angle from the eastern horizon to the northern meridian is divided into three equal sections, which in turn defines three more houses above the western horizon. The houses, twelve in all, are numbered from I starting at the eastern horizon and proceding counter-clockwise.

Astrological house systems are a historical subject with many subtleties. The interested reader is directed to the rich history of astrology and particularly to North [1988].

A similar construction appears on some Islamic astrolabes. These arcs are used in an astrological method called "casting the rays" and relate to a very different system.

Other Graphical Layout Methods

There are several methods for determining the locations and sizes of the plate circles. All of them eventually come down to applying the half-angle theorem, but they reduce the effort required to draw the arcs, particularly when the parameters are taken from tables.

The *Dastūr*

The dastūr, or rule, is a specialized protractor of Persian origin for the laying out of astrolabe plates. The datsūr is divided to reduce the amount of calculation required to locate the various plate arcs. Figure 5-22 is a full sized datsūr that can be copied and used. The dastūr was engraved on a flat and polished brass sheet and was about 10 inches long. It was made with great care since the accuracy of all elements of the astrolabe depended on its precision.

The dastūr scale is normally divided with each half degree shown, and the scale is labeled with a value double the angle from the origin in order to simplify certain operations.

In use, the dastūr origin is aligned with the eastern intersection of a tropic, usually the equator, and the right horizon. The desired angle is located, and a light construction line is drawn on the plate connecting the origin and the angle. In many operations it is not necessary to draw a long line. A prick for the desired line can be made and a straightedge laid from the origin to the prick. The point where the straightedge crosses the meridian is then marked.

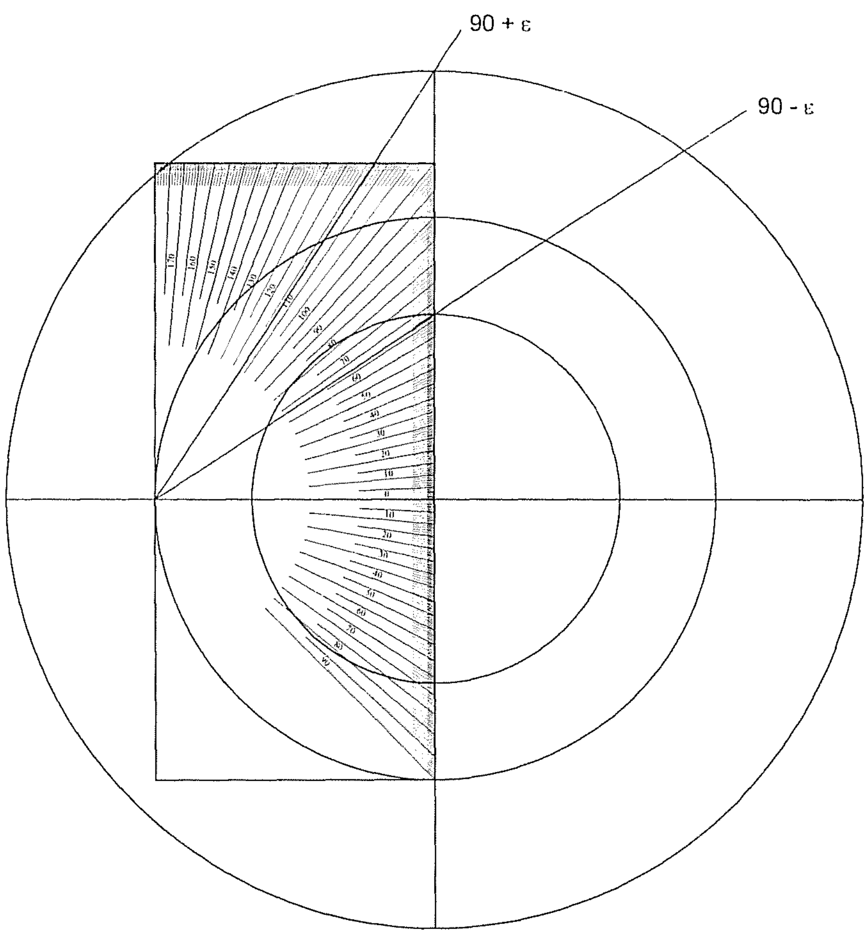


Figure 5-20. Locating the Tropics with the Dastur

You must understand the underlying mathematics of the astrolabe in order to use the dastūr , but it is a handy tool for solving the basic astrolabe equations graphically. The dastūr is very useful tool and is easy to use when taking the plate circle centers and radii from tables.

The theory behind the dastūr is based on solving the fundamental equation:

$$r = R \tan\frac{(90-\delta)}{2}$$

Recall the dastūr is labeled for twice the actual angle subtended from the origin of the dastūr. Therefore, when the dastūr is aligned as shown in Figure 5-20, the distance from the origin of the astrolabe plate to the intersection of a line drawn for an angle a marked on the dastūr from the dastūr origin to the meridian is $\mathbf{R_{eq} \tan (90 - a)/2}$.

Using the dastūr to locate the tropics is shown in Figure 5-20. In this case, the radius of the equator is defined and Cancer and Capricorn are derived. The dastūr is aligned with the eastern point on the equator and an angle of (90-ε), about 66.5°, is located on the dastur. A line is drawn from the dastūr origin to the meridian to define the upper intersection of the Tropic of Cancer. (90-ε) is used on the dastūr instead of (90-ε)/2 because the angles are labeled as double the actual angle. Similarly, the Tropic of Capricorn is located by drawing a line through the dastūr angle of (90+ε), about 113.5°.

Locating the horizon uses the triangle created by the horizon center discussed earlier. The angle of the horizon radius with the right horizon is (90-φ). Therefore, we locate the dastūr angle of 2 × (90-φ), and the point where this line crosses the meridian is the horizon center. The horizon radius is the distance from the center to the east point on the equator. Figure 5-21 shows the horizon construction for latitude 51.5°. The angle on the dastūr is 77°.

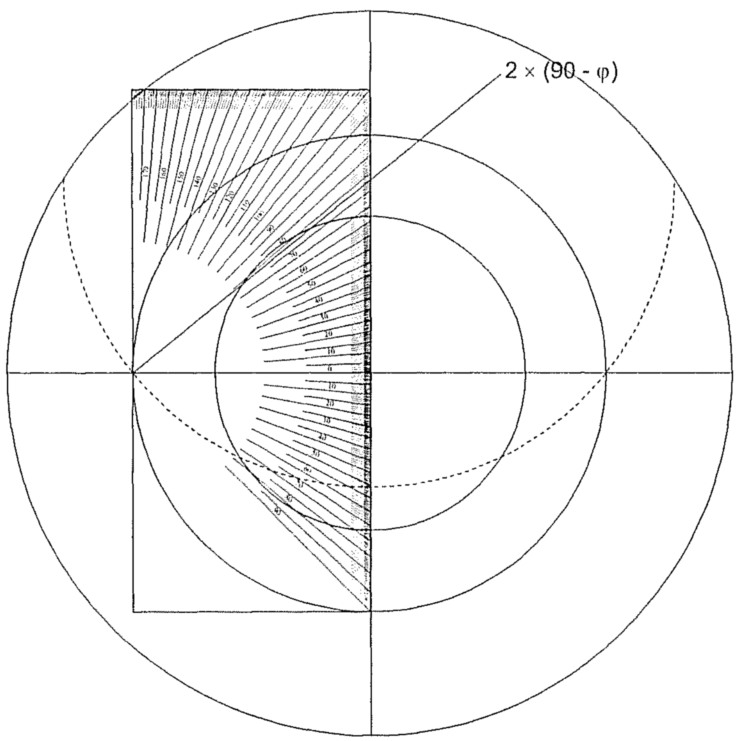


Figure 5-21. Locating the Horizon with the Dastur

Drawing the almucantars with the dastūr is very similar to the usual graphical construction, taking into account how the dastūr is labeled. The dastūr can also be used for the azimuths.

Use of the dastūr reduces the number of construction lines that have to be erased on the plate, but it does not greatly simplify the plate layout process. It would, of course, be possible to create a relatively simple list of instructions for plate layout based on the dastūr that could be used by engravers who are not familiar with the underlying theory. Michel presents complete instructions for laying out a plate using the dastūr.

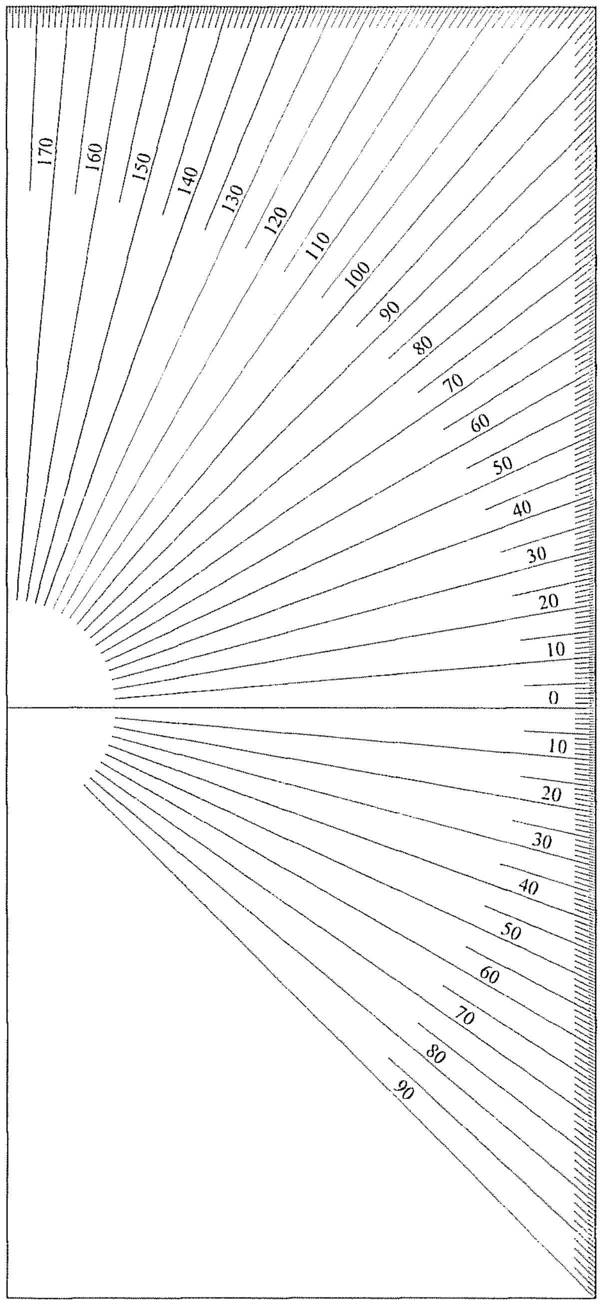


Figure 5-22. Dastur

Hartmann's Method

Georg Hartmann (1489-1564) of Nuremburg presented a slightly different approach in 1527 in his *Practika*[47], which was never published in his time. Hartmann's method uses an auxiliary circle to locate the limits of the horizon and almucantars and significantly reduces the number of work lines needing to be drawn and erased from the final product. This method is quite efficient but requires several auxiliary circles to be drawn and divided, which is more work for a single instrument but is efficient if a number of plates will be made for each latitude. It was ideal for Hartmann who made his instruments in batches.

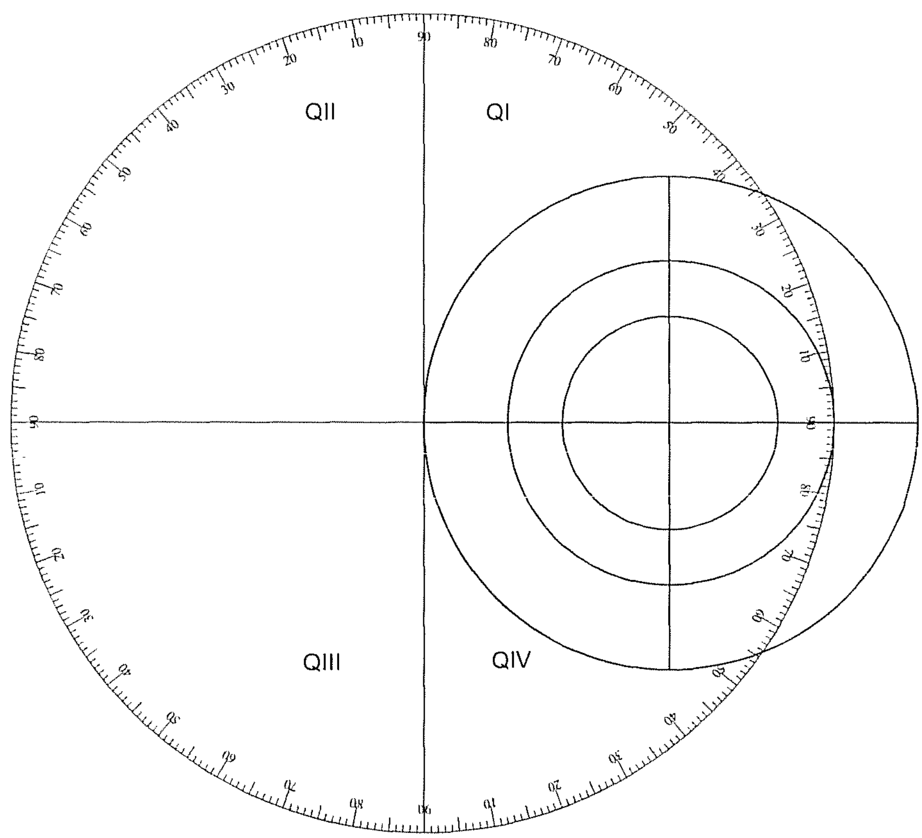


Figure 5-23. Hartmann's Auxiliary Circle

See Figure 5-23. The auxiliary circle is drawn with the center at the east point of the Tropic of Capricorn and its radius as the distance from this point to the intersection of the equator with the right horizon.

This construction is used to locate the zenith, nadir and horizon. See Figure 5-24.

The example is for 40° latitude. First, draw the plate (Tropic of Capricorn), equator, Tropic of Cancer, meridian and right horizon as described above. To locate the zenith, a line is drawn from E to the latitude in QII. The point where this line crosses the meridian is the zenith, Z. Similarly, a line from E through the latitude in QIV locates the nadir, N. The point halfway between these points locates the line of azimuth centers.

[47] Lamprey, John, *Hartmann's Practika*, Bellvue, CO, 2002

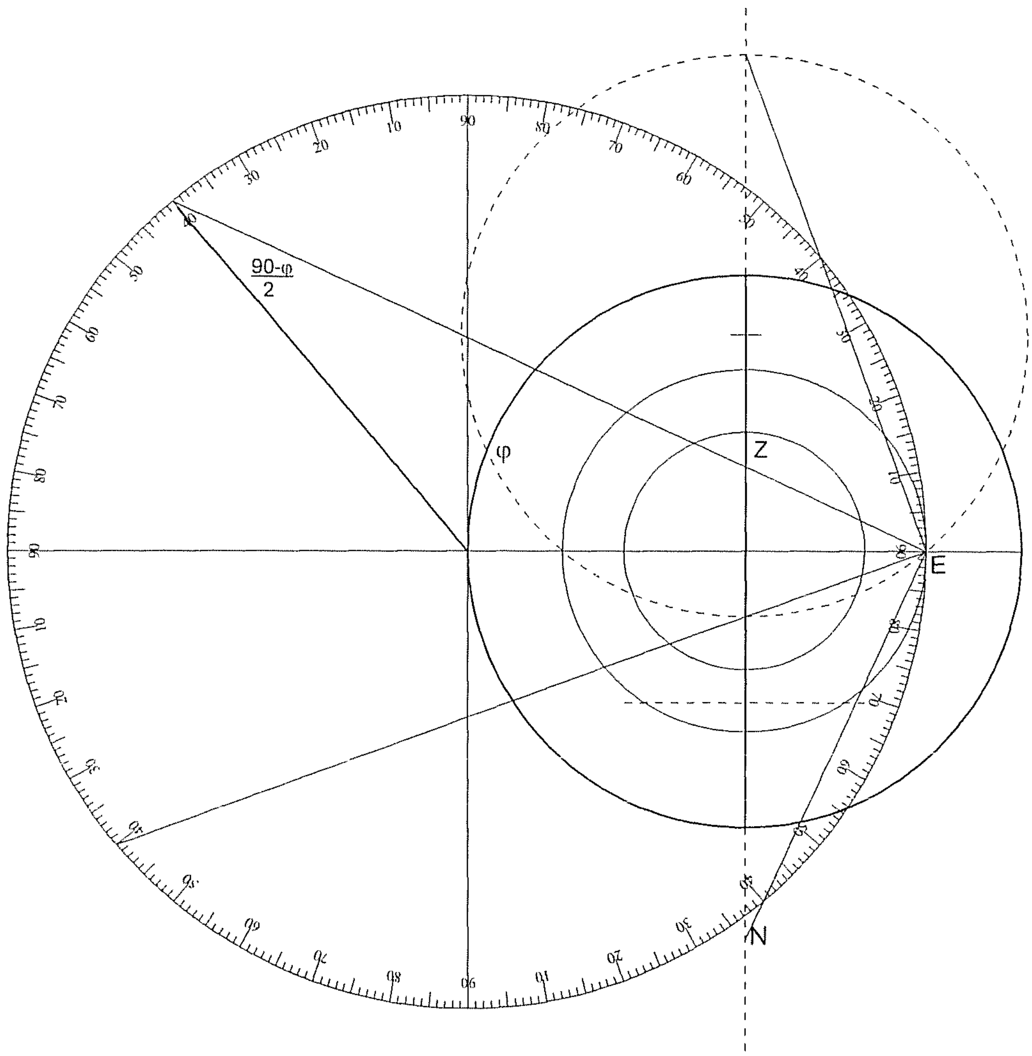


Figure 5-24. Hartmann's Horizon, Zenith and Nadir Construction

The lower intersection of the horizon circle with the meridian is found by drawing a line from E to the latitude in QIII and the upper intersection by a line from E through the latitude in QI. The center of the horizon is halfway between these points. The center of the horizon circle also defines the line of centers of the Houses of Heaven circles.

It is very easy to see this construction is a direct application of the half-angle theorem that simplifies the layout by the clever way the auxiliary circle is labeled. Take, for example, the line defining the zenith. By measuring the angle equal to the latitude in QII we have defined a triangle with base angles of (90 - φ)/2. The distance from the center of the plate to the zenith is R_{eq} tan (90 - φ)/2, which is the desired result. The construction of all of the other lines can be derived using identical logic.

Hartmann does not mention it, but the horizon center can be found directly by drawing a line from E to 2φ in QI and noting the intersection with the meridian. This saves the effort of bisecting the limits of the horizon circle to find its center. Similarly, the line of azimuth centers can be found by measuring down 2φ in QII and drawing a line from E.

Drawing the almucantars requires an additional scale on the auxiliary circle for each latitude (Figure 5-25). The scale begins at the latitude angle in QI and extends 180°, ending at the latitude in QIII. It is divided by the almucantar resolution desired. Hartmann showed this secondary scale as a separate construction, but it is added it to the original auxiliary circle here for clarity.

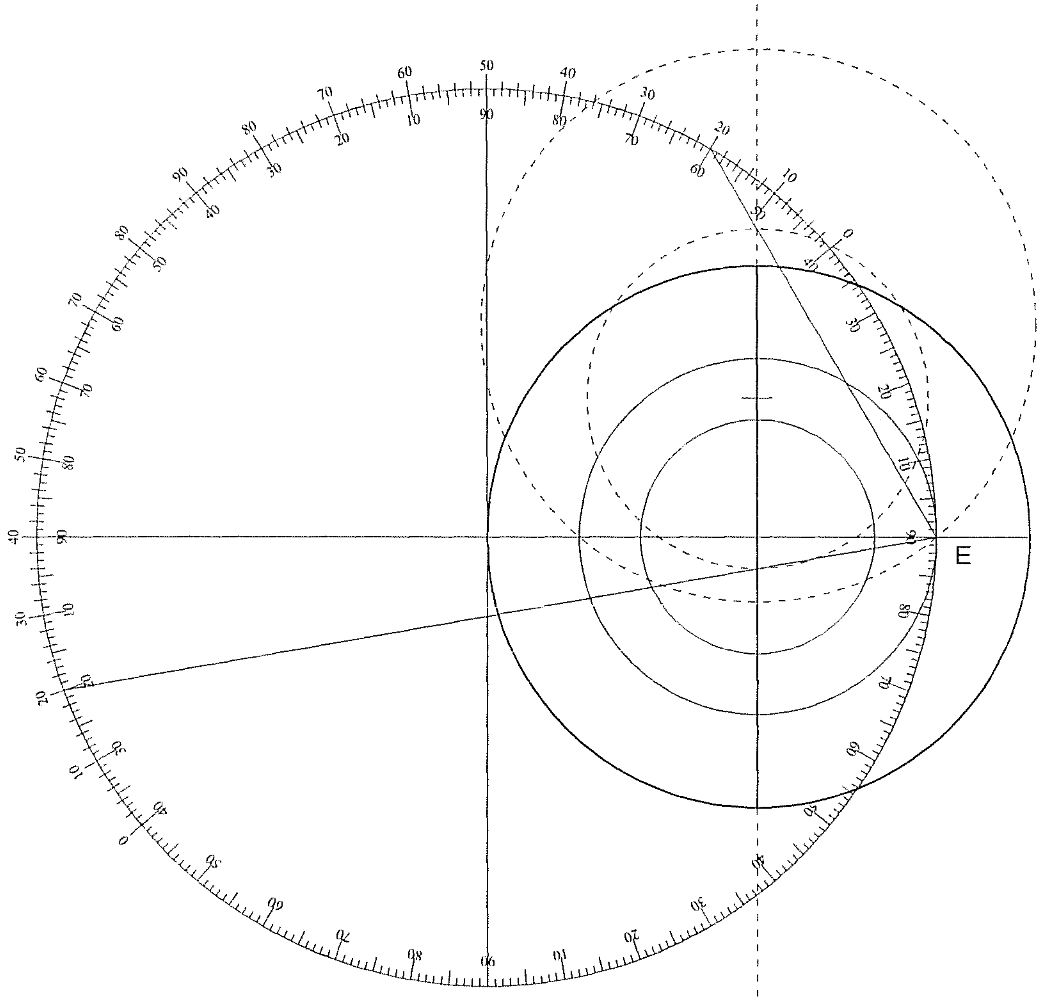


Figure 5-25. Hartmann Almucantar Construction

Figure 5-25 shows an example for latitude 40° with a resolution of 2° (i.e. an almucantar for each 2°). Finer or coarser divisions of the secondary scale can be drawn depending on the desired resolution.

To locate an almucantar, simply draw a line from E to the desired altitude on the secondary scale (20° on the figure). The intersection of the lines with the meridian define the upper and lower intersections of the almucantar circle. The center is halfway between.

It might be possible to make a Hartmann style protractor usable for all latitudes using modern transparent materials. The base circle could be drawn on clear plastic, and a rotating secondary scale with means to lock it in place could be made for the almucantars.

Hartmann also suggests a scale for finding the azimuth centers as shown in Figure 5-26. This scale is a semicircle drawn with the line of centers on the meridian (C) as its center and passing through the zenith (Z) and nadir (N).

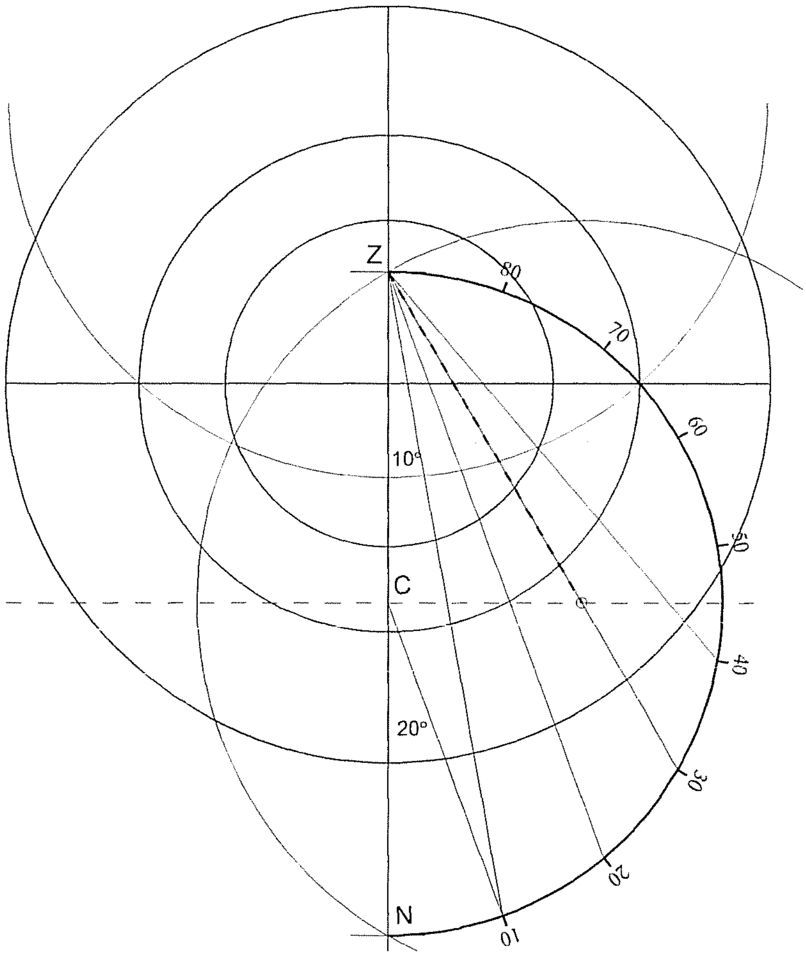


Figure 5-26. Hartmann's Azimuth Center Scale

The scale uses the half-angle theorem to locate the points of the azimuth centers. The scale is divided into as many sections as is desired for the azimuth resolution (10 degrees in the figure). The angles measured from the center of the scale, C, are twice the value labeled on the scale. From the half-angle theorem, the angle from the zenith matches the label. The 30° azimuth arc is drawn on the figure.

Hartmann apparently felt this was an easier or more accurate way to find the azimuth centers. Whether it is easier than simply measuring the angles from the zenith is arguable, particularly since a separate scale is required for each latitude, but it is interesting nonetheless

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Chapter 6 - The Rete

The rete rotates to simulate the movement of the stars in the sky. Everyone knows the stars appear to move in the sky but, in reality, the stars are fixed, and it is the Earth that moves. If the Earth were stationary and only rotated, we would see the same stars in the same positions in the sky at the same time every day. But, because the Earth is also orbiting the Sun, moving a little in its orbit every day, the field of stars also appears to move a little every day.

The rotation of the Earth is used as the basis for keeping time because it is so constant. The average time between the Sun's appearance on our meridian has been defined as a day and the length of the day is defined as 24 hours, a tradition that is about 4,000 years old. But the length of a mean solar day is not the same as one rotation of the Earth. The Earth must rotate a little extra for the Sun to reach a given meridian since the Earth has also moved in its orbit. The length of one rotation of the Earth is called a **sidereal day** (sidereal means star) and is about 23 hours 56 minutes 4 seconds, or about 3 minutes 56 seconds shorter than a mean solar day. It is because of these four minutes or so we see different stars in the sky at different times of the year.

On the astrolabe, the coordinate system where you are (the altitude and azimuth circles) does not move because your location does not move, but the stars move in our sky. The rotation of the rete simulates the rotation of the sky and allows the positions of the stars to be set for any time. The entire field of stars rotates in one sidereal day.

The rete on old astrolabes included pointers indicating the celestial positions of a number of fixed stars. The oldest instruments apparently followed the earliest manuscripts from the 4^{th} to 7^{th} centuries and included 16 or 17 fixed stars. Many later instruments included as many as 30 or more stars.

The position of a star pointer over the altitude and azimuth arcs shows the star's position in the sky when the rete is positioned to a sidereal time. This function can work in two ways. If the star's pointer is positioned to a specific altitude, such as rising, setting or culminating, the time of the event can be read from the astrolabe. If the rete is set to a time, the star pointers show the sky's configuration at that time. The star pointer can also be used as a reference for the star's declination and right ascension. The declination is measured with the rule and the right ascension is found by noting the position of the first point of Aries when the star pointer is positioned at the meridian.

Of all the parts of the astrolabe, the rete offered the maker the most opportunity for artistic expression. Some beautiful and ingenious retes have graced old instruments.

The rete's design presented the maker with both problems and opportunities. The rete must contain the projection of the ecliptic and enough star pointers to ensure some bright stars are always in a readable position regardless of the season. In addition, the rete must have enough open space to allow the plate to be seen while still being robust enough to avoid damage under normal use. The projection of the equinoctial colure is almost always shown by a line, which may use counterchanges, connecting the equinoxes. There are few constraints beyond these and makers exercised considerable ingenuity and artistic inspiration in rete design. On the other hand, a region might adopt a traditional rete style with little variation over centuries of production. Western Islamic astrolabe retes in particular were generally of a common style.

There is no "normal" rete design, but a few general layouts became reasonably widely used. The following figures represent a sample of rete styles without star pointerss. See the pictures on pages 22 and 24 for samples of star pointers. A brief perusal of museum instruments or descriptive astrolabe books will illustrate the variety of rete styles.

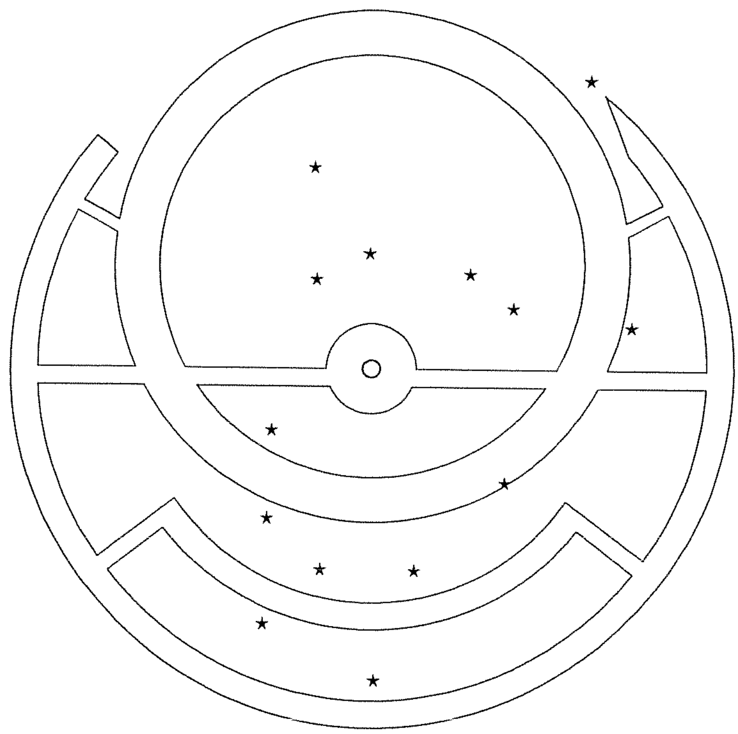

Figure 6-1. Gothic Rete

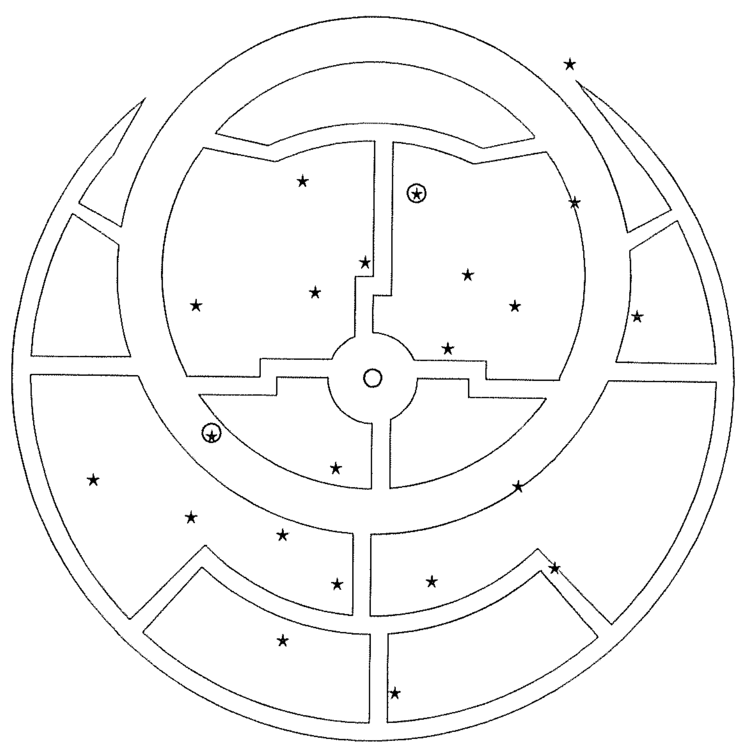

Figure 6-2. Late Gothic Rete

The rete in Figure 6-1 is a somewhat stylized representation of the retes used on the earliest Islamic and western Islamic astrolabes and was also found on early European instruments. The star pointers on this type of rete were usually spikes with a more-or-less ornate base. Some of the pointers were quite long and delicate due to their length. Instruments made with this type of rete generally included fewer than 20 stars. The figure includes 14 stars at their positions in 1062[48].

The relative simplicity of the Gothic rete style had been replaced by a later, more delicate and artistic style by the 14th century. The late Gothic rete in Figure 6-2 is in the spirit of this rete type, which is characterized by several counter-changes in the supporting bars and much lighter construction. Star pointers were generally gracefully curved and unadorned. Considerable variety was employed in the details of this rete type. The rete in the picture of the Fusoris astrolabe in the first chapter is a good example and contains 21 star pointers. The stars in Figure 6-2 are the ones used by Fusoris accurately drawn[49]. Fusoris misplaced two stars. The correct positions are circled in the figure. Note the counterchange on the solsticial colure, which was universal on Andalusian astrolabes although the details vary.

Some Gothic astrolabe retes from Spain, France and Italy employed a full or half quatrefoil (Figure 6-3) as a supporting element.

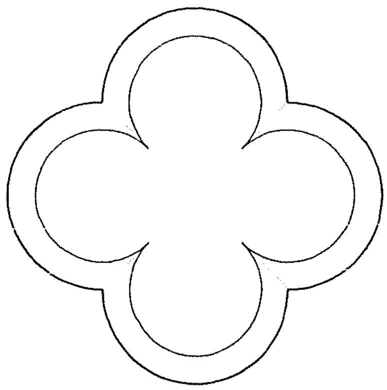

Figure 6-3. Quatrefoil

The quatrefoil as an artisitic element probably originated in Byzantium, but was more popular in Andalusian decorative art from which it found its way into Europe. Quatrefoils and trefoils were common in Gothic art and architecture and various symbolic meanings have been attached to the figure[50].

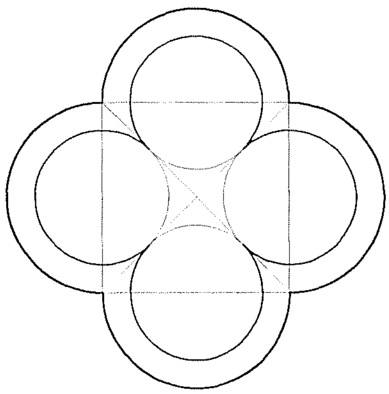

Figure 6-4. Drawing the Quatrefoil

[48] Stautz [1997], p. 178.
[49] Stautz [1997], ;p. 265.
[50] King, David A., *The Ciphers of the Monks*, Franz Steiner Verlag, Stuttgart (2001), Appendix F, pp 380-390.

The quatrefoil is easy to draw either analytically or geometrically. The centers of the circle defining the leaves are centered on the edges of a square and the cusps are the intersections of the circles with the square's diagonals (Figure 6-4). Many variations, some quite lovely, survive in art, architecture, manuscripts and astrolabes. The trefoil is similar but based on an equilateral triangle.

The rete style may provide a clue to place of manufacture for an instrument. For example, Renaissance astrolabes from the Low Countries often used a rete network outlining a tulip. Exploring the range of artistic expression and technical dexterity exhibited in astrolabe retes is one of the great joys in studying the old instruments.

Modern materials offer a major opportunity to improve the utility of the rete by allowing it to be made on transparent material. Transparent plastic or glass retes do not obstruct the plate, and many more stars can be included. Figure 6-5 shows a rete for modern time printed on transparency material. This style will be our model for the following examples.

The rete in Figure 6-5 includes 124 stars. When the stars in the sky are arranged into figures so they are easier to recognize, the resulting figure is called an **asterism**. Most of the stars on the transparent rete are arranged into constellation asterisms to make identification easier.

Notice the constellation shapes are well preserved by the projection, but the sizes of the constellations near the pole and the Tropic of Capricorn are out of proportion to the constellations near the equator. This distortion in the size of constellations is an unavoidable byproduct of the stereographic projection.

When looking at the constellations on the astrolabe it is important to understand that, unlike paper star charts, the astrolabe represents the sky as seen from outside the celestial sphere, which is exactly how you see them when looking at a clear plastic celestial globe. Most star charts are constructed with a projection where the projection plane is tangent to the celestial sphere and you look at the bottom of the projection. Star charts are intended to be held above the head. You look down on an astrolabe, just like you look down on a compass. The altered orientation is most obvious on the handle of the Big Dipper (Ursa Major), which arcs in the opposite direction to what you see in the sky. Most of the constellations look enough like the star chart equivalents to be easily recognized.

Picking the stars to include on the rete is a non-trivial exercise. It is easy to get the impression that stars were sometimes chosen based more on where they are than what they are when examining the retes on old instruments. A few bright stars, such as Regulus, Sirius and Rigel, are almost always included. Some of the other choices appear to have been selected for artistic balance of the rete rather than the utility of the star or to ensure at least one star was always in a good viewing position.

A list of astrolabe stars with their J2000.0 coordinates is included in an appendix.

A point of confusion to many people when first introduced to the astrolabe concerns the moon and planets. It is not possible to represent the motion of the planets or the moon on a static instrument. It is, however, possible to solve problems involving the moon and planets if you have an ephemeris. All you have to do is look up the appropriate right ascension and declination and make a mark on the rete with ink for the position. This can also be done, of course, for deep space objects not represented on the rete. Good candidates are the Messier objects, the galactic center or any other star, galaxy, nubula or other object of interest.

To mark an object on the rete, simply turn the rete until the sidereal time pointer is on the right ascension and then mark the declination on the meridian using the rule.

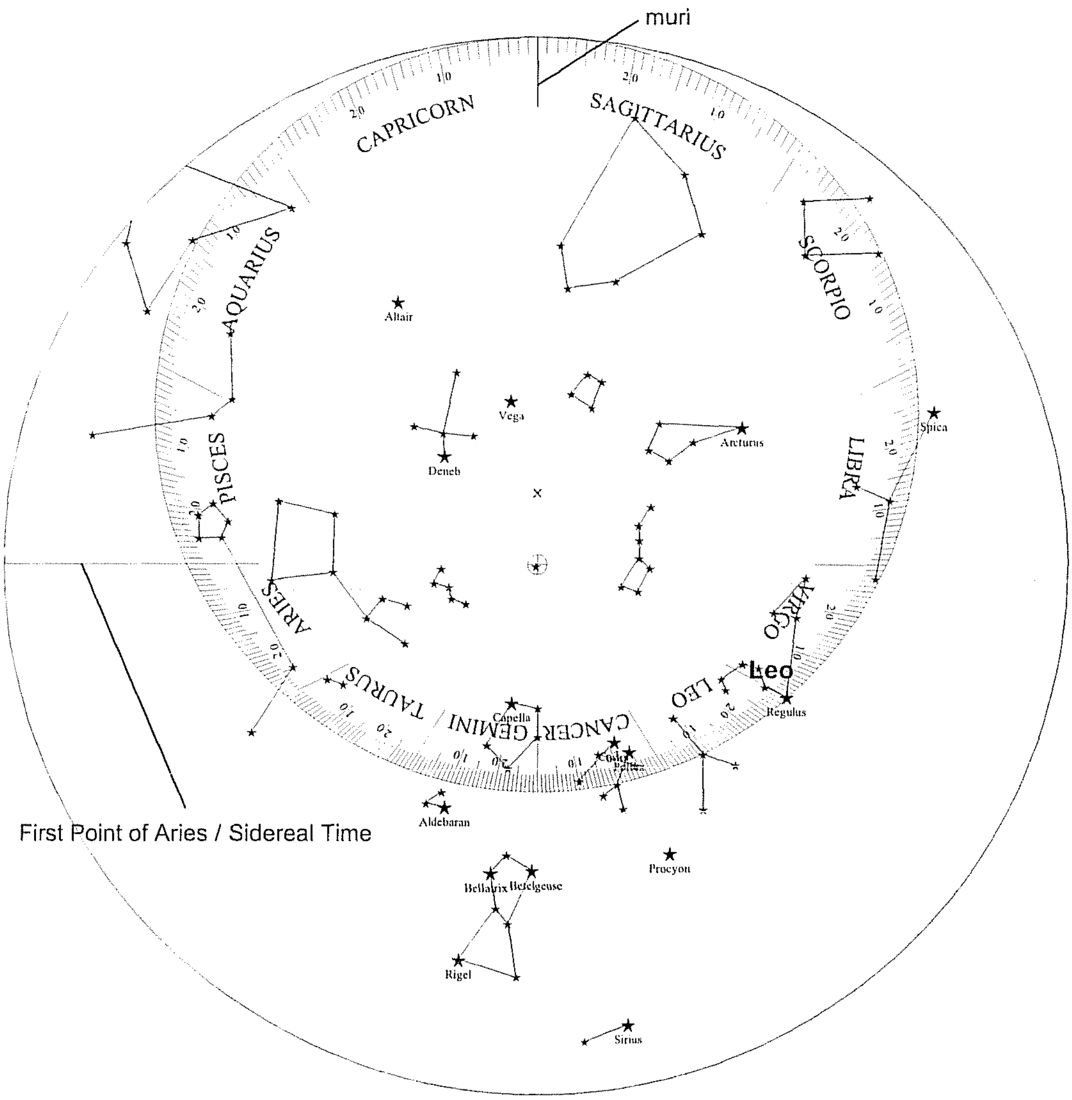

Figure 6-5. Transparent Rete

Precession

The coordinates of the so-called fixed stars change gradually over time and appear to rotate around the ecliptic pole. This phenomenon, called the precession of the equinoxes, is caused by the Earth's equatorial bulge and the inclination of the moon's orbit. The stars will make a complete circuit of the ecliptic in about 26,000 years. This sounds like a long time, and it is, but the change in a star's declination and right ascension is sensible on a much smaller scale. The change in longitude of a star due to precession is a full degree in about 75 years. Your thumb held at arm's length displaces about a half degree, so a star's longitude will change by double this amount in a normal life-span. Precession has a major effect on measurements based on stellar positions and must be taken into account for the rete. Stautz has shown how star positions on old astrolabes depended on the star list used and how precession was applied. The results varied from fairly good to really awful.

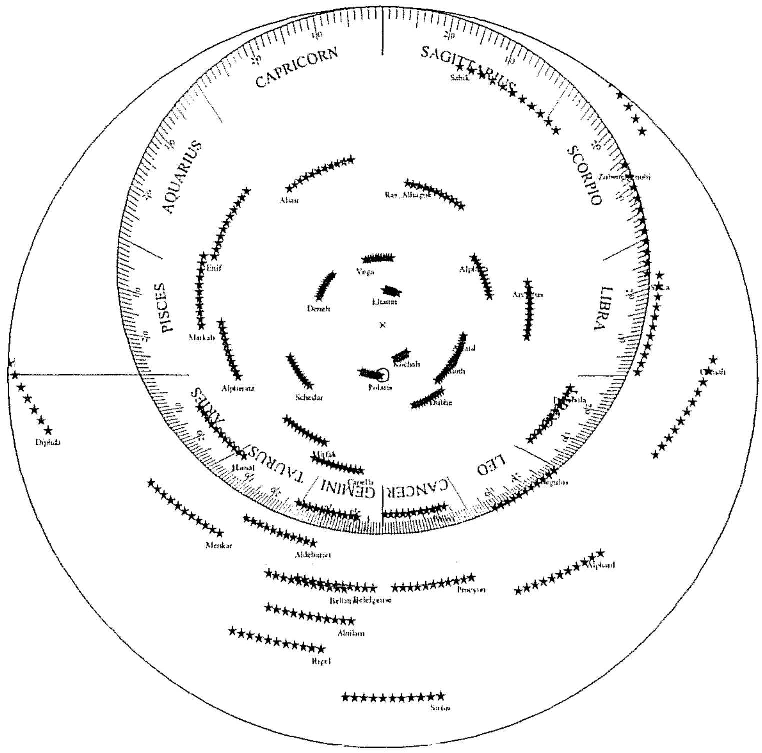

Figure 6-6. Stellar Precession

Figure 6-6 shows how the positions of selected bright stars have changed over time. The figure shows the star positions from 400 to 2000 for each 150 years. Precession is significant enough to affect the accuracy of an astrolabe in about 30 years. Many old retes were mutilated by well-intentioned owners attempting to update the instrument by bending the star pointers to correct for precession. Precession calculations are shown in the chapter on astronomical calculations (Page 364).

The Ecliptic

The ecliptic is the path the Sun follows through the fixed stars in the course of the year. This much is pretty easy. Since the ecliptic is defined by the Earth as it orbits the Sun, then the Earth and the Sun are always on the ecliptic. Therefore, as seen from the Earth, the ecliptic is always where the Sun is. The Tropics of Cancer and Capricorn are defined as the northern and southern limits of the Sun over the course of the year.

The position of the ecliptic on the celestial sphere is always the same (for our purposes), but the rotation of the Earth changes the part of the ecliptic visible at a given time of day. In the daytime it is easy to find the ecliptic; it is where the Sun is. At night, it not as easy to find the ecliptic unless you can locate a planet or have an astrolabe. None of the five classical planets is ever more than about 6° from the ecliptic.

From the Earth, the Sun seems to move along the ecliptic. In ancient times, the ecliptic was divided into twelve sections of 30° each. Each section was named for a constellation close to the section of the ecliptic at that time. This division of the ecliptic is called the zodiac. Due to precession, the constellations which originally gave the zodiac divisions their names are no longer in the same sections, but the traditional names—Aries (♈), Taurus (♉), Gemini (♊), Cancer (♋), Leo (♌), Virgo (♍), Libra (♎), Scorpio (♏), Sagittarius (♐), Capricorn (♑), Aquarius (♒) and Pisces (♓)—have persisted. The ecliptic is traditionally divided by the zodiac on classical astrolabes.

The zodiac begins at the vernal equinox, Aries 0°, where the ecliptic and equator cross. The signs proceed in a counter-clockwise direction from that point. Each degree is shown. The position of a planet on the ecliptic is its geocentric (i.e. Earth centered) longitude. The × near the center is the ecliptic pole.

It is critical to the comprehension of astronomy and its history to understand the zodiac has nothing to do with astrology. The zodiac is merely a convenient way to divide the ecliptic by celestial longitude. The practice of dividing the ecliptic into twelve divisions is undoubtedly tied to the twelve lunar months in a year. Exactly when this was done is still a matter for debate, but there is no doubt the zodiac was used in Mesopotamia by the mid-6th century BC, and possibly several hundred years earlier. Even in its astrological uses, the zodiac was used only to specify the geocentric longitude of the Sun or a planet.

The ecliptic circle is a single arc defined by the path of the Sun passing through the center of the zodiac. The ecliptic band includes the entire range of the classical planet positions and is about 12° wide, centered on the ecliptic circle. In theory, the ecliptic circle on the astrolabe rete covers only the northern 6° of the ecliptic band in order to obscure as little of the underlying plate as possible and to make it easier to set the ecliptic circle on specific points, such as the horizon. In fact, the ecliptic ring on most astrolabe instruments is as wide as needed to hold the divisions and labels with scant regard to its theoretical width.

Look at Figure 6-5. The line extending out from the ecliptic circle to the Tropic of Capricorn originates at the First Point of Aries, which is both the Vernal Equinox and the beginning of the sign of Aries. Local sidereal time is the hour angle of the vernal equinox at an instant. Hour angles are projected as radial lines on the plate and marked on the limb in both degrees and time at 15° per hour. When the rete is set to a date and time, the vernal equinox points to a time on the limb which is the hour angle of the vernal equinox. Therefore, you can find the local sidereal time for any date and time, by simply noting the position of the vernal equinox on the limb.

Also, the right ascension of any celestial object is its hour angle from the vernal equinox. You can find the right ascension of any object represented on the rete by rotating the rete until the object is on the meridian and reading its right ascension from the position of the vernal equinox on the limb.

One point on the ecliptic deserves special mention. Classical astrolabes had a special pointer at the winter solstice (Capricorn 0°). Many old astrolabes did not have the limb graduated in hours and this point, called the *muri*, was used as an index of the rete's position to determine the length of the day. If you align the rete so Capricorn 0° is on the meridian you see Aries 0° (right ascension 0 h) is just on the horizon. Move Capricorn 0° to I on the limb, which represents one hour in sidereal time. Since the rete has moved one hour in sidereal time, the point now rising in the east on the right horizon is 1 h. Hence, the time Capricorn 0° points to on the limb is the right ascension of every celestial object rising at the east point of the horizon at that time. This feature allowed some difficult problems relating to the length of the day at different latitudes to be solved before the invention of spherical trigonometry.

A critical property of the ecliptic for use in finding the time in unequal hours during the day and other applications is that it is symmetrical on the celestial sphere. A point on the ecliptic of longitude λ will intersect an altitude circle, a. The opposite point on the ecliptic with longitude $\lambda+180°$ will intersect the $-a$ altitude circle. This property allows you, for example, to find the Sun's altitude below the horizon at night. Its use for finding unequal hours is discussed later.

Dividing the Ecliptic

The projection of the ecliptic circle is trivial; the limits of the ecliptic are at the Tropic of Cancer and the Tropic of Capricorn. The center is halfway between. See Figure 6-7. The ecliptic radius $\mathbf{R_{Ec} = (R_{Cap} + R_{Can}) / 2}$. The center is located at $\mathbf{R_{Cap} - R_{Ec} = (R_{Cap} - R_{Can}) / 2 = R_{eq} \tan \varepsilon}$.

There are at least six methods of dividing the ecliptic by solar longitude. The simplest and probably the most often used method is to locate the point on the ecliptic circle corresponding to the Sun's right ascension. In practice, the right ascension for each desired division of the ecliptic must be known or calculated. The rete is oriented with the vernal equinox on the south meridian and the angular value of the solar right ascension located on the limb. A line is drawn from the ecliptic edge toward the center of the instrument. This is the method used to divide the ecliptic by the calendar

The equation to do this calculation is:

$$\tan\alpha = \frac{\sin\lambda\cos\varepsilon - \tan\beta\sin\varepsilon}{\cos\lambda}$$

α = right ascension
β = latitude
λ = longitude
ε = obliquity of ecliptic

Since the latitude of a point on the ecliptic is always 0, this equation simplifies to:

$$\tan\alpha = \tan\lambda\cos\varepsilon$$

Simply calculate the right ascension of the Sun for each degree of solar longitude and then mark these points on the ecliptic circle using the time or angle scale on the limb, which is hard to do accurately considering the rather coarse limb degree scale. Some old treatises had tables of longitudes and right ascensions for this purpose.

The second method is similar but uses the Sun's declination corresponding to a given solar longitude. The point on the ecliptic circle intersected by a given declination circle is found and the line drawn to mark the point. This method, while theoretically valid, was probably never used in practice because it is very difficult to locate the points on the ecliptic accurately near either tropic.

A third method described in old treatises is not valid, but was used by some makers nonetheless. In this method the equator is divided by longitude and a line is drawn from the projection center to the point on the equator. The intersection of the line and the ecliptic is taken as the solar longitude. This method is valid only at the solstices and equinoxes and results in errors of up to 2½ degrees near the center of each quadrant. Recommending this method to divide the ecliptic indicates lack of understanding of the stereographic projection, but it was shown in a number of treatises by otherwise respected authors.

Thomson[51] mentions a graphical method shown by Llobet of Barcelona that shortens the arc from the vernal equinox to the summer solstice in a proportion (14/15) allowing the remaining arc to be divided into three parts locating the beginning of the signs with fair accuracy.

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[51] Thomson, Ron B., *Jordanus de Nemore and the Mathematics of Astrolabes: De plana spera*, Pontifical Institute of Mediaeval Studies, Toronto (1978), pp. 63-64.

Thomson[52] also covers a rather obscure ecliptic division technique involving the construction of great circles defining equal arcs on both the equator and ecliptic. These great circles are the circles through the poles of a great circle with half the declination of the ecliptic. This great circle also passes through the equinoxes. The point of division on the ecliptic is found by finding the pole of this great circle and drawing an arc through it and through the point of division measured on the equator, and the point on the equator 180° from this point. This procedure defines an arc on the ecliptic the same as the given arc on the sphere. It is certain this method, which is of theoretical interest only, was never used to construct an instrument. It requires significant effort to define the required circles, which have a large radius, and are very difficult to draw with any accuracy.

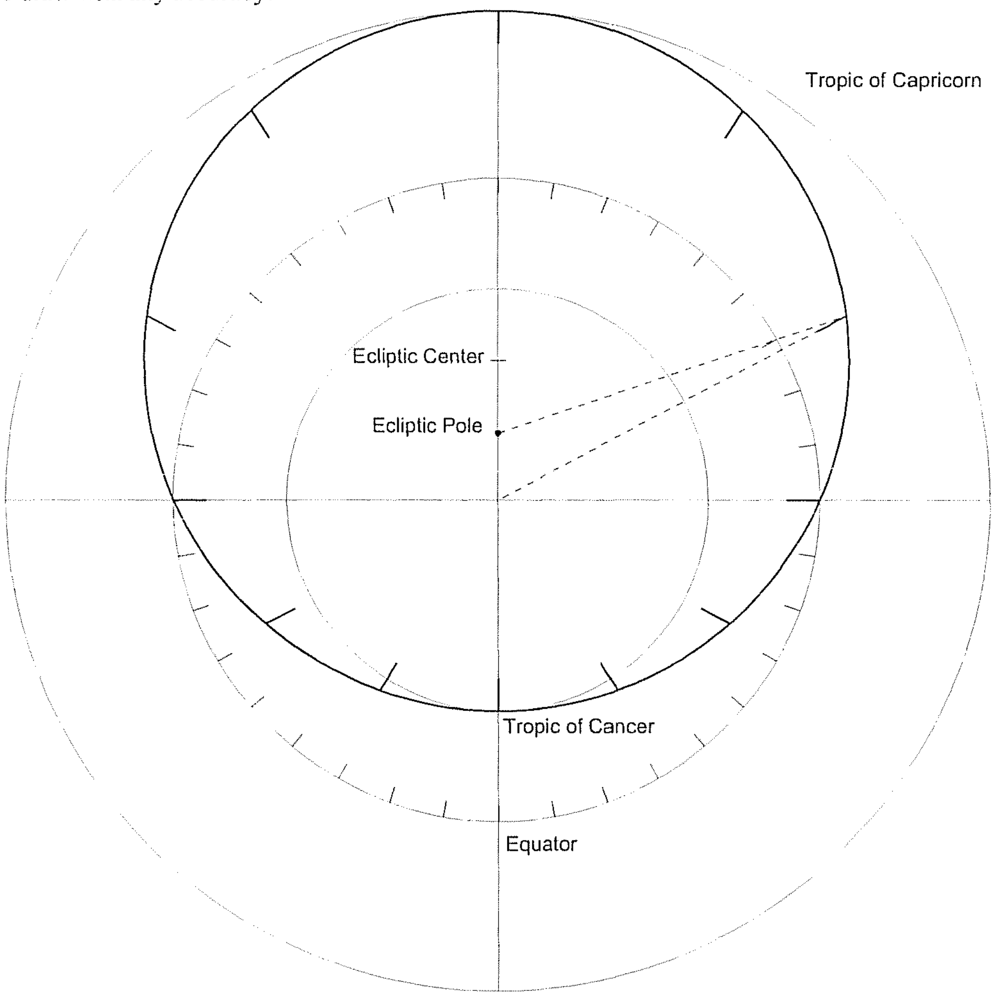


Figure 6-7. Ecliptic Division

The most common method of ecliptic division is both theoretically accurate and easy to execute. It uses a variation of the great circle method mentioned above. Recall any great circle on the sphere passing through the projection origin is projected as a straight line. Therefore, a great circle passing through the ecliptic pole and the south pole will project as a line. The axis of the great circle will be perpendicular to the the plane bisecting the equator and ecliptic plane, and the circles will define equal angles on both the ecliptic and equator. This method has been known since the 12th century at the latest.

[52] Thomson, *ibid.* pp. 65-66.

The procedure for dividing the ecliptic is (Figure 6-7):

1. Locate the ecliptic pole on the meridian at $R_{eq} \tan(\varepsilon/2)$ from the center
2. Divide the equator into equal segments of longitude: 12 divisions of 30° for the entry into each zodiac sign: more divisions depending on the resolution desired.
3. Draw a line from each equator division to the ecliptic pole. The corresponding longitude point on the ecliptic is where this line intersects the ecliptic circle.
4. A tic mark on the ecliptic is drawn toward the center of the instrument.

A more convenient way to determine the ecliptic divisions for a computer generated astrolabe is to calculate the points on the ecliptic circle directly. See the chapter on Computers and Astrolabes for a complete description of how to divide the ecliptic in a computer program.

Star Positions

Locating the star positions is quite easy. Modern star positions are specified by declination and right ascension. The position of a star on the rete is found from the fundamental equation.

The distance of a star from the center of the rete is $r = R_{eq} \tan(90 - \delta)/2$ where δ is the star's *precessed* declination for the date of the instrument.

The angle of the star on the rete is its *precessed* right ascension converted to degrees. This position is found most easily by orienting the rete with the vernal equinox on the meridian and then measuring the right ascension angle clockwise from the meridian. It can also be found by setting the vernal equinox to the right ascension and marking the declination on the meridian.

A list of stars and their J2000.0 coordinates is included in an appendix. See page 364 for precession calculations.

Mediation

Star positions in Medieval star lists and astrolabe treatises are often defined by their declination and "mediation". Mediation (literally *coeli mediatio* = measure of the sky) is a hybrid coordinate probably used to make it easier to lay out the star pointers on the astrolabe rete. A star's mediation is the longitude intersecting the meridian at the same time as the star. That is, the longitude on the ecliptic corresponding to a given right ascension.

To convert from ecliptic to equatorial coordinates:

$$\tan\alpha = \frac{\sin\lambda\cos\varepsilon - \tan\beta\sin\varepsilon}{\cos\lambda}$$

Since the mediation is measured on the ecliptic, $\beta = 0$ so, the mediation **m** is the ecliptic longitude for the given right ascension and:

$$\tan m = \frac{\tan\alpha}{\cos\varepsilon}$$

Mediation is usually expressed as a zodiac position, although it can also be expressed as degrees of longitude.

For example, Rigel has a J2000.0 right ascension of 5 hr 14 min 32.2 sec or 5.2422 hr or 78.6342°. The J2000.0 obliquity of the ecliptic, ε, is 23° 26′ 21.448″ or 23.439291°. Using the equation above, m = 79.55° or about Gemini 19.5°. Mediations for old instruments can be calculated by first calculating the precessed position and then doing the simple conversion. Similarly, right ascensions can be calculated from old star lists.

The use of mediation made it quite easy to locate the tips of the rete star pointers. A scale divided by the stereographic projection of declination or a pair of dividers set to the declination distance is needed. First, draw and divide the ecliptic circle. Then, for each star simply lay the rule from the center to the desired mediation and mark a point at the appropriate declination.

See also page 339.

A Modern Rete

A rete with the ecliptic divided directly by the calendar allows the rete and rule to be set in a single step. It is identical to the classic astrolabe rete except for the ecliptic division. This style rete works nicely in conjunction with the modernized astrolabe back described on page 149. Division of the ecliptic by the calendar is discussed on page 388.

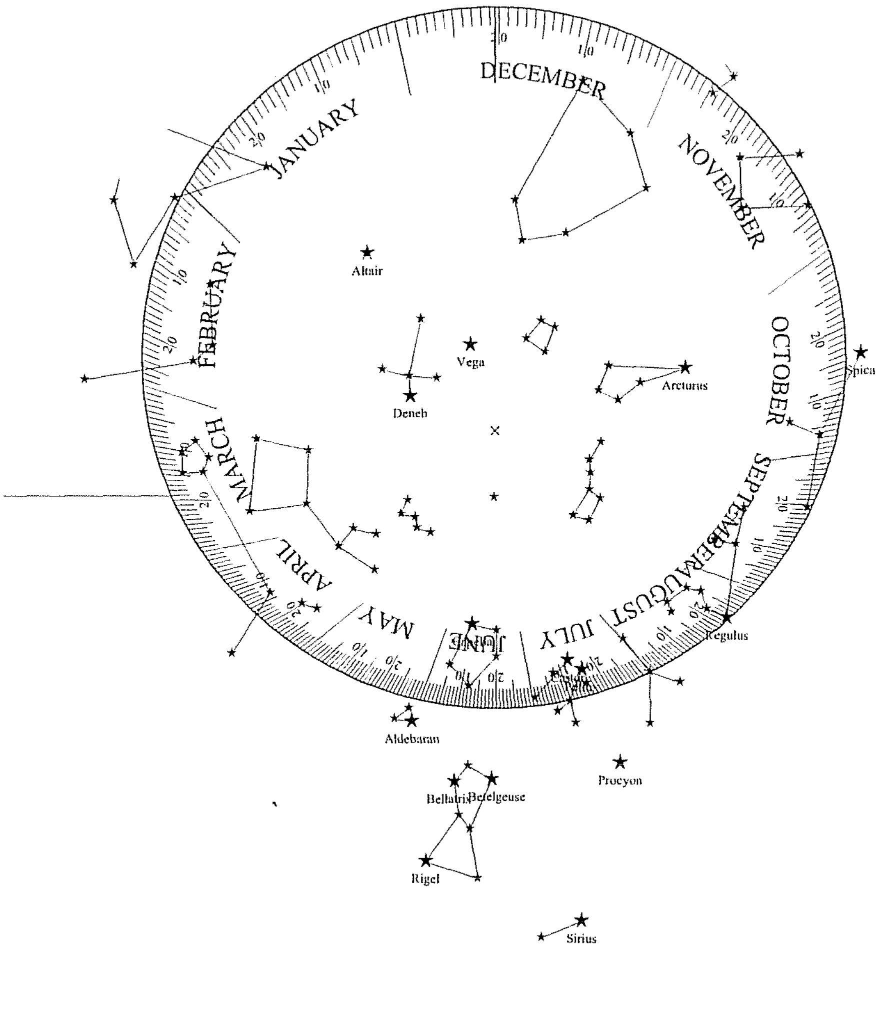


Figure 6-8. Modern Astrolabe Rete

A Celestial Navigator's Rete

The planispheric astrolabe is not a navigational instrument. Is is simply not precise enough. There is, however, one celestial navigation application that an astrolabe can do.

One of the problems faced by the celestial navigator is identifying **navigation stars** when the sky is partially obscured or planning a sighting. The navigator can sight a star visible through a hole in the clouds with his sextant. He then he must decide which star it was before reducing the sight. The problem of identifying a star under these conditions has plagued navigators for centuries and a number of methods have been developed for its solution. The most widely known is the 2102-D Star-Finder and its successor, the CP 300/U (see page 318). The 2102-D has a fixed plate showing the navigation stars and a transparent plate with a altitude/azimuth gride. You go through a rather tedious process to estimate **LHA ♈** for your meridian and set the rotating grid to that position. You can then see which stars are visible.

Figure 6-9 shows a conceptual planispheric astrolabe rete that is intended to accomplish the same purpose. This rete contains the constellation asterisms that already shown, all of the 57 navigational stars that fit on the rete and has an SHA (Sidereal Hour Angle) scale around the perimeter. It is intended to be used with the modern style astrolabe described in the previous section which divides the ecliptic by the calendar.

This rete is capable of providing the same function as the 2102-D if it is implemented on an instrument of equivalent size, but with some tradeoffs. This rete includes constellation asterisms to simplify constellation identification, which are not on the 2102-D. However, the 2102-D covers the entire horizon and the astrolabe goes only to the Tropic of Capricorn, which is a restriction, depending somewhat on the latitude[53].

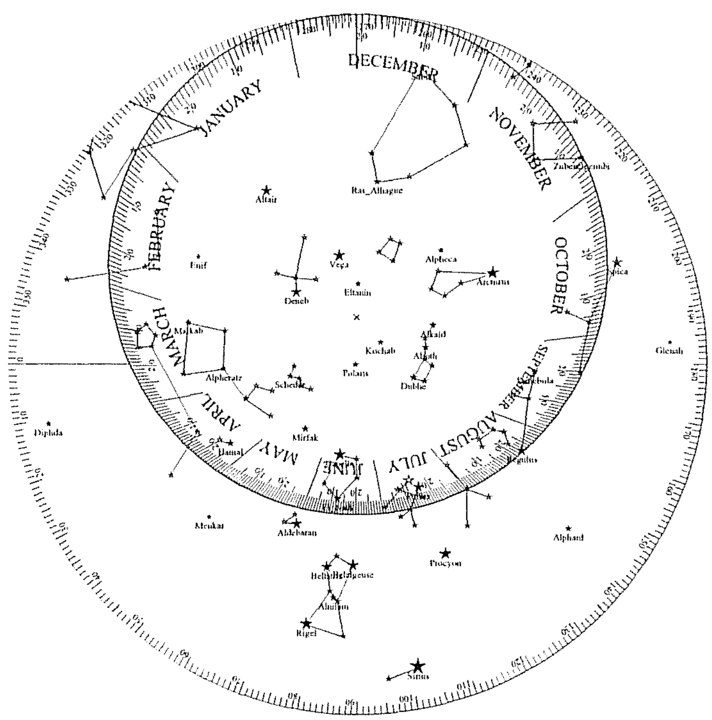


Figure 6-9. Celestial Navigaton Rete

[53] *Sight Reduction Tables for Air Navigation Pub. No. 240* (HO-249) includes stars only to to ±29° declination.

Chapter 7 - The Rule

Most European astrolabes included a rotating rule free to rotate over the rete. The rule is used to locate the Sun on the ecliptic, to indicate the time, and to measure declinations..

When the astrolabe is set to a celestial event, the rule points to the current apparent solar time on the limb. The place where the rule crosses the ecliptic changes daily and is the location of the mean Sun in the ecliptic and, therefore, the date.

The rule rotates at the mean solar rate of once every 24 hours, while the ecliptic and stars rotate once in a sidereal day.

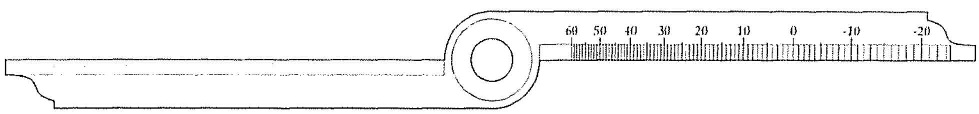


Figure 7-1. Rule

A representative rule is shown in Figure 7-1. There are as many astrolabe rule styles as there are astrolabe makers, and each maker brought his own artistic and technical skills to the task.

As with all astrolabe components, the accuracy of the rule is important. The fiducial edge of the rule must be exactly on a radius of the plate, and it must be perfectly straight. The edge of the rule was usually honed to a sharp edge to reduce parallax.

The edge of the rule is often divided into degrees of declination with each degree shown. Keep in mind negative declination is on the outside of the equator. The Sun's declination can be estimated by reading the point where the rule crosses the ecliptic and the declination of any object on the rete can be measured using the declination scale. The scale can also be used to locate the position of an object, such as the moon or a planet, taken from an ephemeris.

The rule is most useful if it extends all the way across the plate but some rules, even those on very fine instruments, were half length, like the minute hand on a clock.

Making the rule

The rule is always counterchanged. Basically, fabricating the rule is relatively simple once the design is selected, although there is considerable artistic latitude available for the details.

If a declination scale is included, and it should be, the distance, r, of each declination tic from the center of the rule is calculated from:

$$r = r_{eq} \tan\left(\frac{90-\delta}{2}\right)$$

where r_{eq} is the radius of the equator and δ is the declination represented by the tic.

The Rule

Chapter 8 - The Astrolabe Back

The back of the astrolabe provides a canvas on which the *astrolabiste* could demonstrate his technical prowess and artistic range. The only strictly required scale on the back of the astrolabe is the scale for measuring altitudes. A large amount of space remains that was used in a wide variety of ways depending on the cultural heritage of the instrument and the technical sophistication of the designer. The greatest differences between Islamic and European astrolabes were in the scales on the back of the instruments. A scale for solving simple surveying problems were almost always included on all astrolabes from all cultures. Additional scales for solar longitude, unequal hours, mathematical functions, religious customs or astrological information might be included depending on the instrument's origin or when it was made.

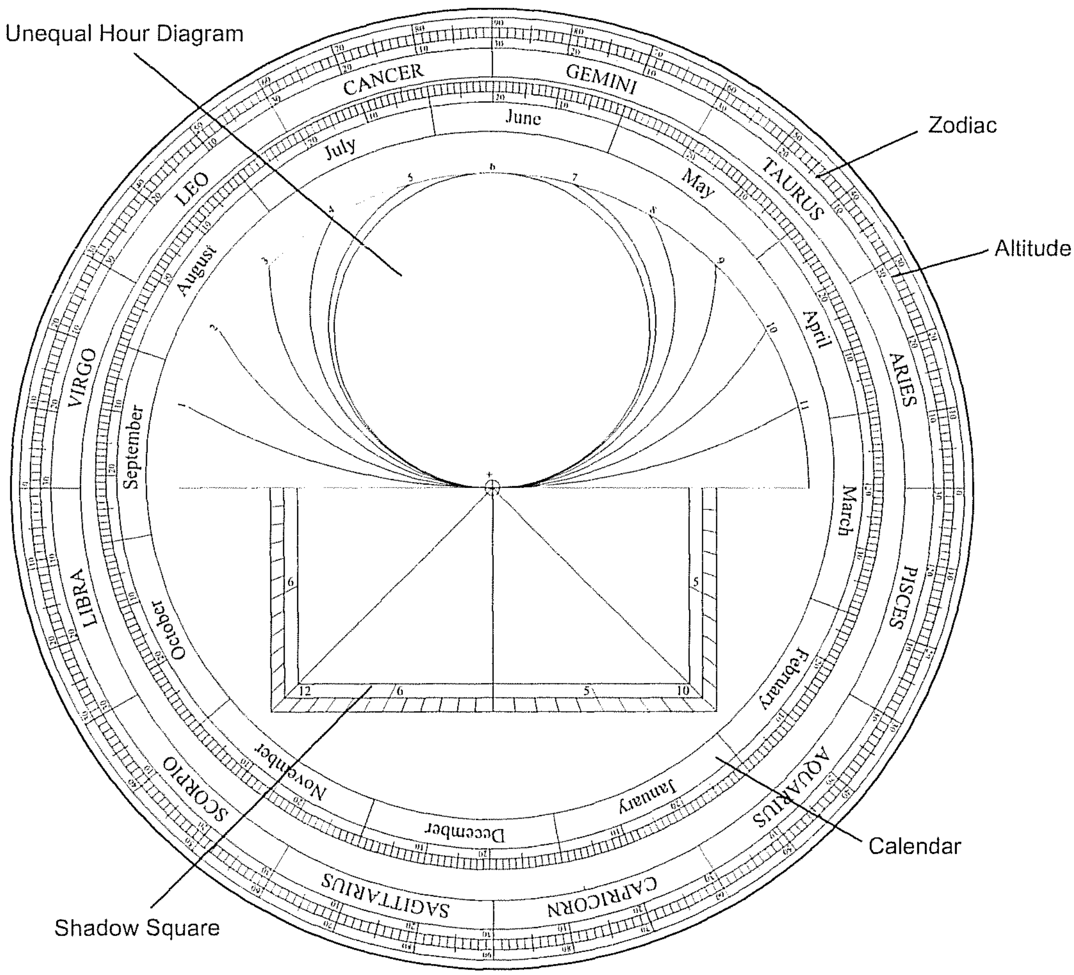


Figure 8-1. Typical European astrolabe back

The Back of European Astrolabes

There is no standard for the scales appearing on the back of any astrolabe, although certain scales were more popular than others. Figure 8-1 shows the back of a European astrolabe representing a fairly common style. This style was used by Fusoris in the late 14th century and was advocated by Stöffler in the 16th century.

Each of these and other less common scales will be discussed. References to the sections of the back by quadrant are in the standard way: QI is the upper right quadrant, proceeding counterclockwise through QII, QIII, and QIV.

TheAltitude Scale

The altitude scale is used to measure the current altitude of the Sun or a star. It is always divided into 90° with every degree shown. The altitude scale is labeled from 0° at the horizontal point to 90° at the vertical. Almost all astrolabes have this scale in QI and QII and it may be continued around the entire back. This scale is also used as a general scale of degrees.

In use, the alidade is used to sight the Sun or a star included on the rete and note is made of the altitude indicated on this scale. This scale is also to indicate degrees for other functions.

Making the altitude scale is trivial; simply divide the outer margin of the upper two quadrants into 90 equal divisions and label each 10° starting at the horizontal. The scale is usually boxed.

The Solar Longitude Scale

One of the primary uses of the astrolabe back is to determine the Sun's geocentric longitude for a date. This value is then used to position the rete and rule on the front. Solar longitude was always expressed in terms of the zodiac on old astrolabes. The zodiac scale is simply a division of the entire margin of the astrolabe back into 360 equal divisions and then labeling the divisions in 12, 30° sections with the signs of the zodiac:

Solar Longitude	Name	Symbol
0°	Aries	♈
30°	Taurus	♉
60°	Gemini	♊
90°	Cancer	♋
120°	Leo	♌
150°	Virgo	♍
180°	Libra	♎
210°	Scorpio	♏
240°	Sagittarius	♐
270°	Capricorn	♑
300°	Aquarius	♒
330°	Pisces	♓

The scale is further subdivided, with every degree shown on the best instruments, but at least with divisions for five and ten degree segments. The zodiac scale is used in conjunction with the calendar scale. Most astrolabes used the zodiac section names although a few used only the symbol for a sign.

Aries 0° is always aligned with the horizontal diameter of the back.

The Calendar

The calendar scale is used with the zodiac scale to find the Sun's longitude for a day. The alidade is positioned over a date, and the corresponding longitude is read from the longitude scale.

The calendar scale can be drawn in two ways: as an eccentric or concentric with the center of the instrument. The eccentric calendar scale was the more popular form, although many fine astrolabes were made with concentric calendars.

Eccentric Calendar

The eccentric calendar, such as the scale shown in Figure 8-2, is an exact recreation of the eccentric model of the Sun from Ptolemaic astronomy. This representation of the Sun is quite accurate by medieval standards and well within the overall accuracy of the astrolabe. See page 339 for a discussion of the Ptolemaic Earth/Sun model and its validity. The Sun moves at a constant speed around the calendar circle in this model as represented on the astrolabe. The Sun is observed from the Earth at the center of the instrument. The Sun's true anomaly for a date is shown on the zodiac scale when the alidade is placed on a date on the calendar scale.

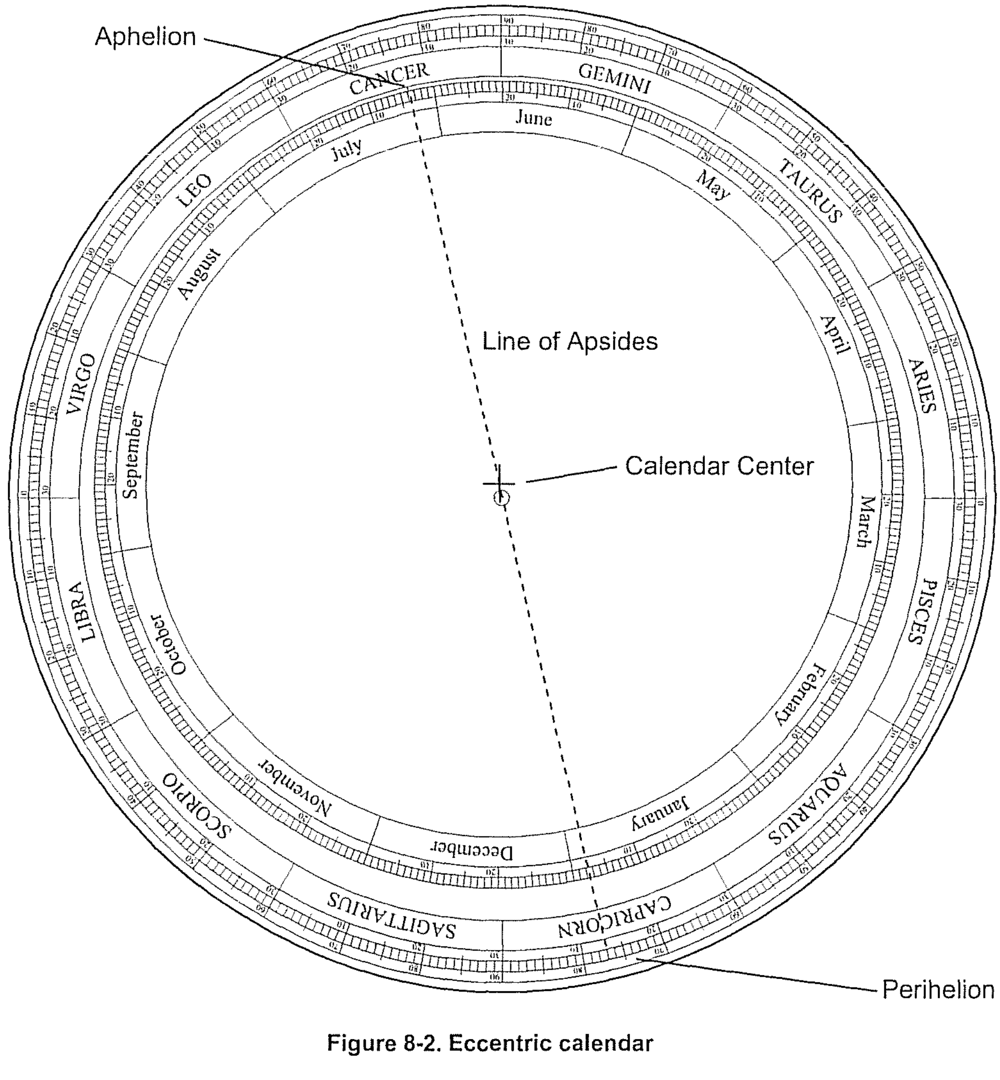


Figure 8-2. Eccentric calendar

The calendar is drawn on a circle offset from the instrument center on the line of apsides by the Ptolemaic eccentricity. Note carefully that the eccentricity in this model is twice the eccentricity of the elliptical orbit. The current eccentricity of the Earth's orbit is about 0.01671. Therefore, the calendar circle is offset from the center by $2 \times 0.0167 = 0.0334 \times$ calendar radius. The value of the eccentricity changes slowly over time, so it must be calculated for the date of the instruments.

An eccentric calendar is shown in Figure 8-2. The line of apsides is the line connecting perihelion and aphelion and can be found in the figure by the small tics just inside the zodiac scale. The tic near Cancer 13° is the aphelion and the tic near Capricorn 13° is perihelion. The small + near the center is the center of the calendar eccentric.

Note carefully the time indicated by the tic in the figure for each day is midnight of the day. Look at January. The long tic dividing December and January is midnight December 31. The first small tic is midnight, January 1, and so on. On some old instruments the tic mark represented noon for the day.

Drawing the eccentric calendar

The eccentric calendar is quite easy to draw in principle, because the day divisions are equal. There are, however, several considerations. Modern calculation methods are used. Old astrolabes were designed using tables of solar longitudes to achieve the same results.

As mentioned above, the center of the eccentric calendar is located on the line of apsides, offset from the center by 2e, where e is the eccentricity of the Earth's elliptical orbit. The first step in drawing the calendar is to determine the longitude of aphelion and perihelion (π) to define the line of apsides. These values can be found in the Astronomical Almanac or calculated from:

$$\pi = 102.937348 + 1.7195269\ T + 0.00045962\ T^2 + 0.000000499\ T^3$$

where T is the time measured in Julian centuries from J2000.0 (see Astronomical Calculations). Although it is not quite accurate, we can take aphelion as π + 180°. Here the view of the universe is Earth-centered, so the longitude is 180° from the heliocentric longitude of the Earth. The perihelion shown on the astrolabe is π + 180°. The longitude of perihelion changes slowly over time and should be calculated for the date of the instrument. The current value is about 283°. If the offset of the calendar center is $\Delta = 2\ e\ r_c$. where r_c is the radius of the calendar, the center of the calendar circle is located at $x_c = \Delta \cos \pi$, $y_c = \Delta \sin \pi$.

The eccentricity of the Earth's (Sun's) elliptical orbit for any date can be calculated from:

$$e = 0.01670862 - 0.000042037\ T - 0.0000001236\ T^2 + 0.00000000004\ T^3$$

Now comes the tricky part; orienting the calendar with the zodiac. In principle this sounds easy. Simply determine the date and time of the vernal equinox and start marking the days from that point. To be accurate you must set the vernal equinox at the local time, which will probably not be the time shown in an almanac. The instant of the vernal equinox in almanacs is shown in UT, and your location may be a meaningful fraction of a day from Greenwich. For example, Pacific Standard Time is 8 hours from UT, a third of a day. Therefore, your calendar must be adjusted for your local longitude. It is difficult to determine where a fraction of a day falls between the day tics, so it is easier to adjust the point where the calendar divisions begin. To find the position of January 0 (i.e., midnight, December, 31), note the mean anomaly is 0° at perihelion. The angular difference from January 0, and perihelion is the mean anomaly of January 0, M_0 (see page 361 for calculating the mean anomaly). Finally, adjust for longitude from $\lambda/360 \times 360/365 = \lambda / 365$.

In summary, the angular distance from the vernal equinox to January 0 = $\pi + M_0 + \lambda / 365$.

The calendar is then divided into 365 equal divisions around the calendar center, with the month divisions marked once the starting point is accurately determined.

Old astrolabe treatises advise, when dividing scales such as this, it is best to first make large divisions, then progressively divide them into smaller and smaller segments to avoid accumulating errors. A critical evaluation of instruments made before dividing engines were available shows how hard this is to do accurately by hand.

Following is a numeric example for 2006 for an eccentric calendar of radius 1:

1. The beginning of the year is midnight, December 31, 2005. The Julian day is 2453736.5. T = 0.06000.
2. Calculate e = 0.01667061 so the offset of the calendar center = 2e = 0.03334122.
3. The longitude of perihelion is calculated as π = 103.041°. Since the view is geocentric, the longitude of perihelion on the astrolabe = π + 180° = 283.041°.
4. The coordinates of the calendar center are: x = -0.00754, y = 0.03255.
5. The mean anomaly at January 0 = 357.472°.
6. For a longitude of 75° 6′ W, the rotation angle of January 0, is -79.282° [(283.041° + 357.472° + 75.1°)/365].

Concentric Calendar

The calendar can also be drawn concentric with the center of the instrument. It is tempting to theorize that concentric calendars were used by makers who were not well enough versed in astronomy to design an eccentric calendar, but there is no evidence this was the case. Some very fine instruments were made with concentric calendars, although with mixed results.

Basically, the fabrication difficulty with a concentric calendar is that the distance between the day tics is not even, but varies depending on the Sun's daily rate of longitude change. Making an accurate concentric calendar is much more difficult than an eccentric calendar and requires many more calculations. Frankly, it is doubtful any makers of old astrolabes with concentric calendars did all of the required calculations but "estimated" many of the day lines instead.

An example of a concentric calendar is shown in Figure 8-3. A careful examination of the figure will show the distance between the date lines is quite a bit narrower near Cancer and larger near Capricorn. The change in width of the days shows the Sun's longitude increases more slowly in the summer and more rapidly in the winter

Making a concentric calendar

The theory behind the eccentric calendar is a bit complicated, but the fabrication is quite easy. By contrast, the theory behind making a concentric calendar is easy, but it is hard to make accurately.

To divide the concentric calendar, calculate the Sun's geocentric longitude for each day of the year and then mark the day line using the accurately drawn zodiac scale. Computation of the Sun's geocentric longitude is covered in the chapter on astronomical calculations. The Sun's longitude can also be found in an almanac. The daily longitude should be adjusted by a constant amount equal to λ / 365 to compensate for the local longitude. A computer is almost required given the large number of calculations needed.

The Sun's geocentric true longitude is calculated from:

True Longitude = Mean Longitude + True Anomaly - Mean Anomaly.

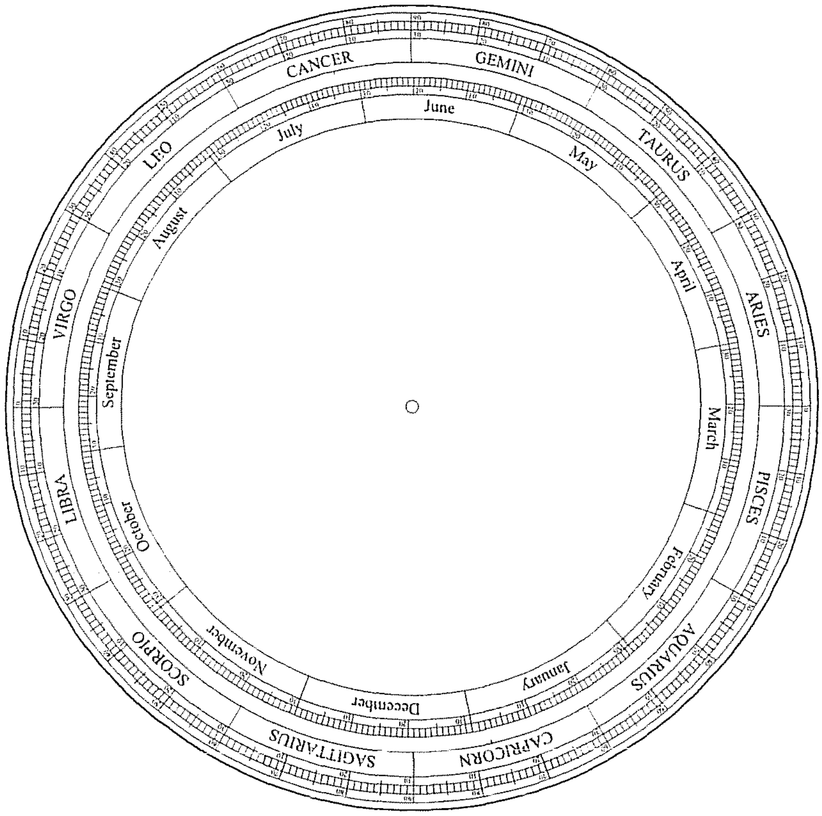


Figure 8-3. Concentric calendar

The Shadow Square

The shadow square is used for simple surveying problems and also has mathematical applications. It was very common and was present on almost all astrolabes from all cultures. Application of the shadow square to determining prayer times is discussed in the section on the back of Islamic astrolabes as are older variations of the same function..

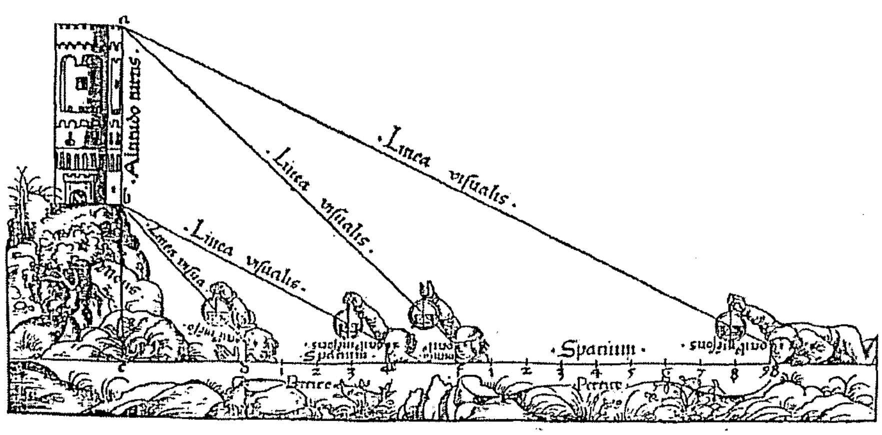


Figure 8-4. Shadow Square Application

It is called the Shadow Square because it is used to solve simple problems in trigonometry most easily described by the shadows cast by a vertical or horizontal stick. The scale is simply a graphical way to find the tangent or cotangent of an angle and solve simple problems involving triangles.

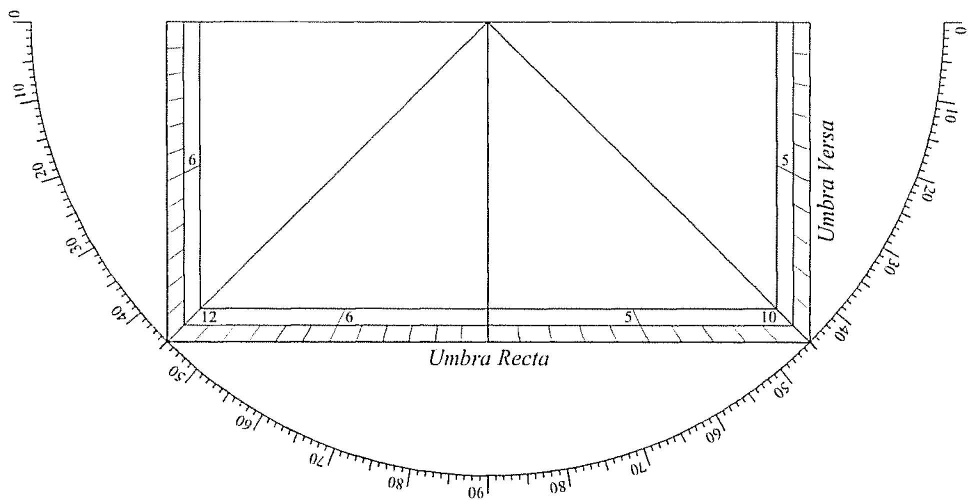
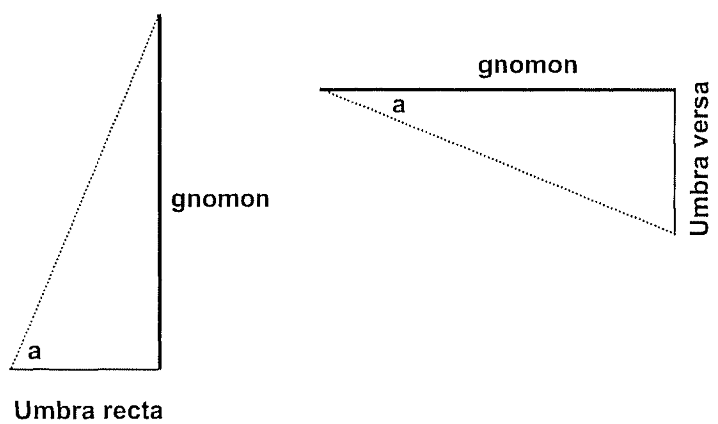


Figure 8-5. Shadow square with defining circle

The sides of the scale are divided into equal parts of whatever units of length were in common use where the astrolabe was used. The scales on Figure 8-5 are divided into ten parts on the right for working with decimal distances and into twelve parts on the left for working with inches. Stöffler recommended dividing the scales into 12 parts. Islamic astrolabes divided the shadow square into 7 and 12 divisions.



If the gnomon length is taken as 1, **Umbra recta = cot a** and **Umbra versa = tan a**. The defining circle in Figure 8-5 shows the angles associated with the division.

The vertical scale is used to solve problems involving the shadow cast by a horizontal gnomon on a vertical surface (*Umbra versa* on European astrolabes). In other words, it provides the tangent of an angle, and the scale can be used to solve problems where the length of the base of a triangle and the base angle are known. For example, if you want to find the height of a tree, pace off a known distance and use the alidade to find the angle to the top of the tree. The alidade will cross the vertical scale on the shadow square at the proportion the height of the tree is to the distance from the tree. For example, if you pace off 100 feet and read the top of the tree at 26°, the alidade crosses the scale at 5 so you know the tree is 5/10 (one half) as high as you are away from it: the tree is 50 feet tall. In order to be accurate for measurement over short distances, you need to add the amount the astrolabe was elevated over the base of the tree when the angle measurement was made. This scale is very easy to use in practice and tolerably accurate.

The horizontal scale provides the cotangent of an angle, and is used to solve problems involving a shadow cast by a vertical gnomon on a horizontal surface (*Umbra recta* on European astrolabes – *recta* is Latin for upright (the gnomon is upright). In practice, this scale was most often used to solve problems where it is required to find the angle when both sides of a triangle are known. For example, if a 50-foot tower casts a shadow 25 feet long, what is the altitude of the sun? To solve the problem, set the alidade at 5 on the horizontal scale (25/50 = 5/10) and read 63.5° on the altitude scale. This scale was particularly useful on Islamic instruments to determine prayers time, and an associated scale of cotangents was sometimes inscribed on an arc to extend the range of useful angles (described later in this chapter).

Figure 8-4 is from Stöffler (1524) and shows the shadow square being used to estimate the height of a tower of unknown distance from the observer. In this case, two readings of the angle of the tower top and bottom and made. The first observation is made at an unknown distance from the tower and the second observation is made at a known distance from the first. The result can then be calculated using simple arithmetic.

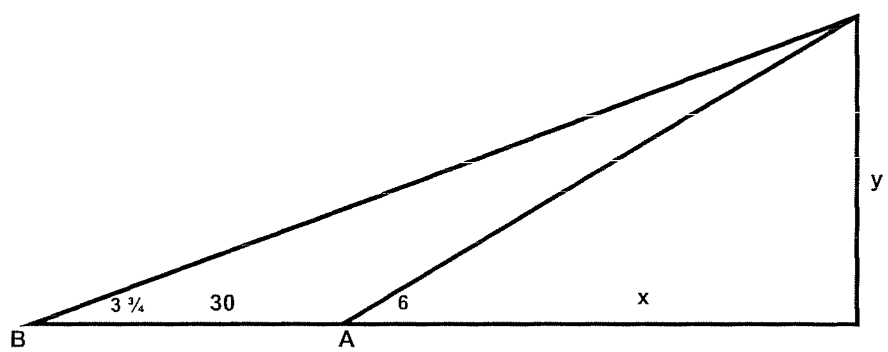


Figure 8-6. Shadow Square Example

See Figure 8-6. y is the height of the tower and x is the unknown distance from the tower for our first observation. The shadow square reading is 6 at A. Then move 30 paces away from the tower to B and read 3 ¾ from the shadow square.

$$\tan B = y / (x + 30), \quad \tan A = y / x, \quad \text{so } (x + 30) / x = \tan A / \tan B$$

Numerically, $(x + 30) / x = 6 / 3\frac{3}{4} = 1.6$ so $x = 50$. From the reading at A, $y = 30$, which can be converted to any convenient measure. A wide variety of problems can be worked by applying simple proportions such as this. Stöffler clearly expects his 16th century readers to be comfortable with decimal arithmetic.

Drawing the shadow square

The shadow square is the easiest astrolabe scale to draw. Simply draw a rectangle defined by two squares with the center at the point where the squares meet and divide the edges into as many divisions as are desired. Ornamentation can be applied to taste. Islamic astrolabes almost always divided the shadow square sides into seven divisions because they used a seven feet tall gnomon. European astrolabes often divided one side into feet with 10 divisions and the other side into "fingers" with 12 divisions.

The Diagram of Unequal Hours

A diagram used to determine the unequal hour from the Sun's meridian and current altitude is often included on astrolabes and astrolabe quadrants from all sources and was the primary scale on the quadrans vetus. This diagram probably originated in the East based on its description in very early treatises of undoubted Islamic origins.

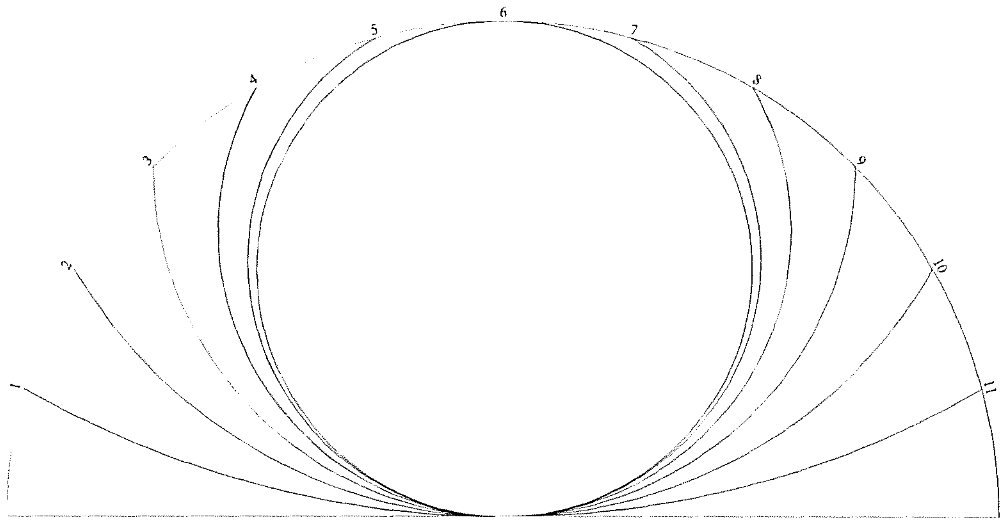


Figure 8-7. Unequal Hours Diagram

This diagram is described in detail in the chapter on quadrants.

The diagram can be drawn to show both the morning and afternoon hours as in Figure 8-7, or since it is symmetrical, can be reduced to a single quadrant. The single quadrant representation is sometimes drawn in conjunction with another scale such as the sin/cos scale.

Using the Unequal Hours Diagram

This diagram is used to estimate the unequal (seasonal) hour from the Sun's altitude. There are two steps:

1. Determine the Sun's noon altitude from the front of the astrolabe or by calculating $\mathbf{h = 90 - \varphi + \delta}$ and set the alidade to this value. Note the point on the alidade where it intersects the 6 hour semicircle.

2. Move the alidade to the Sun's current altitude. The position of the point on the alidade where it intersected the noon circle indicates the current unequal hour.

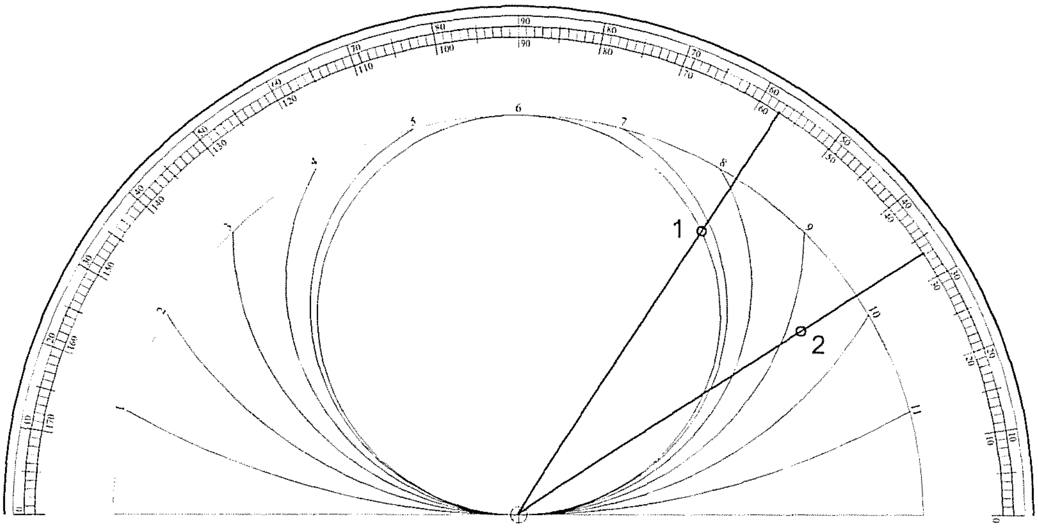


Figure 8-8. Unequal Hour Example

It is easier to use this diagram if the alidade is equipped with some sort of scale serving as an index for noting the noon hour intersection. In fact, the scale becomes fairly flexible if the alidade has a solar declination scale as this allows the unequal hour to be determined for any hour of any day of the year quite easily, but it makes the scale latitude specific.

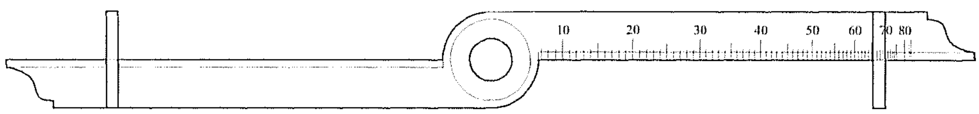


Figure 8-9. Alidade divided for Sun's noon altitude

The alidade in the figure above is divided by the Sun's noon altitude for this scale. The alidade scale is useable for latitudes between about 30° and about 55° for the entire year. Outside this range, the Sun's noon altitude near one of the solstices is unuseable.

Figure 8-8, shows an example of the use of the unequal hour scale. Assume you are in Woodstock, NY at latitude 42° N on April 12, and have measured the Sun's current altitude as 33°. The Sun's noon altitude from the front of the astrolabe as about 57° if you have a plate for this latitude, or from 90° - 42° + 9°= 57°, 9° being the approximate solar declination for the date. Set the alidade on 57° on the the altitude scale and note where the alidade edge crosses the 6 circle (1). Then move the alidade to 33° on the altitude scale (2) and see the point identified in step 1 is about one-fourth of the way into the 9th unequal hour.

The same result can be found from the front of the astrolabe if you have the appropriate plate and it includes the unequal hour arcs.

There is some question about how much use was actually made of this scale despite its wide appearance on astrolabes. North[54] examined a large number of astrolabes and noted that very few of the instruments containing this scale also had an alidade engraved with an index for noting the intersection with the six hour circle. This is clearly not conclusive and may not even

[54] North, J. D., "Astrolabes and the Hour-Line Ritual", Journal for the History of Arabic Science, 5, (1981), 113-114.

be indicative that this diagram was not considered to be vital. Several treatises remommend noting the noon position with a mark on wax or ink dot on an otherwise undivided alidade.

Archinaud[55] discussed the accuracy of this diagram and found it provides a fair estimate of the unequal hour for low latitudes but is increasingly inaccurate for northern latitudes, depending somewhat on the unequal hour and the Sun's declination, with a greater error for positive declinations. The error in the second hour from noon can be nearly a quarter hour for 49° north latitude. The error for the same time and declination is about six minutes at 30° north latitude. It works exactly for all latitudes when the Sun's declination is zero. The low level of precision for this diagram was probably not appreciated when astrolabes were in wider use and would not have been considered unacceptable even if it had been known. See Page 226 for a more thorough discussion of the accuracy of the unequal hour scale.

Drawing the Unequal Hours Diagram

The procedure for drawing the unequal hours diagram is also presented in the chapter on quadrants. It is repeated here.

Graphical construction of the unequal hour scale arcs.

a) Divide the six-hour semi-circle into six equal parts, P_i , i = 1...6 using the degree scale along the limb. Each section is 15 degrees.

b) For each division, draw a chord from the instrument center to the limiting circle.

c) Find the center of this line and erect a perpendicular.

d) The point where the perpendicular to the chord intersects the meridian line is the circle center.

e) The circle radius is the distance from the circle center to the point where the chord intersects the limiting circle. The method is outlined in Figure 8-10 for a single curve.

Analytic construction of the unequal hour scale.

Notice in Figure 8-10, the center of the hour curve is the apex of an isosceles triangle with equal sides of r_i where i is the hour. The base angles of this triangle are **(90° - 15i)**. The perpendicular erected from the chord defines a right triangle with base of **R/2**, where **R** is the radius of the limiting circle. The apex angle of this triangle is 15i.

Solving for r_i, $\sin 15i = (R/2) / r_i$ and $r_i = R / (2 \sin 15i)$. The arc is drawn from a distance rI from the base of the six-hour curve. A similar construction results if the hours are counted from the opposite end of the scale with the result, $r_i = R / (2 \cos 15i)$. The alidade divisions are **2R sin a** from the center.

The origins, history and accuracy of this scale are in the chapter on the universal horary quadrant, beginning on page 226.

[55] Archinaud, M., "The Diagram of Unequal Hours", Annals of Science, 47 (1990), 173-190.

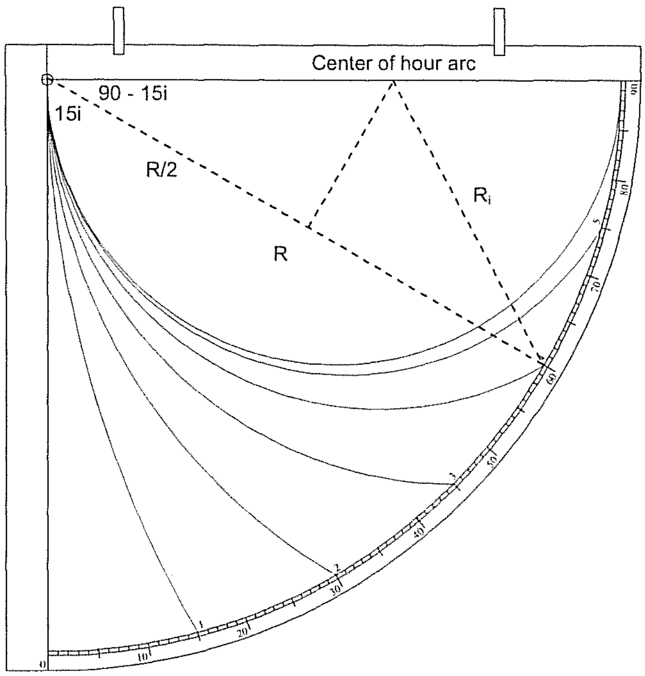


Figure 8-10. Unequal Hour Diagram Construction

Equal Hour Scale

A scale in equal hours used in the same way as the unequal hour scale is possible and is occasionally included. The arcs of this scale are not circles but are drawn as circles on the instrument and the error is not large. It is likely the curves were drawn by calculating three points and then constructing a circle through them. An example scale for 52° latitude is shown in Figure 8-11.

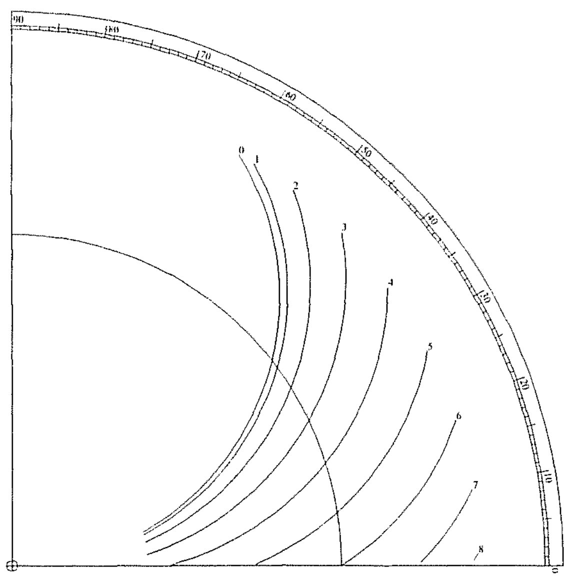


Figure 8-11. Equal Hour Scale

It is easy to see why this form of the hour scale was not more popular. It is latitude specific and thus applies to only one of the instrument plates. It also duplicates a function more easily done using the front of the astrolabe.

The curves are drawn by calculating the Sun's altitude for each even hour (i.e. hour angle multiples of 15°) for the full range solar of declinations and connecting the points. The curves are close enough to circles to be estimated by three points.

Equal / Unequal Hour Conversion Scale

Many later European astrolabes had a scale for converting between equal and unequal (seasonal) hours. This scale would have been useful in a society using unequal hours for everyday life but was adapting to times shown on clocks and sundials. An example of the scale is shown in Figure 8-12. This example shows the scale for both the morning and afternoon hours and is labeled in a representative manner. There was considerable variation in how the arcs and scales were labeled depending on the artistic inclinations of the maker. It was not unusual for only the right half of the scale to be drawn on the astrolabe.

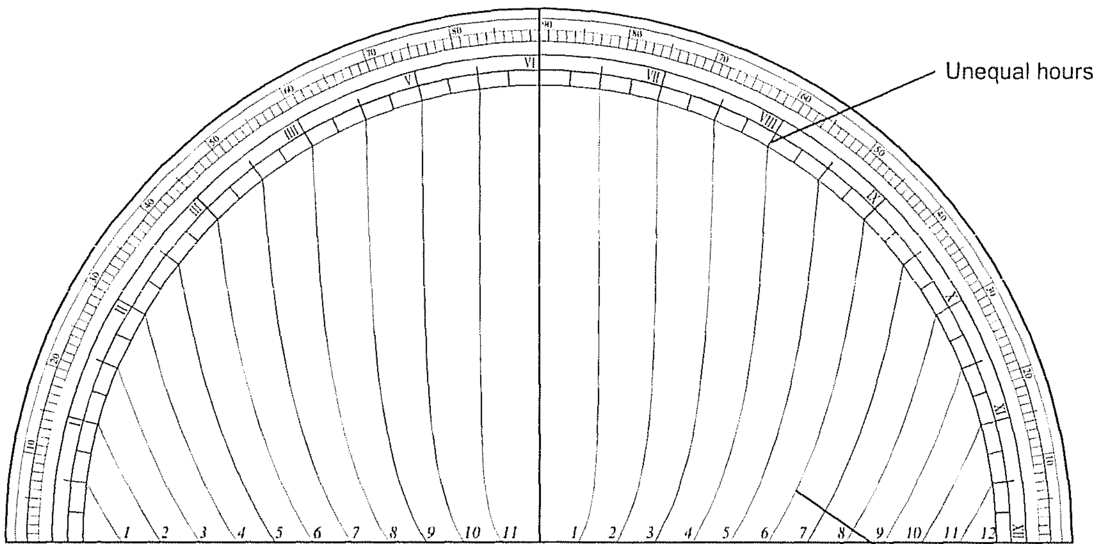


Figure 8-12. Equal / Unequal Hour Scale

The purpose of this scale is to provide the equal hour corresponding to a known unequal hour for a day or vice-versa. In the example shown in Figure 8-12, the Roman numerals around the arc containing the scale represent unequal hours. The Arabic numerals along the horizontal axis label the equal hour curves. Each curve and unequal hour division represent both a morning and afternoon hour if only the right half of the scale is shown.

The alidade is often graduated with the same divisions as the horizontal scale.

Using the Equal / Unequal Hour Scale

The use of this scale requires the time of sunrise or sunset for the day in question, which is easily found using the front of the astrolabe.

1. **Find the equal hour corresponding to an unequal hour**. For this example we will assume it is afternoon. The procedure for morning problems is identical but uses sunrise. Locate the point on the horizontal scale corresponding to the equal hour time of sunset. Rotate the

alidade until it points to the unequal hour. The sunset point on the alidade is on the equal hour line.

For example, find the equal hour corresponding to the end of the ninth unequal hour on a day when sunset is at 7:00 PM. Refer to the line marked 1 on Figure 8-13. The dot is at the distance of the 7:00 curve and lies at about 3:30 PM.

2. **Find the unequal hour corresponding to an equal hour**. As in example 1, note the time of sunset on the alidade. Rotate the alidade until this point is on the equal hour. The alidade points to the unequal hour.

   For example, find the unequal hour corresponding to 2:00 PM on a day when sunset is at 5:00 PM. The line marked 2 in Figure 8-13 points to not quite halfway through the ninth unequal hour.

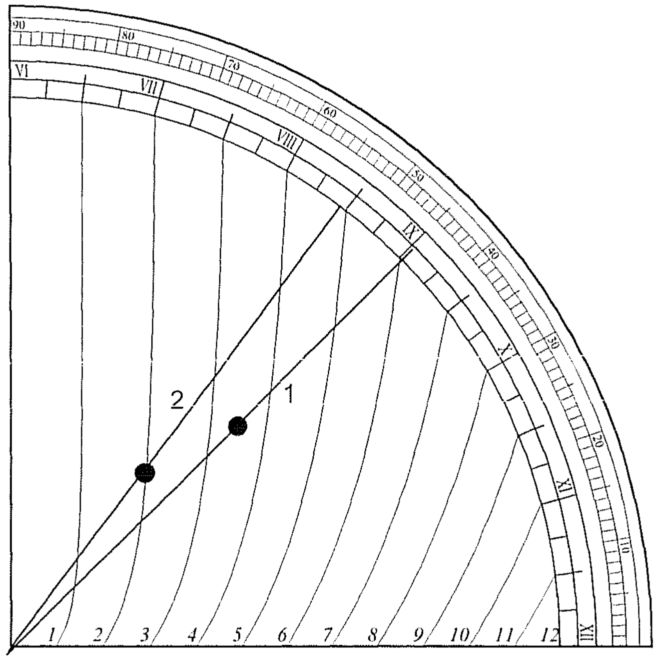


Figure 8-13. Equal / Unequal Hour Examples

This scale works for any latitude as long as the time of sunrise/sunset can be determined. This is easily found on the front of the astrolabe for any location for which there is a plate, or from the latitude included on a plate of horizons.

Drawing the Equal / Unequal Hour Scale

The scale itself is not difficult to draw, but the underlying theory requires a bit of thought. We will attempt to break the logic down into small pieces.

The scale works on the ratios of equal to unequal hours. Both equal and unequal hours have noon in common. The time from noon to sunset is divided into six seasonal hours of equal length. There are, of course, twelve equal hours from noon to midnight.

The number of equal hours from noon to sunset is simply the time of sunset. If E is the time of sunset in equal hours (the time of sunset in unequal hours is always the end of the sixth hour after noon), the number of equal hours for each unequal hour is E/6. For example, if sunset is at 7PM, there are 7/6 equal hours (1 hour 10 min) per unequal hour. Similarly, if sunset is at 5PM, there are 5/6 of an equal hour (50 min.) per unequal hour.

A scale is needed that automatically provides the ratio for a day. Consider Figure 8-14. This figure shows the underlying method for determining the equal hour curves.

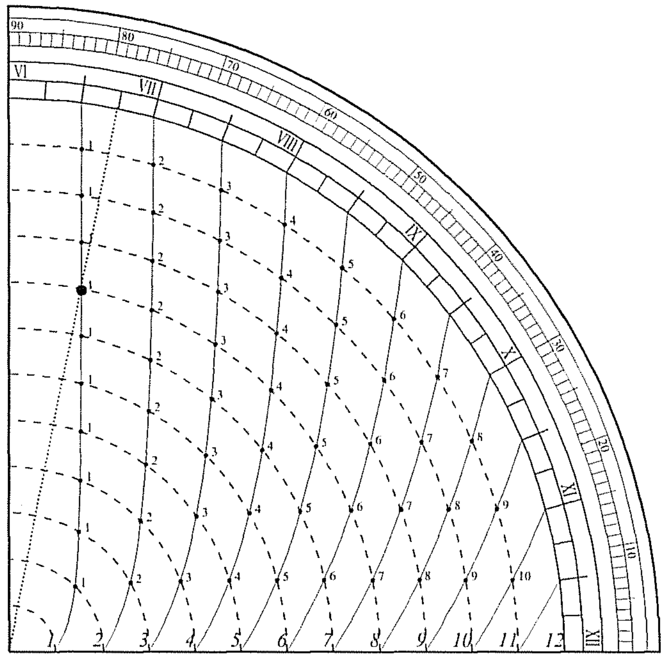


Figure 8-14. Equal Hour Divisions

The horizontal axis is divided into 12 equal divisions numbered 1 to 12. The number of the divisions represents the number of hours from noon to midnight in equal hours. A quarter-circle is drawn for each division. Each quarter-circle is then divided into the same number of divisions as the number of the arc. Look, for example, at arc 6. The six equal divisions of this arc correspond exactly to the six unequal hours between noon and sunset when sunset is at 6 PM. Now look at the arc for 8. When sunset is at 8 PM, the seasonal hours will be longer in terms of equal hours. If you draw a line from the center through the 1 point on the 8 curve to the unequal hour scale, you find it intersects at 3/4 into the 7th unequal hour. This means each equal hour is ¾ of an unequal hour. Similarly, a line through the 2 point intersects halfway through the 8th unequal hour, and so on. For each arc, the length of the divisions represents the number of equal hours per unequal hour. A line drawn from the center through any numbered point on the quarter circles will show the number of unequal hours that have passed since noon for the curve.

A large number of divisions and their corresponding arcs could be drawn and, thus, represent every instant of sunset and the equivalent ratio of equal to unequal hours. The same thing is accomplished by connecting the equal divisions with a smooth curve so all possible sunset times are covered.

To be completely accurate, it would be necessary to draw a great many curves and connect the points. This is not necessary in practice. The smooth-appearing curves in the figures were drawn by calculating the quarter circles for each (equal) 15 minutes. A finer division would not result in appreciably smoother curves.

The Back of Islamic Astrolabes

If it is difficult to define a standard for the scales on the back of European astrolabes, it is impossible for Islamic instruments. A wide variety of scales and annotations was used on Islamic astrolabes depending on the location, inclination, technical proficiency and artistic talent of the maker. Gibbs and Saliba[56] documented the range of scales on Islamic astrolabes in the Smithsonian collection, an investigation dramatically illustrating the wide range of options available.

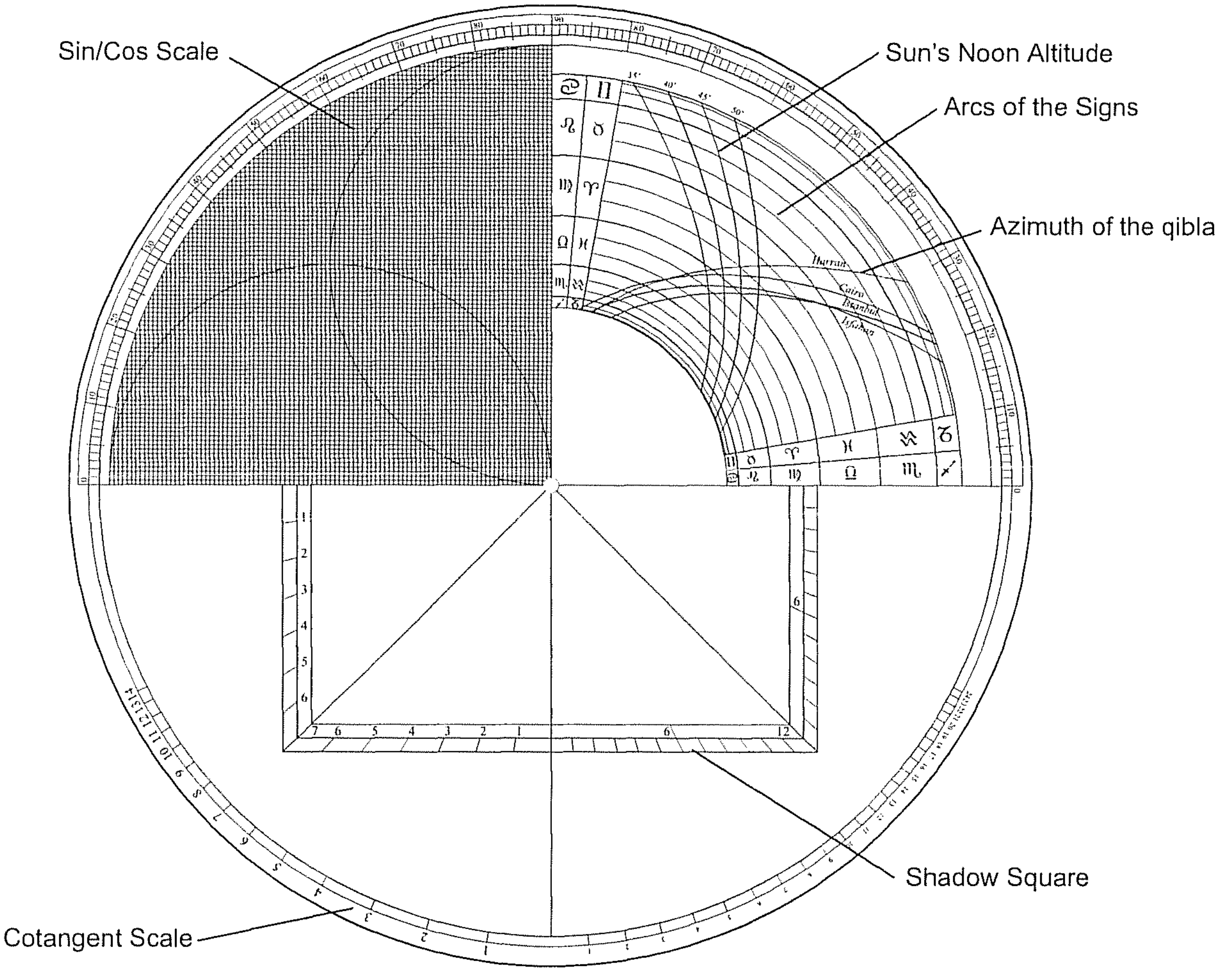


Figure 8-15. Islamic Astrolabe Back

In any case, the backs of Islamic astrolabes were generally wildly different from their European cousins. Several diagrams and scales fairly common on Islamic astrolabes were directly related to prayer rituals that were not relevant in medieval Christian Europe and many Islamic astrolabes were adorned with elaborate astrological material that did not apply to European astrological practices.

A calendar and solar longitude scale were standard on Islamic instruments from Andalusia and

[56] Gibbs, S. and Saliba, G., "Planispheric Astrolabes from the National Museum of American History", Smithsonian Institution Press, Washington, 1984.

the Maghrib. The Islamic lunar calendar was used everywhere in Islam, but a solar calendar was also used for civil and administrative purposes and, of course, astronomy. The specific calendar used was inherited from pre-Islamic tradition. The Roman Julian calendar was used in Andalusia. The Coptic calendar was used in Egypt and Syria and later in Ottoman Turkey. The Sassanid Persian calendar (era of Yazdigird) and the Malikshah calendar were used, but I have never seen an astrolabe from Iraq, Iran or India with a calendar scale[57]. All of these calendars have twelve, 30 day months with 5 days added at the end of the year, but with differing dates for the beginning of the year. Our examples of the scales on Islamic astrolabes are in the Mashriq tradition and do not include a calendar.

Shadow Scales

The shadow scale included on most Islamic astrolabes from the 10th century on is identical to the version already discussed, except the divisions are usually seven on one side, for a seven foot gnomon, and 12 'fingers' in the other part. Sometimes, only half the scale is included. The squares are labeled *al-ẓill al-mabsūt* ("the shadow which is stretched out"), corresponding to the European *umbra recta*, and *al-ẓill al-mankūs* ("the reversed shadow") for *umbra versa*[58].

The earliest reference to a shadow scale on an astrolabe is from al-Khwārizmī in the 9th century[59], a linear scale that is a model of the shadow cast by a vertical gnomon (Figure 8-16).

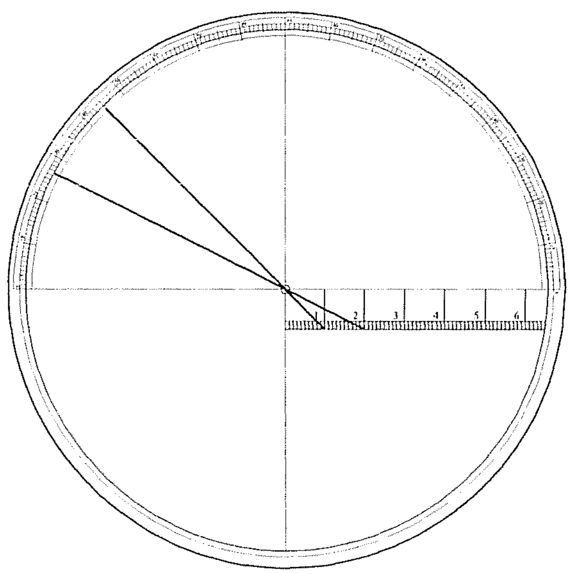


Figure 8-16. Linear Shadow Scale

The scale is divided by gnomon lengths. For example, in the figure the line from 45° intersects the scale at 1, or the shadow is one gnomon length long when the Sun's altitude is 45°. The line from two gnomon lengths intersects the degree scale at about 26.5°, and so on. The scale in the figure is drawn for a gnomon of ten units high. The original version used a 12 unit gnomon.

This scale would have been very useful for finding the solar altitude corresponding to prayer times defined by shadow lengths. For example, if the a□r prayer time begins when the shadow length is the noon shadow length plus the length of the gnomon, the point on the scale for the noon altitude is found. The solar altitude when the prayer time starts will be that point plus the

[57] Charette, François, private correspondence.
[58] King [2005], p. 251.
[59] Charrette and Schmidt [2004]

lengh of the gnomon, 10 units in the figure.

The scale is quite easy to draw. Simply select a convenient distance of the bottom scale line from the horizontal diameter. Divide this distance by the number of units for the gnomon height. Then divide the scale into equal sections of gnomon height and mark the division units.

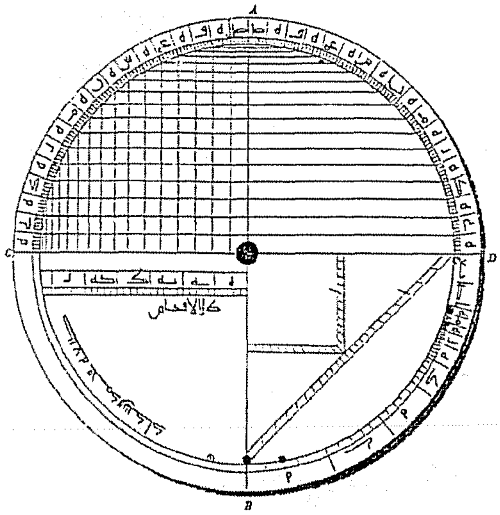


Figure 8-17. Astrolabe with Four Shadow Scales

Other shadow scale forms are possible. Figure 8-17 shows an astrolabe from the 9th or early 10th century with four different shadow scales; the linear scale described above, half a normal shadow square, a diagonal scale and a semi-circular scale in the margin[60].

The alidade sights were sometimes used as a form of universal equatorial sundial on a few Islamic and European astrolabes. In use, the astrolabe is suspended in the plane of the meridian and set to the Sun's noon altitude. The shadow of one of the sights falls on a scale of unequal hours engraved on the alidade. This method, which has been known since antiquity, assumes the Sun's hour angle increases at 15° per hour and gives only an approximate result[61].

The Sine/Cosine Scale

Many Islamic astrolabes have a scale in the upper left quadrant (QII) of the astrolabe back for the solution of trigonometric problems. This quadrant is at least as old as the 9th century. The principle of the scale is very simple as shown if Figure 8-18.

If the alidade is set to an angle a, the proportion of the radius on the vertical axis equals sin a, and the proportion of the radius on the horizontal axis is cos a.

This scale was used on astrolabes and as a stand-alone quadrant using a thread and bead in place of the alidade . It was introduced in the early ninth century by al-Khwārizmī[62] and was commonly used from the early 12th century on. The original version showed only the sine, and the cosine capability was added later. The sine scale provided a sort of trigonometric slide rule for medieval Islamic astronomers who used equations stated in sines and cotangents. For example, a formula from al-Battāni for estimating the qibla, the azimuth of Mecca, was **sin q = sin d / sin c** where **d** is the longitude difference and **c** is the latitude difference[63]. The solution to this equation could be found directly from the scale with no arithmetic. Some other sine quadrant

[60] From Dorn, "Drie arabische Instrumente", p. 116. Courtesy David A. King.
[61] King [2005], pp. 253-255.
[62] Charette and Schnidt [2004]
[63] King [1983]. p. 13.

applications are discussed later in this section. This scale was used to solve some rather complex problems and became a general purpose trigonometric tool.

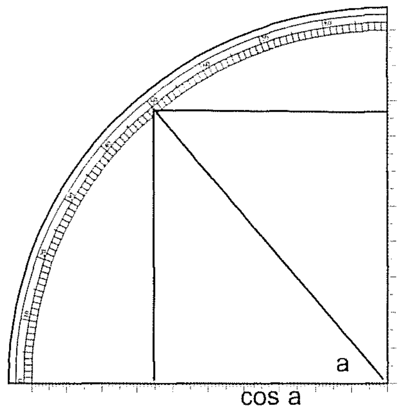


Figure 8-18. Sine/Cosine scale principle

There are two forms of this scale. In the form shown in Figure 8-19, the sines and cosines are indicated for each degree marked on the altitude scale. The scales along the axes are included to illustrate the principle and are not included on the astrolabe but are marked on the alidade. This form is most useful for finding the value of the sine or cosine of an angle, but the lines get rather congested for angles near 90° for sines and 0° for cosines. The scale in the figure includes lines for each degree. This level of accuracy would be provided on only the very best instruments. Most old astrolabes included lines for every 5° or so and, even then, were not always carefully drawn. The best examples are on later Persian astrolabes.

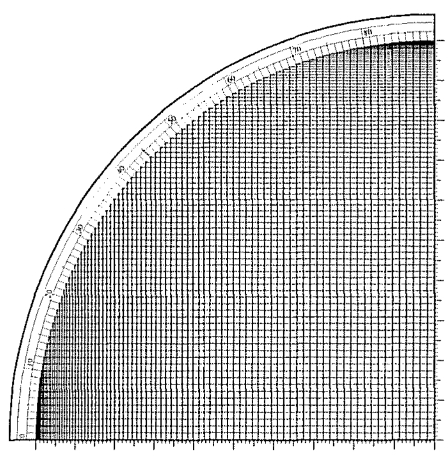


Figure 8-19. Sin/Cos scale for altitude angles

The second form (Figure 8-20) supplies the sines and cosines at even graduations on the axes, 100 in the figure but usually 60 on old astrolabes[64]. The corresponding angle is read from the altitude scale. This form is most useful for finding angles corresponding to a given sine or cosine and also has the advantages of not requiring a graduation on the alidade and the congestion at the ends of the axes is eliminated. The value of the sine or cosine is estimated by counting the divisions along the axis. This was the more popular of the two forms.

Additional curves are sometimes added to this scale. The quadrant in Figure 8-21, adds circles to a sine scale for for each 10° of longitude, which can be used to find **R sin a**.

[64] Some Indian instruments used a scale of 150.

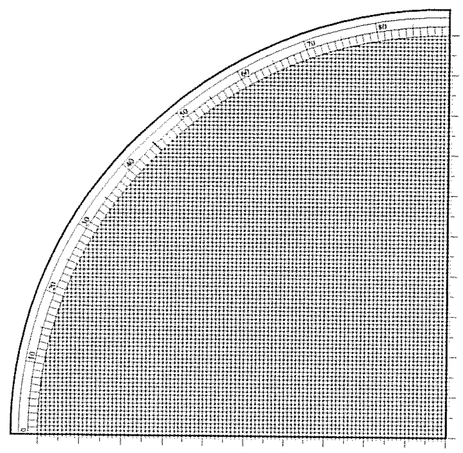


Figure 8-20. Sin/Cos scale with even divisions

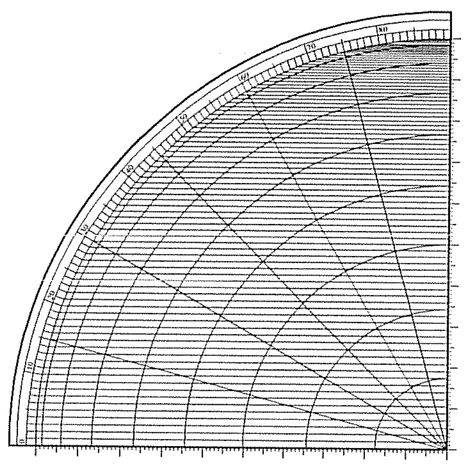


Figure 8-21. Sin/Cos Scale with Circles and Radials

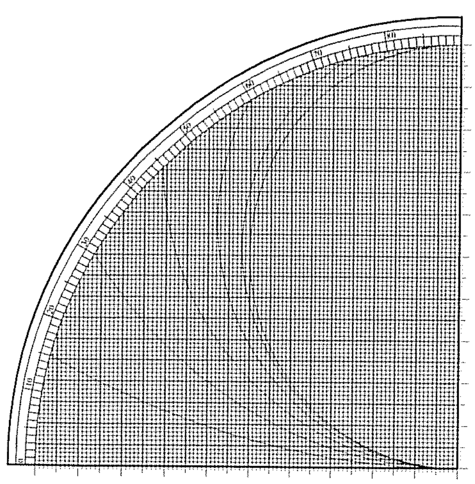


Figure 8-22. Sin/Cos scale with Unequal Hour scale

An unequal hour scale is sometimes added to the sine/cosine scale in order to make the quadrant serve two purposes. The result is shown in Figure 8-22. This scale is discussed in the section on the back of European astrolabes and the horary quadrant.

Other additions to the scale are encountered. Some old astrolabes have radial lines from the origin to the altitude scale (Figure 8-21). Some instruments include only the sine lines. A sine/cosine scale with two semi-circular arcs is sometimes encountered (Figure 8-23). This form is actually a bit redundant because the two semi-circles are also used to find the sine and cosine, but it allows the sine or cosine to be set on the alidade very quickly. The lower semi-circle, the one with its center on the horizontal axis, is used to find the cosine of an angle. The alidade is set to an angle and the corresponding cosine is read from divisions engraved on the alidade. The other curve provides sines in the same way. This scale is also used on the *quadrans novus* described later.

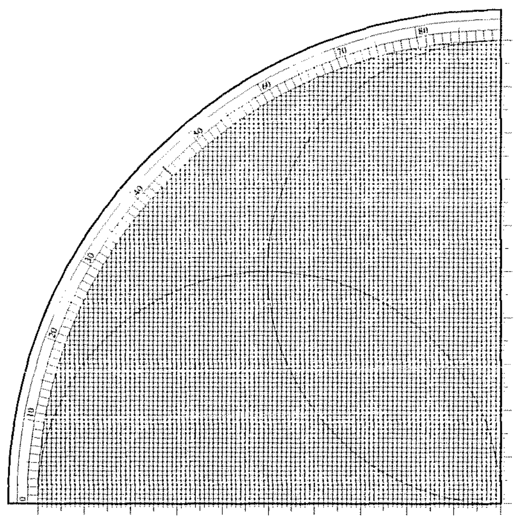


Figure 8-23. Sin/Cos scale with semi-circles

Some old Islamic instruments provided hash marks or other notation on the scale to note values frequently used for specific purposes.

Using the sine/cosine scale

1. **Find the sine of an angle between 0 - 90°**. Rotate the alidade so the edge is on the angle on the altitude scale. Read the sine of the angle from the portion of the vertical radius intercepted by the horizontal line from the angle.

   Example: Find the sine of 56°. Rotate the alidade to 56°. Read the sine as about 83/100 (0.83) on the vertical radius. Values can be estimated to three decimal places on the best instruments.

   The cosine is found in an identical way, but using the portion of the horizontal radius intercepted by the vertical line from the angle. The cosine of 56° is about .56.

2. **Find the arcsine of a sine.** Locate the value of the sine on the vertical radius and follow the horizontal line to the point where it intersects the altitude scale.

   Example: What is the arcsine of 0.33? Locate the horizontal line closest to 33/100. Follow the line to the altitude scale and read about 19.3°. The arccosine is similar using the value on the horizontal radius.

The original intention of the sine scale was probably to solve problems in the form:

Such an equation for finding the approximate qibla was mentioned above. A more common equation for estimating the unequal hour from the Sun's noon altitude and current altitude is:

$$\sin 15T = \frac{\sin h}{\sin H}$$ T = Unequal hour from noon, h = Sun's altitude, H = Sun's noon altitude

15T is the Sun's hour angle. This equation is also discussed in the section on the horary quadrant, page 224.

See Figure 8-24. The example is for Cairo at the vernal equinox. The sun's noon altitude is 60°, the Sun's current altitude in the afternoon is 38°. The sine line for the Sun's noon altitude is found and the position noted on the alidade. The alidade is rotated until the position noted is on the sine line for the current altitude. The radial from the center to the degree scale passing through this point (the edge of the alidade) points to the Sun's hour angle: 45°. The unequal hour is, therefore, 45°/15, three hours from noon. Since it is after noon, the unequal hour is 6 + 3 = 9.

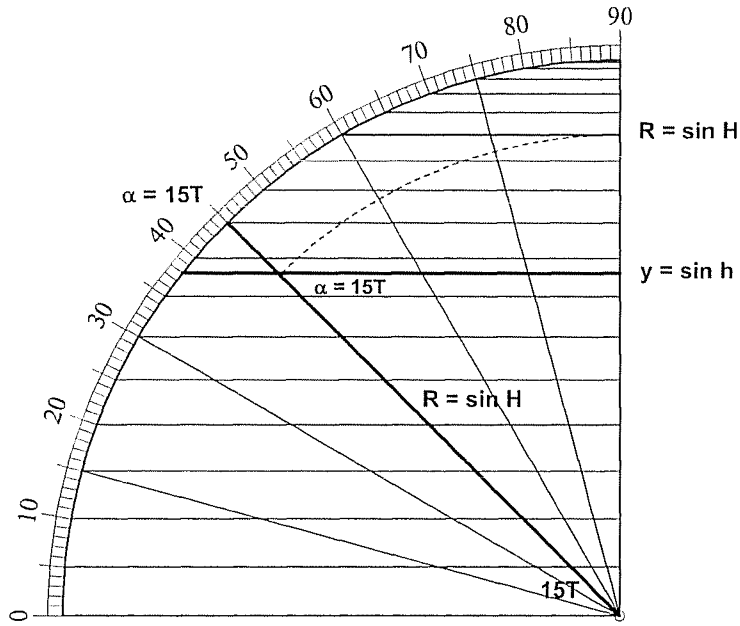


Figure 8-24. Sine Quadrant Unequal Hour Solution

The same procedure can be used to solve any problem of this type. In general, the triangle defined by bold lines in the figure above is the basis for constructing problems of the form $y = R \sin \alpha$[65].

The approximate unequal hour equation is exact at the equinoxes and is less accurate at other times. It gives surprisingly good results for middle latitudes, with less accuracy for farther northern latitudes. See page 228 for a discussion of the accuracy of this equation.

[65] Lorch [2000], p. 252.

The form of the sine quadrant with concentric circles, sometimes called "declination circles" for a reason that is not clear, is used to solve problems involving arcsines of products of sines. For example, the Sun's declination for a given solar longitude is found from **$\sin \delta = \sin \lambda \sin \varepsilon$**.

See Figure 8-25. In this figure, a concentric arc corresponding to the obliquity of the ecliptic is added[66]. To find the solar declination, the alidade is placed on the solar longitude value on the angle scale. The point where the alidade edge intersects the obliquity circle is noted. The horizontal line from that point to the angle scale indicates the declination. In the example, the solar longitude is 30° and corresponding declination is 11° 32′. The solar longitude value used is the difference in longitude from the nearest equinox. For example, a longitude of 30° would be used for the end of Aries, the beginning of Virgo, the end of Libra and the start of Pisces. The triangle formed by the line to the solar longitude and the obliquity circle confirm the equation: **$\sin \lambda = \sin \delta / \sin \varepsilon$**.

A similar procedure can be used to solve other problems of this form. With a little thought, similar procedures can be defined using cosines and combinations of sines and cosines.

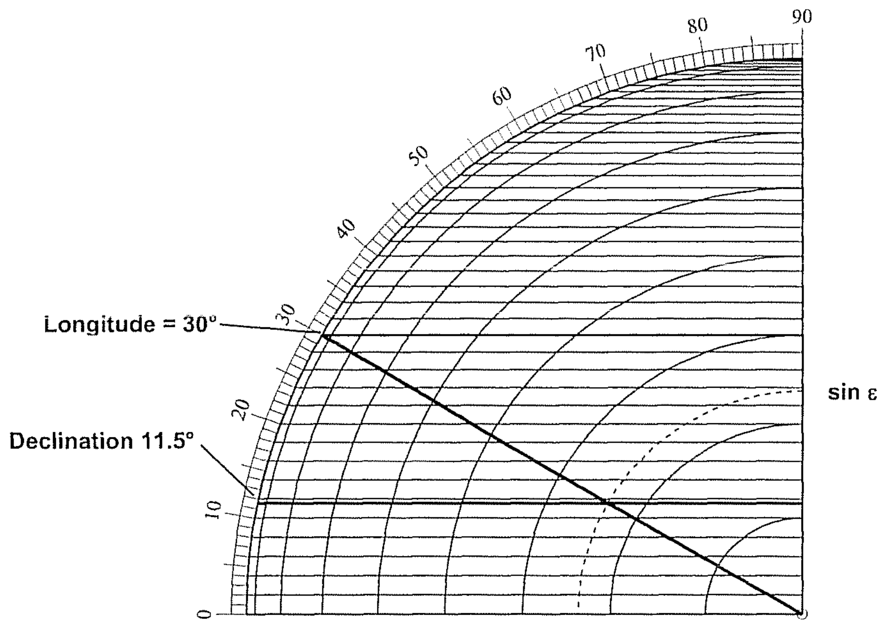


Figure 8-25. Finding Solar Declination with Sine Quadrant

Making the sine/cosine scale

Drawing the sine/cosine scale is very easy. If the scale is drawn to provide a line for each angle on the altitude scale, simply draw vertical and horizontal lines from each degree desired.

If the scale uses even divisions, divide the radii into the appropriate number of divisions and draw vertical and horizontal lines from the radius to the altitude arc from each division. Additional ornamentation can be inserted as desired. None of the shown variations requires any additional explanation.

[66] A value for the obliquity of 23° 35′ is used in the example to be consistent with al-Khwārizmī's treatise.

The Cotangent Scale

Many Islamic astrolabes included a scale of cotangents that extends the range of the cotangent scale on the shadow square. The primary use of scale was to determine what altitude the Sun would have at the time of a prayer.

Figure 8-26 illustrates a form of the cotangent scale.

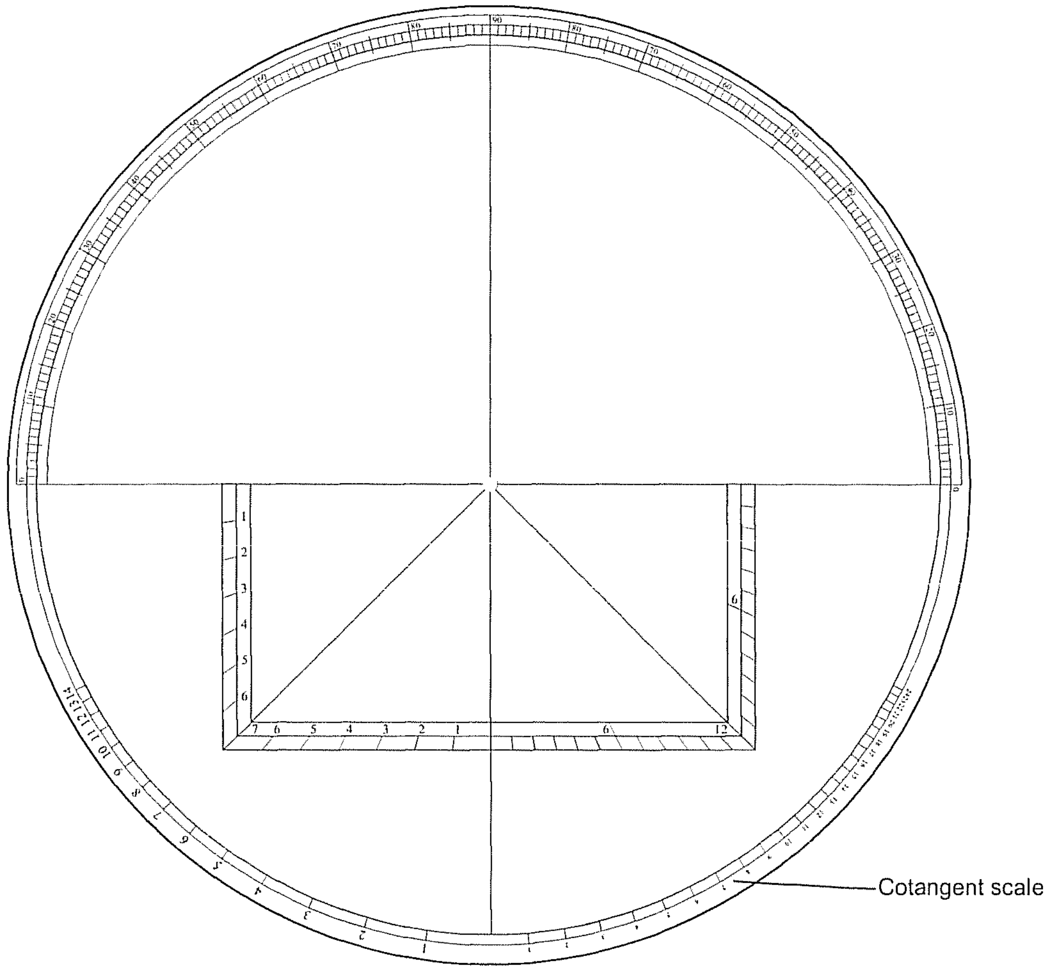


Figure 8-26. Islamic astrolabe back with cotangent scale

The cotangent scale is in the margins of QIII and QIV. There is no mystery about this scale; it is simply the cotangent of the alidade angle scaled to a common gnomon length. The value on the cotangent scale is the length of the shadow cast by a gnomon of the length represented by the scale division. Muslims traditionally used a seven-foot tall gnomon, hence the division of the left half of the shadow square into 7 sections representing feet. The right half of the shadow square was divided into 12 divisions representing "fingers".

Using the cotangent scale

The following example will illustrate a typical use for this scale. Say it is May 5, and youwish to find the Sun's altitude at Harran (latitude 35° 51′) when the shadow cast by a seven-foot gnomon is equal to the shadow length at noon plus the length of the gnomon.

You need the Sun's noon altitude first. This can be found from the front of the astrolabe or estimated from the 36° horizon on a plate of horizons or we calculated from h = 90 - φ + δ. The Sun's declination on May 5 (Taurus 15°), is 16° 17′. The Sun's noon altitude on May 5, at Harran is about 70.4°.

Set the alidade to 70.4° and see the shadow length is about 2.5 feet long from the cotangent scale. We want the Sun's altitude when the shadow length is 2.5 + 7 feet = 9.5 feet long.

Set the alidade to 9.5 feet on the cotangent scale and read about 36.4°. This is the desired result. The steps in the solution of the problem are shown in Figure 8-27.

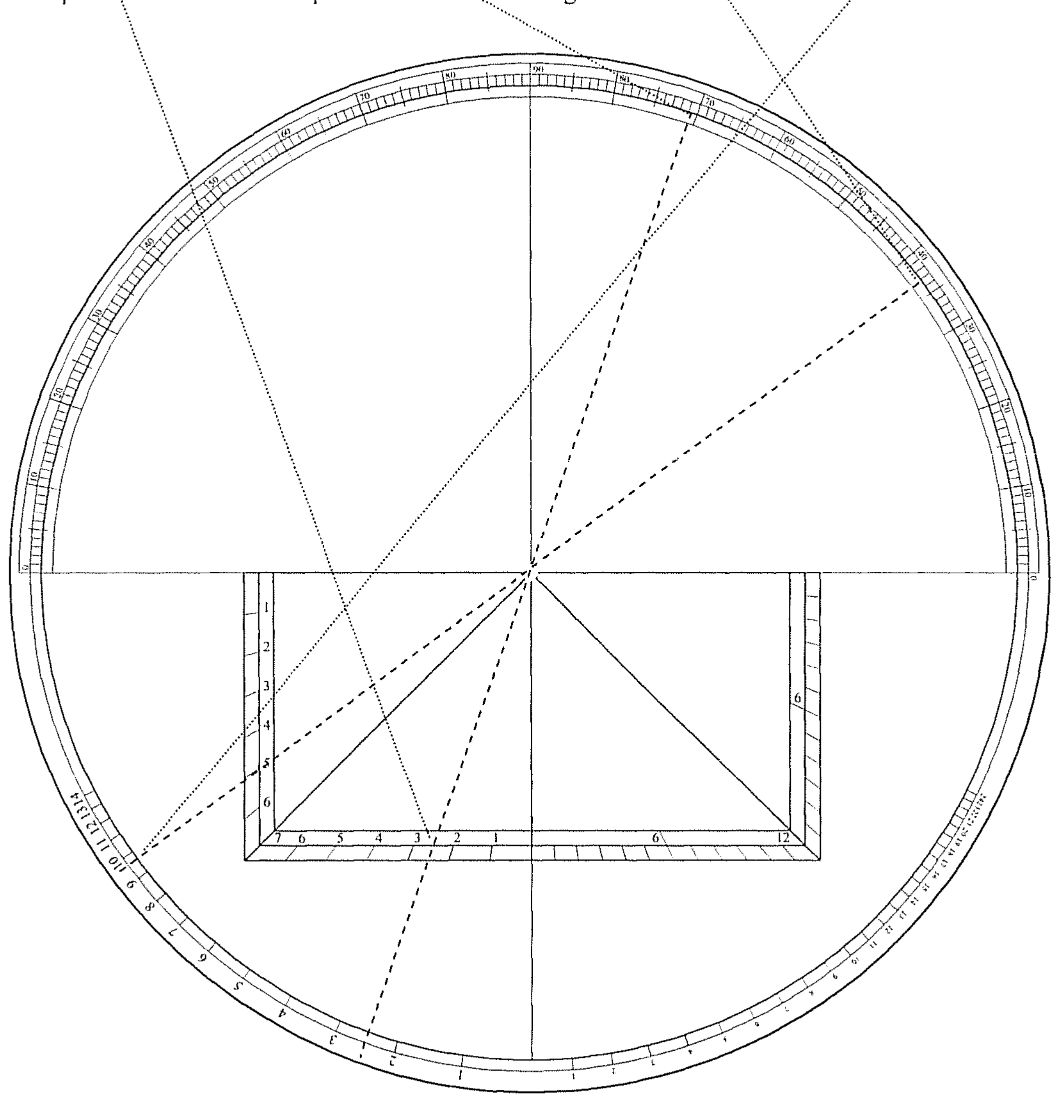


Figure 8-27. Cotangent scale example

Drawing the cotangent scale

The angle of each cotangent scale tic from the vertical diameter of the back of the astrolabe is arccot (N / Div) where N is the number of the tic and Div is the number of divisions of the shadow square. It is easier to calculate the angle on a calculator or computer as:
90 – arctan (N / Div).

The tic is labeled with its number.

The cotangent scale in the example extends the scale to twice the range of the shadow square. This makes the divisions quite close together for the fingers scale and a very small font was required to keep the labels from overlapping. You might want to use a shorter scale for your instrument or label only every other tic.

The Arcs of the Signs

Many Islamic astrolabes include scales in the upper right quadrant (QI) on the back of the astrolabe drawn on a display of the signs of the zodiac. This underlying diagram shows the Sun's declination for each sign of the zodiac and is usually called "The Arcs of the Signs". This diagram does not have value in and of itself, but provides one element of a nomograph for other scales drawn in the quadrant.

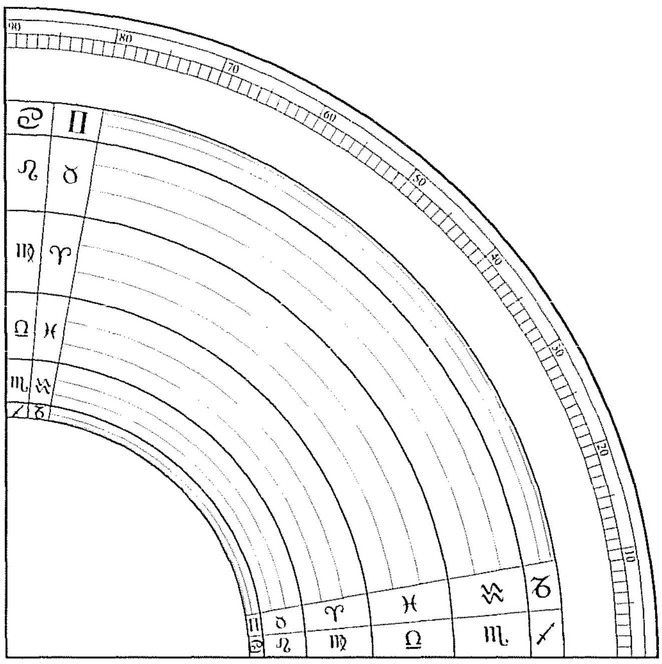


Figure 8-28. Arc of the Signs stereographically projected

The arcs of the signs show the Sun's position in the zodiac and can be drawn in two ways. Figure 8-28, shows the arcs drawn as a stereographic projection of the Sun's declination for each sign. Notice either tropic can be represented by the outer arc and the labeling of the signs can proceed from Aries at the equator outward through Taurus and Gemini to the Tropic of Capricorn. This sequence of labeling is shown along the vertical axis in the figure. The labels can also be done in the opposite order with the outer arc representing the Tropic of Cancer. This sequence is used in the labels along the horizontal axis. Both sequences of labels are commonly used on this scale since two graphs are normally included in this quadrant and one set of labels is more convenient for one scale and the other set is better for the other scale.

The arcs get very close together near the tropics when using the stereographic projection. Therefore, this scale can be constructed with the arcs drawn at even intervals. This form is somewhat easier to use, particularly for dates close to the tropics. The resulting scale is shown in Figure 8-29.

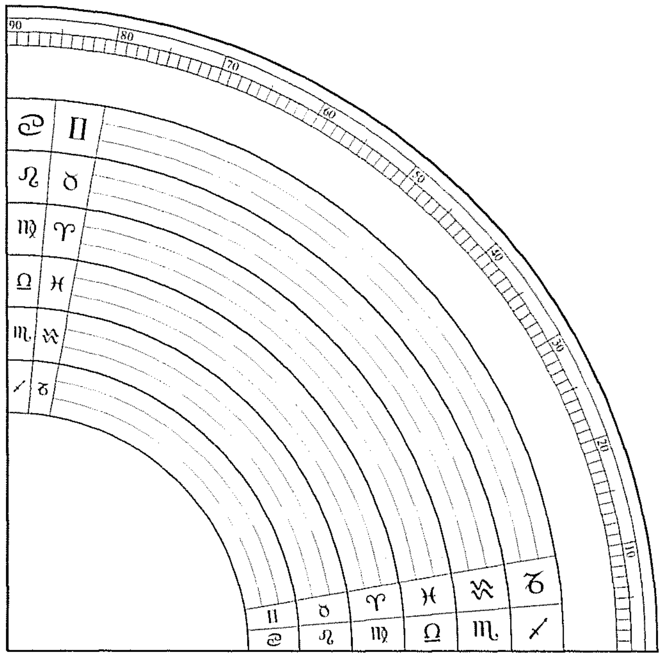


Figure 8-29. Arcs of the Signs evenly separated

The choice of drawing the arcs stereographically projected or even spaced is purely a matter of taste and does not affect the utility of the scales in this quadrant. Very fine astrolabes have been made using both methods, but the best astrolabes used the stereographic projection. The shapes of the curves on the scales superimposed on the Arcs of the Signs do, however, change depending on the method used.

Drawing the Arcs of the Signs

The stereographically projected arcs are drawn using the standard equation for the distance of a point of given declination using the stereographic projection: $r = R_{eq} \tan [(90 - \delta) / 2]$. The declination corresponding to each longitude can be calculated from $\sin \delta = \sin \varepsilon \sin \lambda$. It seems to be usual for the tropics of the stereographically projected arcs to match the sizes of the tropics on the astrolabe plate, but this is not required and any convenient scale can be used. The resulting scale is shown in Figure 8-28.

The evenly divided arcs of the signs is even easier. A cursory examination of pictures of old astrolabes indicates the radii of the tropics match the stereographic projections of the tropics on the plates: R_{Can} and R_{Cap}. This is far from certain, but these radii are good choices. If the inner arc is chosen to be the Tropic of Cancer, the radii of the evenly distributed arcs are simply $r = R_{Can} + i \times (\text{division width})$ where i is the number of the arc and the division width is $(R_{Cap} - R_{Can})/\text{number of divisions}$. The arcs can be drawn for every 10°, 5° or 3° with 5° being somewhat usual. Any finer division puts the arcs too close together to be useful. The arcs are 10° apart in the figures.

The line segments defining the label area are usually 10° wide and are drawn as radials from the center.

On some instruments, the arcs representing the zodiac divisions are simply drawn as quarter circles with labels along the axes.

The Sun's Noon Altitude Curves

Curves showing the Sun's noon altitude for each day of the year for selected latitudes are often superimposed on the arcs of the signs. A different curve is required for each latitude, and the curve is labeled with the latitude. The value found is useful in timekeeping equations.

In use, the edge of the alidade is placed over the intersection of the appropriate curve and the arc corresponding to the date desired. The Sun's noon altitude is then read from the altitude scale. For example, the dotted line in the figure shows the Sun's noon altitude for 35° latitude at the vernal equinox is 55°.

The resulting curves for stereographically projected arcs of the signs are shown in Figure 8-30.

The noon altitude curves are not stereographically projected and are not arcs of circles. They are simply a nomograph calculated to provide the desired result.

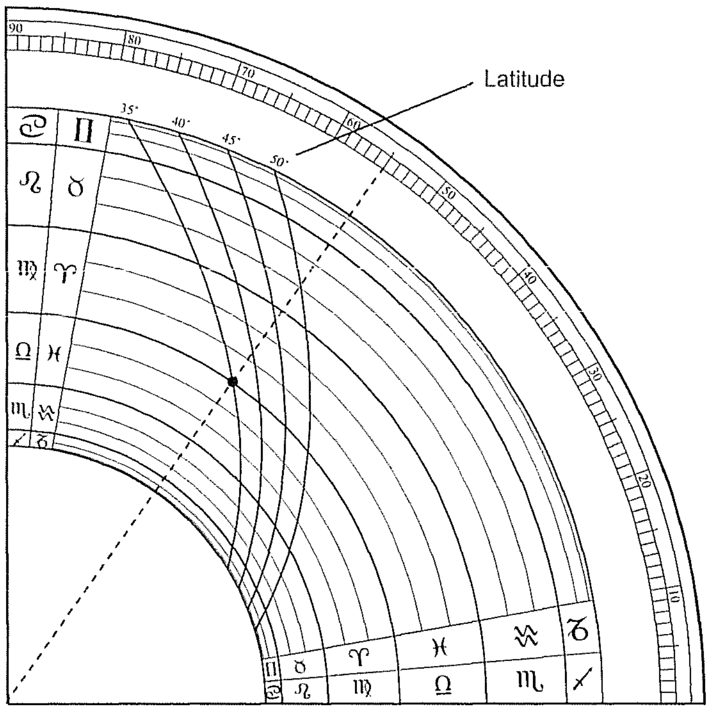


Figure 8-30. Arcs of the Sun's noon altitude for stereographic scale

The curves for evenly spaced arcs of the signs are shown in Figure 8-31. Note the sigmoid (s-shaped) curves. This presentation is actually a bit more informative as it shows the rate of the Sun's daily change in declination. Both forms are found on Islamic astrolabes.

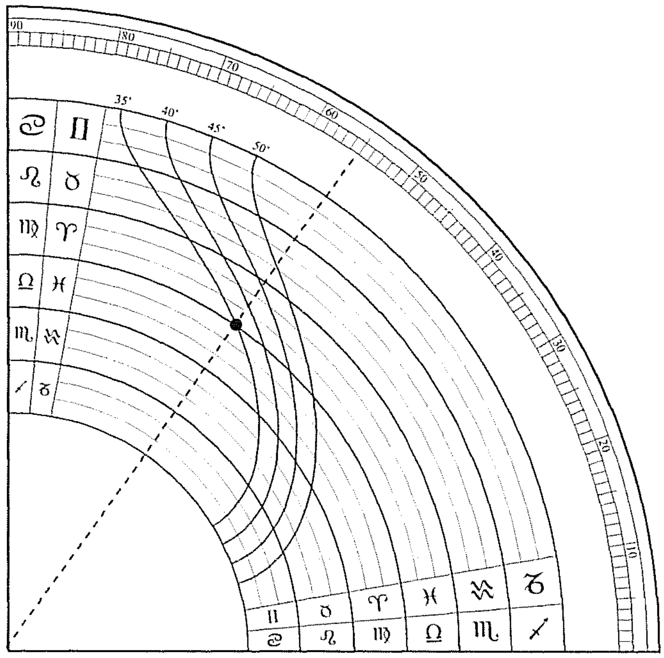


Figure 8-31. Sun's noon altitude curves on even scale

Drawing the Sun's noon altitude curves.

As many altitude curves can be included as seem necessary or useful. The noon altitude curves always use the zodiac labels on the vertical axis, which makes the arcs curve to the left. A series of calculations is required for each curve. For a given latitude, φ, calculate the Sun's noon altitude, h, from:

$$h = 90 - \varphi + \delta$$

where the declination is calculated from $\sin \delta = \sin \varepsilon \sin \lambda$. The polar coordinates of each point will be (h, δ). The interpretation of the declination coordinate depends on the method used to draw the arcs of the signs. If δ_r is the radius of the declination circle for the point, the rectangular coordinates will be:

$$x = \delta_r \cos h,\ y = \delta_r \sin h$$

Mark each calculated point and connect the points with a smooth curve. The points may be connected with line segments if a sufficient number of points are calculated.

The Graph of the Azimuth of the Qibla

These curves are used to find the qibla for selected cities. Quoting from King[67],

> "The Ka'ba is a shrine of uncertain historical origin which served as a sanctuary and center of pilgrimage for the Arabs for centuries before the advent of Islam. It was adopted by the Prophet Muhammand as the focal point of the new religion, and the Koran advocates prayer towards it. For Muslims it is a physical pointer to the presence of God. Thus, since the early seventh century Muslims have faced the Sacred Ka'ba in Mecca during their prayers. Mosques are built with the prayer-wall facing the Ka'ba, the direction being indicated by a *mihrab* or prayer-niche. In addition, certain ritual acts such as reciting the Koran, announcing the call to prayer, and slaughtering animals for food, are to be performed facing the Ka'ba. Also Muslim graves and tombs were laid out so the body would lie on its side and face the Ka'ba. (Modern burial practice is slightly different but still Ka'ba-oriented.) Thus the direction of the Ka'ba – called *qibla* in Arabic and all other languages of the Islamic commonwealth – is of prime importance in the life of every Muslim."

Due to its importance, methods for determining the qibla received serious attention from some of the most famous Muslim scientists including al-Khwārizmī (780-850), al-Battāni (858-929), Abū al-Wāfa al-Būzjānī (940-997), Ibn al-Haitham (965-1040), al-Bīrūnī (973-1048) and al-Tūsī (1201-1274) and others[68].

The qibla was defined as the direction to the Ka'ba from a locality along the great circle on the terrestrial sphere passing through both places[69]. Approximate methods for determining the qibla were developed in the 7th century, and exact methods were developed by the 9th century. Observations were conducted to determine the exact coordinates of Mecca and other cities in order to compute the qibla accurately. In fact, qibla determination inspired much of the activity of early Muslim geographers[70]. The coordinates of the Ka'ba are 21° 25′ 24″ N, 39° 49′ 24″ E. Of course, the latitude and longitude in this form was not used when astrolabes were in common use since standards of longitude were not accepted until the late 19th century. Rather, the difference in longitude from a given place and Mecca was used.

The subject of qibla determination has become somewhat more complicated and controversial within the Muslim community in modern times because Muslims now live all over the world and the great circle definition produces results that are not as intuitive as other methods, such as **rhumb lines**[71]. This was not a problem in the age when astrolabes were being constructed and most Muslims lived in the same hemisphere.

There are several accepted methods for determining the qibla based on both observation and calculation[72]. Observational methods include observing the azimuth of the Sun when it is directly above the Ka'ba, which occurs on approximately May 28 (Gemini 8°) and July 16 (Cancer 23°) at local noon in Mecca. The local apparent time when this event occurs can be calculated from the difference in longitude of the place and Mecca. The qibla can then be noted from the shadow of a vertical gnomon at this instant. This method cannot be used at a great

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[67] King, David A., "Science in the service of religion: the case of Islam", *Impact of science on society, no. 159,* UNESCO, 245-262. pg. 253.
[68] Abdali, S. Kamal, "The Correct Qibla", http://www.patriot.net/users/abdali/ftp/qibla.pdf and personal correspondence with Dr. Abdali.
[69] King, David A, *Astronomy in the Service of Islam*, Collected studies series, CS416, Valorium, Aldershot, Hampshire, UK, 1993.
[70] King [1993]
[71] Abdali
[72] For a discussion of medieval qibla concepts see King, D. A., "The Orientation of Medieval Islamic Religious Architecture and Cities", Journal for the History of Astronomy", xxvi, 1995, pp. 253-274.

circle distance from Mecca greater than 90°[73].

The qibla for selected locations was included on astrolabes in two forms. Some astrolabes contained a gazetteer engraved in the *umm* of the instrument giving the coordinates and qibla for selected cities. The information in the gazetteer was of uneven accuracy depending on the source used for the data[74].

The qibla for selected cities might also be shown by an engraved curve. The astrolabe graph of the qibla allows the user to determine the qibla for any day of the year from selected locations. The graph shows the altitude of the Sun when it is in the direction of Mecca for each day. In use, the graph would be consulted to find the Sun's altitude when it is in the direction of the qibla. The Sun's altitude is then measured continuously until it reaches this value. The Sun is then in the required direction.

This graph has the disadvantage of being usable only at locations where a great circle on the terrestrial sphere passes through Mecca, the local place, and the Sun at some point in its daily path. Otherwise, the Sun is not visible at the required time or never passes through the required direction. Therefore, this method of qibla determination cannot be used for much of the year for many locations, if at all. This restriction was not a factor in the Islamic world when astrolabes were common.

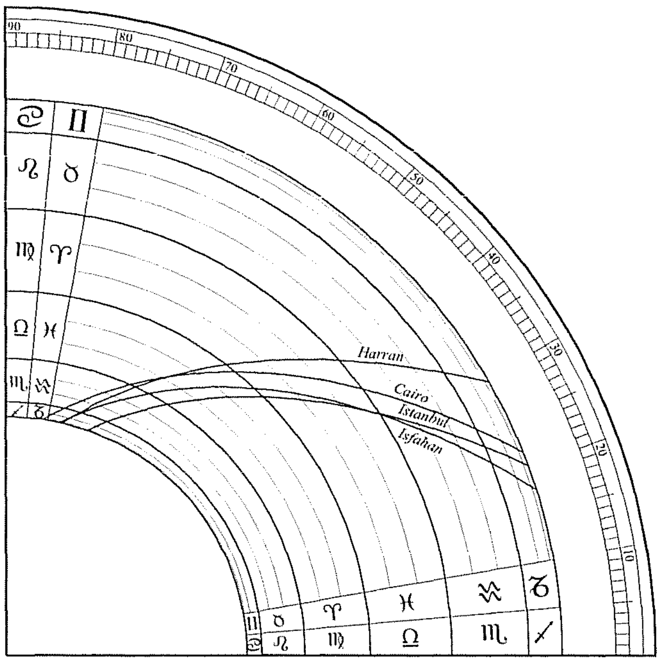


Figure 8-32. Graph of the qibla on stereographic scale

The resulting graph is shown in Figure 8-32 and Figure 8-33. Figure 8-32 shows the qibla graph on the stereographic arcs of the signs. This is the form almost always shown on old astrolabes.

The qibla arcs in Figure 8-32 are for Isfahan (32° 40′ N, 51° 38′ E), Istanbul (41° 01′ N 28° 58′ E), Cairo (30° 03′ N, 31° 15′ E) and Harran (36° 51′ N, 39° 0′ E). In Isfahan, which is east of Mecca, the Sun would be observed in the afternoon. In Cairo and Istanbul, which are west of

[73] Abdali

[74] King, David A., *World-Maps for Finding the Direction and Distance to Mecca*, Brill, Leiden (1999) pp. 170-186.

Mecca, the Sun would be observed in the morning. Harran is almost due north of Mecca so the observation would be made near local apparent noon.

The computed values for the figure are:

City	Latitude	Longitude	qibla
Istanbul	41° 1′ N	28° 58′ E	151°.609
Harran	35° 51′ N	39° 0′ E	177°.121
Isfahan	32° 40′N	51° 38′E	-134°.055
Cairo	30° 3′ N	31° 15′ E	136°.215

Figure 8-33 shows the same curves on an evenly divided arc scale. The figure is included only for completeness and was seldom, if ever, used on old instruments.

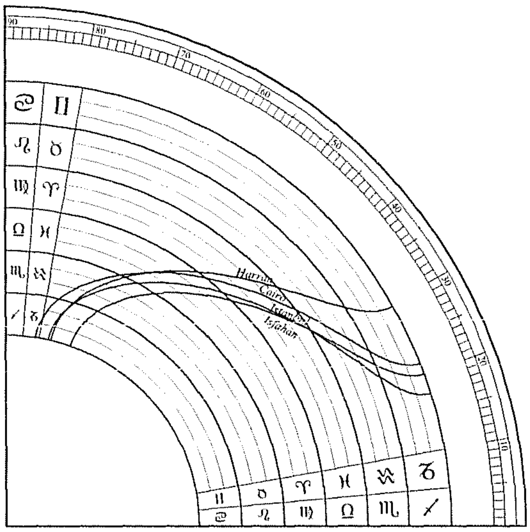


Figure 8-33. Graph of the qibla on an evenly divided scale

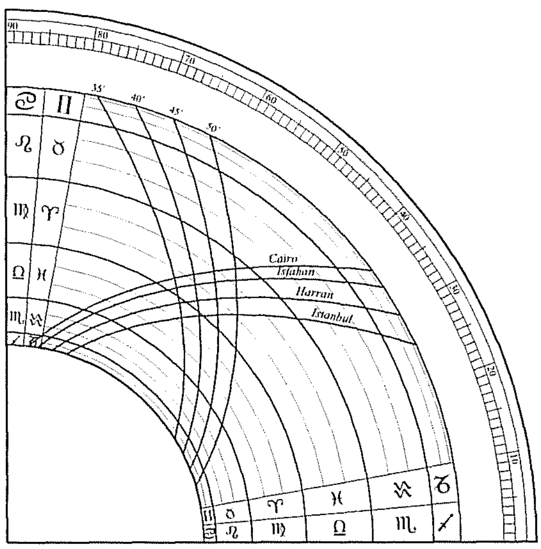


Figure 8-34. Complete quadrant on stereographic scale

The noon altitude curves and the graph of the qibla are almost always shown together in this quadrant. The resulting complete quadrant is shown in Figure 8-34. Figure 8-34 includes slightly different character styles and shapes for the curve labels to illustrate one design option.

Drawing the graph of the qibla

To construct the graph of the qibla, the azimuth is calculated or measured for a location. The Sun's altitude when it reaches that azimuth is determined for each day represented on the arc of the signs. The points are plotted and connected with a smooth curve. The curve is not stereographically projected and is not the arc of a circle, although circular arcs were often used on old astrolabes as the difference is not dramatic and within the accuracy of the instrument. The zodiac labels on the horizontal axis are always used for the graph of the qibla, which makes the arcs curve to the right, thus balancing the curves for the Sun's noon altitude.

Note carefully that the following discussion is for an astrolabe that might be made today and does not represent the method used to make medieval instruments. For a discussion of the methods used for old instruments see King [1999].

The astrolabe itself was often used to find the required solar altitudes. The plate for the location under consideration was placed in the astrolabe and the rete rotated so the Sun's position in the ecliptic for the day was on the qibla azimuth. The Sun's altitude was then noted and plotted on the graph. This method is still advocated in modern astrolabe literature because it is awkward to calculate the Sun's altitude for a given azimuth, declination and latitude.

The equation to calculate the qibla, q, was known in the 9th century and was equivalent to[75]:

$$\cot q = \frac{\cos \varphi_K \sin \varphi \cos(\lambda_K - \lambda) - \cos \varphi \sin \varphi_K}{\cos \varphi_K \sin(\lambda_K - \lambda)}$$

Where the coordinates of the Ka'ba in Mecca are (φ_K, λ_K) and the coordinates of the place under consideration are (φ, λ). Northern latitudes and eastern longitudes are taken as positive. This equation gives the direction <u>from</u> the Ka'ba <u>to</u> the city from north, with west positive.

$$\varphi_K = 21° 25' 24'' \text{ N} = 21°.4233333 \text{ and } \lambda_K = 39° 49' 24'' \text{ E} = 39°.8233333$$

This equation is usually written in the modern form as a tangent, and reduced by dividing the numerator and denominator by cos φ_K:

$$\tan q = \frac{\sin(\lambda_K - \lambda)}{\cos\varphi \tan\varphi_K - \sin\varphi \cos(\lambda_K - \lambda)}$$

q is the angle from north with east positive and must be adjusted by 180° to represent the azimuth from south. When solving this equation on a computer, it is best to use the `atan2` function which returns an angle between ±180° or a polar arctangent routine to return an angle 0-360°. The apparent sign difference between the two equations is due to the reversed orientation and the direction of positive angles.

The Sun's altitude when it reaches the azimuth q can be calculated directly using a method developed by Edmond Gunter in 1624. This method, which was never used to make old instruments and is of interest only to a modern maker who needs to determine the scales analytically, is described in detail in the chapter on Gunter's quadrant. A summary follows.

[75] King, David A., "World-Maps for Finding the Direction and Distance of Mecca. A brief account of recent research", Symposium on Science and Technology in the Turkish and Islamic World, Istanbul, 3-5 June, 1994.

There is a standard equation from spherical trigonometry that at first glance appears to solve the problem:

$$\sin \delta = \sin \varphi \sin h - \cos \varphi \cos h \cos A$$

However, this equation cannot be solved directly for h, although it can be solved by successive approximations. Gunter solved the problem in a clever way. For a given azimuth A, calculate the Sun's altitude, h_0, when the declination is zero, i.e. the Sun is on the equator, from:

$$\tan h_0 = \cos A / \tan \varphi$$

Here A is the azimuth from south. Then calculate the additional altitude from the equator, a, at the desired solar declination from:

$$\sin a = \cos h0 \sin \delta / \sin \varphi$$

Calculate the Sun's altitude, h, for the azimuth and declination:

$$\text{If } A < 90°,\ h = a + h0$$

$$\text{If } A > 90°,\ h = a - h0$$

The calculated points are plotted on the graph and connected with a smooth curve or approximated with connected line segments if a large number of points are calculated. The coordinates of the points are calculated as for the curve of the Sun's noon altitude above, but using the horizontal zodiac scale.

The graph of the qibla is often (usually?) drawn as circles determined from the altitudes calculated for the tropics and the equator. While this is not strictly correct, the result is within the accuracy of the instrument as the maximum error is in the order of a degree or so. Figure 8-35 shows the graph of the qibla accurately calculated and approximated with circles derived from the three aforementioned points.

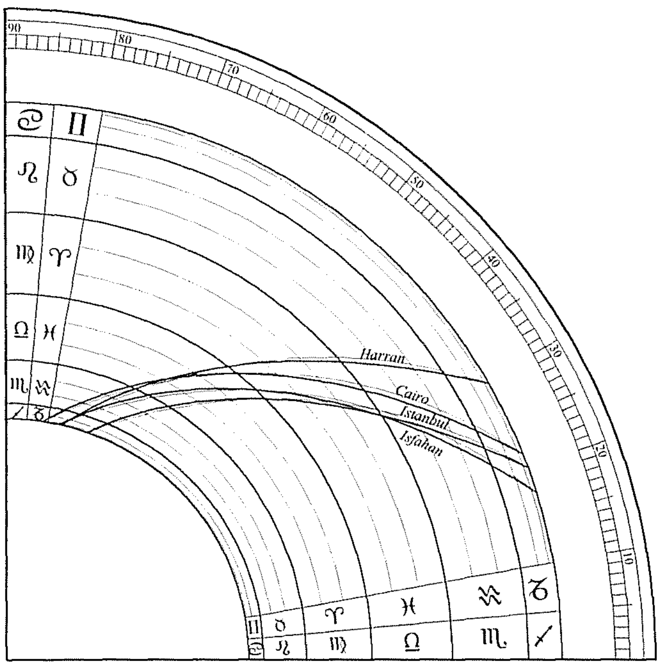


Figure 8-35. Graph of the qibla with approximate circles

An equally divided arcs of the signs quadrant was sometimes used for a scale of prayer times. The time of the 'asr prayer is astronomically determined to begin at the end of the ninth unequal hour of the day. The most common method suggested for finding this time is to note when the shadow of a vertical gnomon is equal to the length of the shadow at noon, plus the length of the gnomon. Curves were drawn in this quadrant relating the Sun's altitude at this time for the day of the year. The resulting sigmoid curve is shown on the scale. The same information is available from the scale of cotangents.

Other Scales on the backs of Islamic Astrolabes

Islamic astrolabs are often impenetrable by English speakers. Even if you read Arabic, the variations in style can be a barrier and the content of the scales may require specialized knowledge to interpret.

There is no such thing as a "typical" Islamic astrolabe back, which is one point that makes them so interesting to historians. There are also meaningful differences between instruments from the Mashriq and Mahgrib or Western Islam. The additional scales often relate to astrological practices but may include geographic or calendric information.

Lunar Mansions

Lunar Mansions (nakshatra – constellation), which are very ancient and of Indian origin, were often included on Islamic astrolabes from all regions. The ideas behind the Lunar Mansions were adopted in Mesopotamia and by the Chinese (called Shiu). In concept, the Lunar Mansions are somewhat like the Egyptian decans associating the season with heliacal rising of stars. Their original purpose was calendric, but they later became mainly associated with astrology.

The Lunar Mansions divide the ecliptic into 28 sections which represent lunar motion in the sidereal month[76]. Thus, the mansions are 12° 51′ 25.7″ of longitude wide and 2 1/3 mansions are the width of one zodiac section. Each mansion is associated with one or more bright star, therefore the positions of the mansions change over time due to precession.

The original use of the Lunar Mansions was to provide common people with a method for estimating seasonal changes[77]. al-Bīrūnī included a description of the mansions in *Chronology of Ancient Nations* and states that poems were used as a mnemonic device to aid in the interpretation. For example, "If the moon is in Thurayya (Pleiades) on the third day, winter is going away". Since the longitude of the Pleides at that time was about 40° and the moon is about 40° from the Sun on the third day of the lunar month, then the Sun must be near the beginning of Aries, so spring is starting.

Later use of Lunar Mansions associated the mansions with the months of the Julian calendar. Most applications of Lunar Mansions to astrolabes was astrological.

Figure 8-36 shows the Lunar Mansion and zodiac scales on the astrolabe on page 25, Figure 1-21.

This scale is often a circular or semi-circular scale outside the shadow square.

[76] Neugebauer, O., *A History of Ancient Mathematical Astronomy*, Springer-Verlag (1975), p. 6.
[77] The Greeks used a similar method dating from the fifth century BC called "parapegma" where a peg was moved in a public calendar to show the advance of the seasons.

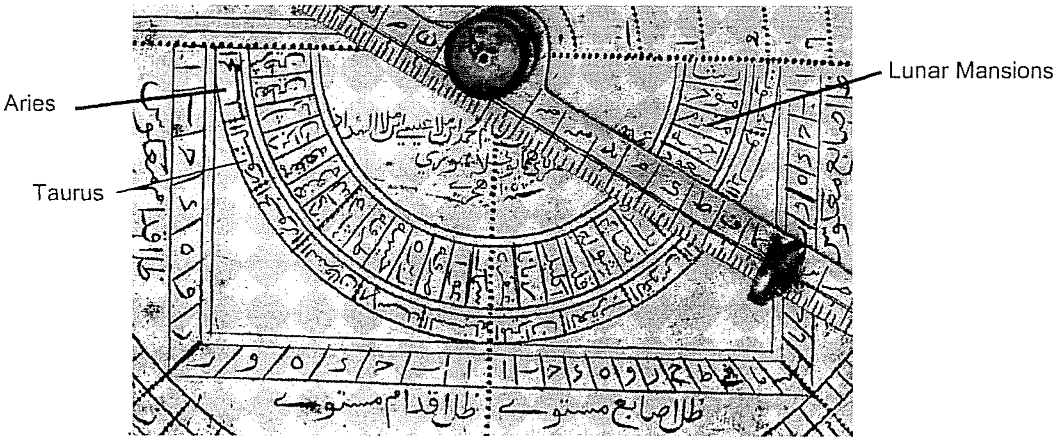


Figure 8-36. Lunar Mansions

Calendars

Some astrolabes from the Maghrib and Middle East include a circular calendar scale with Arabic month names. Monthly calendar scales were never included on old astrolabes from Eastern Islam.

Astrological Scales

Scales and tables related to astrology were common on astrolabes from the Mashriq. A wide variety of scales were used and the back of the instrument may contain lists of planetary influences. One relatively common table listed "triplicities" associating the signs of the zodiac with signs forming equilateral triangles and the planets governing the relationship[78].

Qibla tables

Rather than curves showing the azimuth of the qibla, some astrolabes had a list of qibla directions for a number of cities engraved in the *umm*.

[78] Gibbs, Sharon with Saliba, George, *Planispheric Astrolabes from the National Museum of American History*, Smithsonian Institution Press, City of Washington (1984), p. 38,

A Modern Astrolabe Back

Classic astrolabes have a certain charm and dignity and open many doors to appreciating cultures. They were clearly quite useful in the eras when astrolabes were widely used. However, not only do times change but timekeeping itself has changed in the last few hundred years. Unequal hours are now a historical curiosity, and the use of standard time zones makes apparent solar time an untidy relic.

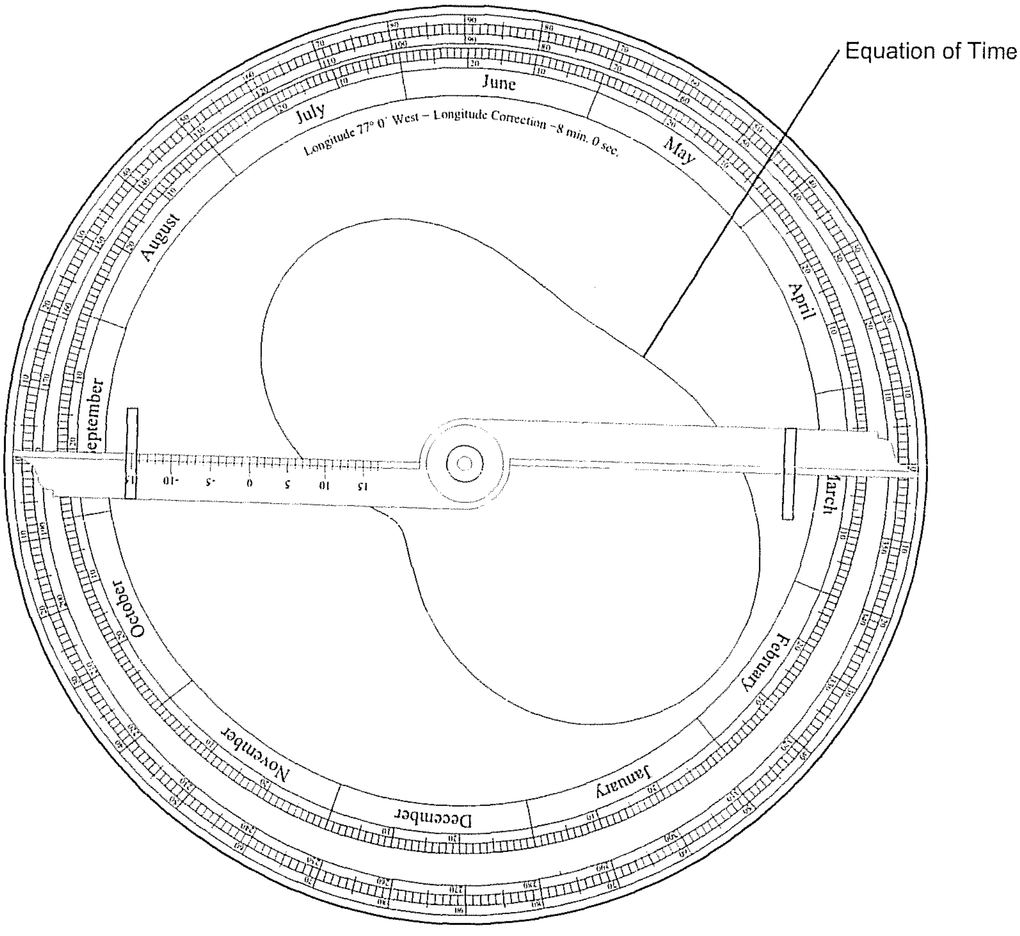


Figure 8-37. Modern Astrolabe Back

The picture above presents one approach to modernizing the astrolabe back. This approach replaces the scales on the old instruments with a single scale allowing easy conversion to zone time from the apparent solar time found on the front of the instrument. The kidney-shaped figure on the back is the equation of time[79], and the alidade is divided into minutes of correction for the equation of time. The longitude and longitude correction are printed on the back as a reminder.

Notice also there is no zodiac scale on this rendition. Many students have difficulty separating the idea of the zodiac from astrology. Removal of the zodiac scale from the back of the astrolabe simply removes this point from the discussion. Solar longitude is shown from 0-360°, and the rete on the front of the instrument is divided directly by the calendar.

---

[79] See the Astronomical Background chapter for a basic discussion of the equation of time.

The motivation behind this approach is to make the astrolabe more useful in our world by making it easy to find standard time from the altitude of a Sun or star and to introduce the concepts of longitude correction and the equation of time.

Using the Modern Astrolabe Back

Find the apparent solar time using the front of the astrolabe. Subtract the longitude correction from this time. Position the alidade to the date and read the value of the equation of time from the scale on the alidade. Subtract this value to get the zone time. Correct for Daylight Savings Time if necessary.

For example, we find the time at Washington, DC to be 11 AM apparent solar time on September 21. We wish to find the zone time.

1. Subtract the -8 min. longitude correction read from the circular text above the city name from 11:00 AM to get 11:08 AM.
2. Rotate the alidade to near noon on September 21, and read 7 minutes from the point where the alidade scale intersects the equation of time curve.
3. Subtract the 7 min. equation of time from 11:08 AM to get 11:01 AM..
4. Daylight savings time is in effect for this example so add one hour to get the zone time of 12:01 PM.

Models of similar astrolabes have been used successfully by hundreds of hikers, sailors and outdoors people, and many students ranging from fourth grade to university graduate students have been introduced to astronomy using this type of astrolabe.

Making the Modern Astrolabe Back

The solar longitude scale divisions are exactly the same as the classic astrolabe back except they are labeled from 0° to 360° continuously.

The calendar is identical to the classic astrolabe calendar.

Calculate the equation of time for each day of the year and plot the value in polar coordinates. These two subjects are covered in reverse order.

Plotting the equation of time "kidney" is a bit tricky, and a wide range of results is possible. You can control the shape and size of the curve depending on how it is defined. The values used in the figure are described. You can experiment with different values.

Let the value of the equation of time for a day be E. Two circles must be defined, one to define the outer limit and one for the inner limit. Let r_O be the outer circle which is chosen to be some value just inside the calendar scale. Let r_I be the radius of the inner circle. It was chosen as $r_O / 4$ in the example. Let r_0 be the radius of the circle when E = 0 and is .625 r_O in the example, which puts the 0 circle midway between the other two circles.

The equation of time ranges from about +17 minutes to about -15 minutes. The value of E must be scaled to fit into the circle dimensions. The scaling factor is $\mathbf{S = (r_O - r_I) / 34}$ which scales the values linearly between $-17 < E < 17$.

The values of E are then linearly scaled to a radial value from the center with $\mathbf{r = r_O - E \times S}$. The minus sign places positive values of E toward the inside of the back. Let λ be the Sun's geocentric longitude for the day. The point is plotted at:

$$x = r \cos \lambda, \quad y = r \sin \lambda$$

The scale on the alidade is simply divided by r.

Calculating the equation of time looks harder than it is. There are many ways to calculate the equation of time with varying degrees of accuracy. The following method is from Smart[80] and is adequate for this application. Three values are needed for the calculation, all of which are discussed in the chapter on astronomical calculations (page 359):

ε = obliquity of the ecliptic

M = Sun's mean anomaly (radians)

L = Sun's mean longitude (radians)

$y = \tan^2 \varepsilon/2$

$$\mathbf{E = y \sin 2L - 2e \sin M + 4ey \sin M \cos 2 L - 1/2\, y^2 \sin 4 L - 5/4\, e^2 \sin 2M}$$

The value of E in the above equation is in radians. Convert E to minutes with $4 \times E \times \pi/180$.

This scaling is not the only possible way to show the equation of time. It was chosen to allow the most precise values to be read while retaining artistic balance.

Other methods of calculating the equation of time are mentioned in the glossary.

[80] Smart, W. M., *Textbook on Spherical Astronomy*, Cambridge University Press, New York, 1977.

Chapter 9 - Sample Problems

This chapter includes several solved sample problems graduated in difficulty for the planispheric astrolabe. Many more applications of the astrolabe will occur to you once you have mastered these fundamentals. Astrolabes for different latitudes and longitudes are used to illustrate differences in the plates for different locations. The appropriate plate for the latitude of the problem is assumed to have been inserted in the astrolabe.

1. **Find the time and direction of sunrise on November 15, in Washington, DC at 38° 53′ N, 77° W.**

   Find the time of sunrise by finding the time when the Sun is exactly on the eastern horizon.

   First, find the Sun's longitude using the back of the astrolabe. The alidade is set to November 15 on the back. We note the date tics are for midnight for each date. Therefore, the alidade is set to about 1/4 of the space between the tics for November 14 and 15. The alidade points to Scorpio 23° (Figure 9-1).

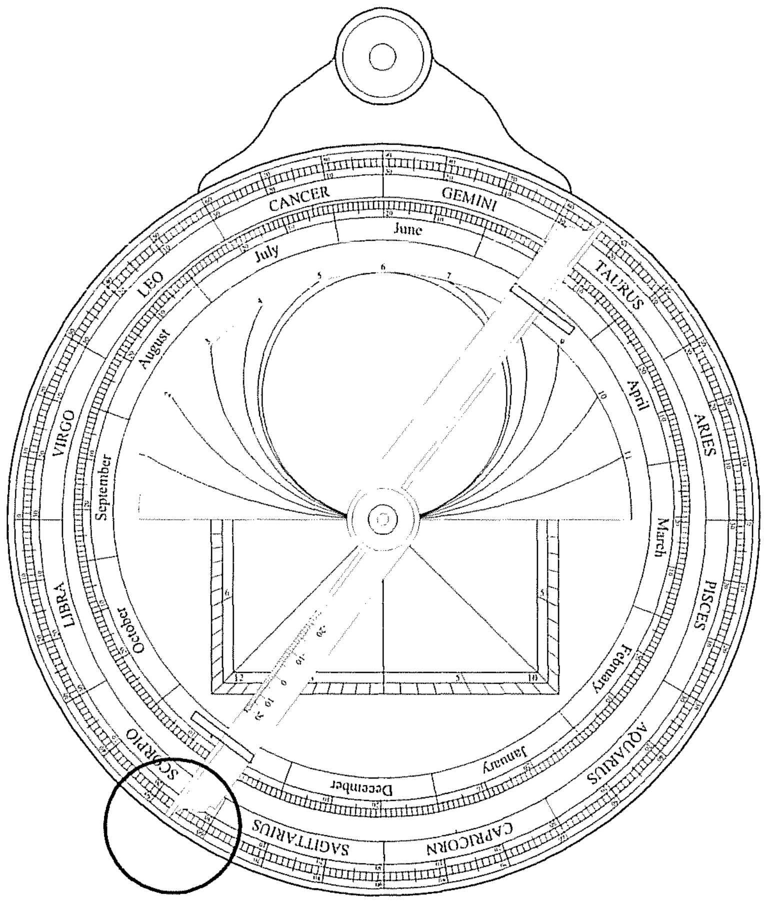


Figure 9-1. Back set to November 15

Turn the astrolabe over and use the front for the rest of the problem. Set the rule to Scorpio 23° on the ecliptic. The intersection of the rule and Scorpio 23° on the ecliptic is the Sun's position for November 19. Move the rete and rule together until the intersection point is exactly over the horizon in the east.

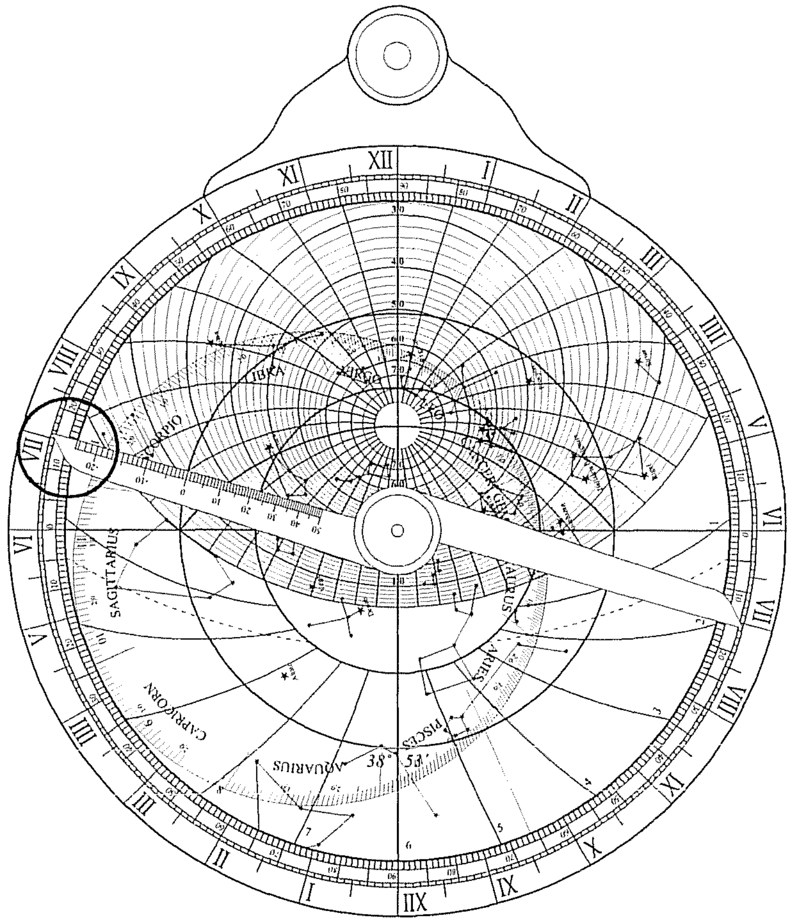


Figure 9-2. Astrolabe set to sunrise on November 15.

The rule points to 7:03 AM on the limb. This is the apparent solar time of sunrise. We now need to convert apparent solar time to local zone time (EST).

Our longitude is 77° W. The center of the time zone is 75° so we are 2° west of the center of the time zone. The Sun must travel an additional 2° from the time zone center to our location. The Sun moves 15° in one hour or 1° in four minutes. Therefore, it takes the Sun an additional 8 minutes to travel from the center of the time zone until it is due south of us. This 8 minutes is called the *longitude correction*. We are west of the time zone center so the Sun is on our meridian later. The sign of the longitude correction is negative to show the Sun is late for our location. The longitude correction is **subtracted** from apparent solar time to get zone time. Therefore, the local zone time of sunrise is 7:03 AM - (-8) minutes = 7:11 AM. This time could also be corrected for the equation of time, which will be covered in a later problem. To be perfectly accurate, sunset and sunrise are defined as the time when the upper limb of the Sun is tangent to the horizon. This occurs when the center of the Sun is about 0.833° below the horizon. The difference in time is about 5 min. for this example. One could put an additional altitude arc at 0.833° below the horizon for working this problem, but it would not enhance the accuracy of the instrument.

The direction of sunrise is shown by the azimuth arcs. The Sun's position is about 4° east of the azimuth arc 20° south of east. Azimuth angles are measured from north, increasing to the east. The azimuth of sunrise on November 15, in Washington, DC on November 15, is about 114°.

2. What is the Sun's maximum altitude on March 7, in New York City (40° 43′ N, 74° W)?

This problem can be solved in two ways on the astrolabe. Both methods require finding the Sun's longitude. We use the alidade on the back to find the Sun's longitude on March 7, to be Pisces 17°.

The Sun's maximum altitude will be at local apparent noon so set the rule to Pisces 17° on the ecliptic and rotate the ecliptic so the Sun's position is exactly on the meridian. The Sun is almost exactly on the 44° almucantar.

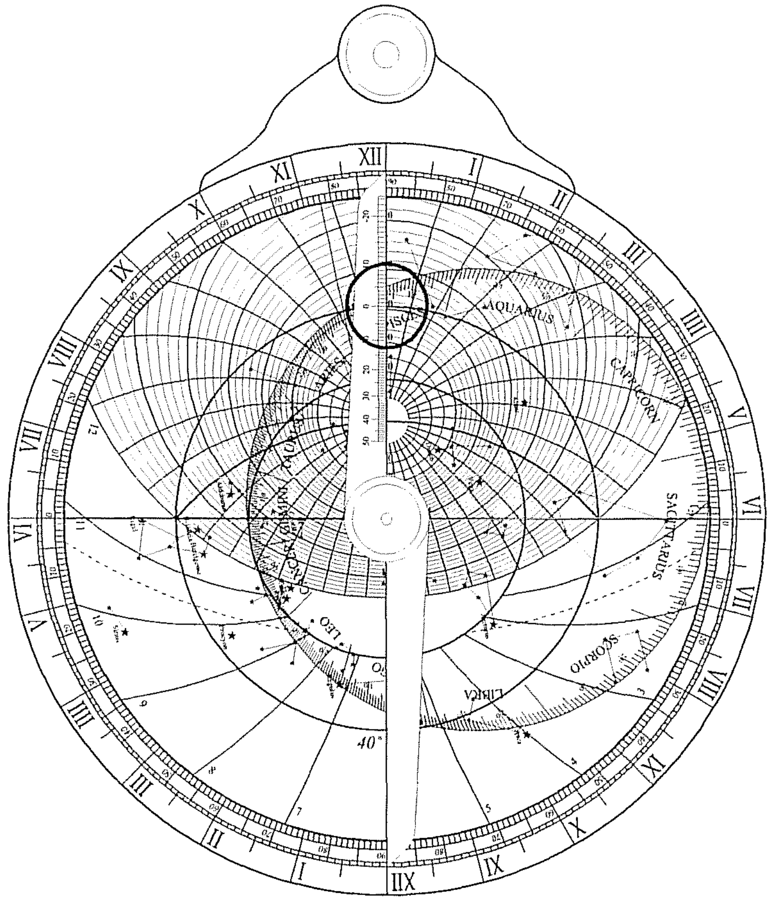


Figure 9-3. Sun's maximum altitude on March 7, in New York

This problem can also be solved arithmetically to get slightly greater accuracy. The Sun's maximum altitude for a day is $a = 90 - (\varphi + \delta$. Set the rule to Pisces 17° and note its declination is -5° (actually 5° 8′). Therefore, a = 90 - 40° 43′ + (-5°) = 44° 17′ (44° 9′).

3. What is the time when we measure the altitude of Betelgeuse as 50° in the west in Dallas (32° 49′ N, 96° 47′ W) on February 24.

After measuring the altitude of Betelgeuse with the alidade we find the Sun's longitude on February 24 from the back of the astrolabe is Pisces 6° and a little. Set the rete so Betelgeuse (the upper right star in Orion) is on the 50° almucantar in the west. Then set the rule to Pisces 6°+ on the ecliptic. The rule points to 9:35 PM on the limb.

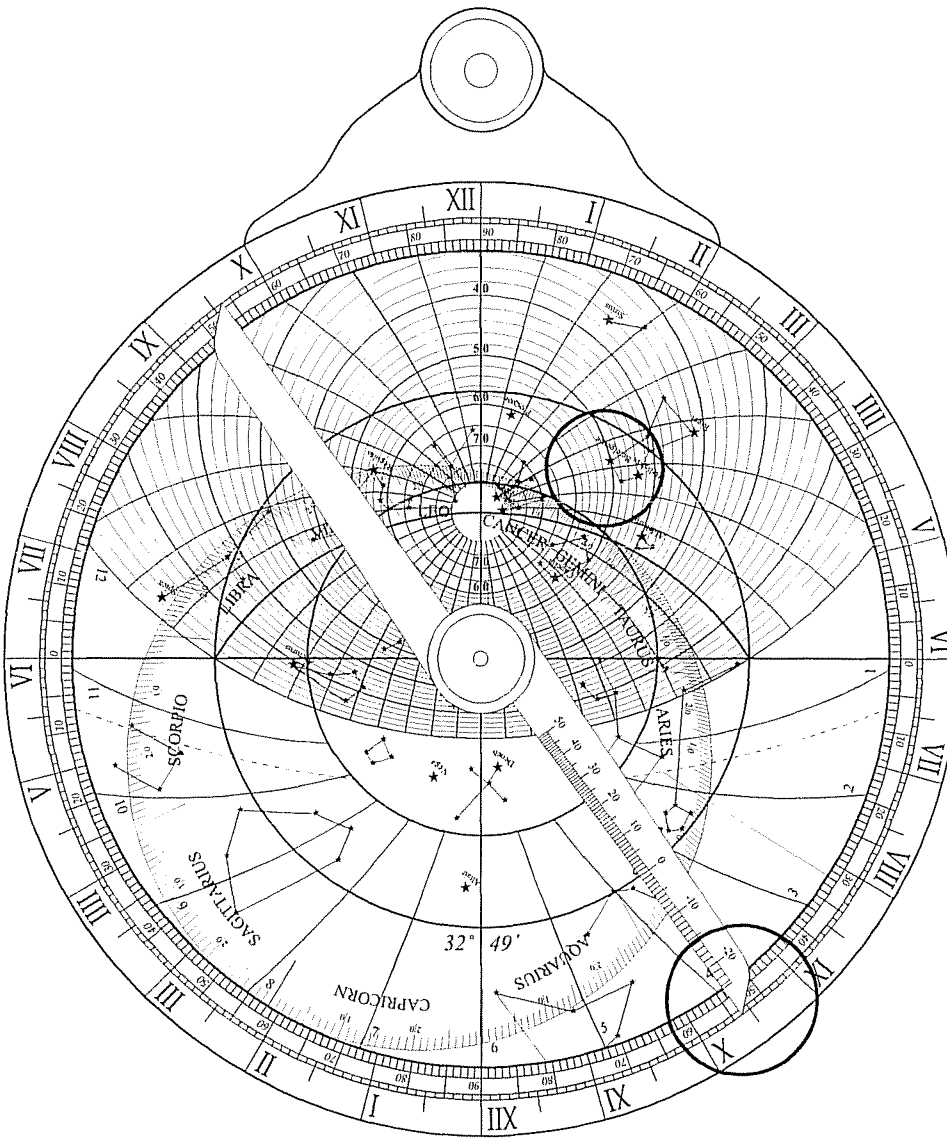


Figure 9-4. Betelgeuse at 50 in west on February 24, in Dallas

The apparent solar time must be corrected for longitude. Dallas is at 96° 47′ W longitude and the center of the Central Standard Time zone is at 90° W. 90° - 96° 47′ = -6° 47′ × 4 min./degree = -27 min 8 sec. Subtract the longitude correction to get zone time: 9:35 - (- 0:27:08) ≅ 10:02 PM.

4. What is the approximate right ascension and declination of Sirius?

Only the front of the astrolabe is needed to solve this problem. Set the rete so Sirius is on the meridian. Place the rule so the edge is on Sirius. Read the declination from the scale on the rule as a little less than -17° (-16° 43'). The first point of Aries points to the right ascension on the limb: 6 hr. 45 min.

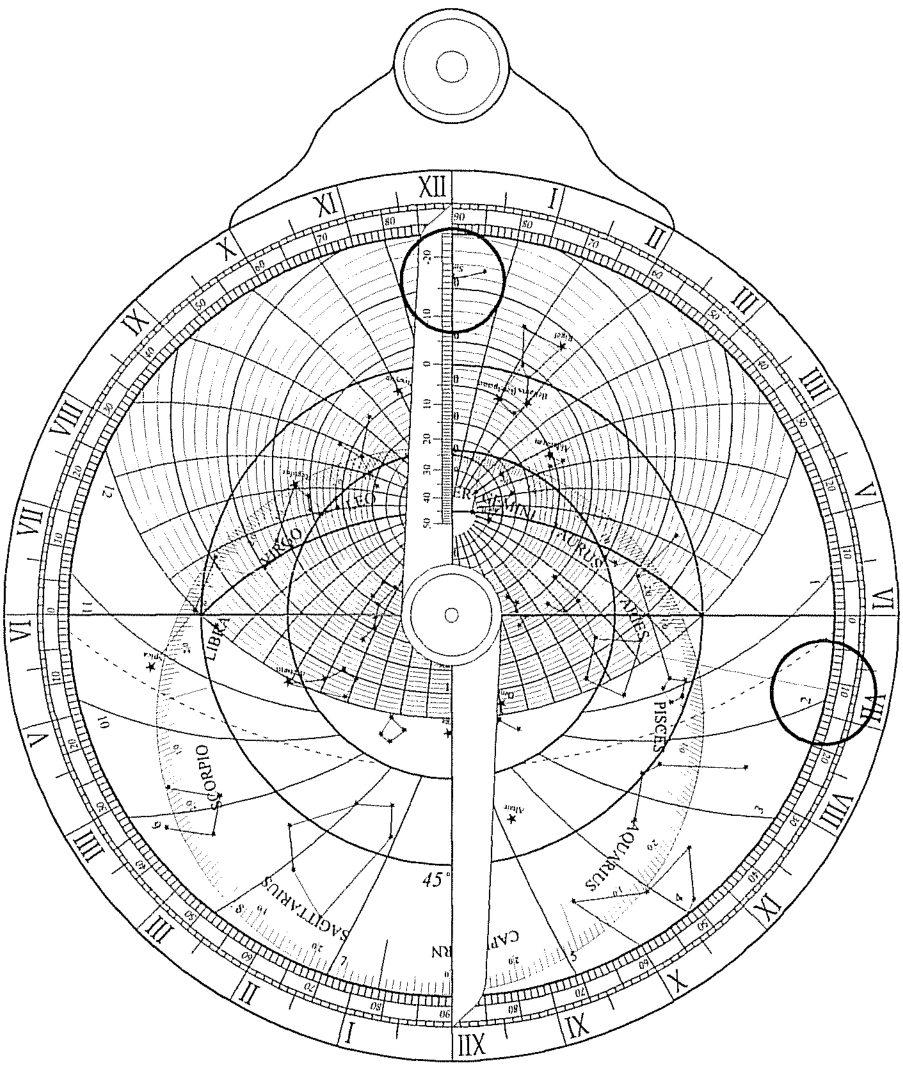


Figure 9-5. Sirius' coordinates

The right ascension and declination of any celestial object represented on the rete can be found in this way. In addition, objects such as the moon or planets can be added by looking up the coordinates and marking the position on the rete. Problems can then be solved using the added object. For example, it is actually possible to predict lunar eclipses using an astrolabe. You must find dates when there is a full moon and the moon's latitude is near zero. You can then mark the moon's position on the rete and test whether it is opposite the Sun. McClusky gives an example[81]. Astrologers would find the coordinates of a planet from an almanac, note the position on the rete and use the astrolabe to find the house and horoscope elements.

[81] McClusky, Stephen C., "Astronomies and Cultures in Early Medieval Europe", Cambridge University Press, 1998. p. 180.

5. What is the sidereal time at Paris (48° 52′ N, 2° 20′ E) at 10:00 PM on February 5?

We will use a modernized astrolabe for this problem to illustrate how to use an astrolabe to work with modern timekeeping. The modern astrolabe has the ecliptic divided directly by the calendar to allow the solar longitude to be set in a single step on the front of the astrolabe, and the back includes the equation of time.

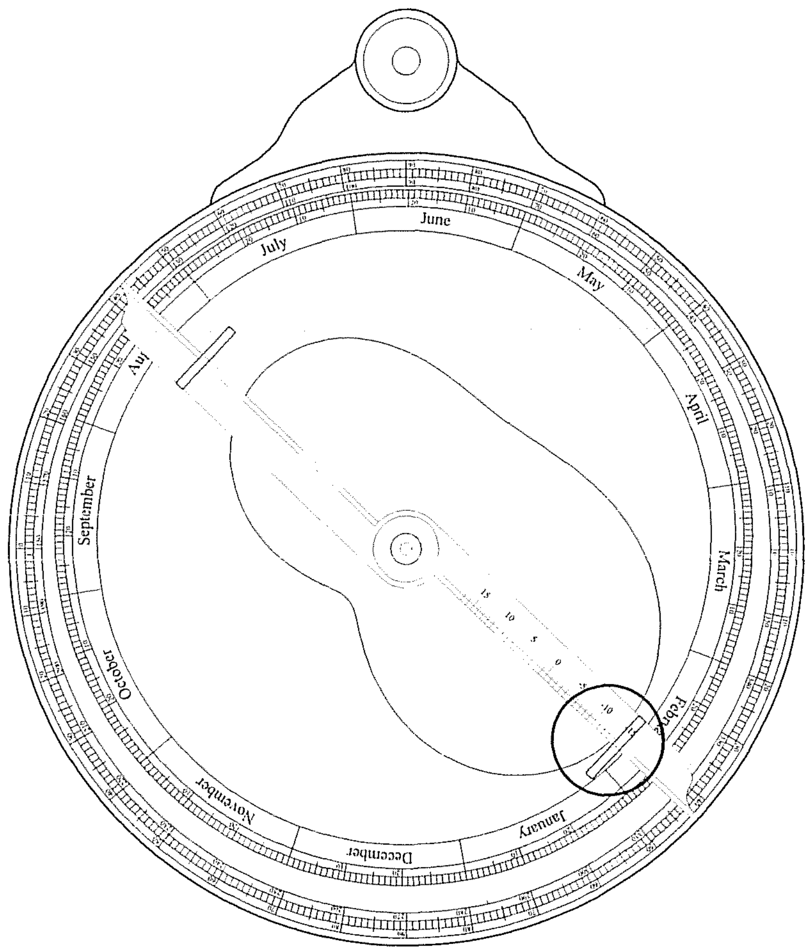


Figure 9-6. Equation of Time for February 5

The astrolabe shows local apparent time, so we must work backwards from zone time. Add the longitude correction and equation of time to the zone time to find the local apparent time. Paris is in the Central European time zone centered at 15° E longitude. The longitude correction is nearly an hour (-50 min 40 sec.), because Paris is located just a little east of Greenwich but uses CET for political reasons (longitude corrections in Spain and Portugal are more than hour, because much of Spain and all of Portugal are actually west of Greenwich but use CET).

The value of the equation of time for February 5, is found from the back of the modern astrolabe. Align the edge of the alidade with February 5, and read -14 min. from the scale.

Thus, the local apparent time in Paris at 10:00 PM on February 5, is: 10:00 + (-0:51) + (-0:14) = 8:55 PM.

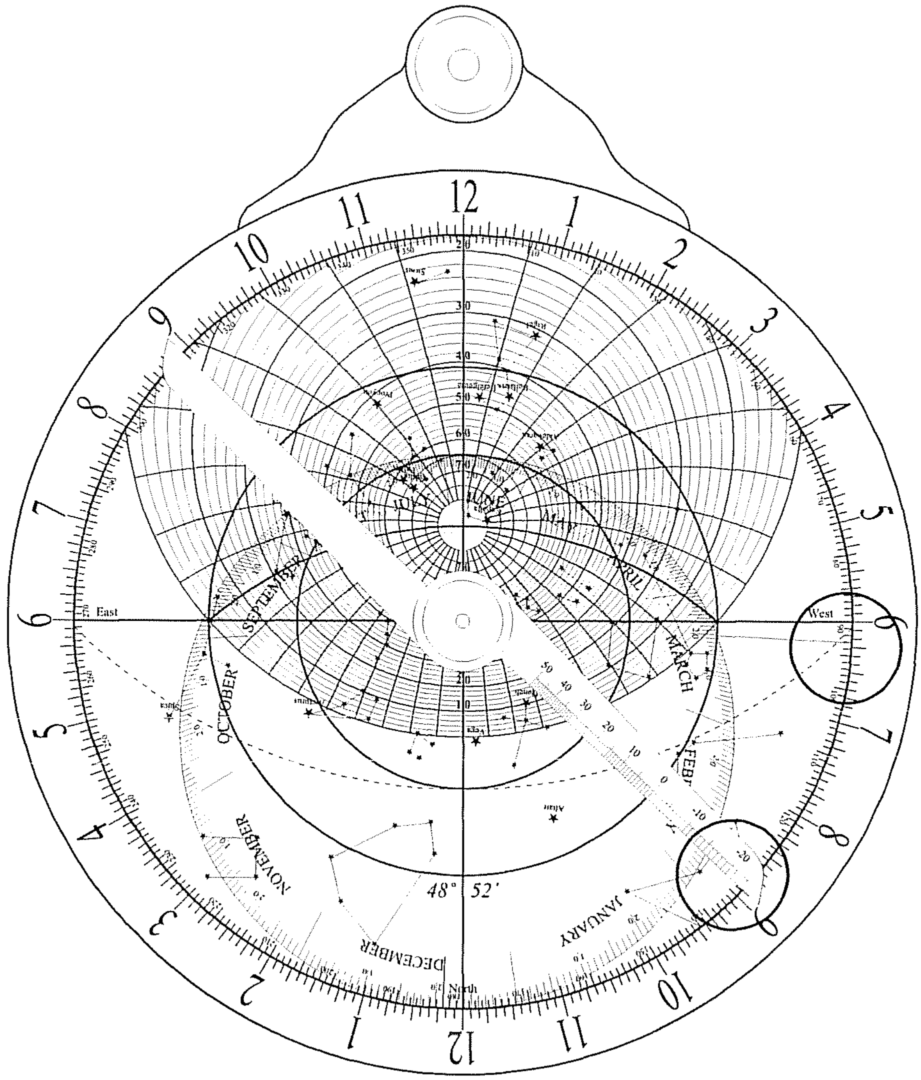


Figure 9-7. Paris, 8:55 PM, February 5

Set the rule at February 5, on the ecliptic and rotate the rete and rule together until the rule points to 8:55 PM. Read the sidereal time from the First Point of Aries as about 6:12 (6:12:16.2).

This problem illustrates the necessity of correcting apparent times for longitude and the equation of time for some places and dates, since the correction here was an hour and five minutes

Chapter 10 - The Astrolabe for Southern Latitudes

The principles of an astrolabe made for southern latitudes are identical to those of an instrument made for the northern hemisphere, but there are several detailed changes that must be considered. These changes may seem a little confusing at first, but they are very easy to get used to and the intuitive appeal of the astrolabe is not diminished in any way once one has adapted to the orientation.

It is interesting to wonder what changes in everyday life would have evolved if ancient civilizations had originated in the southern hemisphere instead of the north.

Virtually all astronomical and timekeeping conventions originated in middle northern latitudes. This is easy to justify, because this was where civilization was centered when these conventions originated. An excellent example is the convention of "clockwise" and "counterclockwise".

Consider the sundial. The Sun is always to the south when it is observed from any latitude north of the Tropic of Cancer. Therefore, a sundial for middle northern latitudes is oriented so the style (the edge of the plate casting the shadow) points to the north celestial pole (the style is parallel to the Earth's axis on most types of sundials). The sundial's noon line is along your North-South meridian. When you face north, east is to your right. When the Sun rises in the east, the shadow on the sundial will fall to the left of the meridian and move "clockwise" through the day until sunset. There is no question our tradition of clockwise motion is a direct result of the movement of the shadow on a sundial made for northern latitudes. The Sun reaches its maximum altitude when it is due south in the northern hemisphere.

The situation is reversed in the southern hemisphere. For latitudes south of the Tropic of Capricorn (which is just south of Rio de Janeiro and passes a little north of South Africa), the Sun is always to the north. Therefore, the style of the sundial points to the south celestial pole. When the Sun rises in the east, which is to your left when you face south, the shadow will be to the right of the meridian and will move counterclockwise through the day. By implication, if sundials had been invented in the southern hemisphere, our sense of clockwise and counterclockwise would be reversed,and clocks hands would move in the opposite direction to what we consider normal.

This reversal in orientation also applies to the astrolabe, since it is based on the movement of the Sun. The following sections describe the differences in an astrolabe designed for northern and southern latitudes.

The Plate

The basic design of the plate is identical for northern and southern latitude instruments except for the orientation and the numerals around the limb. See Figure 10-1, which shows an example of an astrolabe front for Auckland, New Zealand. On an astrolabe for southern latitudes the top of the instrument represents north with east to the right. In addition, the numerals around the limb proceed in a counterclockwise direction. Therefore, sunrise is shown by the Sun's position on the ecliptic rising in the east, which is the right side of the instrument, and the stars will rise and set as the rete rotates counterclockwise. Note that azimuth is measured from the north, increasing to the east in both hemispheres, so the azimuth angle is counted from the top of the instrument, increasing in a clockwise direction.

The positions of the tropics are also reversed on a southern astrolabe, so the outer limit of the plate is the Tropic of Cancer and the Tropic of Capricorn is the inner circle. The location of the equator does not change.

The direction of the numbering of the unequal hours and "Houses of Heaven" is also reversed.

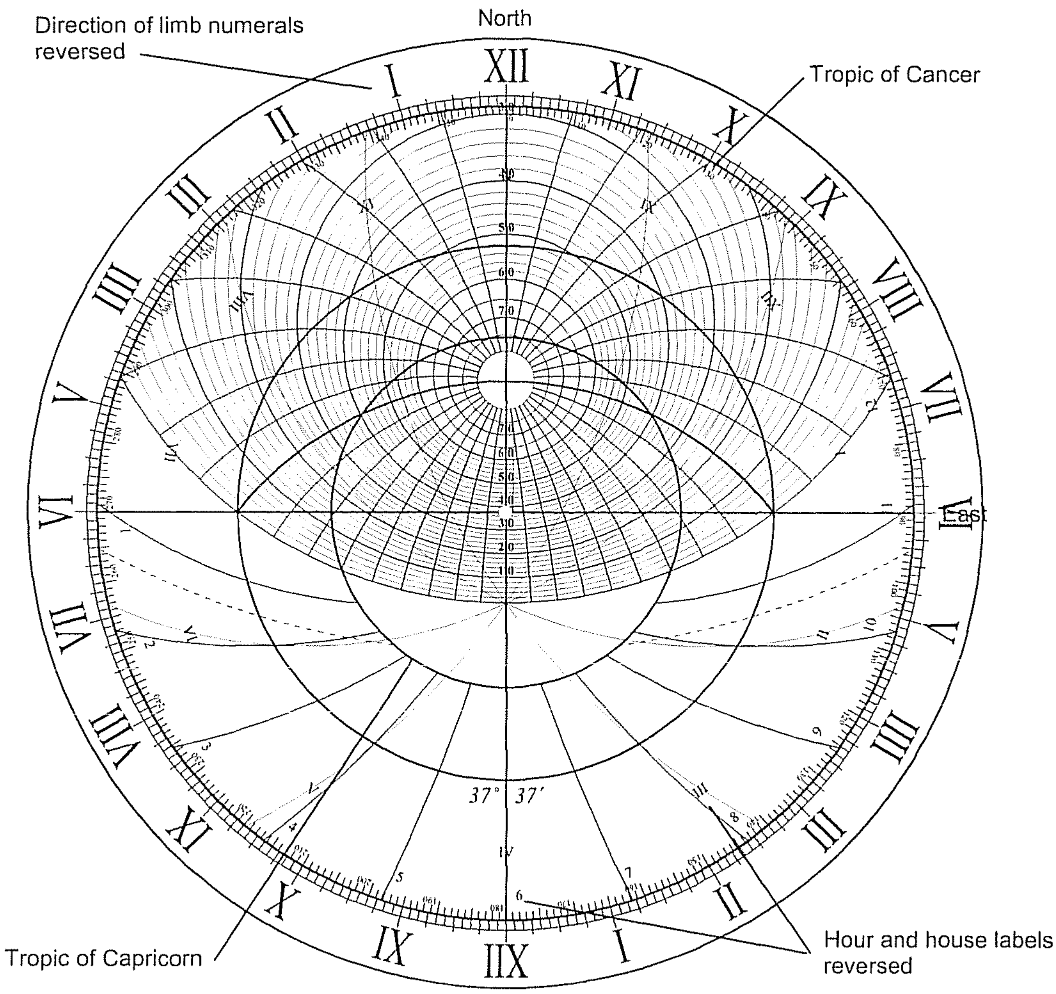


Figure 10-1. Astrolabe Front for Southern Latitudes

The Rete

The rete has the most difference between northern and southern astrolabes (Figure 10-2). First, the rotation of the rete is reversed so it rotates in a counterclockwise direction. The sense of declination is reversed so positive declinations are now to the outside of the instrument and the entire ecliptic is reversed. Of course, the rete contains different stars for southern latitudes, particularly those south of the Tropic of Capricorn. The southern latitude version of the rete in the figure contains 154 stars. Notice the distortion in size for northern constellations caused the projection.

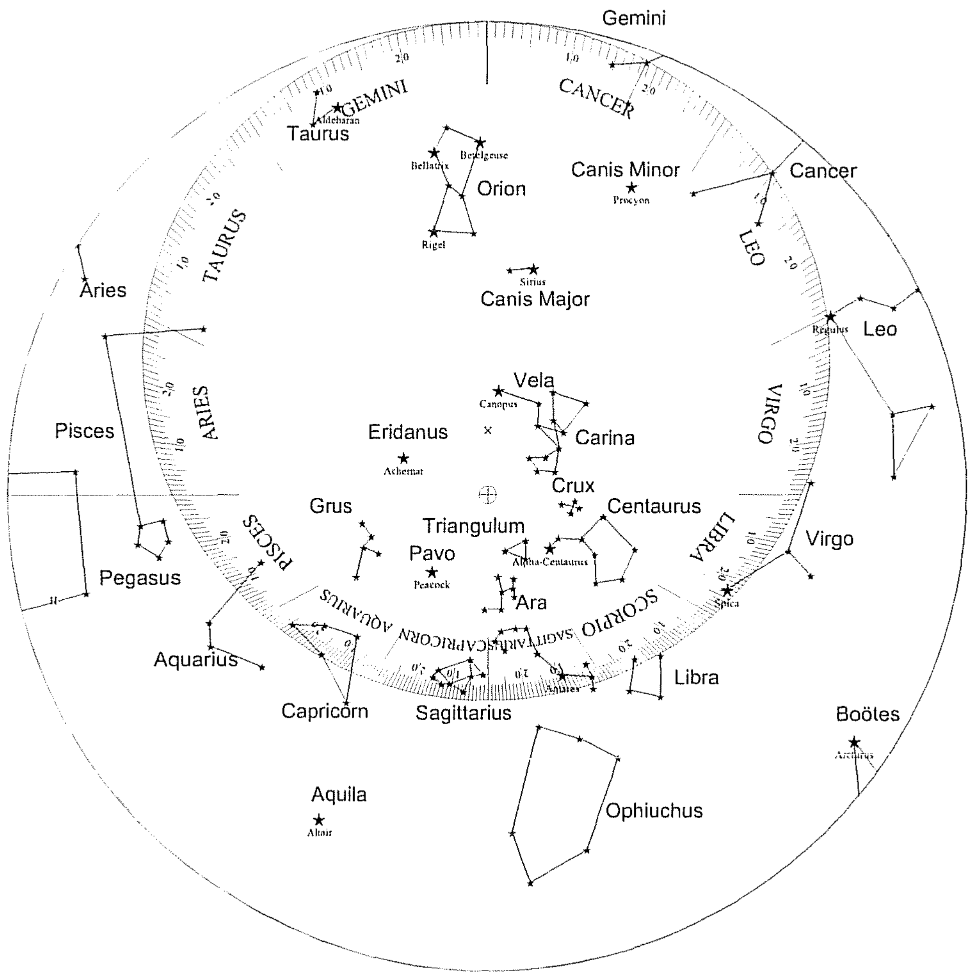


Figure 10-2. Astrolabe Rete for Southern Latitudes

The Rule

The only change to the rule is the sense of declination with positive declinations being toward the outside of the instrument for southern latitudes.

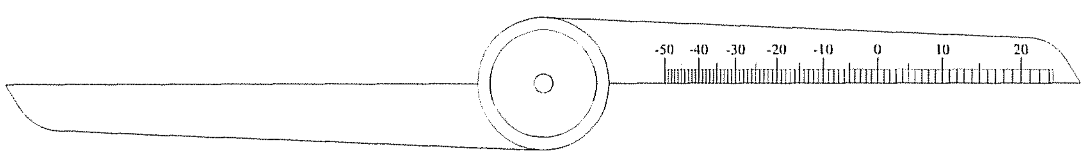


Figure 10-3. Astrolabe Rule for Southern Latitudes

The back of the astrolabe is the same as for northern latitudes.

The principles for using the astrolabe to solve astronomical problems is identical for the two instruments. The only major difference in working problems is the counterclockwise rotation of the rete and rule.

Chapter 11 - Calculation Summary

This chapter contains a summary of the calculations required to draw a planispheric astrolabe plate. Figure 11-1 shows the astrolabe plate elements. The planispheric astrolabe plate has many interrelated elements, and the figure annotation gets somewhat congested. Therefore, a subset of the figure is shown for each element.

Following the equations is a table of calculated values that can be used to confirm your own calculations. These examples are dimension free. It does not matter whether the measurement is in inches, centimeters or some other system. All examples are for unit dimensions (e.g. equator radius = 1 unit). You can multiply the values in the tables by the actual size to get a value for your instrument. Results are shown to five decimal places, which is much more precise than needed, in order for you to compare your results.

A listing of the astronomical values for a complete year is in an appendix. These values can be used to determine required constants such as the obliquity of the ecliptic.

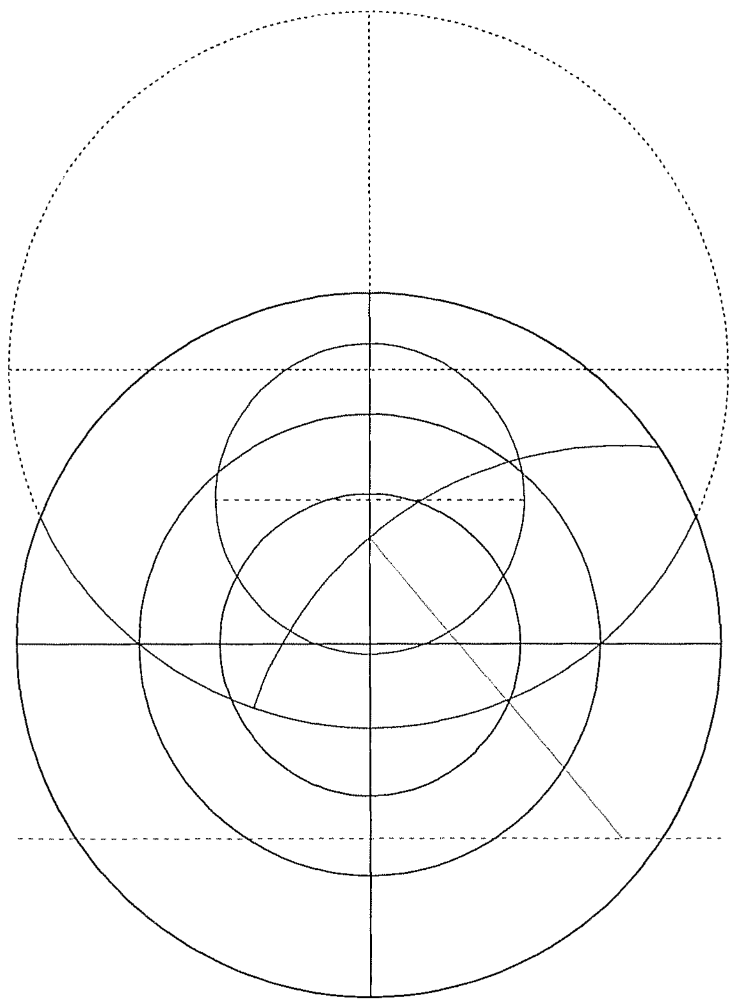
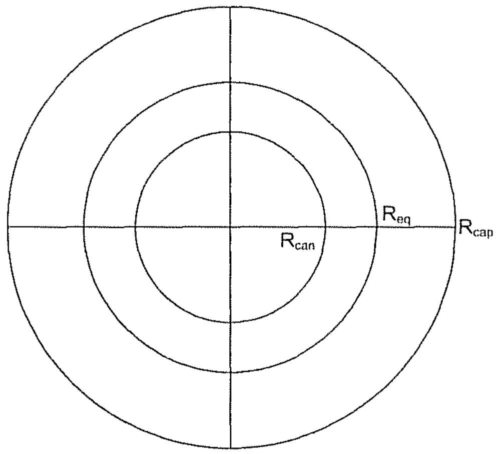


Figure 11-1. Astrolabe Plate Elements

Tropics



Radius of Equator when Tropic of Capricorn radius given: $\mathbf{R_{eq} = R_{cap} \tan(\frac{90-\varepsilon}{2})}$

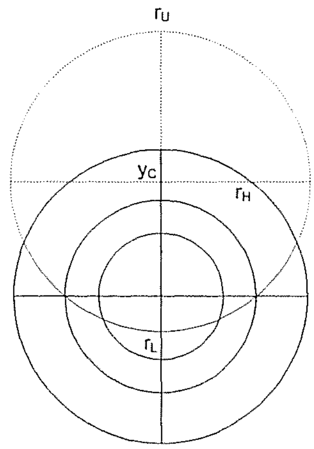
Radius of Tropic of Cancer: $\mathbf{R_{can} = R_{eq} \tan(\frac{90-\varepsilon}{2})}$

Radius of Tropic of Capricorn when Equator radius given: $\mathbf{R_{cap} = R_{eq} \tan(\frac{90+\varepsilon}{2})}$

Example: Year = 2006, ε = 23° 26′ 18.4″ = 23.438446°

R_{cap}	R_{eq}	R_{can}
1.52347	1.00000	0.65640
1.00000	0.65640	0.43086

Horizon



Upper meridian intersection: $r_U = R_{eq} \cot\frac{\varphi}{2}$

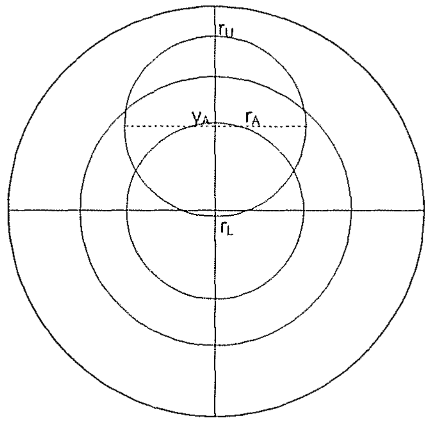
Lower meridian intersection: $r_L = -R_{eq} \tan\frac{\varphi}{2}$.

Center of horizon from origin: $y_C = \frac{R_{eq}}{\tan\varphi} = R_{eq}\cot\varphi = R_{eq}\tan(90-\varphi) = \frac{r_u + r_l}{2}$

Horizon radius: $r_H = \frac{R_{eq}}{\sin\varphi} = \frac{r_u - r_l}{2}$

Lat	r_U	r_L	y_C	r_H
25	4.51071	-0.22169	2.14451	2.36620
30	3.73205	-0.26795	1.73205	2.00000
35	3.17159	-0.31530	1.42815	1.74345
40	2.74748	-0.36397	1.19175	1.55572
45	2.41421	-0.41421	1.00000	1.41421
50	2.14451	-0.46631	0.83910	1.30541
55	1.92098	-0.52057	0.70021	1.22077
60	1.73205	-0.57735	0.57735	1.15470

Almucantars



Upper meridian intersection: $r_U = R_{eq} \cot \frac{(\varphi + a)}{2}$

Lower meridian intersection below origin: $r_L = -R_{eq} \tan \frac{(\varphi - a)}{2}$

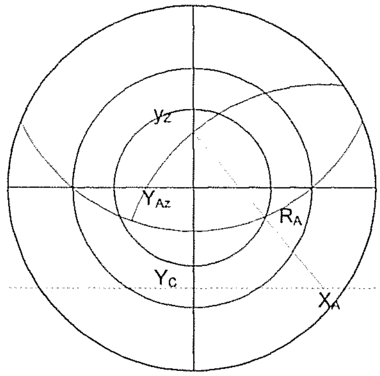
Center of almucantar from origin: $y_A = R_{eq} \frac{\cos \varphi}{\sin \varphi + \sin a}$

Almucantar radius: $r_A = R_{eq} \frac{\cos a}{\sin \varphi + \sin a}$

$\Phi =$ 40

a	r_U	r_L	y_C	r_H
-18	5.14455	-0.55431	2.29512	2.84943
-12	4.01078	-0.48773	1.76152	2.24926
-6	3.27085	-0.42447	1.42319	1.84766
0	2.74748	-0.36397	1.19175	1.55572
5	2.41421	-0.31530	1.04946	1.36476
10	2.14451	-0.26795	0.93828	1.20623
15	1.92098	-0.22169	0.84964	1.07134
20	1.73205	-0.17633	0.77786	0.95419
25	1.56969	-0.13165	0.71902	0.85067
30	1.42815	-0.08749	0.67033	0.75782
35	1.30323	-0.04366	0.62978	0.67344
40	1.19175	0.00000	0.59588	0.59588
45	1.09131	0.04366	0.56748	0.52382
50	1.00000	0.08749	0.54374	0.45626
55	0.91633	0.13165	0.52399	0.39234
60	0.83910	0.17633	0.50771	0.33139
65	0.76733	0.22169	0.49451	0.27282
70	0.70021	0.26795	0.48408	0.21613
75	0.63707	0.31530	0.47618	0.16089
80	0.57735	0.36397	0.47066	0.10669
85	0.52057	0.41421	0.46739	0.05318

Azimuths



Coordinate of zenith on meridian: $y_Z = R_{eq} \tan[(90-\varphi)/2]$

Coordinate of nadir on meridian: $y_N = -R_{eq} \tan[(90+\varphi)/2]$

Coordinate of line of azimuth centers from origin: $y_C = y_Z + y_N / 2$

Distance from line of azimuth centers to zenith: $y_{Az} = \dfrac{R_{eq}}{\cos\varphi}$

Azimuth circle center from meridian on line of azimuth centers: $x_A = y_C \tan A$.

Radius of azimuth circle: $R_A = y_C / \cos A$

Each azimuth arc represents two azimuths. The table below shows the azimuths produced for each value of A. 'West Az' represents the arcs with centers to the west of the center. The azimuth arcs for 'East Az' have the same dimensions with the center moved to the east side of the meridian.

$\Phi =$ 40 $y_z =$ 0.46631 $y_n =$ -2.14451 $y_C =$ -0.83910 $y_{Az} =$ 1.30541

West Az	East Az	A	XA	RA
270	90	0	0.00000	1.30541
265/85	95/275	5	0.11421	1.31039
260/80	100/280	10	0.23018	1.32555
255/75	105/285	15	0.34978	1.35146
250/70	110/290	20	0.47513	1.38919
245/65	115/295	25	0.60872	1.44036
240/60	120/300	30	0.75368	1.50735
235/55	125/305	35	0.91406	1.59361
230/50	130/310	40	1.09537	1.70409
225/45	135/315	45	1.30541	1.84612
220/40	140/320	50	1.55572	2.03085
215/35	145/325	55	1.86431	2.27591
210/30	150/330	60	2.26103	2.61081
205/25	155/335	65	2.79945	3.08886
200/20	160/340	70	3.58658	3.81676
195/15	165/345	75	4.87185	5.04371
190/10	170/350	80	7.40333	7.51754
185/5	175/355	85	14.92087	14.97787

Rete

Distance from rete center to star of declination δ: $\mathbf{r = R_{eq} \tan (90 - \delta)/2}$

The following table shows rete calculation results for the J2000.0 star positions of selected bright stars. Declination is shown in degrees, minutes, seconds and decimal degrees. Right ascension is in hours, minutes, seconds. The J2000.0 mediation is shown by zodiac position.

r is the distance of the star on the rete from the center for unit radius of the equator.

θ is the angle of the star on the rete measured counter-clockwise from the vernal equinox and is the right ascension converted to degrees at 15° per hour.

Star	**Declination**				**Right Ascension**			**Mediation**		**r**	**θ**
Altair	8°	52′	6″	8.86833°	19h	50m	46.9s	Capricorn	25.72°	0.8561	297.69542°
Arcturus	19	10	57	19.18250	14	15	39.6	Scorpio	6.23°	0.7109	213.91500°
Capella	45	59	53	45.99806	5	16	41.3	Gemini	20.05°	0.4040	79.17208°
Sirius	-16	42	58	-15.28389	6	45	8.9	Cancer	10.38°	1.3099	101.28708°
Procyon	5	13	30	5.22500	7	39	18.1	Cancer	23.00°	0.9127	114.82542°
Deneb	45	16	49	45.28028	20	41	25.8	Aquarius	7.94°	0.4114	310.35750°
Castor	31	53	18	31.88833	7	34	35.9	Cancer	21.89°	0.5556	113.64958°
Regulus	11	58	2	11.96722	10	8	22.2	Virgo	0.00°	0.8103	152.09250°
Vega	38	47	1	38.78361	18	36	56.2	Capricorn	8.48°	0.4793	279.23417°
Betelgeuse	7	24	25	7.40694	5	55	10.3	Gemini	28.89°	0.8784	88.79292°
Rigel	-8	12	6	-7.79833	5	14	32.2	Gemini	19.55°	1.1463	78.63417°
Bellatrix	6	20	59	6.34972	5	25	7.8	Gemini	21.99°	0.8949	81.28250°
Antares	-26	25	55	-25.56806	16	29	24.4	Sagittarius	9.05°	1.5870	247.35167°
Aldebaren	16	30	3	16.50083	4	35	55.2	Gemini	10.58°	0.7467	68.98000°
Spica	-11	9	41	-10.83861	13	25	11.5	Libra	23.02°	1.2096	201.29792°

How Accurate is an Astrolabe?

The astrolabe is theoretically very accurate. A perfect astrolabe will give results as accurately as you can read the scales. However, as with any astronomical instrument, the results you get depend on two types of errors: systematic errors and observational errors. At the most basic level, systematic errors are caused by defects in the instrument itself. Observational errors are due to not using the instrument correctly[82].

Even a well made astrolabe will have many systematic errors. The scales must be accurately drawn, the rete, rule and alidade must be correctly made and mounted exactly in the center of the instrument, with no slack in the movement. The scales themselves are not divided with high precision, even on the finest astrolabes. Even the best astrolabes have significant systematic error. This was not particularly worrisome in the Middle Ages when time was not considered to be of great importance and astronomy was much less of a science. In fact, the astrolabe was considered to be a model of precision.

We are not satisfied with such a casual approach today, and we demand any instrument perform to its capability. We expect our astrolabe to be able to find the time to within a minute or two and to give answers limited only by our ability to read the scales.

There is, however, one systematic error out of our control: the plate latitude. Even if we have an astrolabe made for our precise location, we may want to use it somewhere else. If our astrolabe is equipped with several plates, it is unlikely there is one for our precise location.

Consider the example on page 19 where we found the time from the altitude of Altair for a latitude of 38° 58′. The altitude of Altair was measured as 40°. Figure 11-2 is a graph of the error in the time found by the astrolabe if you use a plate that is not for the precise location.

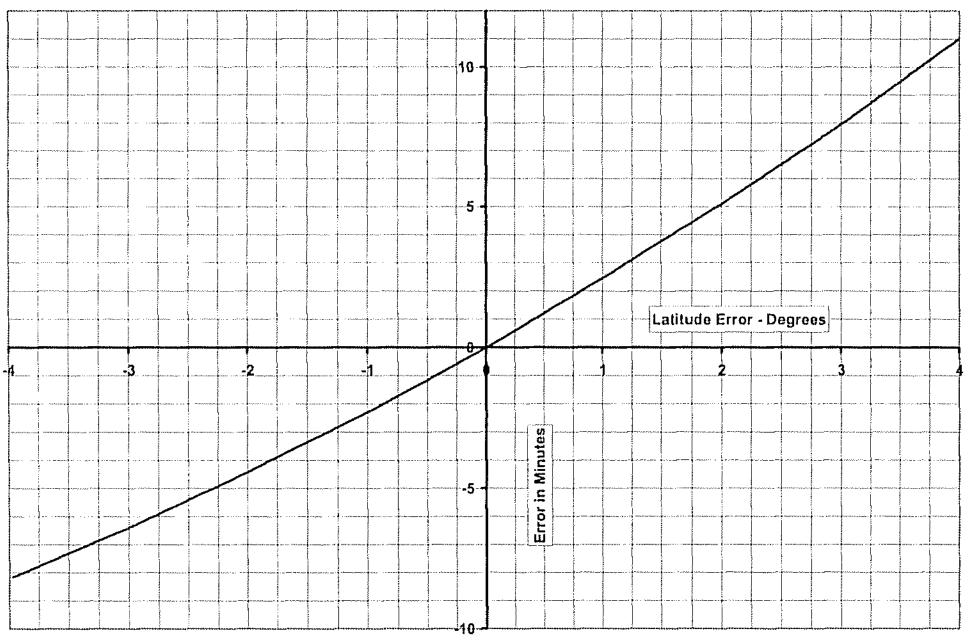


Figure 11-2. Plate Latitude Error

[82] Knowledgeable Medieval astrolabe users considered the astrolabe to be accurate to about 15 min., which seems reasonable for their instruments. A well made modern instrument can do better.

This graph shows that if, for example, this exercise had been done with an accurately made plate for 40°, which is a little over one degree from the actual location, we would find a time about 2.5 minutes later than the correct value. Using a plate for 35° gives a time over eight minutes too early. Whether this is an acceptable error depends on your point of view. In general, the graph shows you can use your astrolabe at a latitude within a couple of degrees of the plate latitude with moderately acceptable results.

Observational error is something else entirely. The following graph shows the error in the resulting time if the altitude measured is incorrect.

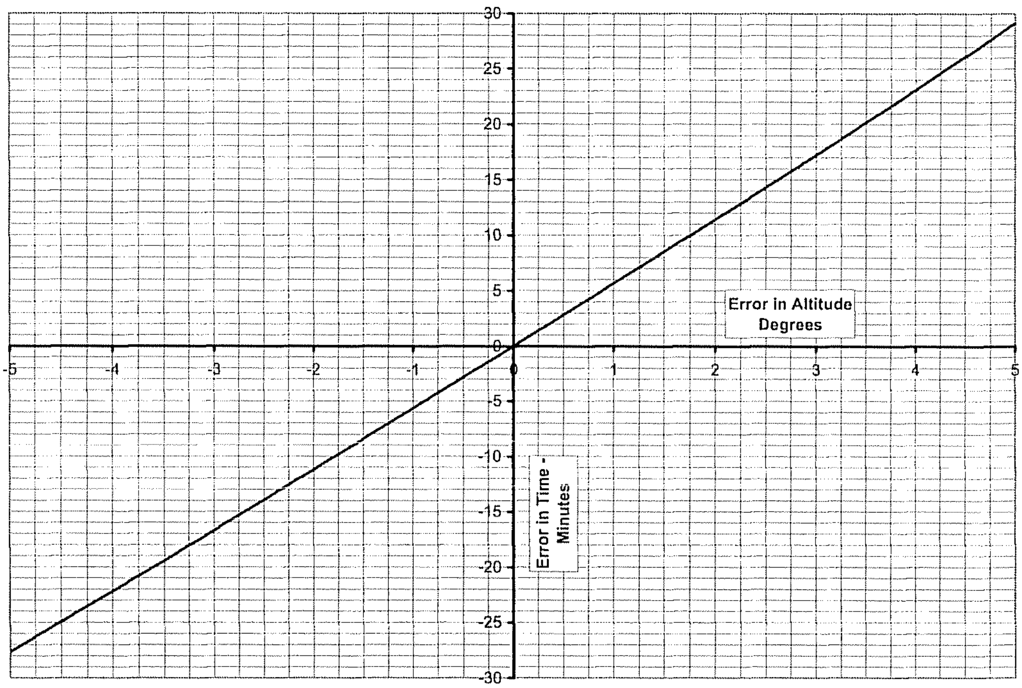


Figure 11-3. Altitude Error

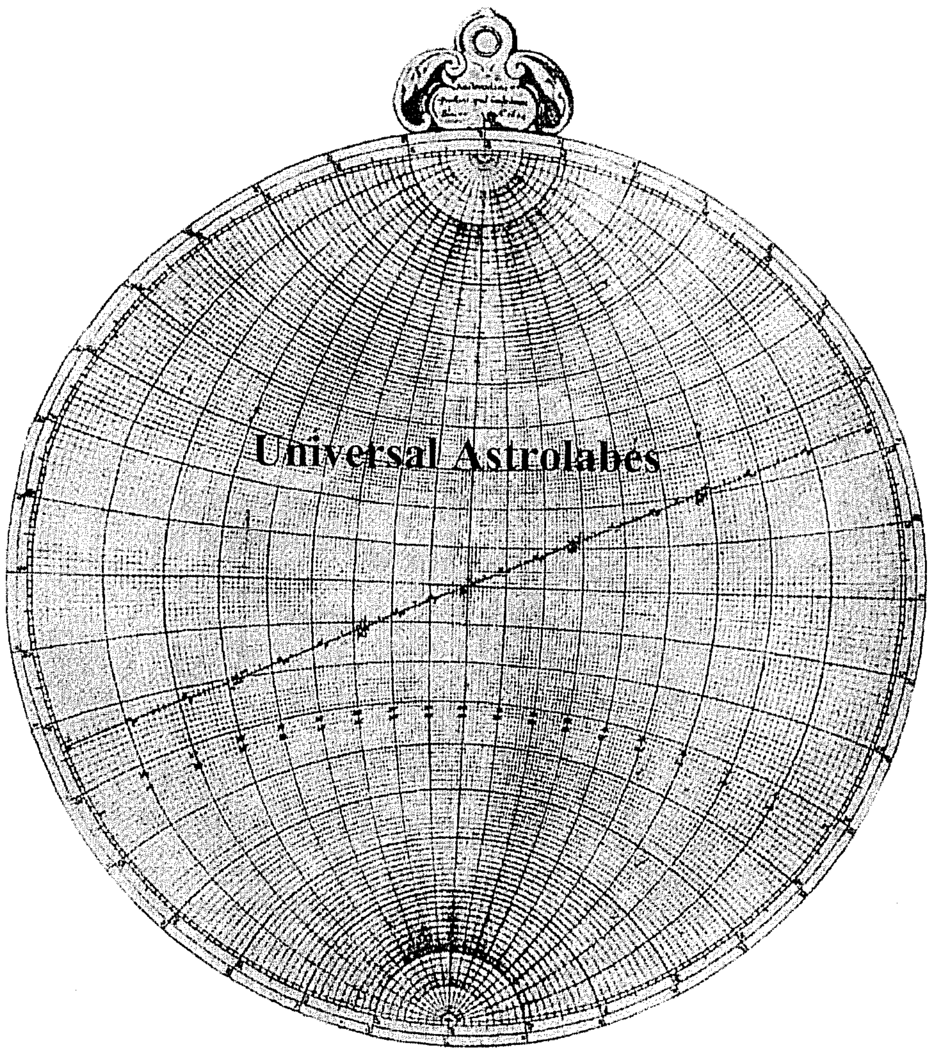
An error in measuring the altitude of Altair of only one degree results in a time over 5½ minutes in error. A five degree error in measuring the altitude results in a time error of nearly half an hour.

Clearly, it is important to make your altitude readings as accurately as possible. Your thumb held at arms length covers about ½ degree in the sky. You should be able to make altitude readings to that level of accuracy. Of course, then you have the problem of setting the rete to the exact altitude you measured, which requires a well made plate and rete, and you must interpolate to the measured altitude.

Even with all these considerations, you should be able to solve problems related to time at a level of accuracy meeting most of your everyday needs.

Different error curves will result from different stars, altitudes and latitudes. You can create your own error analysis with a relatively simple spread sheet by using the method on page 366.

Universal Astrolabes



Chapter 12 - Universal Astrolabes

The 16th century was an awkward period in the history of astronomy and timekeeping in Europe. The Renaissance was in full flower, and humanist education was well established. Printed books were becoming more available, and as a result, dissemination of both ancient and contemporary knowledge expanded rapidly. In addition, technological innovations in many areas were stimulating industrial growth. However, the 16th century was at the beginning or caught in the middle of revolutionary ideas that had not yet had an opportunity to permeate the educated public's awareness.

The transitional nature of the 16th century was perhaps most pronounced in science and mathematics, and astronomy in particular. Astronomical observations were quite crude by our standards, analytical techniques of positional astronomy were in their infancy, and the cosmology of the time relied on the Ptolemaic tradition. Copernicus' revolutionary, *On the revolutions of the heavenly spheres* (*De revolutionibus orbium coelestium*) was published in 1543 and required a number of years to become widely discussed. Kepler [b. 1571] and Newton were still in the future. Trigonometry was in its infancy to all but a few experts. Even algebra was just finding its place in Europe. Throughout the 16th century, the scientific world was rooted in the geometry of Euclid, Roman numerals and medieval ideas of the nature of the world. Mysticism was still a dominant cultural force and the practice of astrology was virtually universal in Western Europe.

The astrolabe was approaching its peak in popularity in the 16th century, and, as more people used them, there was an undercurrent of criticism about the "cargo" of plates needed to use an ordinary astrolabe over a large geographic area. Such criticism was unavoidable in "the Age of Discovery" as more people traveled over greater distances. Several astrolabe variations were perfected in this era that could be used for any latitude and could be used for increasingly complicated problems.

It is not coincidental that these sophisticated instruments emerged over a relatively short period of time from the late-15th century to the middle of the 16th century. As primitive as it seems to us now, this era was in the heart of the scientific and technical Renaissance, when progress in European mathematical sophistication was meteoric compared to previous centuries. It is beyond the scope of this work to consider all of the factors that came together at this time to transform the making of astronomical instruments from a relatively obscure trade to an industry, but we can mention a few of the more significant factors and how they came together in one city.

The greatest influence of all was the invention of printing in 1424, and the rapid growth of the printing and publishing industry. Nowhere was the concentration of printing presses greater than in the Low Countries, which in 1600, had perhaps 40% of all the printing capacity in Europe. Clearly, access to printed material on a wide range of subjects was more easily available in the Low Countries than anywhere else. Astrolabe information was prominent among the first printed technical manuals. In particular, a notable astrolabe treatise, *Elucidatio fabricae ususque astrolbii,* by Johannes Stöffler, who was professor of mathematics at the University of Tübingen, was published in 1512. This work was a model of clarity and completeness and made astrolabe design available to any educated person. Many other older astrolabe treatises and astronomical works, both printed and copied, were eagerly sought out by practitioners.

Did this interest in astronomical instruments and calculations signal a revolution in scientific thinking? Perhaps, as witnessed by the revolutionary ideas flowing from Copernicus and, somewhat later, Kepler. In addition, great strides were being made in both plane and spherical trigonometry as applied to astronomy and navigation. But of more significance to most people was astrology. Astrology had long been a strange bedfellow with fundamental Christianity, but it reached its peak in influence and popularity in Europe between about 1550 and 1650. As interest in astrology grew, so did interest in astronomical instruments, particularly those with astrological applications. The astrolabe was one of the best available astrological devices.

Instrument production was not a large industry in the Middle Ages and early Renaissance. Such production as there was came mostly from nameless artisans or from guild-dominated centers such as Augsburg. Innovation could scarcely be expected to flourish in isolation or under the rigid guild standards. A fortunate combination of circumstance led to a remarkable evolution of instrument design and manufacturing in Louvain, Belgium.

As a base, Louvain was home to a respected university that attracted excellent scholars who stayed current with the latest scientific and mathematical thinking.

Another critical element was the availability of raw materials. Brass was an expensive alloy due to its scarcity. Traditional clock and instrument making had prospered in Augsburg and Nuremberg in southern Germany due, in large part, to the availability of copper and copper-based alloys from the mines in the Austrian Alps. Instrumental in the control of copper sources was the Fuggers banking family, who developed a virtual monopoly throughout Europe. The Fuggers maintained a distribution center in Antwerp. Nearby Louvain had easy access to the required raw materials.

Louvain was also a center of metal and goldsmith work, which provided a source of skilled artisans.

Thus, Louvain had it all: access to learned writings, raw materials and skilled labor. All it took was for these factors to come together. Two styles of universal astrolabes leapt in prominence from this environment, and Gemma Frisius (Reinier van den Steen, "The Frisian", 1508-1555) was influential in the development of both. Gemma was directly responsible for popularizing the stereographic universal astrolabe he called the "Astrolabum Catholicum" (literally, "Universal Astrolabe" with an unorthodox spelling[83]), and was a teacher of Juan de Rojas y Sarmiento, who documented a form of universal astrolabe using the orthographic projection, which became known as the "Rojas astrolabe", and is discussed in detail in the next chapter.

Gemma Frisius was a professor of medicine in Louvain who apparently became interested in astronomy and astronomical instruments through the astrological aspects of medicine as it was practiced at that time. He wanted to create an instrument that would be attractive to users ranging from astronomers and astrologers to surveyors and sailors and named it the "Astrolabum Catholicum" to convey the idea he had succeeded. Whether he did, in fact, succeed in creating a truly universal astrolabe is arguable, but the quality, flexibility and ingenious design of the instrument assured success for the makers and fame for the designer.

An ordinary astrolabe requires a separate plate for each latitude, which makes it impractical to produce an instrument usable anywhere at reasonable cost and convenience. In addition, the ordinary astrolabe, as flexible as it is, is not well suited for certain types of problems, particularly those expressed in celestial latitude and longitude. In order to overcome these shortcomings, Gemma designed an instrument with an ordinary astrolabe on one side and adopted a form of astrolabe that can be used at any latitude for the other side and included a magnetic compass in the throne. This form makes a lot of sense, since different problems are better suited for one type of astrolabe or the other.

It can never be known where Gemma got the information inspiring him to devote the considerable effort required to introduce his version of the instrument. The latitude independent stereographic astrolabe was originally described in Toledo in the 11th century, probably in about

[83] Astrolabe is usually represented by "astrolabium" in Medieval Latin. Some speculate that Gemma did not misuse the name but was attempting to bring classical purity to the usage.

1048, by Alī ibn Kalaf and Ibn al-Zarqālluh (known as Azarquiel in the Latin west[84]) and had been discussed in several treatises in the Middle Ages. The implementations described by Alī ibn Kalaf and Ibn al-Zarqālluh were not identical, and it is unclear which was first. Alī ibn Kalaf 's version used a rete and is known as the *lamina universal*. al-Zarqālluh's did not use a rete and became the more popular form. It was called the *Saphea Arzachelis by* Profat Tibbon (Jacob ben Makir), better known under the name of Prophatius Judaeus, in a treatise dated 1263. The name, which derives from *al-Ṣafīḥat* (the plate) of al-Zarqālluh, has endured, although it is often shortened to simply *saphea*. A number of treatises on the saphea arzachelis could have been available to Gemma. John Blagrave virtually reinvented the *lamina universal* for his "Mathematical Jewell", which is discussed below. For a more complete discussion of the history of this technique see Turner[85].

In addition, Peter Apian's (Peter Bienewitz aka Petrus Apianus), *Astronicum Caesareum*, which was largely based on the work of Johannes Werner (1468-1528), discussed the mathematical uses of the projection and was published in 1540. It is not known whether Gemma used this or any other specific source, since he did not cite any references in his *Astrolabo Catholicum* published in 1550, and co-authored with his son, Cornelius. Gemma did, however, publish another work of Apian's. It is likely Gemma had access to any notable material on astronomy in general and astrolabes in particular published within the previous few decades. John Blagrave references Gemma often in *The Mathematical Jewell*, which was published in 1585, and mentions he had first read a borrowed copy of Gemma's book some years before. If Gemma's book made it to England in less than a decade, then surely the continental sources could have been available in Louvain in no less time. Further, the Benedictine friar Hans Dorn (1440-1509) of Vienna was a student of Regiomontanus, who owned treatises on the saphea, and produced an astrolabe in 1486 with virtually the same format as the Astrolabum Catholicum, including a compass in the throne. There is no way of knowing whether Gemma ever saw or heard of Dorn's astrolabe, and simultaneous inventions in a time of rapid progress are not unusual. However, it is likely the projection came to Gemma by this route: Ibn al-Zarqālluh → John of Lignères → Regiomontanus → Dorn (→ Apian?). The last step in the sequence is highly speculative and is based on the fact that Apian studied in Vienna where Dorn was based which would have given him the opportunity to tap into Dorn's work.

In any case, as Henri Michel[86] put it, "The "Astrolabum Catholicum" was definitely not Gemma Frisius' invention, and with the typical lack of concern of the time, the learned cosmographer "forgot" to say that this instrument had been contrived five centuries earlier." Regardless of the facts, Gemma Frisius was usually credited with the invention in that era. There is certainly no argument the instruments he inspired had a profound effect on instrument making.

Gemma's *Astrolabum Catholicum* is, indeed, a very flexible instrument with applications ranging from simple positional astronomy problems to problems in geography. Advanced users were able to solve a wide variety of spherical trigonometry problems.

The original instruments are identified with the workshop of Gemma's nephew, Gualterus (Walter) Arsenius. Gemma Frisius probably did not actually make any instruments himself, but he apparently designed them. It is not clear whether he managed a workshop or inspired his nephew to establish one, but the resulting instruments set a new and enduring standard for quality and artistic workmanship. Arsenius clearly ran his own operation after Gemma's early death at the age of 47,and capitalized on his relationship to his famous uncle.

In addition to technical sophistication, some of the most beautiful instruments ever made were executed in this style.

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84 King [2002], p. 23.
85 Turner, A. J., *Astrolabe and Astrolabe Related Instruments*, The Time Museum, Rockford, IL, 1985, 155-166.
86 Michel, H., *Traitè de l'Astrolabe*, Librarie Alain Brieux, Paris, 1976. Translated by James E. Morrison, 1992.

Some of the workers from the Arsenius' shop revolutionized calligraphic engraving and artistic design and raised European map and instrument to an esthetic level that is still admired. Thomas Gemini, a Protestant who worked with Arsenius in Louvain as a journeyman, migrated (or was banished for heresy) to England, where he had enormous influence on the English instrument making tradition. The beautiful maps, globes and instruments made by Gerard Mercator, who began his career in Louvain, are still admired as among the finest ever made. The elegant and artistic Louvain style introduced by Arsenius was adopted by instrument makers in England, Spain, Italy and Germany. A notable practitioner was the German Erasmus Halbermel, whose beautiful and delicate instruments rivaled and even exceeded the Louvain products in quality and style. Artisans such as Gemini, Halbermel and Humphry Cole had dramatic influence on the future of European instrument making.

It is interesting that the same forces brought together in Lovain to bring the astrolabe to its pinnacle of development also conspired to bring the astrolabe to the end of its long reign. Demand for accurate and specialized instruments grew as truly scientific interest in astronomy developed. Instrument manufacture grew from a trade to an industry, and the astrolabe was left behind, leaving us with few vestiges of the romance of the old devices.

Chapter 13 - The Saphea Arzachelis

Some terms from spherical geometry are needed to introduce the saphea projection. A sphere can be cut by a plane in only two ways. Every plane that cuts a sphere creates the outline of a circle on the surface of the sphere. If the plane that cuts the sphere goes through the center of the sphere, the circle generated on the surface of the sphere is called a *great circle*. The diameter of a great circle is the same as the diameter of the sphere. Examples of great circles are the equator, which cuts the sphere through the center and is perpendicular to the axis of the sphere, and a meridian, which is a great circle passing through the poles and a specific longitude on the sphere. If the plane cutting the sphere does not go through the center of the sphere it generates a *small circle*. The diameter of a small circle is always less than the diameter of the sphere. The Tropic of Cancer and Tropic of Capricorn are both small circles parallel to the equator.

A ***colure*** is a great circle passing through the poles of the celestial sphere. Thus, the meridian circle is a colure. Recall the vernal equinox is a fixed point on the celestial sphere where the equator and ecliptic meet and the Sun crosses from south to north. The summer solstice is 90° east of the vernal equinox, the autumnal equinox is 180° from the vernal equinox, and so on. The colure passing through the poles of the sphere and the equinoxes is called the *equinoctial colure*. The colure passing through the solstices is called the *solsticial colure*.

Gemma Frisius' universal astrolabe was based on the saphea arzachelis. The saphea uses a clever variation of the stereographic projection to project the celestial sphere on a plane. The celestial sphere is projected on the plane of the equator on a conventional planispheric astrolabe. On a universal astrolabe, the celestial sphere is projected on a colure.

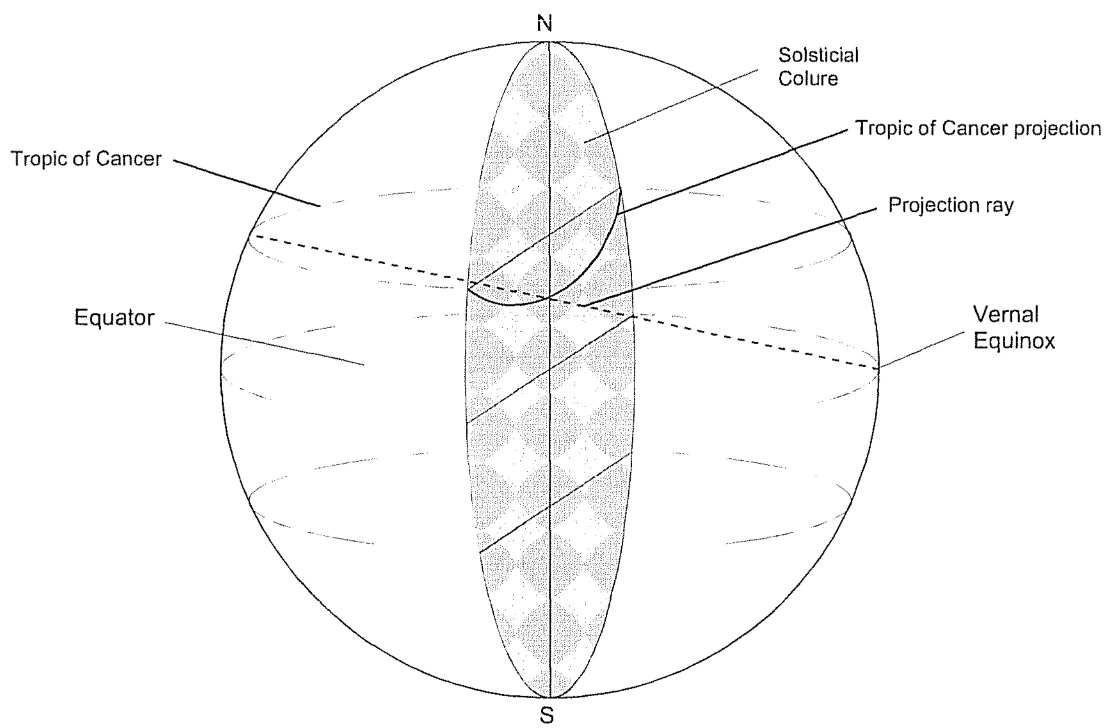

Figure 13-1. Saphea Projection

The projection is discussed first, and then how it is applied to an instrument.

The definition of the saphea projection is something of a mouthful: "The plate of the saphea is the stereographic projection of a hemisphere of the celestial sphere on the solsticial colure with the origin at an equinox." It is really not as difficult as it sounds, and once you understand the definition, you understand the idea. Figure 13-1 illustrates the principle of the projection.

Figure 13-2 shows the result of the projection. The north celestial pole is at the top, and the south celestial pole is at the bottom. The autumnal equinox is projected at the center. East is to the right and west to the left. The horizontal diameter of the projected hemisphere is the equator.

Recalling that circles are preserved in the stereographic projection, the circular arcs above and below the equator are the projection of the circles of declination on the celestial sphere and circles of terrestrial latitudes. These arcs are referred to as *parallels* since they can represent several measurements. The parallels for the Tropics are shown as dashed arcs.

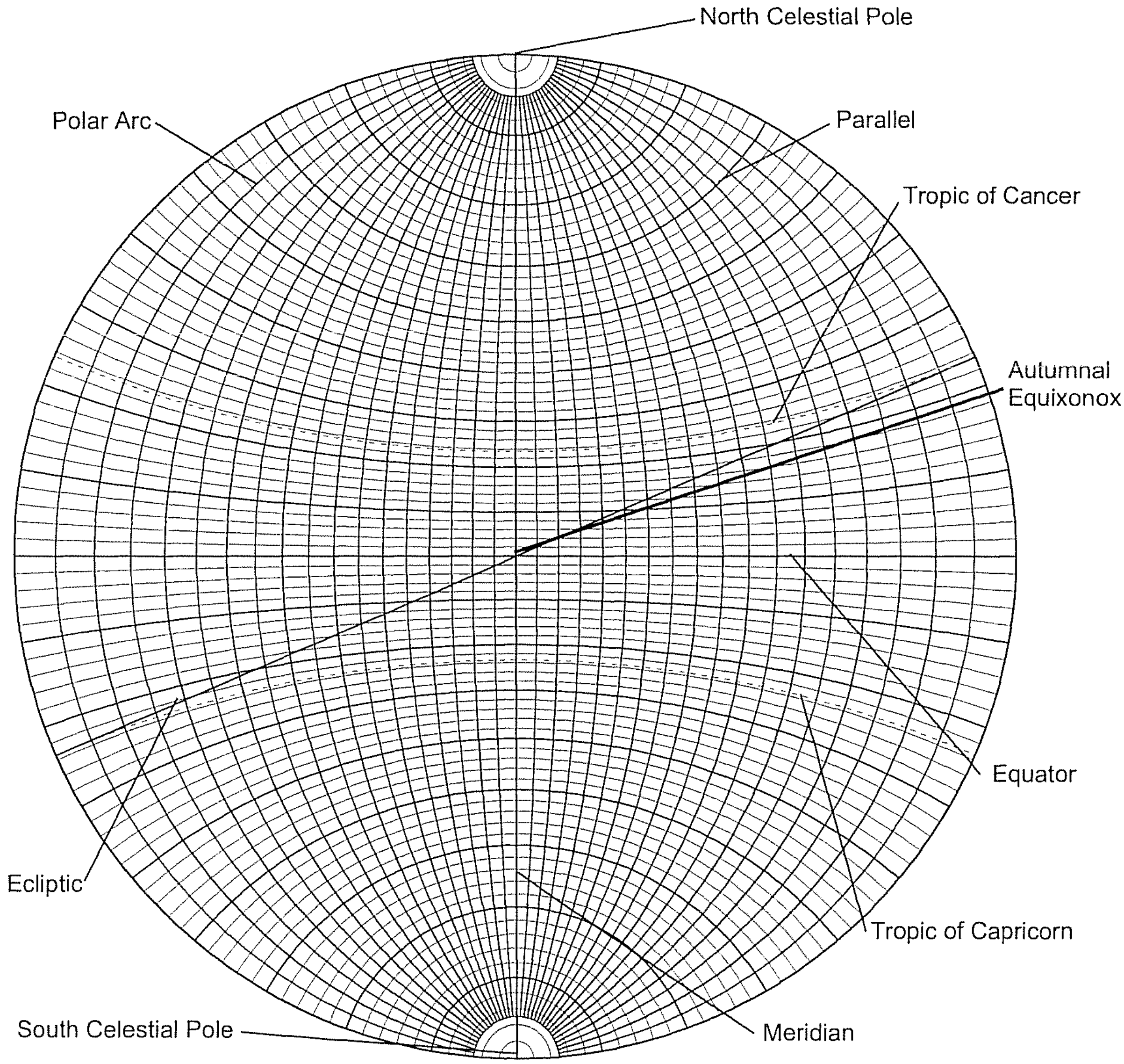

Figure 13-2. Saphea Projection

The arcs connecting the poles are circles of right ascension on the celestial sphere and circles of the terrestrial meridians. These arcs are referred to as *polar arcs*, again because they can represent multiple measurements.

The horizontal diameter is the projection of the equator. It can also represent other elements. It is called the *equinoctial line* here to give it a unique name consistent with the literature.

The vertical diameter can represent the equinoxes or a local meridian. It is always called the *meridian* in saphea literature.

The diagonal diameter is the projection of the ecliptic, which passes through the equinox and meets the circumference at the tropics. The Sun moves back and forth along the ecliptic over the course of a year. Thus, the line representing the ecliptic can be divided by longitude in degrees, the zodiac or both. The ecliptic can also be divided by the calendar.

At this point, note the projection would have exactly the same arcs if the origin of the projection were at the autumnal equinox instead of the vernal equinox. Thus, the ecliptic can be divided for the entire range of solar longitudes and simply imagine the sphere is being viewed from one side or the other depending on the time of year.

The next point is the really clever part of this application of the stereographic projection. If the projection is rotated so the ecliptic is horizontal, the equinoctial line represents the ecliptic, the arcs above the ecliptic represent arcs of celestial latitude, and the polar arcs represent celestial longitude. If the horizontal diameter through the center (the equinoctial line) is considered to be the ecliptic, the poles are the ecliptic poles and the arcs represent celestial latitude and longitude.

Similarly, if the projection origin is moved to a point on the celestial sphere on the extension of the local horizon, the equinoctial line is the horizon, the poles represent the zenith and nadir of the place, and the polar arcs represent angles of altitude and azimuth or hour angles.

Thus, the projection can represent the celestial coordinates of a point in space in three different coordinate systems at the same time. In fact, the simplest applications of the saphea involve converting between coordinate systems. For example, if the celestial latitude and longitude are known, it is very simple to find the declination and right ascension or the altitude and azimuth of the same point, and vice versa.

The Saphea Universal Astrolabe

The stereographic projection of the celestial sphere onto the solsticial colure becomes the plate of the saphea arzachelis when it has the appropriate labels . See Figure 13-3.

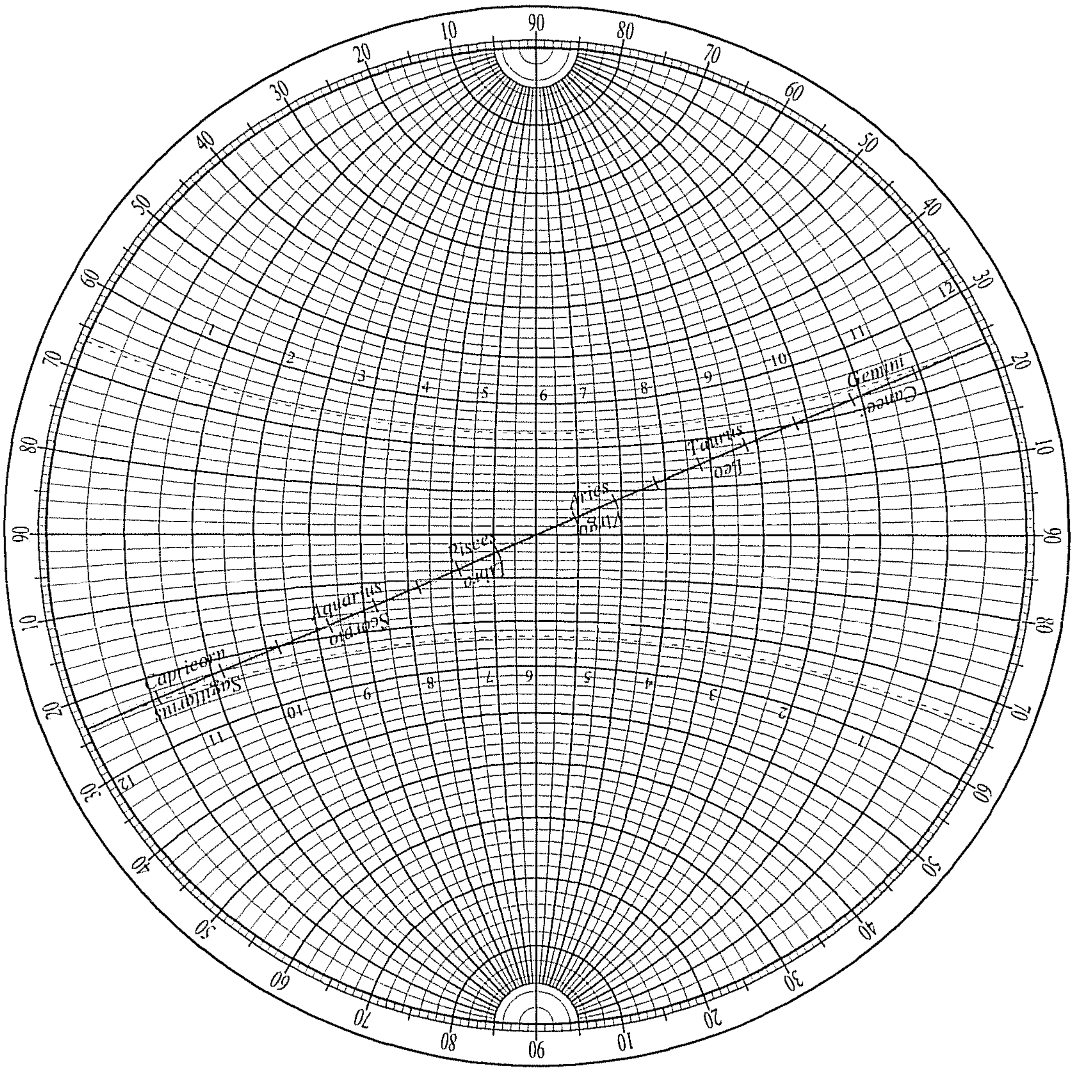

Figure 13-3. The Saphea Plate

The ecliptic is divided by the zodiac on most saphea instruments. These divisions represent the position of the Sun on the ecliptic and require a little imagination to visualize. Recall the projection arcs are identical whether the projection origin is at the vernal equinox or the autumnal equinox. Therefore, the half of the ecliptic under consideration depends on the origin. We see the half of the ecliptic from Aries 0° to Gemini 30° and Capricorn 0° to Pisces 30°, when the origin is the autumnal equinox. These signs are printed above the ecliptic line. We see the half of the ecliptic from Cancer 0° to Sagittarius 30°, when the origin is the vernal equinox. These signs are printed upside down and below the ecliptic line. Thus, the Sun can be visualized as moving back and forth along the ecliptic over the course of a year if we mentally shift our view of the projection origin. It is much more difficult to describe than it is to do. The ecliptic can also be divided directly by the calendar.

The margin of the plate is divided into four 90° quadrants. The sequence of the numbering for a quadrant depends on the use. The numbering in QI proceeds from 0° at the equator to 90° at the north pole. This set of divisions is used when the arcs are interpreted as declinations or latitudes, and the limb divisions show the value of each parallel. QII can be divided in the same way or in the reverse order so 0° is at the north pole, increasing in the counterclockwise direction to 90° at the 9 o'clock position. This set of divisions shows the *polar distance*[87] and are used to orient the local horizon. Instrument makers were not particularly consistent in the labeling of the saphea limb. A mental subtraction is needed for certain problems if QII and QIV are not labeled with polar distance.

The parallels in the figure are drawn for each two degrees. Parallels for each degree would be used on a larger instrument.

The polar arcs are generally not labeled along the equinoctial line for two reasons: the labels would have to be very small in order to be consistent, and the polar arcs can have a variety of meanings. Among the meanings are right ascension, hour angles, equal hours, longitudes and azimuths. It would not be possible to label all of these uses in a coherent way. The hours (equal hours, hour angles, right ascensions) are labeled near the tropics since they are used rather often and would otherwise be difficult to locate. One hour is 15° on the celestial sphere. The labels along the ± 30° parallels identify the equal hour associated with each 15° of hour angle and can be converted to right ascension. The labels on the +30° arc are used in the morning, and the labels on the -30° arc are used in the afternoon. The polar arcs in the figure are drawn for each 5°. Three arcs define one hour so, when used in the context of time, each arc delimits 20 minutes.

Two additional accessories are required to solve problems with the saphea plate. A rule free to rotate around the center of the instrument having one edge as a diameter is called the *regula*. *Regula* is Latin for "rule," and the term is used to differentiate the device from the rule used on the ordinary astrolabe. The regula may be divided by degrees. Connected to the regula is an articulated arm of two or three sections called the ***brachiolus*** (Latin for "little arm"). Some old instruments did not have a brachiolus but had an auxiliary rule called the **cursor** oriented 90° to the regula that could slide up and down the regula and point to any location on the plate. Others had both, with the brachiolus attached to the cursor. There is no particular use advantage of one system over the other, although a cursor with a regula is much harder to make.

Some old instruments showed some stars on the saphea plate even though they were of limited use. Stars shown on the projection plate were located in the equatorial coordinate system by their declination and right ascension. Stars visible from the vernal equinox were sometimes shown as a solid star figure with stars visible from the autumnal equinox shown as a star outline.

Saphea Scales

The greatest advantage of the universal astrolabe is its flexibility. It is this same flexibility that makes it rather complicated when compared to the planispheric astrolabe. Specifically, the saphea divisions can represent so many different positional astronomy concepts it is confusing at first to keep them all straight.

The equinoctial line can be used in several ways: as the equator, the ecliptic or the horizon. In most cases, the scale meanings are defined by the function assigned to the equinoctial line.

[87] Polar distance is the angle of a celestial object from the pole. The polar distance of the local horizon is equal to the latitude of the place.

Parallels

The meaning of the parallels depends on the meaning of the equinoctial line. If the equinoctial line is used as the celestial equator, the parallels represent declination. Since there are two sets of parallels representing positive and negative values, they are consistently numbered with positive declinations above the equator and negative declinations below.

When the equinoctial line is used as the ecliptic, the parallels represent celestial latitudes and are numbered the same as declinations, with positive values above the ecliptic and negative values below.

The parallels represent altitudes, and the polar arcs represent azimuths when the equinoctial line is used as the horizon. In general, only positive altitudes are relevant, but there are problems relating to altitudes below the horizon.

The parallels can also be used as a convenient angle scale for problems involving spherical trigonometry.

Polar Arcs

The polar arc meaning also depends on the definition applied to the equinoctial line.

When the equinoctial line is defined as the equator, the polar arcs can represent several concepts. If they are used to represent right ascension, they are numbered from 0h at the meridian to 6h to the right. They are renumbered from 6h to 18h from right to left across the front of the instrument, and then from 18h to 24h from the left edge, proceeding to the right to the meridian. The labeled equal hour arcs are helpful in finding an even hour, although a bit of mental arithmetic is needed to convert from equal hours to right ascension.

The polar arcs can also represent hour angles. As hour angles they are numbered starting with 0 at the meridian and proceeding to 6h at either limb, or starting at 18h with a 6 hour difference to either limb.

When the equinoctial line is the ecliptic, the polar arcs represent celestial longitude, and are numbered from 0° to 90° proceeding to the right from the meridian, 90° to 270° from the right limb to the left limb, and 270° to 360° back to the meridian.

The polar arcs represent azimuths when the equinoctial line is the horizon. They are numbered from 90° (left limb) to 270° (right limb) when facing south and 270° to 90° when facing north.

The ecliptic projection also has two sets of division, but they are marked on the scale.

Like the parallels, the polar arcs are also used as simple degree divisions when working with spherical triangles.

The Regula and Brachiolus

All saphea-based universal astrolabes had a rule divided by projected degrees pivoting around the center of the instrument. The regula is used to define the horizon and to take measurements from the plate. The regula represents the local horizon when it is rotated to the polar distance for the latitude under consideration. Once in position, the time of sunrise and sunset for any day of the year can be read directly from the plate.

Refer to Figure 13-4, which shows the regula and brachiolus. The rule is positioned for a latitude of 38° 42′ as shown by the degree scale in QII. The rule is divided by degrees of longitude.

The brachiolus in the figure is made in two sections, each of which is half the radius of the plate in length. Some old instruments had a three segment brachiolus. The tip of the brachiolus pointer can be positioned to any position on the plate. This brachiolus pivots around the center of the plate in the figure, but this is not the only way to position it. The connection of the brachiolus to the rule must be firm so they will rotate together, and mechanical considerations might govern how the brachiolus is anchored. It is likely Regiomontanus was the first to suggest the use of this device.

The brachiolus in Figure 13-4, is oriented to the point representing a declination of 20° and an hour angle of 30°. The day is May 20 (Taurus 29°) in Figure 13-4.

The regula represents the local horizon for a latitude of 38° 42′. On this day, the Sun will follow the 20° declination curve from the horizon (the regula) to the edge of the plate at noon and then back to the regula.

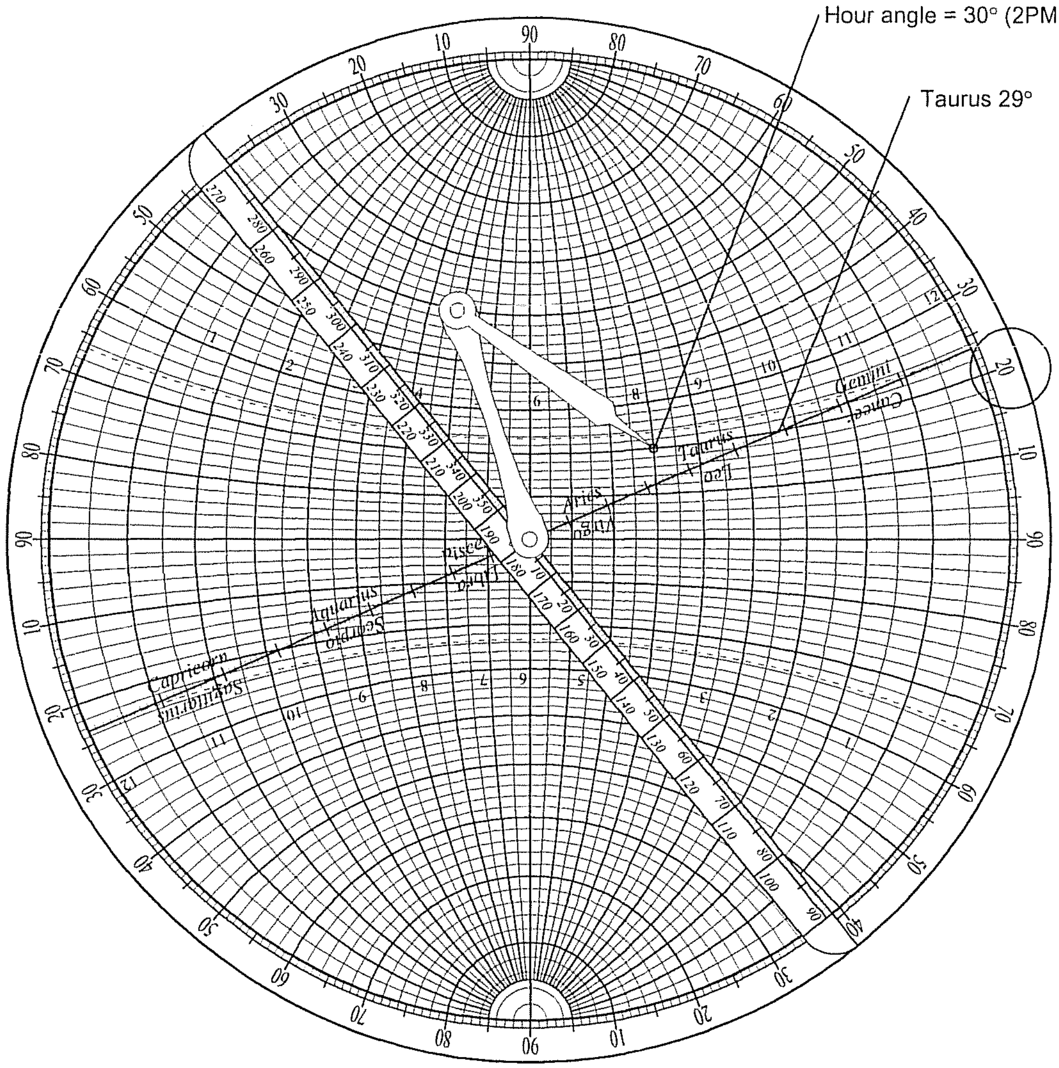

Figure 13-4. Saphea Plate with Rugula and Brachiolus

Uses of the saphea

The saphea is a very versatile device. Problems involving conversion of coordinate systems are very easy. Thus, if the user has an ephemeris listing celestial positions by latitude and longitude, which are very awkward on a planispheric astrolabe, it is very easy to convert the positions to declination and right ascension, which are more convenient.

The saphea can be used to find the time of sunrise and sunset, and thus the length of the day, for any latitude. It can also be used to find the latitude from the Sun's meridian altitude, although this is such a simple problem arithmetically it is unlikely anyone would go to the trouble to use an astrolabe to solve it (Sun's noon altitude = 90° + Sun's declination - latitude).

A few sample problems are presented below.

1. **What is the Sun's declination and right ascension on any date?**

   This problem requires only the saphea plate. Simply find the Sun's longitude in the zodiac from the date/calendar conversion scale. Then locate that point on the ecliptic and read the solar declination from the declination arc crossing that point.

   For example, to find the Sun's declination on April 30:

   Read the Sun's longitude on April 30, from the calendar/zodiac scale as approximately Taurus 10°.

   Find Taurus 10° on the ecliptic line.

   Read the declination as approximately 15° (14° 52′) from the declination arcs. Read the right ascension as approximately 38° or 2 ½ hours (2:31) from the polar arc.

   You can confirm this answer using the front of the astrolabe by setting Taurus 10° on the meridian and reading the declination from the rule and the right ascension from the position of the first point of Aries on the limb.

2. **Find the maximum altitude of the Sun for any latitude given the Sun's declination.**

   Set the regula to the equinoctial line. Set the tip of the brachiolus to the arc equal to the Sun's declination on the circumference of the saphea plate.

   Rotate the regula and brachiolus together until the regula is at the horizon for the place in question. Recall the horizon is found when the regula is pointing to the zenith distance (i.e. 90° - latitude).

   Read the Sun's meridian altitude from the tip of the brachiolus. All that was done here was to represent the equation $90 - \phi + \delta$ with the brachiolus pointer.

   The inverse of this problem is similar. To find the latitude of a place given the meridian altitude of the Sun, set the regula to the equator and point the tip of the brachiolus to the Sun's noon altitude. Rotate the regula and brachiolus together until the tip points to the declination. The regula is in the position of the horizon in question (you will need to calculate the zenith distance from the position of the regula).

   Example: Assume a latitude of 40° and a solar declination of +15°. Set the regula to the equinoctial line and set the tip of the brachiolus to 15° on the circumference. Rotate the regula and brachiolus to the 50° declination arc (the zenith distance for 40° latitude = 90° - 40°). Read approximately 65° from the tip of the brachiolus on the circumference.

You would normally work such a problem in two steps: find the Sun's declination for the date and then find the maximum altitude for a given latitude.

3. **Find the longitude and latitude of a celestial object given its right ascension and declination.**

Conversion between ecliptic and equatorial coordinates is one of the easiest problems to solve with the saphea, and one requiring significant computation without it. Conversely, the declination and right ascension given the latitude and longitude can also be found.

The equinoctial line represents the base for one of the coordinate systems, and the ecliptic line represents the base for the other. You always start with the regula aligned with the equinoctial line. This is the base for the coordinate system you are converting **from**. Set the tip of the brachiolus to the coordinates for the coordinate system you are converting from. Then rotate the regula to the ecliptic line and read the coordinates of the system you are converting to from the tip of the brachiolus.

For example, to convert from latitude/longitude to declination/right ascension, set the regula to the equinoctial line and set the tip of the brachiolus to the latitude and longitude. Rotate the regula to the ecliptic line and read the declination and right ascension from the tip of the brachiolus.

To convert from ecliptic coordinates to equatorial coordinates, set the regula on the equinoctial line and set the tip of the brachiolus to the latitude and longitude. Rotate the regula to the ecliptic and read the declination and right ascension from the tip of the brachiolus.

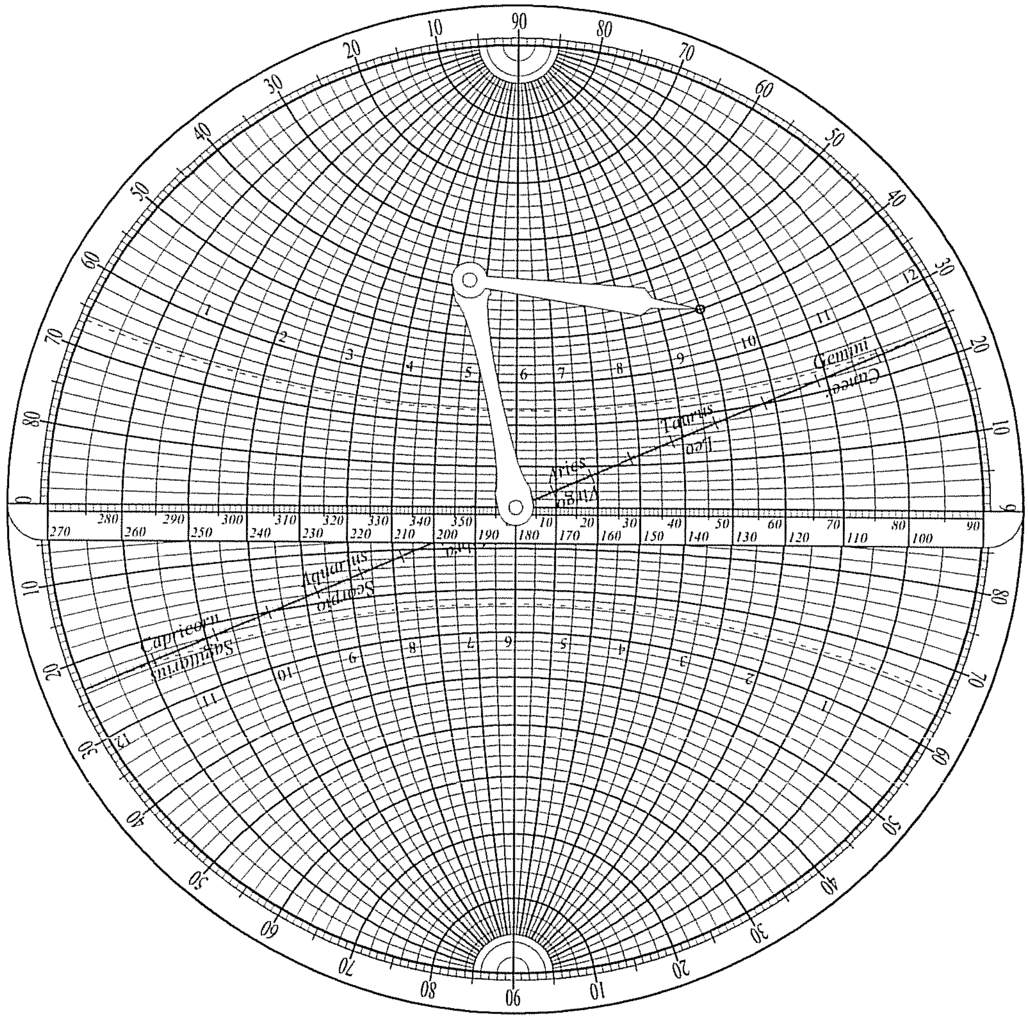

Figure 13-5. Set to latitude 40°, longitude 50°

As an exercise, convert latitude = 40°, longitude = 50° to declination and right ascension. The answer is δ ≅ 55° (55.4°) and RA ≅ 30° (29.9°) ≅ 2h (1h 59m). See Figure 13-5 and Figure 13-6.

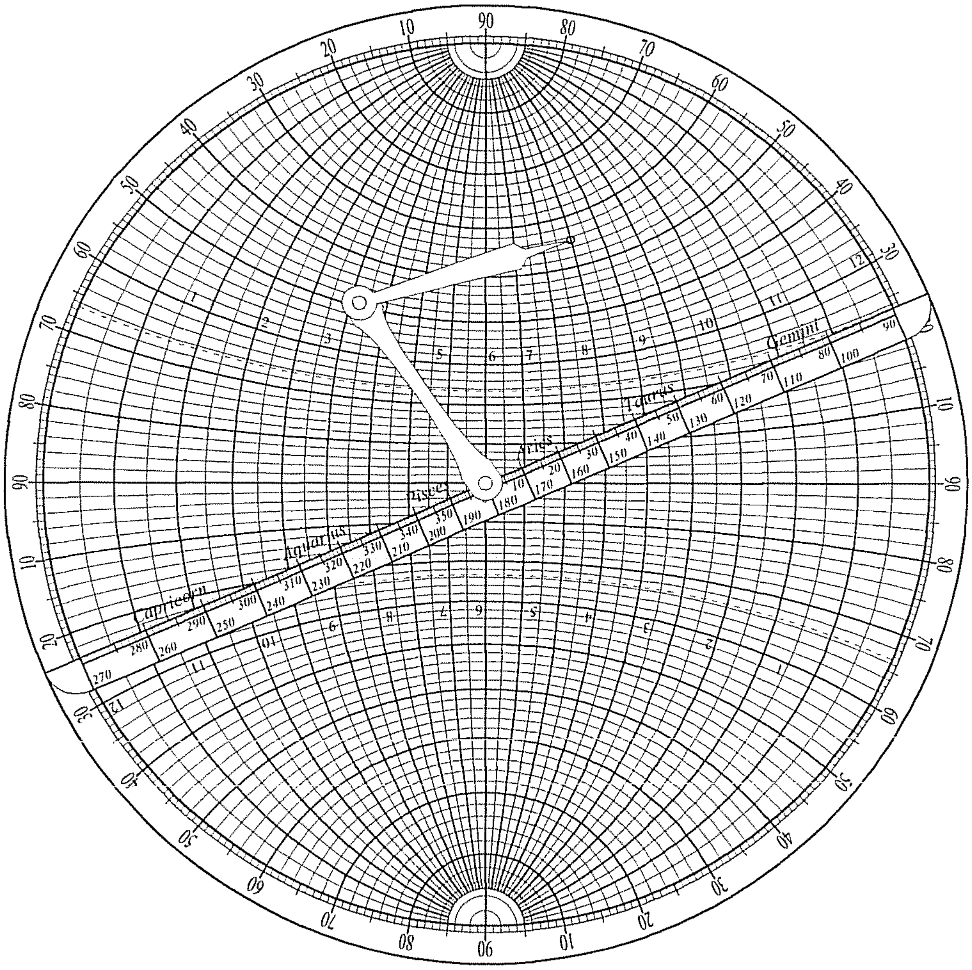

Figure 13-6. Set to read declination, right ascension

4. Find the time of sunrise and the Sun's path for any day for any latitude.

This problem does not use the brachiolus. Position the regula for the horizon in question by moving the regula to the polar distance of the place. Find the Sun's declination for the day from the calendar/zodiac conversion scale and locate the declination arc passing through that point. The declination arc represents the Sun's path for the day, and the point where it intersects the horizon (the regula) is sunrise. The polar arcs also represent time. Read the time of sunrise from the polar arc.

For example, fine the time of sunrise on May 20 at a latitude of 40°. Read the Sun's longitude as Taurus 29°. Locate Taurus 29° on the ecliptic and see the declination arc for 20° passes through that point. Set the regula to the horizon for 40°. The point where the 20° declination arc intersects the regula is the same point as the polar arc for about 4:50 AM. This is the time of sunrise.

The time of sunset is read from the lower set of labels as about 7:10 PM. Remember, each polar arc represents 20 minutes in the figure.

Notice the length of the declination arc from the horizon (the regula) to the circumference is the amount of time from sunrise to noon. The length of the day is twice the number of hours represented by the daily declination arc, 14:20 in the example.

See Figure 13-7.

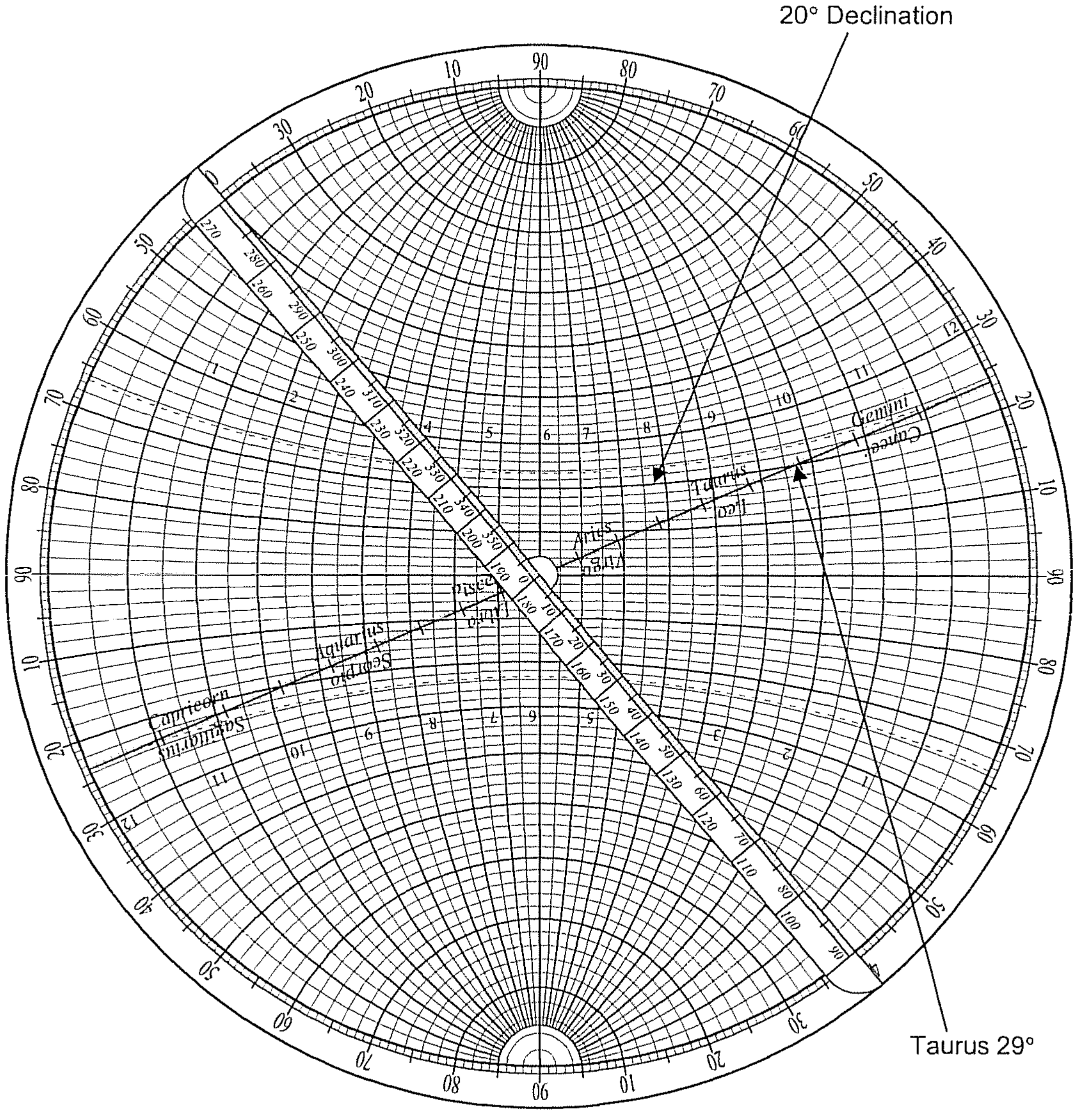

Figure 13-7. Regula set for 40° latitude

5. **Find the Sun's altitude for a day and time.** Set the regula to the horizon position. Set the brachiolus pointer to the hour angle on the declination arc for the day. Rotate the regula to the equinoctial line. The point is on the parallel for the Sun's altitude.

   The Sun follows the declination curve for the course of a day passing through each hour angle. **The declination curve becomes the altitude curve when the regula is rotated.**

6. Find the time from the altitude of the Sun or a star.

Although this is one of the easiest and most basic applications of the common astrolabe, it is rather complicated on the saphea and cannot be solved in a single step. The solution uses the principle shown in the previous problem.

a) Measure the altitude of the Sun or a bright star.
b) Orient the regula to the polar distance for the latitude.
c) Point the tip of the brachiolus to a point on the parallel corresponding to the declination of the Sun or star and on an hour arc near to the time.
d) Rotate the regula to the equinoctial line and note the position of the tip of the brachiolus.
e) If the tip of the brachiolus is on the parallel corresponding to the measured altitude, then the time is the time selected in step c.
f) If the tip of the brachiolus is not on the measured altitude, move the tip of the brachiolus and repeat steps d and e.

Thus, finding the time with the saphea is not very straightforward. By implication, the conventional astrolabe was included with the Astrolabum Catholicum for this type of application.

Why can't the time be found directly on the saphea plate? Time is determined by the Sun's altitude. The Sun's altitude cannot be related to an hour angle on the saphea directly, because the only fixed linear measurement on the projection is for great circles passing through the center of the plate. The distances measured from the center of the plate are projected as R tan $\alpha/2$. There is no fixed relationship for distances on the plate measured from other points. Therefore, the time cannot be found directly from the Sun's altitude, because the altitude would be measured from a position on the regula that is not in the center of the plate.

The Planispheric Astrolabe

The other side of almost all saphea instruments included a planispheric astrolabe with some differences from a normal instruments. The rule on the planispheric astrolabe is replaced by an alidade with sights but is divided like a rule for making altitude measurements. Depending on the specific instrument, there may also be a calendar/zodiac conversion scale included. The alidade/rule was used as both an alidade as on the back of a common astrolabe and as the normal rule.

Some provision is required for the calendar/zodiac scales somewhere on the instrument. These scales could be around the edge on the back, which reduces the size of the saphea plate, as scales on the limb of the planispheric astrolabe side or the calendar scale can be included on the ecliptic.

Making the Saphea

Despite the fact that the saphea plate looks somewhat complicated, it is rather easy to draw.

The diameter of the plate should be as large as possible. The arcs on the plate get quite close together, and the larger the plate, the easier it is to read positions. The finest astrolabes of this type from Arsenius were around 15 inches in diameter. It is difficult to print an instrument of this size on a normal printer, but there are professional services that can print larger sizes.

Draw the outer diameter and scales in the normal way leaving ample space for the interior scales.

Divide and number the degree scale around the limb considering the discussion above.

Draw the equinoctial and meridian lines as orthogonal diameters.

The stereographically projected distance of any point on the plate from the center can be derived easily by using logic identical to that for the astrolabe as:

$$D = R\tan\frac{\alpha}{2}$$

Where D is the distance from the center and α is the angle a great circle makes with the center of the sphere.

Drawing the parallels. The parallels can be drawn to any desired resolution, but each 2° seems to be typical. The parallels should be drawn for each 2° up to 86° and then a single parallel for 88°. The parallels can be treated mathematically exactly like the astrolabe horizon. It does not matter what the parallels represent when they are drawn. Assume they represent latitude for this discussion. Given the radius of the plate, R, and the latitude, β, the center, c, and radius, r, of the parallel circles are calculated from:

$$c = \frac{R}{\sin\beta}$$

$$r = \frac{R}{\tan\beta}$$

The latitudes are drawn up to 85° and then farther apart up to 90°.

To construct the parallels graphically, draw a line from the west point on the plate to each angle division on the opposite limb. The point where this line crosses the meridian is the intersection of the parallel with the meridian. Draw a circle passing through the degree tics on each side of the limb and the intersection point.

Drawing the polar arcs. The polar arcs are most easily drawn by defining the circles from three points, the intersection with the equinoctial and the poles. The intersection of a longitude polar arc with the equinoctial line is:

$$x = R\tan\frac{\lambda}{2}$$

Then construct a circle using x and the poles. The polar arcs must be clipped within a circle at about 85° of latitude. Otherwise, they are just an undifferentiated mass near the poles.

The polar arcs can be constructed graphically quite easily. Draw a line from the south point on the plate to the limb division corresponding to the longitude. The point where this line crosses the equinoctial line is the point of intersection of the polar arc. Simply construct an arc of a circle that passes through the two poles and this point.

The Lamina Universal *and the "Mathematical Jewell"*

Several variations of the basic saphea based instrument have been described. Some have used two sets of grids with one grid rotated to the ecliptic to make it easier to convert coordinate systems. Such an instrument would be drawn on paper or parchment so the two grids could be of different colors in order to sort out the maze of curves.

The use of the stereographic projection onto a colure for an instrument was first described by Alī ibn Kalaf using a rotating rete with a projected grid (the *lamina universal*). Such a variation was also suggested by Regiomontanus and others. The use of a rotating rete makes a lot of sense. It simplifies the solution of problems involving altitudes, particularly finding the time from the Sun's altitude, and it allows problems of coordinate conversions to be solved in a single step.

John Blagrave reinvented the lamina rete in his "Mathematical Jewell" published in 1585. Blagrave's instrument was briefly popular in England, but his treatise describing the instrument was not particularly clear, even for the 16th century. Other accounts describing the instrument were published by Thomas Blundevill (1594) and John Palmer (1658). Three instruments have survived, and it is known that others were made from paper and cardboard.

Figure 13-8 shows a representation of the rete. The rete consists of a stereographically projected grid of half the sphere and is free to rotate around the center of the plate. The rete shows altitudes when the equinoctial line of the rete is oriented to the horizon or latitudes and longitudes when it is rotated to the ecliptic.

Blagrave's rete also included a number of stars in the lower, undivided semi-circle.

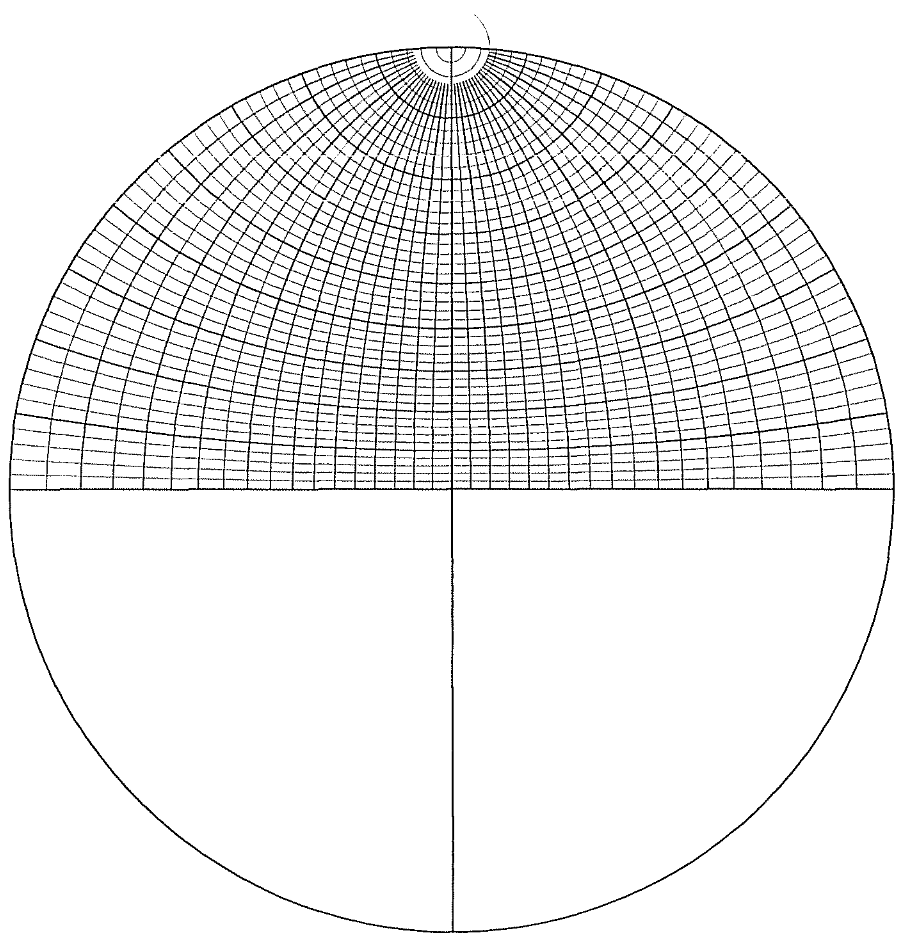

Figure 13-8. *Lamina Universal* Rete

Such retes on the old instruments were made of brass and blocked a good portion of the underlying grid. Regiomontanus suggested making the rete of transparent horn, and in 1504, Jacob Ziegler suggested making the rete with thread. No example survives to demonstrate whether either of these suggestions was actually implemented.

A modern version of the lamina rete would be printed on transparency material. The *lamina universal* rete is made using exactly the same methods as the saphea plate.

Figure 13-9 shows an example of the saphea with a lamina rete, and also illustrates why this form of the astrolabe requires an expert to use. The overlapping grids do, in fact, simplify the operation of the instrument, but it is rather difficult to sort out the grids due to the large number of curves. A contrasting color for the rete might help.

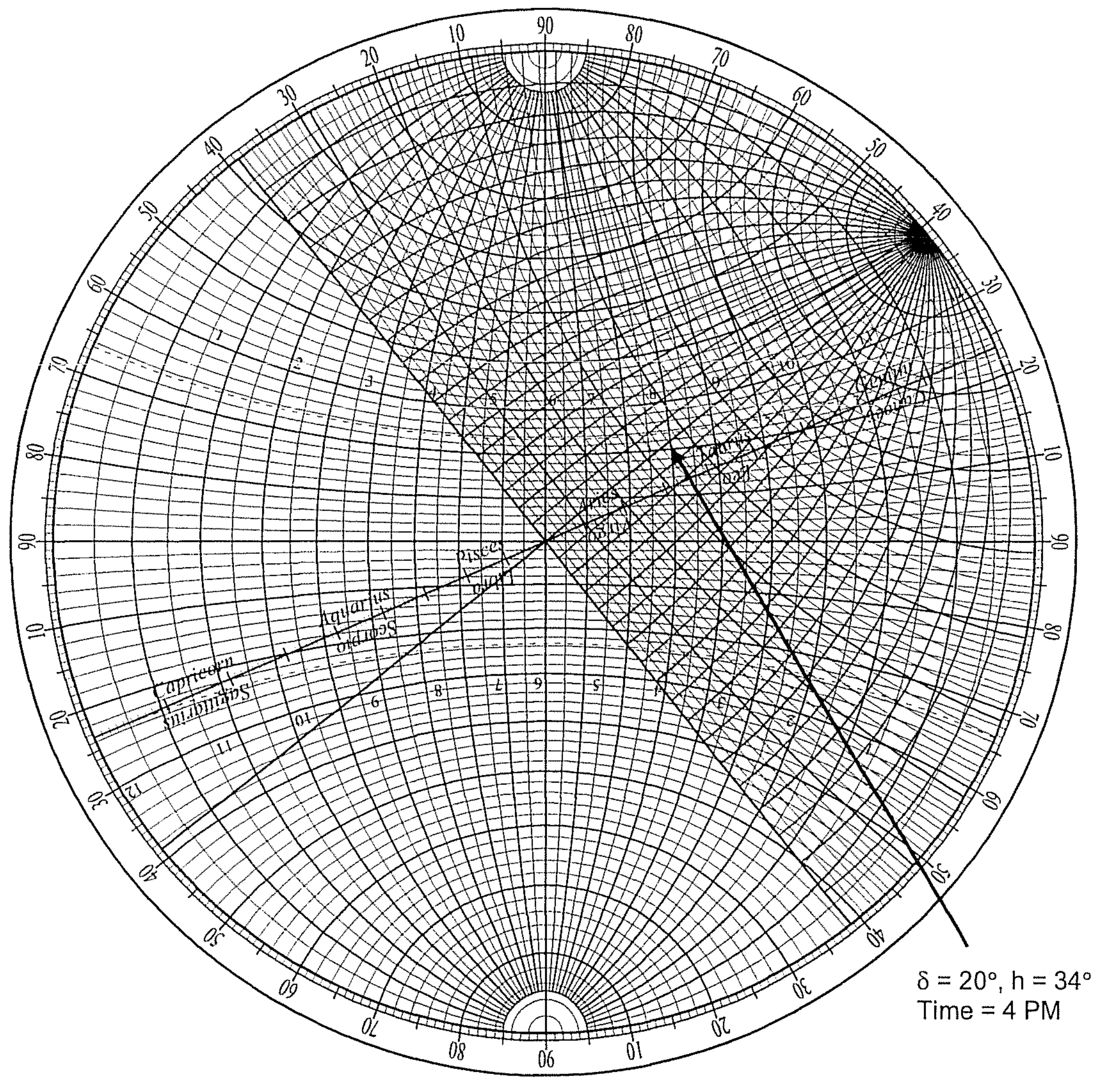

Figure 13-9. Saphea Plate with Lamina Rete

The plate in the figure also shows the ecliptic divided directly by the calendar to illustrate this design option. Also, such instruments would also have a divided rule, which is not shown in the figure.

The astrolabe in the figure is set for a latitude of 50°. The time of day for any solar altitude for any day of the year can be read directly from this setting.

For example, the Sun's altitude in the afternoon of May 20, is read as about 34°. The Sun's declination for May 20 is found to be about 20°. The time is found to be about 4 PM by locating the intersection of the 20° declination curve on the plate with the 36° altitude curve on the rete.

Peter Apian's "Meteoroscope"

"Meteoroscope" is the name given by Peter Apian to an device intended to allow the user to solve spherical trigonometry problems without calculations[88]. The name seems a bit obscure, but was apparently understood at the time to mean an instrument used to for determining celestial distances. If this is the correct context, it is appropriate for this construction, since the primary use of spherical trigonometry in the 16th century was astronomical.

Apian's meteoroscope is nothing more than a quadrant divided by the same stereographic projection used on the saphea plate. The meteoroscope is not an astronomical instrument, so the projection should not be described in astronomical terms. Therefore, the definition is altered to being a stereographic projection from a point on the surface of a sphere onto a great circle of the sphere orthogonal to the projection origin. The resulting figure is shown in Figure 13-10, which shows the similarity to the saphea plate. Each degree is drawn for both the parallels and polar arcs, and there are no labels except on the axes.

The quadrant is equipped with a thread anchored at the origin, and a sliding bead to mark distances on the thread. It is used to find the unknown side of a right spherical triangle, i.e. a spherical triangle with one interior angle of 90°.

This projection is suited for this application because all three sides of a right spherical triangle can be represented. For example, a right spherical triangle **ABC** is shown in Figure 13-10, with sides **a** = **40°**, **b** = **50°** and **C** = **90°**. The problem is to find the length of **c** and the angles **A** and **B**.

What is not immediately obvious about the projection, is that **c** is an accurate representation of the side of a spherical triangle. It is because the straight line, of which **c** is a segment, is the projection of a great circle passing through the origin.; the circle is viewed "sideways" or edge-on, thus a straight line.

Recall that the distance of any projected point from the origin is projected as **tan** $\alpha/2$. Therefore, if the thread is stretched from the origin to B and the bead set at that length, the thread can then be moved to one of the scales in the margins and the length read from the scale. The angle **A** is found by extending the thread to the scale on the limb and reading the angle. The angle **B** is found by reorienting the spherical triangle with **B** at the origin and **C** on the vertical axis. Thus, all three sides and all three interior angles of the right spherical triangle can be found from the meteoroscope.

Spherical triangles that are not right spherical triangles can also be solved by reducing them to two right spherical triangles.

According to North, Apian appropriated this use of the projection from Johannes Werner (1468-1528). Apian apparently considered this application to be significant, since it allowed spherical trigonometry problems to be solved by someone with no knowledge of spherical trigonometry.

[88] North, J.D., "Werner, Apian, Blagrave and Meteoroscope", British Journal for the History of Science, iv, 1966, 57-65.

North also points out it never gained much notoriety since few people who know nothing about spherical trigonometry want to solve spherical trigonometry problems.

It does, however, provide an additional application of the stereographic projection as used on the saphea.

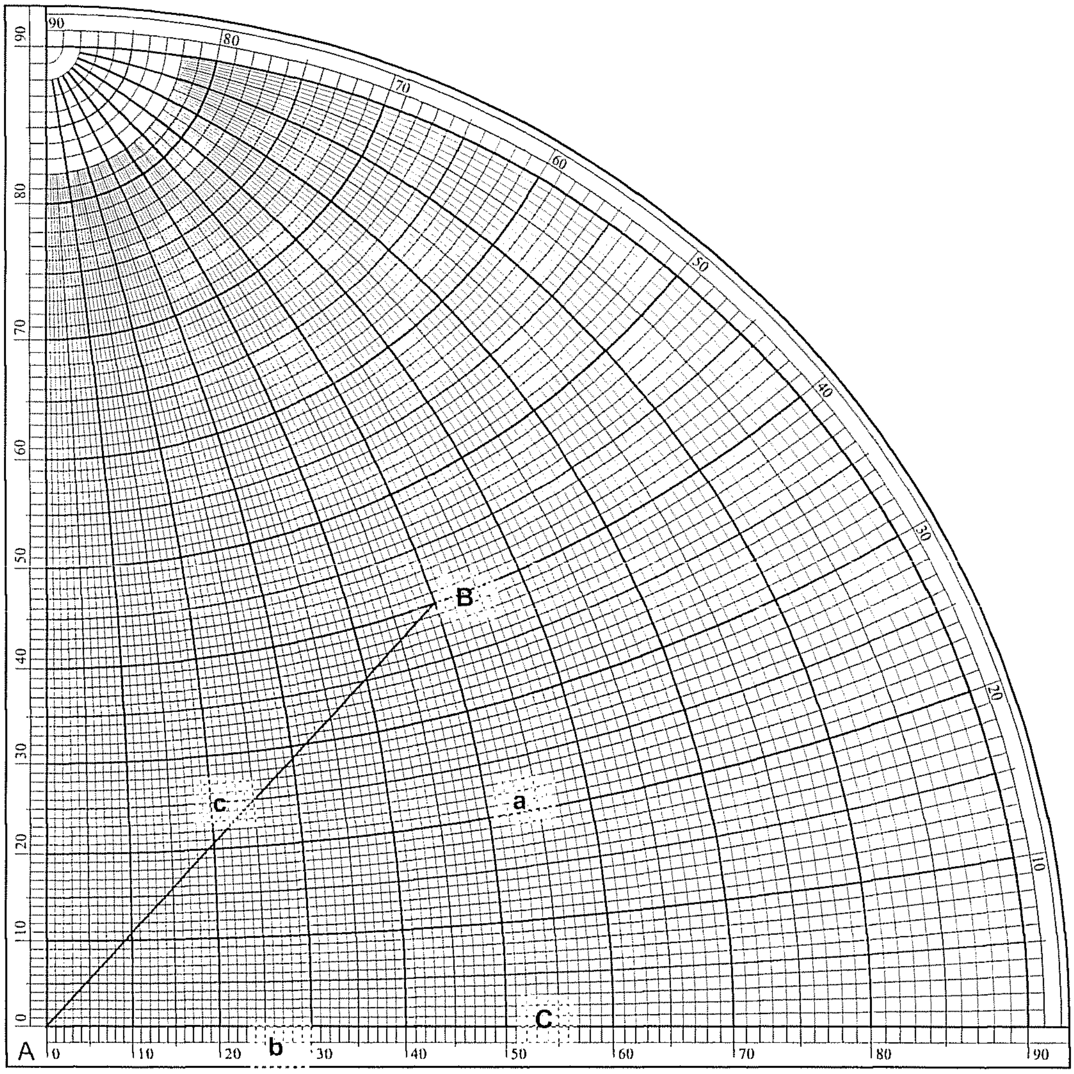

Figure 13-10. Peter Apian's Meteoroscope

The Quadratum Nauticum

Gemma Frisus' astrolabes included a *Quadratum Nauticum* on the inside of the mater. This construction was apparently an invention of Gemma Frisius, and he was quite excited about it, although it does not accomplish his original intent. It is included in this discussion, since it was often associated with saphea instruments inspired by the Louvain style.

The Quadratum Nauticum is intended to tell a sailor or traveler the direction between places of known latitude and longitude. See Figure 13-11.

The edges of the Quadratum Nauticum represent differences in latitude and longitude. The top and bottom edges represent longitude, and the sides represent latitude. Each edge is divided into two sections from 0° to 90°. The upper half of the sides are for a positive latitude difference, and the lower half is for a negative latitude difference. Similarly, the upper and lower edges represent positive longitude differences to the right of center, and negative longitude differences to the left. It might be simplest to treat the nautical square as a simple graph with East longitude difference as the positive x-axis and latitude difference as the y-axis.

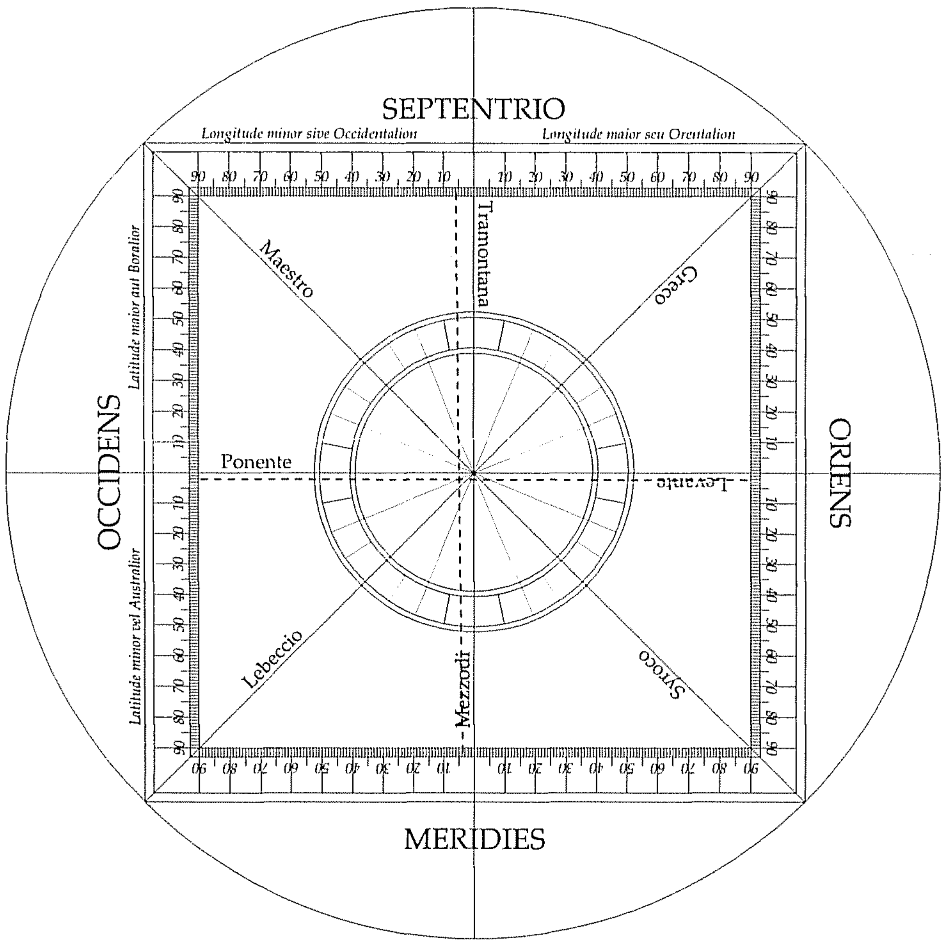

Figure 13-11. Quadratum Nauticum

The wind rose in the center is divided into eight directions for each quadrant representing the compass points. The labeled directions name the winds commonly used by Renaissance sailors in the Mediterranean Sea, and name the direction that the wind comes **from**. The wind names were fairly commonly accepted, although different spellings are to be found. The names on the figure are typical of those shown on this decoration.

In use, the user is considered to be at the center of the square. The latitude and longitude of the place are known. To find the bearing to another place of known coordinates, you subtract your latitude and longitude from the latitude and longitude of the place you want to go. Then a rule or thread is laid across the differences, and the point of intersection shows the bearing.

For example, if you are at Cagliari in Sardinia (39°N, 9° E) and want to go to Algiers (37° N, 3° E). Subtract your latitude from the latitude of Algiers (39° - 37° = -2°), and subtract your longitude from Algiers' longitude (3° - 9° = -6°). Since the longitude difference is negative, a thread is stretched from the 6° tic mark to the left of the vertical axis (vertical dashed line in Figure 13-11). Similarly, since the latitude difference is also negative another thread is stretched across the square from the 2° tic mark below the center (horizontal dashed line in Figure 13-11). The lines cross near the line on the wind rose two divisions below the x-axis. Each wind rose division represents one-eighth of the circle or 11¼°. Therefore, the nautical square tells you the bearing from Cagliari to Algiers is about 112 ½° W (90° + 22 ½°).

There is little reason to believe the Quadratum Nauticum was used very much. First, it doesn't work for large differences in latitude and longitude, since it does not account for the fact that a degree of longitude has different lengths depending on the latitude. A degree of longitude at the equator is many more miles long than a degree of longitude, say 45° from the equator (they are proportional to cos φ). It does give a fair approximation of the bearing between locations that are fairly close together, but the fine scale divisions make it very difficult to position the rule or thread accurately. Finally, it takes four hands to work it in any case.

In summary, the Quadratum Nauticum is a pretty diagram and makes a nice astrolabe decoration. It is not very useful.

Chapter 14 - Orthographic Astrolabes

Other projection techniques can be used to project the celestial sphere besides the stereographic projection. The orthographic projection (also called orthogonal) was applied to astrolabes very early in the development of the instrument, but its use was never widespread and orthographic methods were apparently never imported from Islamic sources. European astrolabes based on the orthographic projections were developed in the 15^{th} and 16^{th} centuries.

The Orthographic Projection

An orthographic projection of a sphere uses projection rays originating at infinity, and thus all rays are parallel. This projection technique is familiar, as it is used in drafting to show front, back and side views of a three dimensional component to preserve the dimensions.

When applied to the projection of the celestial sphere, declination circles are seen edge-on and are projected as straight lines parallel to the equator. Circles of right ascension are seen at an angle and are projected as half ellipses. The principle of the orthographic projection is shown in Figure 14-1. The result of the projection of the celestial sphere as used in the orthographic astrolabe is shown in Figure 14-2.

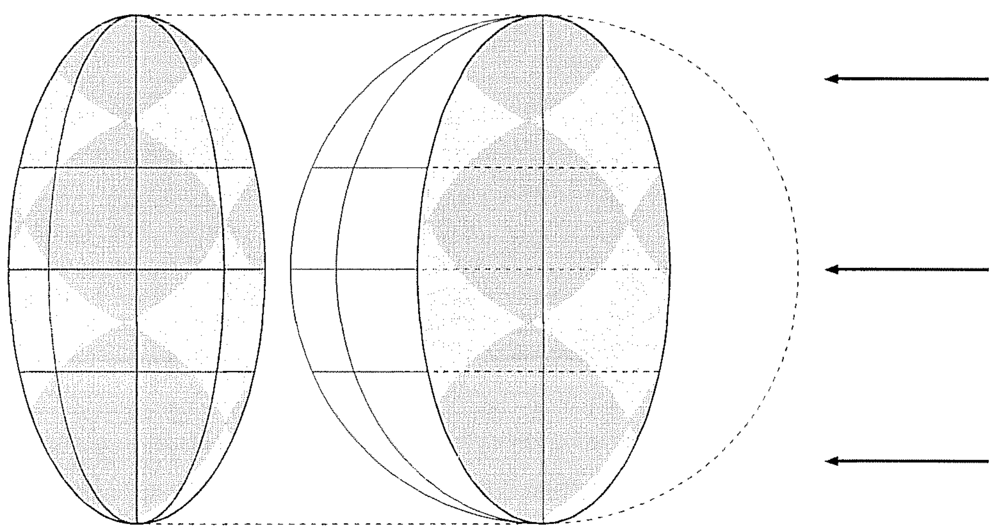

Figure 14-1. The Orthographic Projection

The mathematical relationship of a projected point is quite simple for this projection. Consider any point on the sphere. The point will be at some angle θ from the equatorial plane. The point forms a right triangle with hypotenuse equal to the distance of the point from the center, **r**. The distance of the point from the equator will simply be $\mathbf{r = R \sin \theta}$. A valuable property of the orthographic projection for use on an astrolabe is that the distance on the projection between any two points separated by angle θ is also $r \sin \theta$. We will see later how this property simplifies the uses of an astrolabe.

The projection shown in Figure 14-2, includes each degree of hour angle (right ascension) between the tropics and each three degrees of declination.

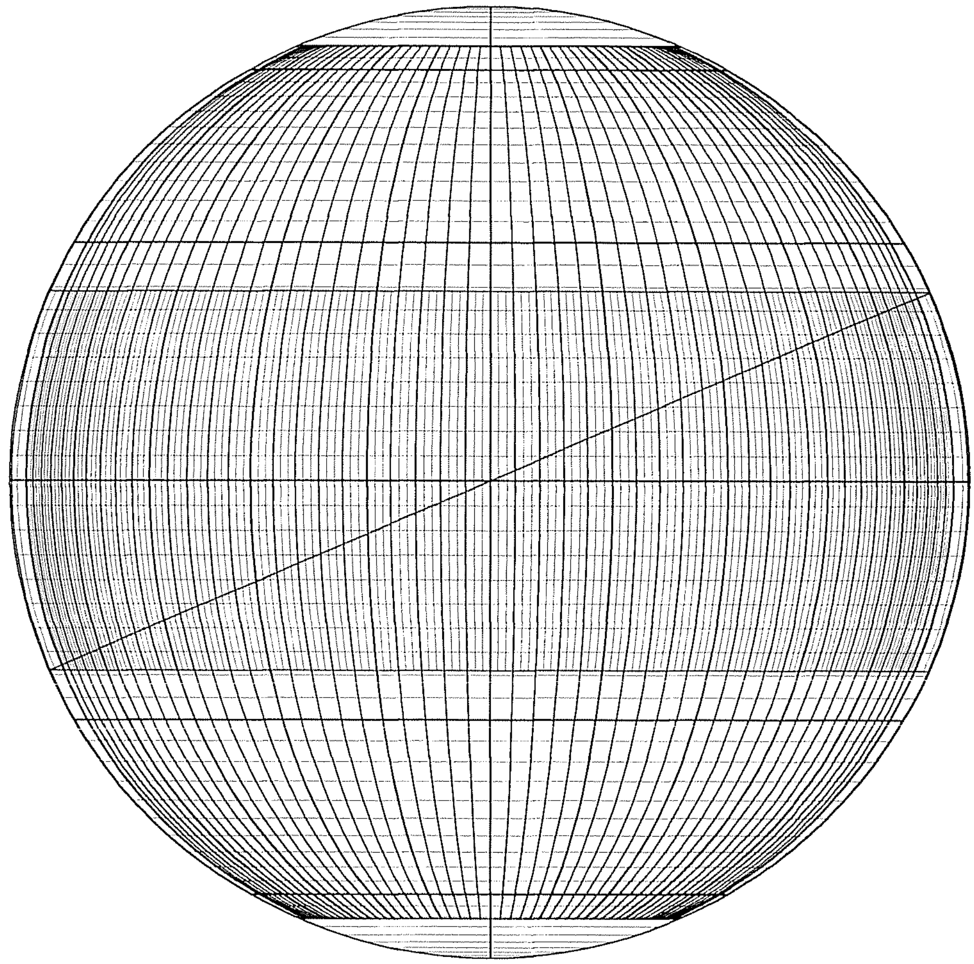

Figure 14-2. Orthographic Projection of the Celestial Sphere

The Organum Ptolemei

The first European application of the orthographic projection to astrolabes was probably made by Regiomontanus, and was either his invention or an invention of his teacher, Georg Peurbach[89]. Regiomontanus does not claim to have invented the grid and knew Ptolemy had touched on the theory[90]. This form of the display is known as the *organum Ptolemei*, which is literally translated as "Ptolemy's instrument". *Organum* is an instrument of any kind, including musical instruments. Regiomontanus was fond of Latin puns, and it is tempting to speculate that the name could also relate to celestial music inspired by Ptolemy.

The *organum Ptolemei* was engraved on a plate free to rotate on the back of the astrolabe. The plate could be oriented to the polar distance of a location in order to represent the local horizon.

[89] King, D. A., "Some Medieval Astronomical Instruments and their Secrets", in Renato Mazzolini, ed., *Non-Verbal Sources in Science before 1900*, Florence: Leo S. Olschki, 1993 pp. 20-52.
[90] King, D.A. and Turner, G. L'E., "The Astrolabe Presented by Regiomantanus to Cardinal Bessarion in 1462", *Nuncius: Annali di Storia della Scienza* (Florence) 9:1 (1994), p. 178.

Once in this position, the time of sunset and sunrise could be found using the alidade as the equinoctial and noting the time when the declination parallel for a day intersects the edge of the alidade. Thus, the *organum Ptolemei* was an astrolabe accessory for solving certain problems for any latitude. It could not be used to find the time from the Sun's altitude without an additional accessory; a rule with a graduated scale would be needed to measure the Sun's altitude on the grid. There is no evidence such rules were supplied with the instruments incorporating this diagram, so its use for finding time was probably little used if at all.

An example of an *organum Ptolemei* oriented to a latitude of 40° is shown in Figure 14-3. The parallels on the organum were for the declinations corresponding to zodiac longitudes of the Sun, and were drawn only between the tropics. The parallels in the figure are for each 3° of longitude. Examples on surviving instruments included division for the zodiac sections in the margin that are not included in the figure.

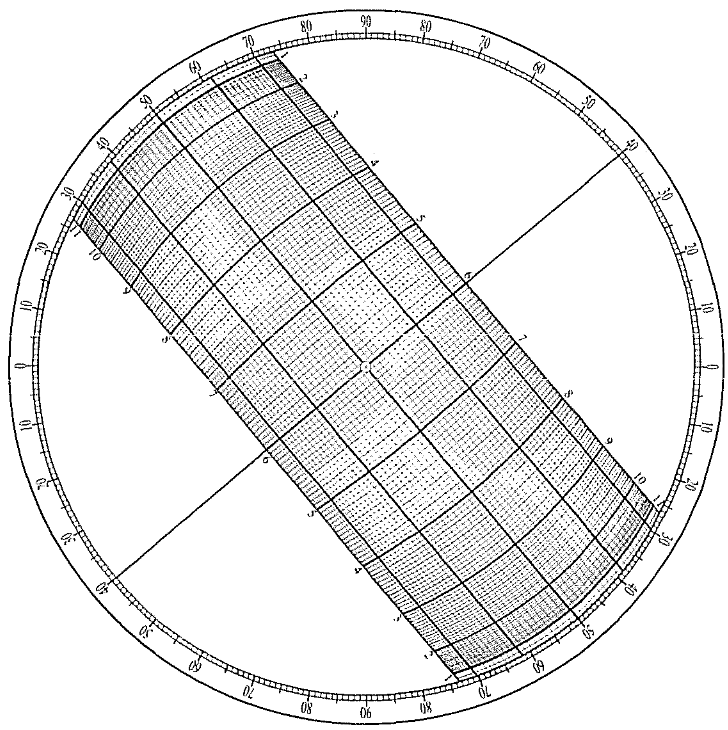

Figure 14-3. Organum Ptolemei

The Orthographic Astrolabe

Gemma Frisius' work on the *Astrolabum Catholicum* led directly or indirectly (probably directly) to the investigation of using other projection methods for universal astrolabes. Other projections had been independently studied in the Islamic world earlier, but it is not known whether this work was available in Louvain, although its existence was surely known. An obvious projection to investigate was the orthographic projection. Juan de Rojas, who was a student of Gemma, combined all of the relevant theories into a form of universal astrolabe using the orthographic projection that is traditionally associated with his name. The Rojas astrolabe can be used for many of the same problems as Gemma's universal astrolabe, and is much simpler to use to find the time. It is, however, more difficult to make, and beyond the skill of many makers who would have been intimidated by the use of elliptical arcs[91]. Any brass instruments would have been very expensive, and Rojas recommended making an instrument out of wood and paper.

The Rojas astrolabe never achieved the notoriety of the common astrolabe or Gemma's universal astrolabe, and only about 30 examples are known to exist today. It is highly likely many more specimens were made of wood and paper and do not survive. It was, however, studied by all serious astrolabe students.

The Origins of the Rojas Astrolabe

The Rojas astrolabe is so called after Juan de Rojas y Sarmiento. Juan de Rojas did not invent the astrolabe form carrying his name, nor was he the first to apply it to astrolabe, both of which he readily admits. He did, however, publish a beautiful and widely distributed book on the instrument. The instrument came to bear Rojas' name due of the popularity of the book.

Almost nothing is known about the life of Juan de Rojas, not even the dates of his birth and death. We can guess he was born 1520-1525. It is known he was from an old and distinguished noble Spanish family. He was the second son of the first Marqués de Poza and apparently showed academic promise at an early age, as it is said he was thought to be a potential bishop[92].

Rojas went to the Netherlands with the Emperor Charles V and Prince Philip, where he stayed in Louvain, site of the famous university. He was probably not a regular student at the university, but he studied under Gemma Frisius.

The exact dates Juan de Rojas spent in Louvain are not known, but he was certainly there in 1544, based on a published eulogy for his brother-in-law. Rojas apparently returned to Spain soon after, where he wrote his book on the orthographic astrolabe at his father's home in Monzón. Some of the examples in the book use dates in 1545 and 1546, so it is inferred he was back in Spain working on the book at that time.

Rojas' book was published in Paris in 1550 and dedicated to his patron, Emperor Charles V. It is a quarto volume of 160 leaves. The title page begins, *Illustris uiri D. ioannis de Roias Commentariorium in Astrolabum quod Planisphaerium vocant, libri sex nunc primum in lucem editit...* It is illustrated with 63 exquisite diagrams on the construction and use of the astrolabe described in the text. In addition to the astrolabe description, the book includes material on making sundials, tables related to sundials, practical geometry, tables of shadow lengths, squares and square roots, quotes from Gemma's work on surveying, and tables of the Sun's position. Rojas was apparently very well read, and the work contains many exact citations from classic sources, including some in classical Greek. It was reset and printed in a second edition the following year.

Unlike Gemma, Rojas does not claim credit for the invention of the instrument he describes and states quite clearly he did not define the projection used and was not the first to use it on an astrolabe. It is possible Rojas' interest in this specific instrument originated with a part of *Libro del Saber* called *Libro del la açafeha,* which included a picture of an astronomical device (definitely not an astrolabe) using an orthographic projection that was published only in Spanish. Rojas cites this work in his book. Other influences are possible, as there were other works describing astronomical devices based on the orthographic projection available. Most sources lead one to believe Rojas learned about astrolabes from Gemma, got interested in universal astrolabes, and decided to write a book on the instrument using the orthographic projection to fill

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[91] John Blagrave rejected the orthographic projection on the grounds that it used "geometrical crooked lines called ellipses".
[92] This section relies heavily on Maddison, Francis, "Hugo Helt and the Rojas Astrolabe Projection", *Rivista de Fgaculdade de Ciencias,* xxxix, (Agrupamento de Estudo de Cartografia Antiga XII Secção de Coimbra), Coimbra, 1966.

a gap in the literature. This may not be fair to this intelligent, well read, honorable man since he was, in fact, the first to put all the pieces together and write it down, making a permanent contribution to the art in the process.

On the other hand, there were some actual astrolabes made using the orthographic projection before Rojas. Surviving instruments by Hans Dorn of Vienna, dated 1480, anticipate the Rojas form in most regards. Dorn also made an instrument virtually identical to Gemma's *Astrolabum Catholicum* long before Gemma. Is it possible Gemma knew about Dorn's work, adopted the saphea arzachelis for himself and directed Rojas to the orthographic instrument? We will never know.

Rojas did not work alone on this instrument or book. The section of the book devoted to the instrument's construction was written by Hugo Helt, a Frisian who Rojas met in Louvain. Helt was from a wealthy but not noble family and studied in Louvain. He gained some local notoriety through his bravery during a siege of the city in 1542. Nothing is known about how Rojas and Helt became acquainted, but Helt returned to Spain with Rojas and lived in his home in Monzón, where he worked with Juan de Rojas on his book. Helt published a small book on a form of sundial he had designed in 1549, and it is clear Rojas thought him to be unusually gifted in both mathematics and literature. Little is known for certain about Helt's life after the astrolabe project was complete. It is likely he never returned to the Netherlands, preferring Spain, and may have died in 1594, possibly in rather poor circumstances.

As for Rojas, he seems to have abandoned scientific pursuits after publishing his book and to have gone into military service, as befitted a young nobleman. He is said to have died on a journey to Thrace, but it is not known when or why he was there. His estate included very a well-equipped library.

The Rojas Astrolabe

The orthographic projection of the celestial sphere becomes the plate of an astrolabe when a divided limb and appropriate labels are applied. The following figure shows a Rojas type astrolabe plate. The names of the plate elements are the same as the saphea.

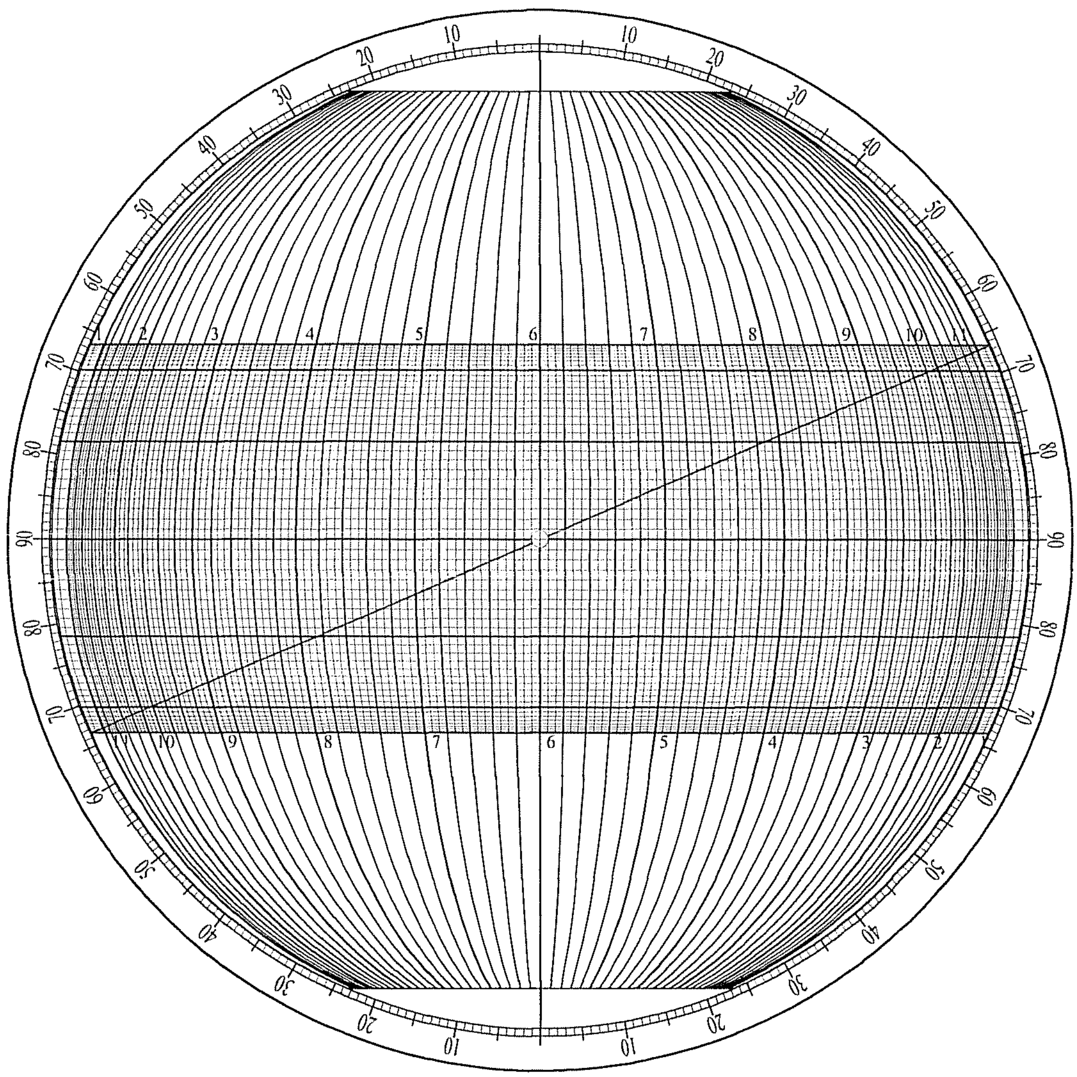


Figure 14-4. Rojas Astrolabe Plate

As on the saphea, the horizontal diameter (the equinoctial line) represents the equator, ecliptic or horizon. The center of the plate is the equinox. The vertical line represents the equinoctial colure or the local meridian. It is always called the meridian line.

The parallels represent degrees of declination, latitude or altitude. The traditional Rojas instrument shows the declination parallels for degrees of the zodiac and, as such, do not represent even values of declination. For example, the first bold parallel above the equinoctial represents the Sun's declination at the entry to Taurus or a declination of about 11° 29 ′. This style of division is convenient when working with celestial longitude described in terms of the zodiac. Problems involving declinations and altitudes are solved using the cursor to be described later. It is clearly possible, and maybe desirable, to draw the parallels for decimal values of declination. Variations on the Rojas instrument will be discussed later.

The Rojas astrolabe plate does not usually show polar arcs and declination lines for the entire celestial sphere. As described by Rojas, the intended use of the instrument is to solve problems relating to time and the position of the Sun. Therefore, the only portion of the celestial sphere shown in detail is between the tropics.

For timekeeping applications, the hour angle arcs are labeled with equal hours. Rojas instruments often have the phrase *Horæ ante meridian* in the top half of the plate between the Tropic of Cancer and the Arctic circle to emphasize that the hour labels on the top part of the plate represent hours before noon. *Horæ post meridian* is below the equinoctial to point out the afternoon and evening hours, and may be printed upside down.

The diagonal line connecting the tropics is the projection of the ecliptic. It is not divided on Rojas instruments because the zodiac divisions are supplied by the parallels, which represent the zodiac divisions. The Sun is at the center of the plate at the vernal equinox, moves up the ecliptic to the Tropic of Cancer, then reverses, moves back to the center at the autumnal equinox, continues down the ecliptic to the Tropic of Capricorn, reverses again and moves back up the ecliptic to the vernal equinox.

Fitted over the Rojas astrolabe plate is a regula that rotates around the center of the instrument. The regula is usually divided in degrees. When the regula represents the ecliptic, the divisions represent degrees of longitude. When it represents the equator, the divisions represent degrees of right ascension.

Connected to the regula, and free to slide along its length, is a cursor. The cursor is divided in degrees. The scale of the cursor can be interpreted as declination, latitude or altitude depending on the position of the regula.

The degree scale around the limb is divided in polar distance for setting the regula for a latitude. The angle of the horizon for a specific latitude is the polar distance of the latitude (90° - latitude). The scale is divided in this way to make it easy to set the regula for the latitude of a place.

Some Rojas instruments include the projected positions of a few stars for use in finding the time at night.

Like the saphea, the lines on the Rojas astrolabe plate can represent multiple coordinate systems. If the equinoctial line represents the equator, the parallels and polar arcs represent declination and right ascension. If the equinoctial line represents the ecliptic, the parallels and polar arcs represent latitude and longitude.

Figure 14-5 shows the complete Rojas astrolabe with the regula and cursor. The mechanical problems associated with making a regula accurately aligned with the plate center and having a means for the cursor to slide smoothly along its length and still be able to be secured are meaningful. There are not a great many Rojas astrolabes surviving, and the existing examples show a variety of approaches to solving the fabrication problems.

The regula in the figure is divided ± 90°. Other Rojas type instruments divided the regula 0-90° to the right of the center, and then 90-180° back to the center and so on, completing 360°. The arrows show only that the cursor slides back and forth along the regula and are not part of the instrument.

A close examination of the parallels will show the cursor is divided by hour angle degrees, and the parallels between the tropics are drawn for the zodiac angles. For example, the declination of Taurus 0° is about 11° 29′, which is read from the cursor.

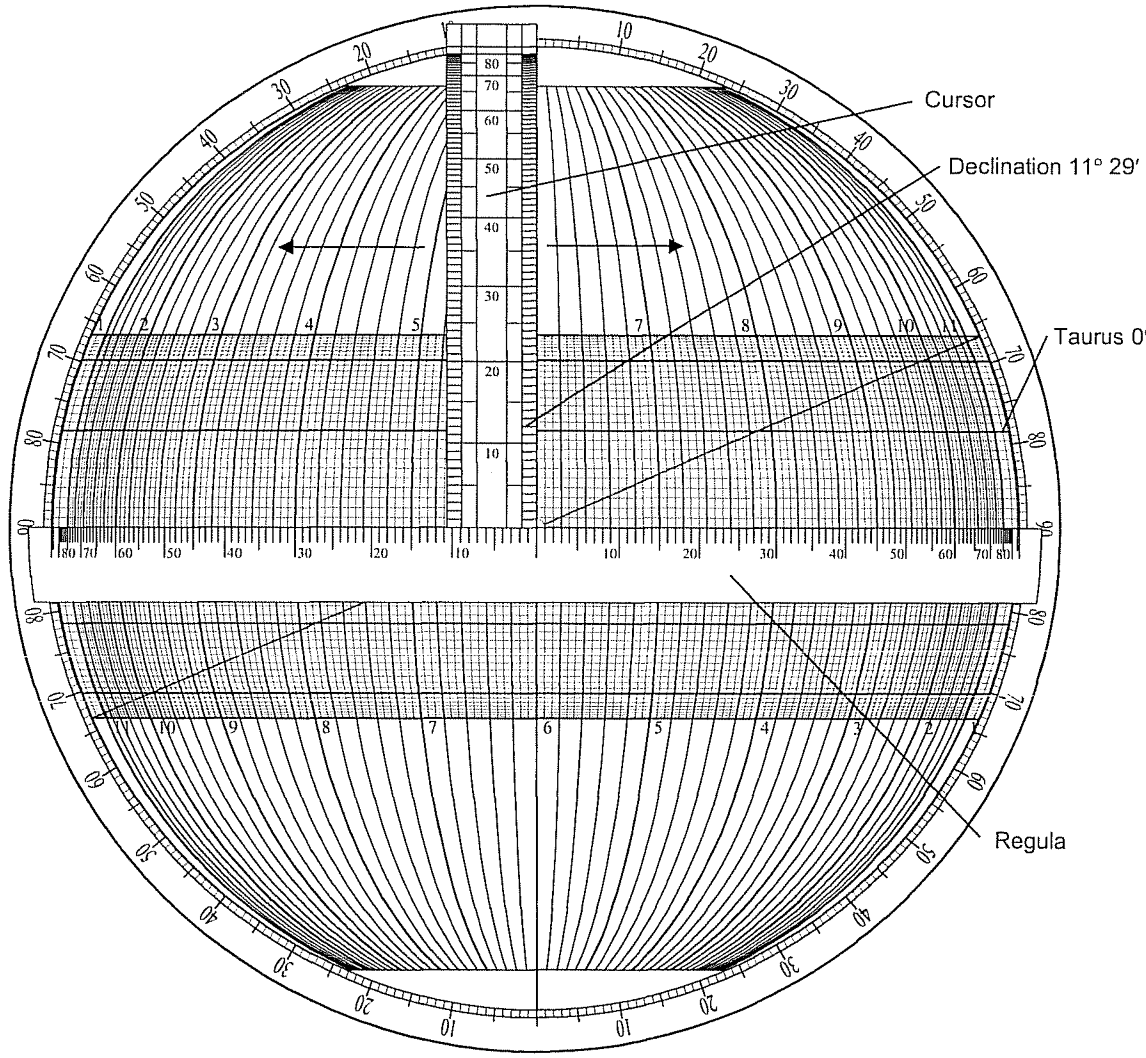


Figure 14-5. Rojas Astrolabe with Regula and Cursor

Using the Rojas Astrolabe

The uses of the Rojas astrolabe are identical to the saphea, with the notable exception of telling time. The astrolabe in Figure 14-6 is set to find the time on April 20 (Taurus 0°), at the Adler Planetarium in Chicago (latitude 41° 51′) when the Sun's altitude has been read as about 30° in the afternoon.

The regula is set for the latitude and the cursor is positioned on the regula, so 30° is on the declination parallel corresponding to Taurus 0°. Since it is afternoon, the time is read from the labels on the lower half of the plate as about 4 PM.

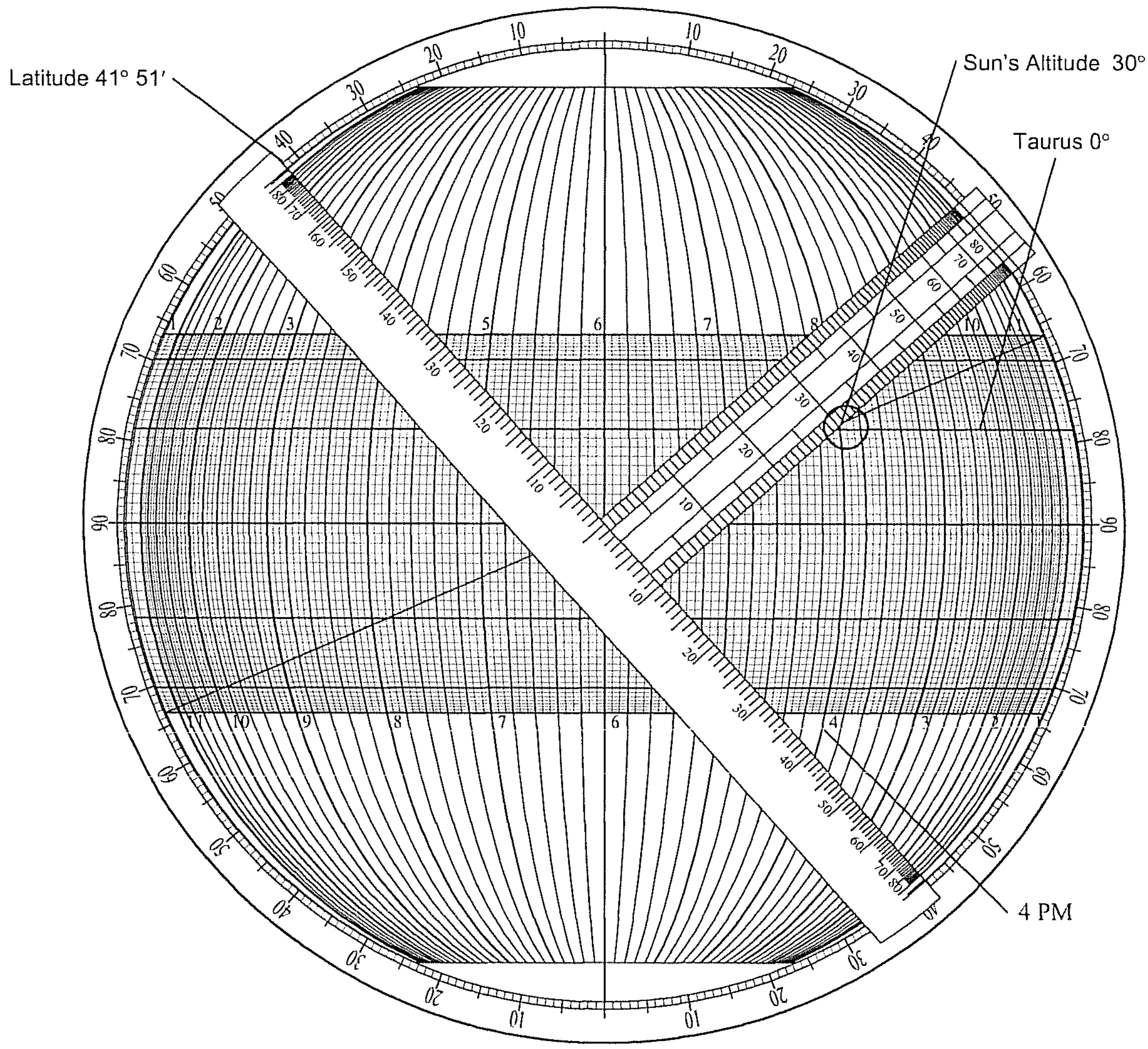


Figure 14-6. Rojas Astrolabe Set to Time

It is possible to find the time directly from the Sun's altitude on the orthographic plate, because all distances on the plate separated by angle α, regardless of the starting point, have the same distance relationship, namely $\mathbf{r} \sin \alpha$.

Making the Rojas Astrolabe

Like the saphea, a Rojas instrument should be made as large as possible.

A number of decisions are needed before beginning the layout. First, the method of dividing the limb needs to be determined. The division scheme in the examples is a good one, because it allows for very easy setting of the regula and for determining angles.

A classic Rojas instrument would include declination parallels corresponding to the zodiac longitudes, which restricts them to the area between the tropics. A modern instrument might include a full range of declinations with decimal divisions (see the note below).

The hour labels on the polar arcs are pretty standard.

Drawing the parallels.

Drawing the parallels is quite simple. For each desired line, calculate:

$$y = R \sin \delta$$

y is the distance of the line from the center, R is the radius of the plate, and δ the declination. Simply draw a line across the face of the plate at a distance y from the center. The upper and lower halves are symmetrical, so the calculation needs to be done only once. In fact, no calculations are needed at all if the limb has already been divided. Simply draw horizontal lines connecting the angle tics on the limb.

Drawing the parallels corresponding to the zodiac can be done by either calculating the declination for each degree desired, or using a geometric technique that simplifies the process (Figure 14-7). The geometrical method uses an auxiliary circle constructed at the tropics. The radius of the auxiliary circler, is $r = R \sin \varepsilon$. For each longitude λ, calculate $y = r \sin \lambda$. The line for λ is then drawn at a distance y from the center. The intersection of the line with the circle is at $x = \sqrt{(R^2 - y^2)}$.

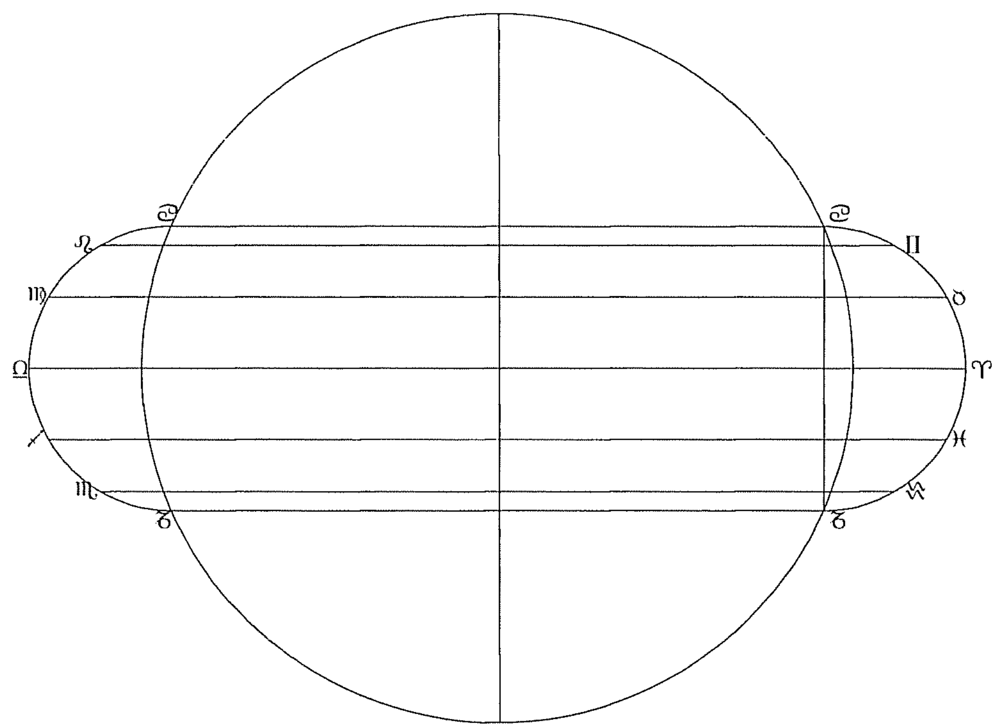


Figure 14-7. Drawing the Zodiac Parallels

The same procedure can be done graphically by dividing the auxiliary circle into even divisions and drawing a horizontal line from each division.

It is difficult to draw the lines any closer together than every two or three degrees. The parallels in the figures are drawn for each three degrees of longitude.

Drawing the polar arcs.

Each polar arc is a half ellipse with semi-major axis **R** and semi-minor axis **R sin** α centered at the center of the plate. The ratio of the ellipse's minor axis to the major axis is **cos** α.

Drawing an accurate ellipse can be quite difficult. Drawing an ellipse on a computer is easiest with PostScript which has commands for drawing ellipses of any eccentricity, although some additional advanced steps are needed to draw a wide line of uniform width. There is an excellent ellipse drawing technique called "The Mid-Point Algorithm"[93] that will draw ellipses very fast on a computer. This algorithm requires considerable programming skill to implement. The result is an ellipse drawn from a great many individual points which will not yield a smooth curve except at very high resolution, and the resulting ellipse is only one pixel wide. There are several ellipse drawing tools available for drafting. Other ellipse drawing methods for graphics artists are described in textbooks. Most ellipse drawing tools use the basic definition of an ellipse: the sum of the distances of a point from the foci of the ellipse is a constant.

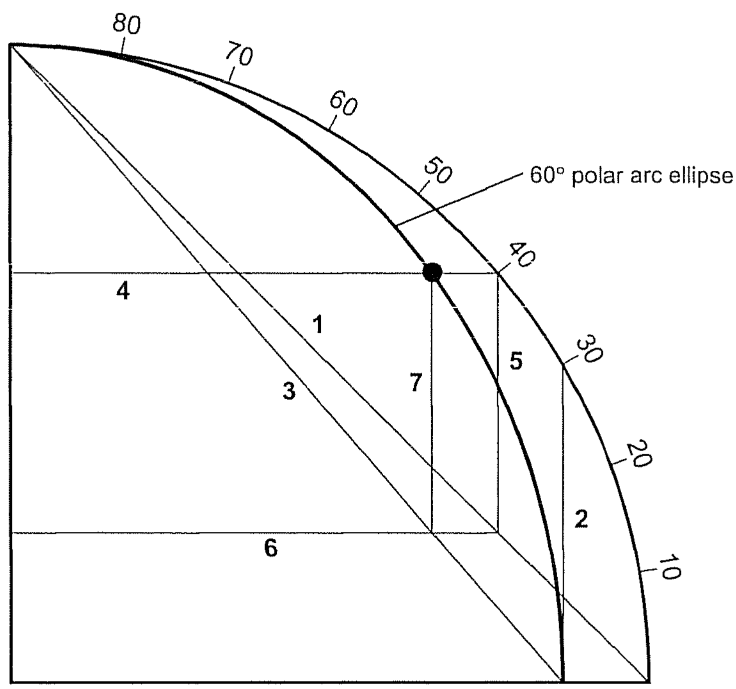


Figure 14-8. Ellipse Construction

Michel[94] describes the method used by Rojas to construct the ellipses. This method divides the parallels by a common ratio. The steps for constructing points on the ellipse as described by Rojas are shown in Figure 14-8. The steps are numbered in the figure.

The objective in the figure is to construct the ellipse for the 60° polar arc. We start with a blank quadrant.

1. Draw a line from the top of the quadrant to the lower corner of the quadrant.

[93] Van Aken, J. R., "An Efficient Ellipse-Drawing Algorithm", IEEE Computer Graphics and Applications, Sept. 1984, P.23 and Dappel, M. R., "An Ellipse Drawing Algorithm for Raster Displays", *Fundamental Algorithms for Computer Graphics*, R.A. Earnshw [ed.], Springer-Verlag, 1985, p. 257.
[94] Michel, H., *Traitè de l'Astrolabe*, Librairie Alain Brieux, Paris, 1976, translated by J. E. Morrison, p. 97

2. Locate the intersection point of the 60° polar arc with the equinoctial line by drawing a vertical line from (90 - λ) = 30°. This is equivalent to calculating the distance from the center as R sin λ.
3. Draw a line from the top of the quadrant to the intersection point.
4. Select any parallel. The 40° parallel is used in the figure, but the procedure is the same for all parallels. Draw a horizontal line for the parallel.
5. Draw a vertical line to the intersection with line 1.
6. Draw a horizontal line from the intersection.
7. Draw a vertical line the point where line 6 intersects line 3.
8. The point where line 7 intersects line 4 is a point on the ellipse.

This procedure is repeated as many times as possible using a different parallel for each point. The points are then connected with a smooth curve. Repeat the procedure for the lower quadrant and then for both quadrants on the other side. Repeat for all polar arcs. It works, but is very tedious.

For completeness, note the coordinates of the point on the ellipse for polar arc λ and parallel δ are:

$$x = R \cos \delta \sin \lambda, \; y = R \sin \delta$$

Note also it was common on Rojas astrolabes to mark the location of parallels outside the tropics that were not drawn with a little pip on the polar arcs.

The Regula and Cursor.

The divisions of the regula and cursor are calculated the same as the points on the plate. The mechanical construction of a working regula/cursor mechanism is beyond the scope of this book. It would probably be wise to examine photos of old instruments to see how their makers approached the problem. The cursor must slide smoothly along the length of the regula and have a method to lock it in position. The edge of the regula must lie exactly on the equinoctial line, which means the body of the regula will have a small connection to the pivot. Making a strong connection to such a small pivot is difficult.

Some instruments used a beveled edge to hold the cursor flat against the regula. This is an excellent method in theory, but the bevel must not hang over the regula edge and obscure the line beneath.

It probably makes more sense for a modern instrument to divide the orthographic plate by declination instead of the zodiac. Figure 14-9 shows the result.

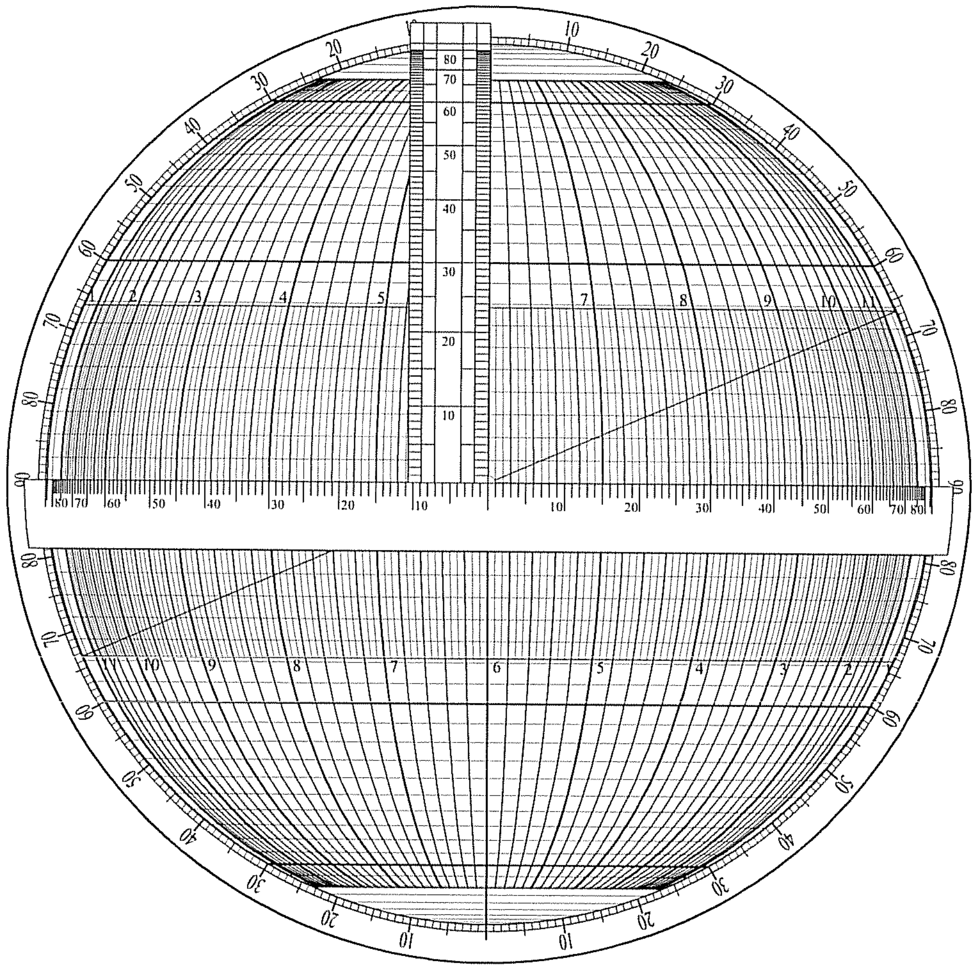


Figure 14-9. Orthographic Plate Divided by Declination

Chapter 15 - De la Hire's Astrolabe

As flexible as they are, the saphea arzechelis and orthogonal astrolabes have shortcomings resulting from the projection techniques. The parallels on the saphea become very close together near the equator, and the parallels near the poles on the orthogonal astrolabe are so close together they are usually omitted entirely. Phillipe de la Hire (1640-1718) suggested a revision to the universal astrolabe projection to solve this problem in his lectures to the French *College Royale* that caught the attention of Nicholas Bion (1652-1733), a prominent instrument maker. Bion produced a number of astrolabes on paper and cardboard using de la Hire's projection, only one of which survives (at the British National Maritime Museum in Greenwich). The de la Hire projection astrolabe never gained much popularity as it was introduced too late and interest in the astrolabe had already declined in favor of more specialized and accurate instruments. It is unlikely it would have gained a wide following in any case, because it is very difficult to draw.

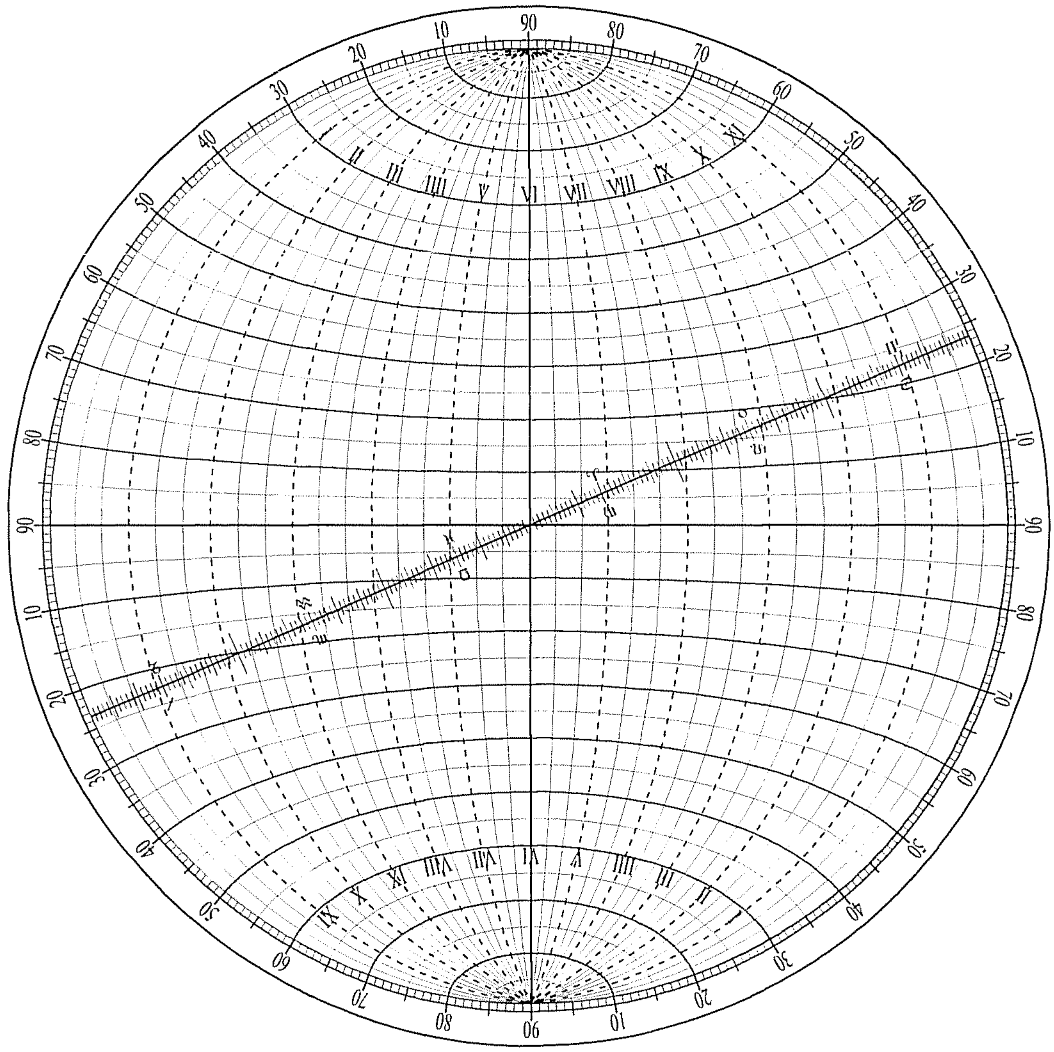


Figure 15-1. de la Hire Astrolabe Plate

De la Hire's approach was to set the projection origin at a point between infinity (the orthographic origin) and the circumference of the celestial sphere (saphea). The point is chosen so the parallel for 45° declination is midway between the equator and pole. This projection origin results in a plate on which the parallels are more evenly spaced for all declinations. The parallels are, however, arcs of ellipses that are less than the semi-ellipses as on the Rojas astrolabe. A good understanding of conic sections is required to draw a de la Hire plate.

The plate in the figure follows the surviving example and is divided for each five degrees in both latitude and hour angle. The nature of the projection would allow a denser resolution if desired without undue clutter. An actual instrument would be equipped with a rule and brachiolus as on the saphea.

Using the de la Hire astrolabe

The use of the de la Hire astrolabe is identical to the saphea and has the same requirement for finding the time by trial and error. It is a marginal improvement of the saphea and the additional manufacturing complexity would be difficult to justify for the slight benefit.

Making the de la Hire astrolabe

The layout of the meridian and equinoctial lines is the same as the saphea, intersecting perpendicular diameters.

It is useful to understand the derivation of the de la Hire projection in order to draw the curves.

Projection Origin

First, the location of the origin of the projection will be derived.

The projection origin is chosen to place the 45° parallel midway between the center and the circumference on the meridian.

See Figure 15-2. The view of the projection is from the side. The circle is the celestial sphere.

H is the projection origin. The vertical line represents the solsticial colure, which is the projection plane. The ray from the projection origin cuts the projection plane at P, halfway up the radius of the sphere. The objective is to find the distance of H from the center.

$$\cot\beta = \frac{R\sin 45}{R\sin 45 - R/2}$$

$$H = \frac{R}{2-\sqrt{2}} = 1.707106781\ R,\quad H = hR$$

$$H = \frac{R}{2}\cot\beta$$

$$\cot\ \beta\ =\ \frac{\frac{R}{\sqrt{2}}}{\frac{R}{\sqrt{2}} - \frac{R}{2}} = \frac{2}{2-\sqrt{2}} \qquad \sin 45 = \frac{1}{\sqrt{2}}$$

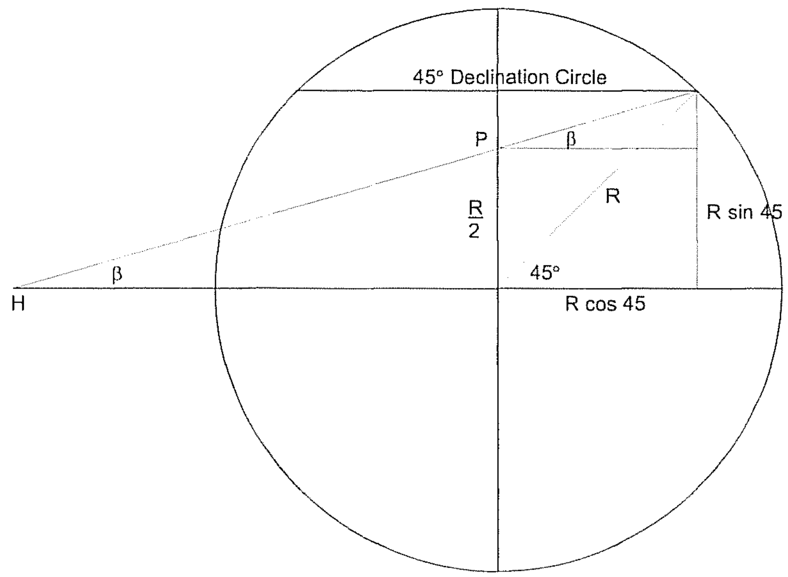


Figure 15-2. de la Hire Projection Origin

Parallels

The parallels are projected as ellipses that are "clipped" by the circumference of the sphere. The center of the ellipse and the lengths of the ellipse's axes must be calculated.

Figure 15-3 shows the celestial sphere from the side with the projection rays for the extremes of a declination circle (40° in the figure). The projections of the points C and D, C′ and D′, define the upper and lower limits of the projected ellipse and, thus, the minor axis.

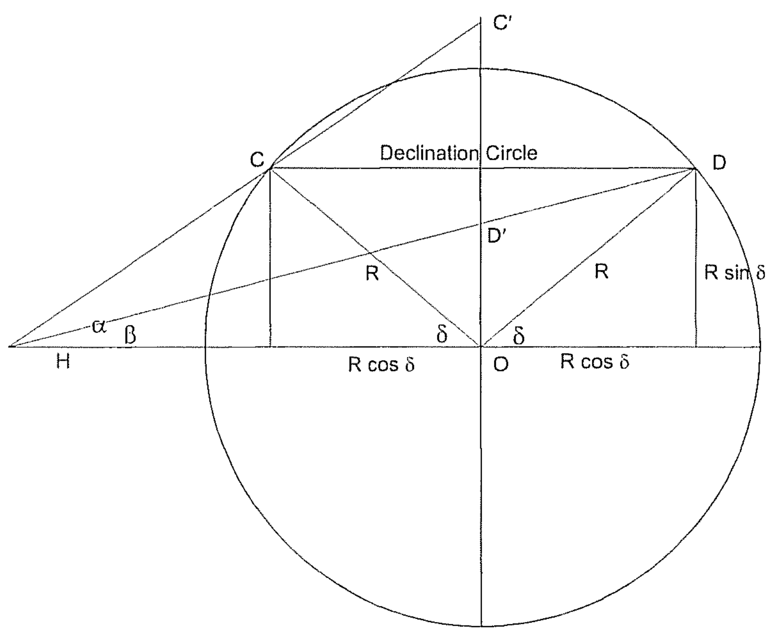


Figure 15-3. de la Hire Parallels - Ellipse Minor Axis

$$\tan\beta = \frac{R\sin\delta}{hR + R\cos\delta} = \frac{OD'}{hR}$$

$$OD' = \frac{hR^2\sin\delta}{hR + R\cos\delta} = \frac{hR\sin\delta}{h + \cos\delta}$$

Similarly, for C',

$$\tan\alpha = \frac{R\sin\delta}{hR - R\cos\delta} = \frac{OC'}{hR}$$

$$OC' = \frac{hR\sin\delta}{h - \cos\delta}$$

C'D' = OC' - OD' = ellipse minor axis = 2b. The ellipse center is at OD' + b.

The major axis of the parallel ellipse is found from similar logic, viewing the sphere and projection from above (Figure 15-4).

In this case, the size of the major axis cannot be found directly as a function of the declination, but there is a closed solution. A' is the projected end of the ellipse. a is the semi-major axis, OA'.

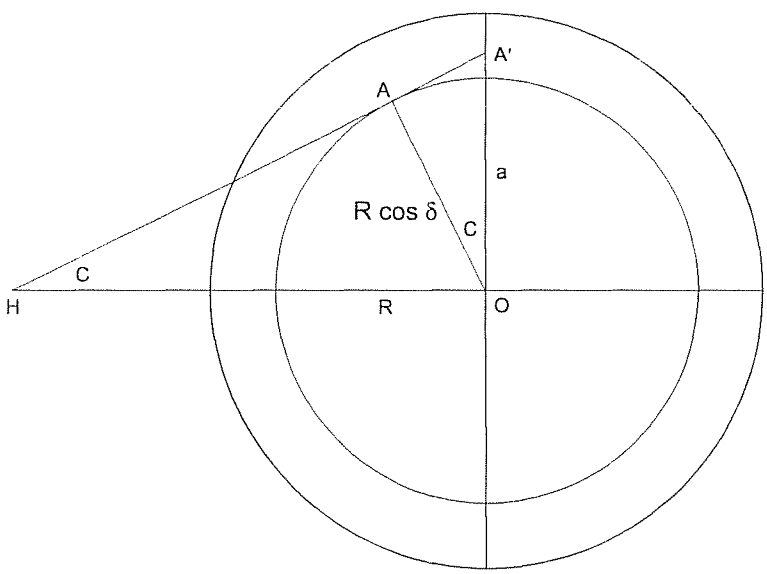


Figure 15-4. de la Hire Parallels - Ellipse Major Axis

$$\tan C = \frac{a}{H}$$

$$\sin C = \frac{R\cos\delta}{H} = \frac{R\cos\delta}{hR}$$

$$a = H\tan C \quad \text{where } C = \arcsin\left(\frac{\cos\delta}{h}\right)$$

The resulting semi-ellipses calculated using the above method are shown in Figure 15-5 for each 10°. Note only the part of the ellipse falling inside the plate is actually drawn on the plate.

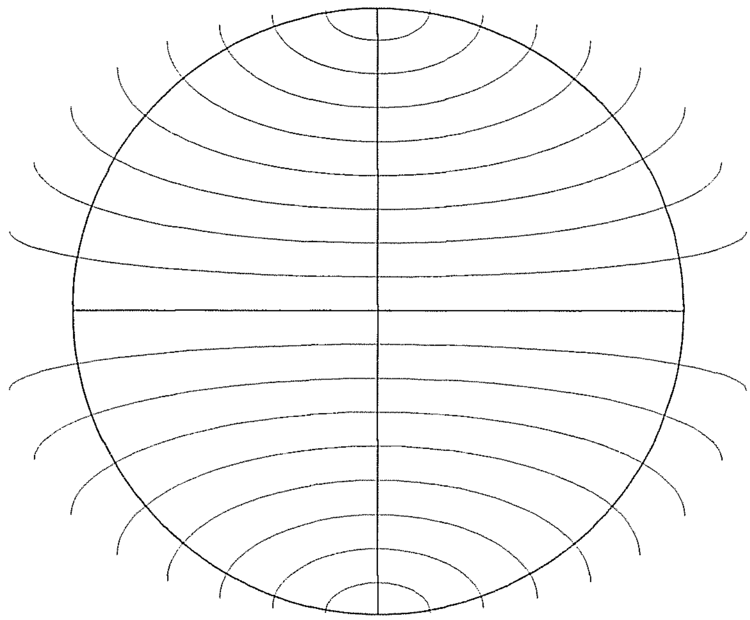


Figure 15-5. de la Hire Parallel Ellipses

Polar Arcs

The parameters of the polar arcs are also easy to derive.

The minor axis construction is similar to the minor axis for the parallels. Figure 15-6 shows the construction of the polar ellipse minor axis for 40° longitude looking down on the projection from above.

$$\tan\beta = \frac{R\sin\lambda}{hR + R\cos\lambda}$$

$$OB' = hR\tan\beta = \frac{hR\sin\lambda}{h + \cos\lambda}$$

Similarly,

$$OA' = \frac{hR\sin\lambda}{h - \cos\lambda}$$

The minor axis of the ellipse is:

$$b = \frac{OA' + OB'}{2}$$

The y-coordinate of the center is 0 and the x-coordinate of the center of the ellipse is:

$$x = \frac{OA' - OB'}{2}$$

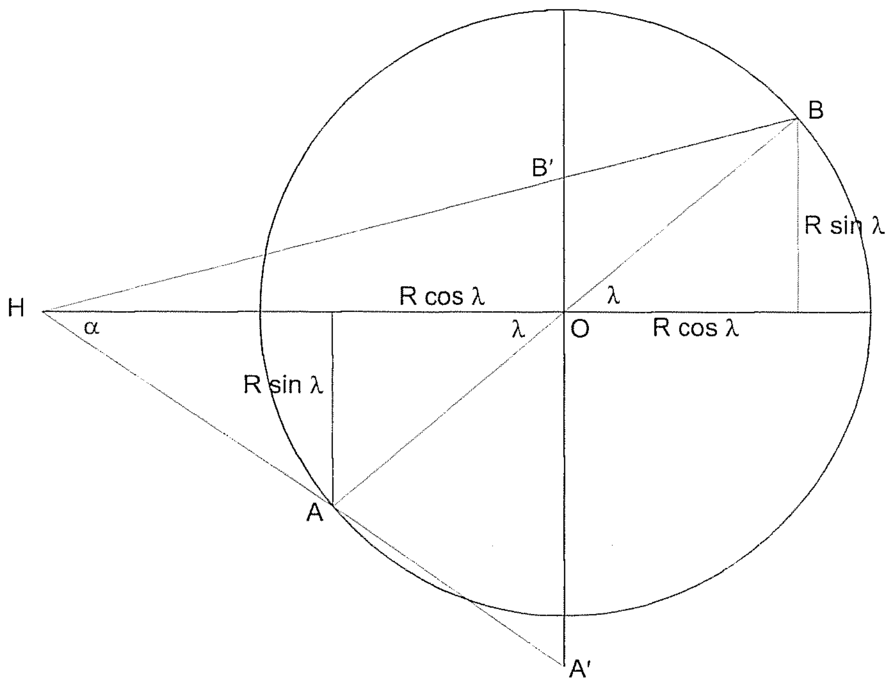


Figure 15-6. de la Hire Polar Ellipse - Minor Axis

Finding the semi-major axis of the polar ellipse requires some basic analytical geometry.

Figure 15-7 shows the polar ellipse for 40°. Note all of the polar ellipses pass through both the North and South poles (**N** and **S**). This gives us a point of reference for finding the semi-major axis, **a**. There are two way to find **a**, the semi-major axis of the ellipse.

The first method is to calculate **a** directly. The equator for an ellipse is:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

For all polar arcs, the point on the ellipse passing through **N** has the coordinates, $x = R$ and $y = x_C$ (the coordinate system is rotated to maintain the convention of the semi-major axis termed **a**).

Solving the equation defining the ellipse for **a**:

$$a = R\,b\sqrt{\frac{1}{b^2 - x_C^2}}$$

All of the parameters required to draw the polar ellipses are now available.

A second method uses an ellipse drawing technique. A rule of length (**a+b**) is drawn and positioned with the ends of the rule on the axes of the ellipse. The point of the division between the **a** and **b** parts of the line locates a point on the ellipse.

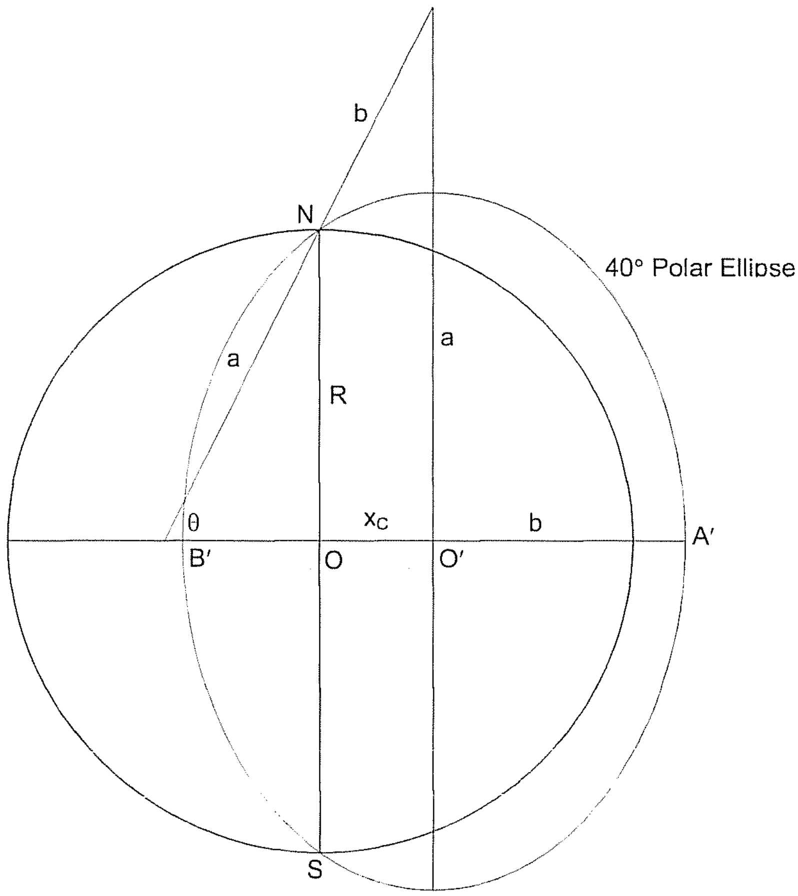


Figure 15-7. de la Hire Polar Ellipse - Major Axis

This line is shown in Figure 15-7 above. Note the point dividing the line into length of **a** and **b** falls on **N**, the point where the ellipse intersects the projection circle. The line makes an angle θ with the horizontal axis. Note also **sin θ = R / a** so **a = R / sin θ**. The value of θ is found from **cos θ = x_C / b**. Therefore, **a = R / sin [arccos (x_C / b)].**

The method to use to calculate **a** depends only on whichever is more convenient.

Drawing the ellipses is another matter entirely. Methods of drawing ellipses is beyond the scope of this work, but methods are described in drafting and graphic design books. The figures in this section were drawn using fairly simple PostScript programs and accurately represent the theory.

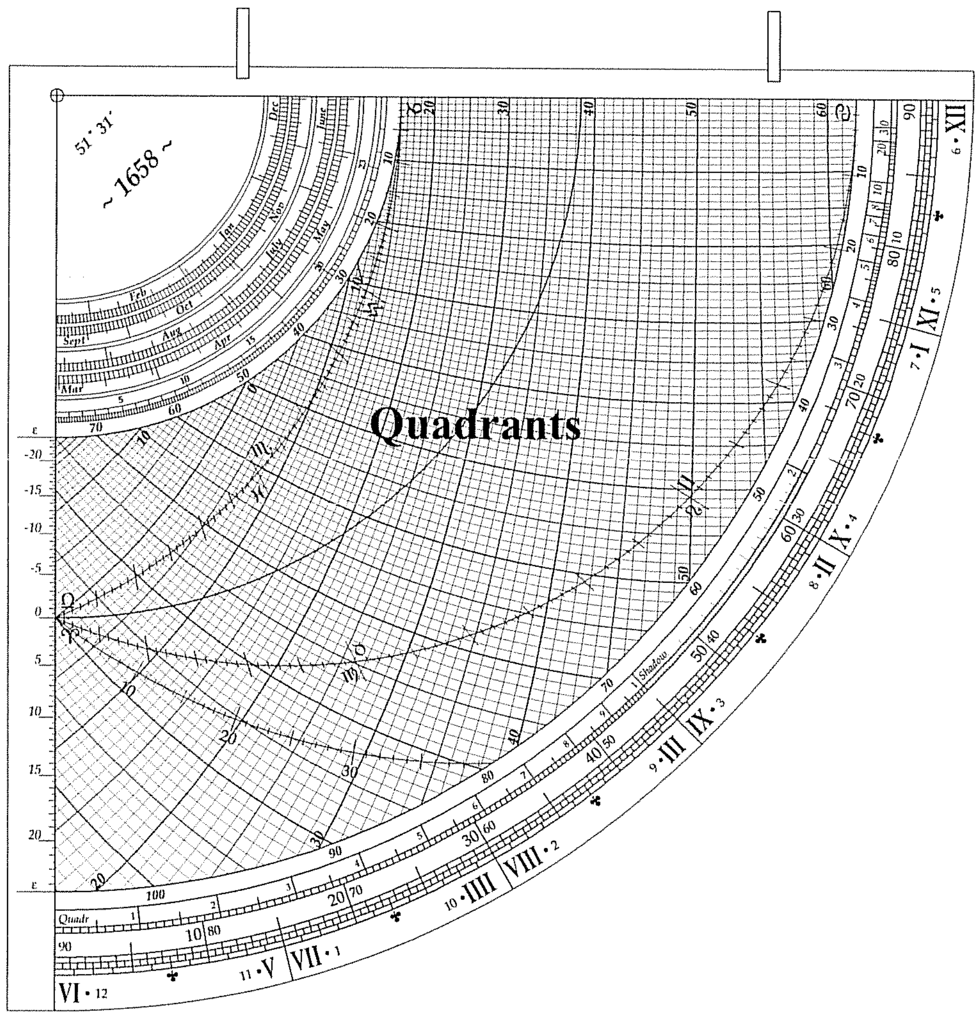
Calculation Example

Following is a table of calculated values for a de la Hire astrolabe of unit radius. All that is needed to draw the parallels and polar arcs are the lengths of the semi-major and semi-minor axes of the ellipses and the center coordinate. The centers of the polar ellipses are all on the vertical diameter, so only the y-distance from the instrument center is needed. Similarly, the center of the polar ellipses is the x-distance from the instrument center.

de la Hire Astrolabe

r = 1 h = 1.707107 H = 1.707107

	Parallels				Polar Arcs		
Φ	a	b	Center	λ	a	b	Center
5	1.22673	0.07712	0.13216	5	1.23142	0.13216	0.07712
10	1.20565	0.15014	0.26026	10	1.22425	0.26026	0.15014
15	1.17149	0.21541	0.38071	15	1.21282	0.38071	0.21541
20	1.12557	0.27011	0.49071	20	1.19780	0.49071	0.27011
25	1.06947	0.31243	0.58849	25	1.18003	0.58849	0.31243
30	1.00494	0.34156	0.67327	30	1.16041	0.67327	0.34156
35	0.93366	0.35756	0.74515	35	1.13979	0.74515	0.35756
40	0.85720	0.36117	0.80486	40	1.11899	0.80486	0.36117
45	0.77689	0.35355	0.85355	45	1.09868	0.85355	0.35355
50	0.69385	0.33609	0.89260	50	1.07945	0.89260	0.33609
55	0.60898	0.31025	0.92340	55	1.06172	0.92340	0.31025
60	0.52293	0.27745	0.94729	60	1.04587	0.94729	0.27745
65	0.43620	0.23902	0.96548	65	1.03213	0.96548	0.23902
70	0.34910	0.19614	0.97899	70	1.02070	0.97899	0.19614
75	0.26185	0.14989	0.98865	75	1.01170	0.98865	0.14989
80	0.17455	0.10122	0.99510	80	1.00521	0.99510	0.10122
85	0.08727	0.05099	0.99880	85	1.00131	0.99880	0.05099


51° 31'
~ 1658 ~
Quadrants
Shadow
Quadr
VI • 12
11 • V
VII • 1
10 • IIII
VIII • 2
9 • III
IX • 3
8 • II
X • 4
7 • I
XI • 5
6 • XII

Chapter 16 - Quadrants

Introduction

A quadrant is one-fourth of a circle, or 90°. Normally a quadrant instrument is made as a solid pie shaped wedge engraved with scales appropriate for its use.

Many types of quadrant instruments have been used, some for very specific applications. A simple quadrant made of wood or engraved metal with a scale of degrees has been used since antiquity to measure angles (Figure 16-1). A typical quadrant of this type would be equipped with sights along one edge and a thread with a weighted bob or an articulated rule. In use, the user would view the object of interest, such as the top of a tower or a star, through the sights while the weighted thread was allowed to swing freely. Note would be made of the point on the scale cut by the thread when the sighting was complete. The angle was then used to solve a specific problem in navigation, surveying or other application.

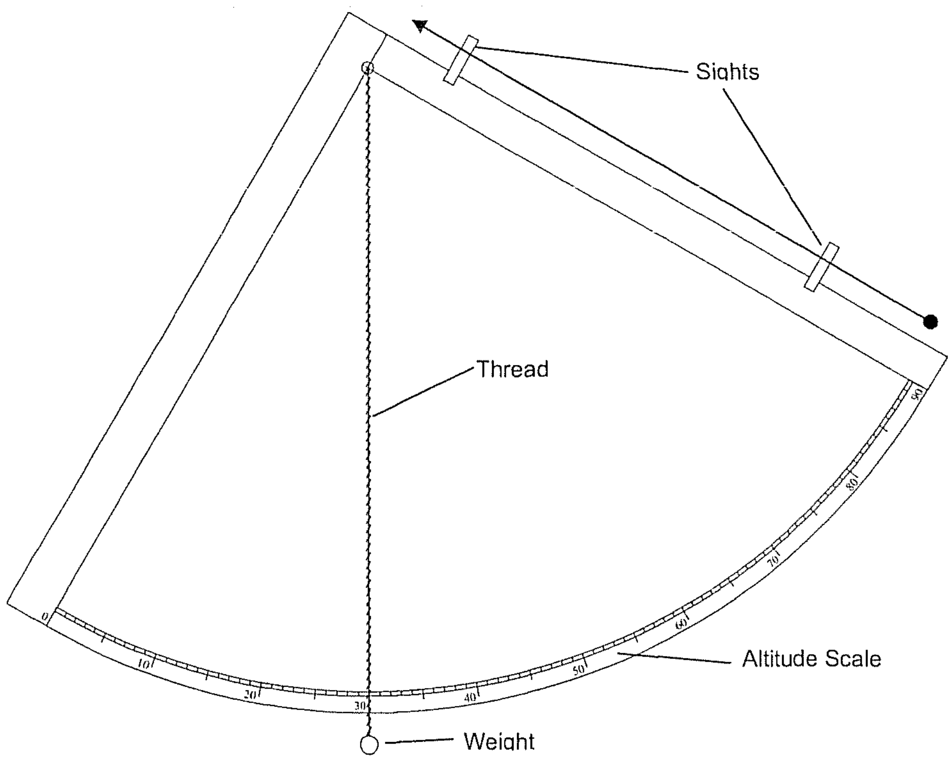


Figure 16-1. Simple Quadrant

The quadrant was very popular because it was easy to make and use, relatively inexpensive, and adequately accurate for most contemporary applications.

A hand-held quadrant is not easy to use accurately. It is difficult to find a star in the sights, hold the quadrant steady, press the thread against the scale and get a reading closer than a degree. Errors of five degrees were common. Nonetheless, such instruments were a staple of astronomers and were also used for navigation, artillery, construction, surveying and other applications where it was necessary to measure angles of large objects or objects that were far away.

When a quadrant is permanently mounted and aligned with the meridian, it is called a *mural quadrant*. Later mural quadrants used by astronomers, Tycho Brahe in particular, were permanently mounted, quite large, corrected for systematic instrument errors, and included very accurate scales capable of interpolation to a few arc minutes. Tycho's mural quadrant was not of the same order of precision of modern instruments, but it was accurate enough to give Kepler the empirical data required to revolutionize positional astronomy. Astronomical mural quadrants equipped with telescopic sights and very accurate nodial scales, and verniers later, were used in serious observatories through the 19th century, and some examples are true works of art

The Horary Quadrant (quadrans vetus)

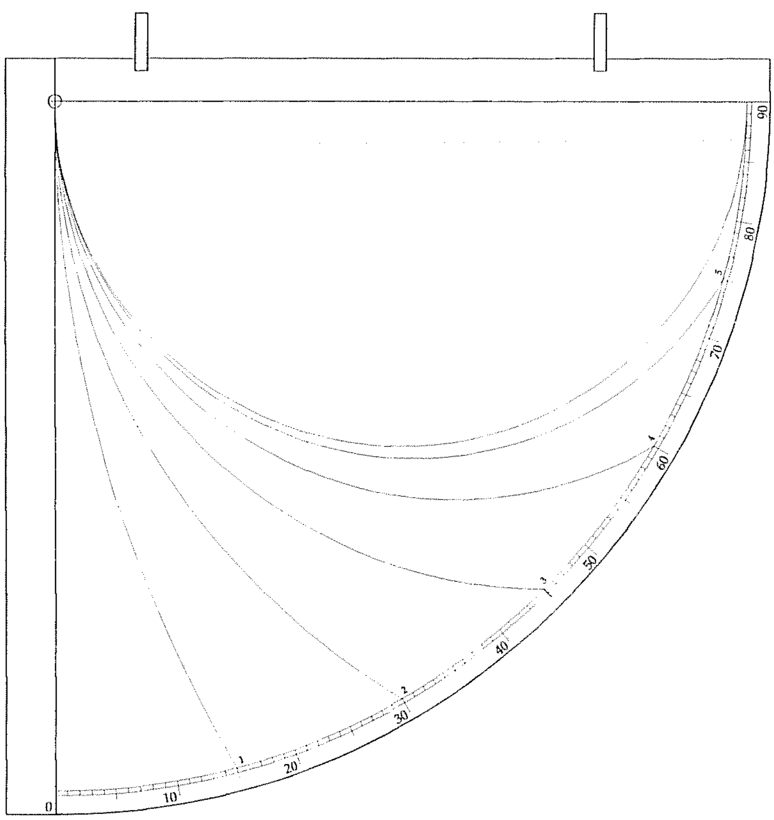


Figure 16-2. Horary Quadrant (*quadrans vetus*)

A *horary quadrant* (Figure 16-2) is a quadrant used to find the time from the altitude of a celestial object. One type of horary quadrant, often called the *quadrans vetus* (old quadrant) after the introduction of the astrolabe quadrant, was of Islamic origin and apparently enjoyed some medieval popularity. Its sole use was to find the unequal hour of the day from the Sun's noon altitude and current altitude. A simplified example is shown in Figure 16-2. A shadow square might also be included. King refers to this construction as a *universal horary quadrant* because the single scale can be used at any latitude, although with declining accuracy for more northerly locations.

A thread, traditionally of silk, with a weighted bob on the end is suspended from the hole in the upper left corner. A pierced bead or seed pearl that will slide on the thread, but hold its position, is strung on the thread.

The scale is constructed and used exactly like the unequal hour scale on the back of many European astrolabes. The semi-circle represents noon, and the other arcs represent unequal hours from noon. The appropriate correction is required, depending on whether the time is before or after noon. The thread is positioned at the angle of the Sun's noon altitude for a date and the bead is set on the six-hour semi-circle. The thread is then positioned at the Sun's current altitude and the time is read from where the bead lies in the hour arcs. For example, if the bead falls near the second arc from noon and it is the morning, it is the fourth hour. If it is afternoon, it is the eighth hour. Note this scale works fairly well in low latitudes, which includes most of the medieval Islamic world, but is less accurate the farther north the latitude.

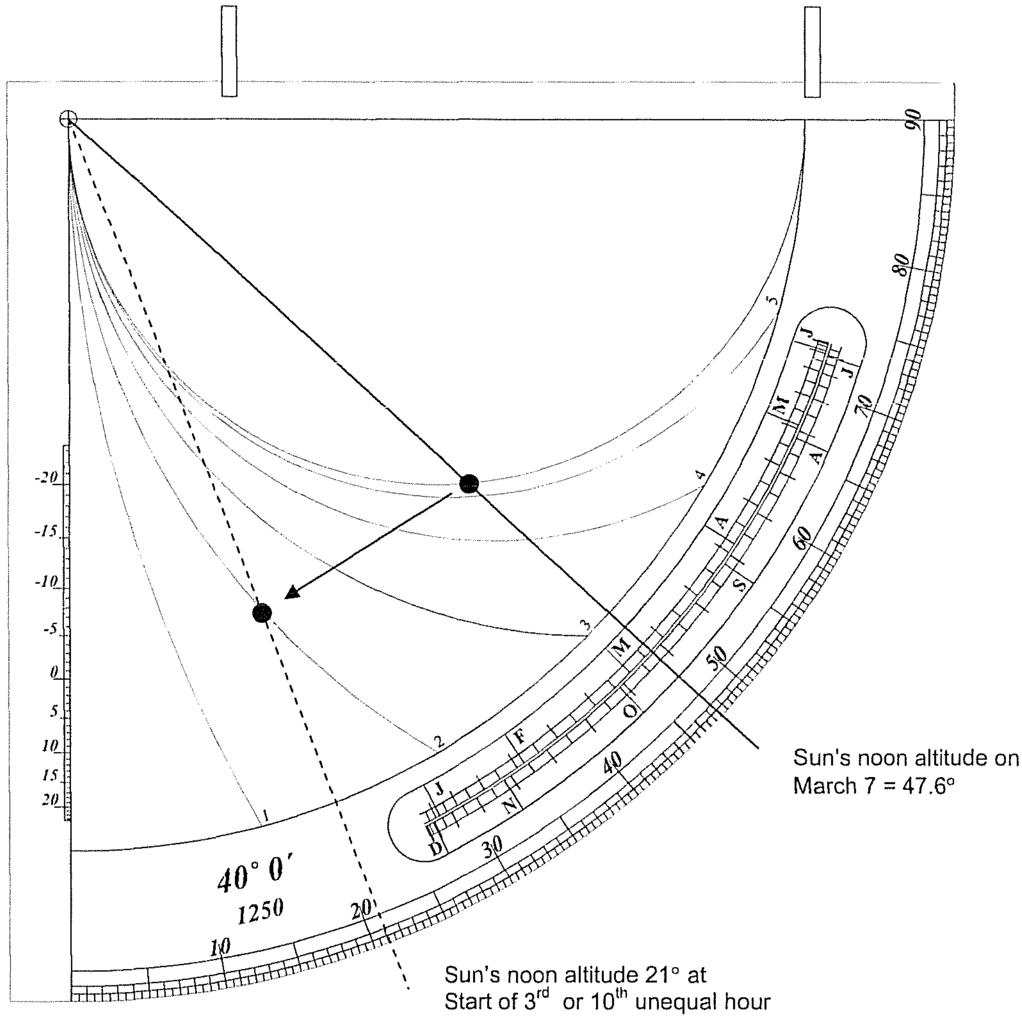


Figure 16-3. Horary Quadrant with Calendar Scale

This instrument is quite portable if the Sun's declination for the day is known. The Sun's meridian altitude is calculated from: **h_6 = 90° - Latitude + Sun's declination**. Therefore, the bead can be positioned on the six-hour arc using the calculated altitude angle and the current time found from an observation of the Sun.

Medieval manuscripts suggest this type of quadrant should be supplied with a calendar scale giving the Sun's meridian altitude for each day of the year to eliminate the need for access to an ephemeris of the Sun's daily declination. The quadrant in Figure 16-3 includes a calendar scale for the latitude of the quadrant. In use, the thread is laid on the date, and the Sun's noon altitude is shown on the degree scale. An example for March 7, 1250, is shown on Figure 16-3. The bead is moved to the six-hour circle and the current time determined as before. It was suggested the calendar scale should be inserted in a curved groove so it can be adjusted for different places. Several such instruments survive.

The cursor shown in manuscripts and surviving examples include the zodiac along with the calendar, but this makes the scale quite large and bulky. If a zodiac scale is desired, it would be more convenient to put it on one of the instrument margins. Of course, it would also be possible to replace the calendar scale with the zodiac.

Figure 16-3, which does not represent an actual surviving instrument, also includes a scale of declination along the left margin. This scale is rather obvious, and while it is unlikely any old instruments included such a device, it does improve the utility of the quadrant. This scale can be used to find the Sun's meridian altitude or to find the declination if the meridian altitude is known. If the declination is known, the thread is placed over the declination scale and the bead is set to the day's solar declination. The thread is then moved to the Sun's current altitude and the hour is read from the arcs. To find the declination, the bead is set to the six-hour curve for the day's meridian solar altitude and then moved to the declination scale, and the declination for the day is read from the scale.

Many variations are possible. For example, it would be possible to divide the six-hour circle by the calendar or zodiac directly, although this would be a rather dense scale.

History of the Horary Quadrant

The universal horary quadrant is based on an approximate equation for the unequal hour of the day of Indian origin:

$$\sin 15\mathbf{T} = \frac{\sin \mathbf{h}}{\sin \mathbf{H}}$$, **T** = the unequal hour, **h** = Sun's altitude, **H** = Sun's noon altitude

The horary scale is of Arabic origin, possibly as early as the 9th century, from anonymous Baghdad astronomers as confirmed by a recently discovered manuscript[95].

The earliest Latin manuscript describing this instrument is from the early thirteenth century by Johannes de Sacrobosco, or John Halifax, or Jean de Hollywood, about whom little is known except he was born in England and was a professor at the University of Paris. Historians have inferred his date of his death as 1244 or 1256. Other manuscripts describing this type of quadrant have been attributed to Robertus Angelicus, also of the 13th century, around 1271. It is generally assumed the *quadrans vetus* has an Islamic origin based on the appearance of this scale on an instrument attributed to Arzachel in a manuscript dated 1231.

Interested readers are referred to Archinard for a more complete discussion of the history and accuracy of this scale.

Making a Horary Quadrant

The only justification for making a reproduction of this type of instrument would be for historical interest, as it has virtually no modern use. It is, however, instructive to consider how such a device might be constructed.

[95] King, David A., *World-Maps for Finding the Direction and Distance to Mecca*, Brill, Leiden (1999). p 367.

Before beginning, it is essential to decide what features your quadrant will include. A basic quadrant needs only a divided scale along the edge and the unequal hour scale. Either a smaller unequal hour scale or a larger instrument will be required if a calendar scale is to be included. The declination scale or a zodiac scale is completely optional.

1. **Basic design parameters.** Determine the size of the quadrant. The side dimension of most old quadrants was about six inches (15 cm), which gives a very useable instrument. A slightly larger instrument of, say, eight inches (20 cm), would be a bit more accurate.

2. **Margins.** Margins outside the quadrant proper are needed for the declination scale if included, and to provide for the center hole. About 1/2 inch (13 mm) is sufficient. The two margins need not be equal, and it may be necessary for the margin on the side with the sights to be a bit larger to keep the sights from intruding on the scales.

   The intersection of the margins defines the center of the limb and tropic arcs and the hole for the thread. Note the limb, equator and tropics are drawn from this center. The radius of the limb is less than the length of the sides. Therefore, there must be a straight section the length of the margin from the tangent to the limb at the margin to the edge of the quadrant.

3. **Limb division.** Divide the limb by degrees. It is possible to show half degrees on most quadrants. It is customary to box the degree scale for this type of instrument. Almost all old instruments used boxed scales.

4. **Draw the six-hour circle.** Determine the size of the six-hour circle based on whether a calendar scale will be included between the hour scale and the limb. Draw a circle equal with radius equal to the diameter of the six-hour circle to define the limits of the scale. The radius of the six-hour circle is half the size of the limiting circle.

5. **Graphical construction of the unequal hour scale arcs.**

   f) Divide the six-hour semi-circle into six equal parts, P_i , i = 1...6 using the degree scale along the limb. Each section is 15 degrees.

   g) For each division, draw a chord from the instrument center to the limiting circle.

   h) Find the center of this line and erect a perpendicular.

   i) The point where the perpendicular to the chord intersects the meridian line is the circle center.

   j) The circle radius is the distance from the circle center to the point where the chord intersects the limiting circle. The method is outlined in Figure 16-4 for a single curve.

6. **Analytic construction of unequal hour scale.**

   Each unequal hour arc intersects the altitude scale at 15i° where i is the unequal hour. Notice in Figure 16-4 that the center of the hour curve is the apex of an isosceles triangle with equal sides of r_i. The base angles of this triangle are **(90° - 15i)**. The perpendicular erected from the chord defines a right triangle with base of **R/2**, where **R** is the radius of the limiting circle. The apex angle of this triangle is 15i. Solving for r_i, $\sin 15i = (R/2) / r_i$, so $r_i = R / (2 \sin 15i)$. The arc is drawn from a distance r_i from the base of the six-hour curve. A similar construction results if the hours are counted from the opposite end of the scale with the result,

   $$r_i = R / (2 \cos 15i).$$

7. **Declination scale.** The positions of the tics on an optional declination scale are calculated from:

   $$x = R \sin (90 - \text{latitude} - \text{declination})$$

where x is the distance of the declination tic from the center. This is easily derived by noting x is the length of a chord of the six-hour circle define by a line from the instrument center to the noon altitude on the limb. This chord defines the base of a right triangle inscribed in the six-hour circle with a base angle of (90 - h), where h is the meridian solar altitude.

h = 90° - latitude - declination. cos (90 - h) = x / R, therefore, **x = R sin h**.

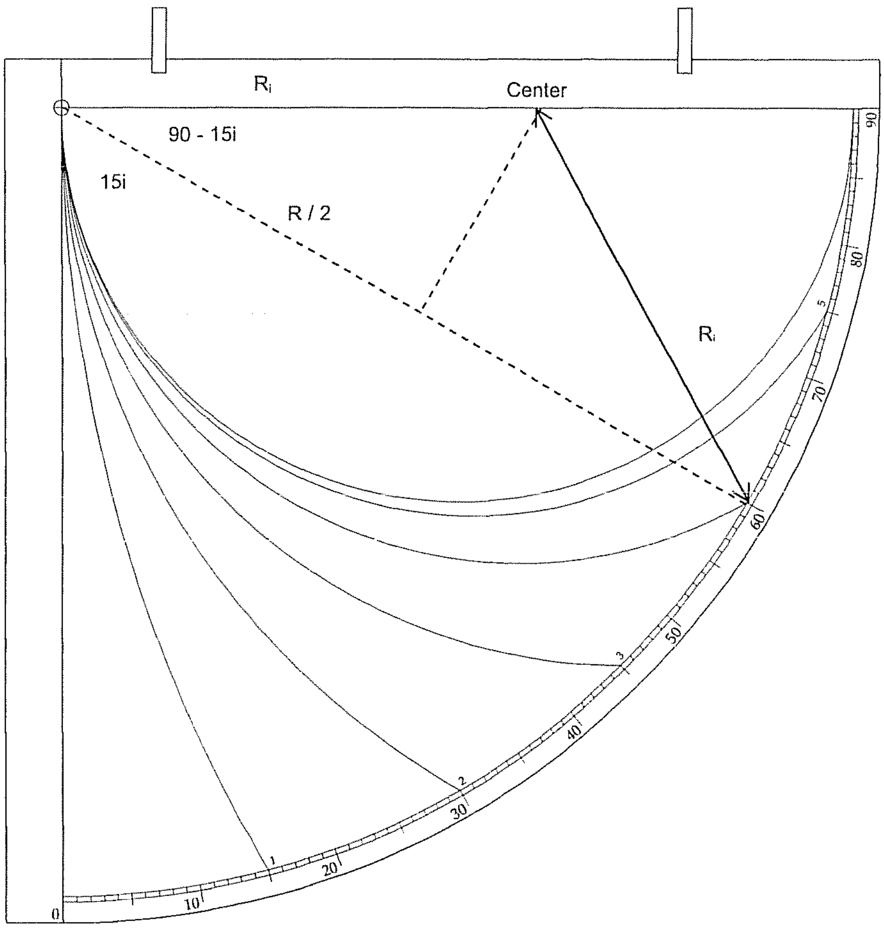


Figure 16-4. Construction of Unequal Hour Scale

Accuracy of the Horary Quadrant

It is clear the scale is not very accurate by modern standards, although it was certainly accurate enough for the standards of the Middle Ages. In fact, given the casual attitude toward time in those days, this scale would have been considered perfectly acceptable even if its failings had been appreciated.

The unequal hour scale is exact at the equinoxes for all latitudes, when the Sun's declination is zero and is less accurate at non-zero declinations. It is also exact for a latitude of 0°, i.e. at the equator. The hour angles of the unequal hours are at 15° intervals in both of the these cases. The length of the unequal hours is either greater or less than 15° in all other cases, and there is some divergence from the scale.

The amount of the error depends on both the latitude and the solar declination. The accuracy is fair for low and middle latitudes and becomes worse for higher latitudes, particularly for high values of solar declination.

For the case of zero declination, we need only consider how the arcs are constructed. Assume the Sun's altitude has been measured as a. The thread from the origin makes an angle a with the vertical side of the quadrant.

For any noon altitude, $H = 90° - \varphi + \delta$, the lengh of the chord from the center to the noon curve is: $c_6 = 2r \sin H$, where **r** is the radius of the six hour arc. The length of a chord for an unequal hour arc is easily shown to be $c_i = 2\ r_i \sin h$. The use of the scale specifies the length of the noon chord is used to find the unequal hour arc. In other words, $r \sin H = r_i \sin h$.

From the construction of the arcs, $r_i = R / (2 \sin 15i)$ and $r_6 = R/2$. Combining, $\sin 15i = \sin h / \sin H$, confirming the underlying equation. In the cases when $\varphi = 0$ or $\delta = 0$, this is also the solution of the spherical trigonometry equation for converting from equatorial to horizontal coordinates. The construction of the unequal hour scale curves matches the actual unequal hours at the equator or when the Sun is on the equator, but not at any other time.

Archinaud[96] develops a closed relationship for calculating the error for cases where the latitude and/or declination are not zero. It is, perhaps, easier to appreciate the magnitude of the inaccuracy visually. Figure 16-5 to Figure 16-7 show the curves required to determine the unequal hour accurately from the Sun's noon altitude and current altitude for all integer hours and solar declinations. The extra quarter circle is a reference arc showing zero declination. The heavy arcs are the required curves.

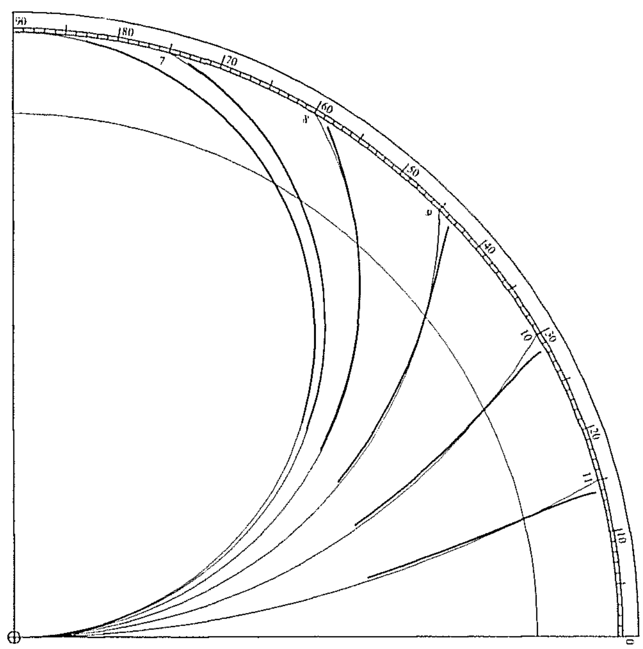


Figure 16-5. Unequal Hours - 30°

[96] Archinard, Margarida, "The Diagram of Unequal Hours", Annals of Science, 47 (1990), 173-190

The heavy curves in the figures show the annual range of solar altitude for the latitude. Negative declinations are toward the center. Notice the required curves cross the scale curves at zero declination in all cases.

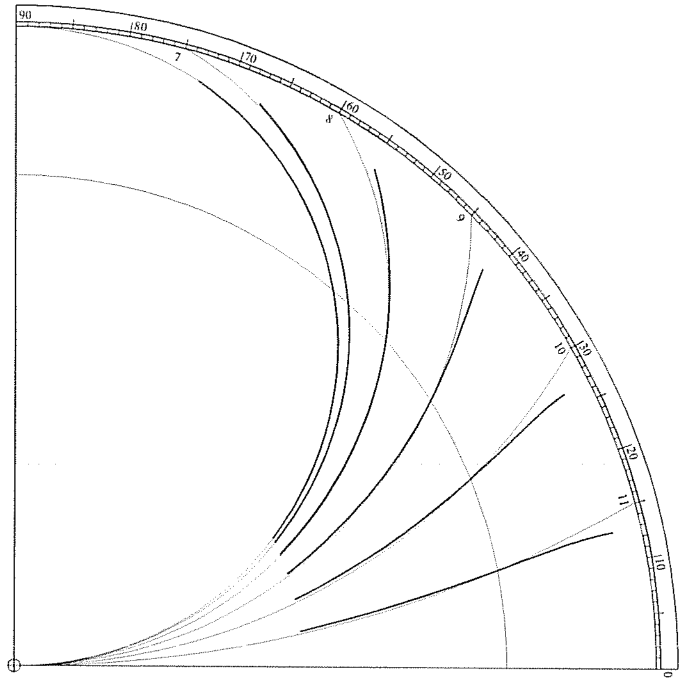


Figure 16-6. Unequal Hours - 40°

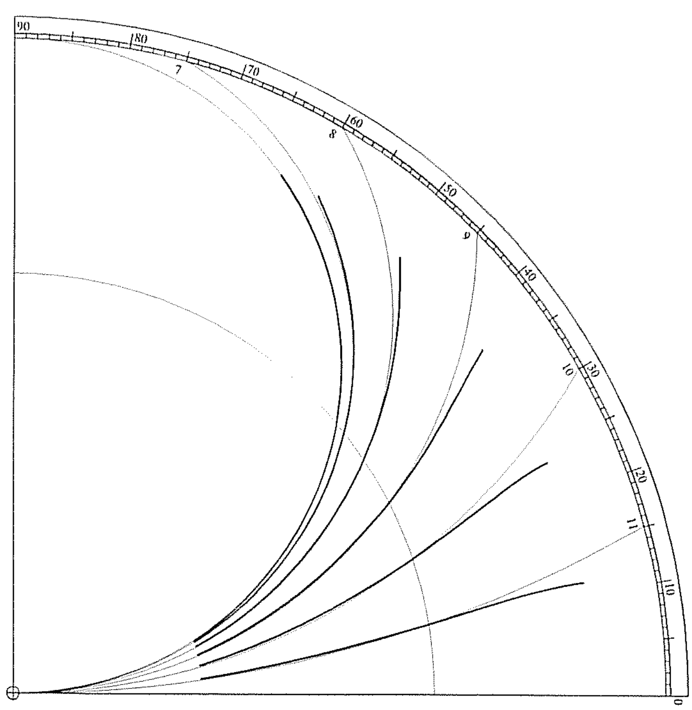


Figure 16-7. Unequal Hours - 50°

Chapter 17 - The Astrolabe Quadrant

Introduction

The astrolabe quadrant is a specialized form of quadrant instrument that greatly extends the uses of the *quadrans vetus*. Classic planispheric astrolabes were very expensive. The mere fact that they were made of brass, which was very costly in the Middle Ages, made them affordable by only the wealthy. The large number of parts and the amount of skilled hand work required to finish a quality instrument added significantly to the price. It was natural that attempts were made to reduce the cost of astrolabes to make them available to a population other than the very rich. It is likely the astrolabe quadrant grew from such considerations. The astrolabe quadrant is also lighter and smaller than a complete astrolabe of comparable precision, and therefore easier to carry. Having no moving parts except the thread, it is more rugged.

However, finding the time with the *quadrans novus* is much more complicated than with a normal astrolabe and the large number of scales is confusing to the novice user.

The *quadrans novus* would have been attractive mainly to experts. Other types of astrolabe quadrants that were much easier to use for finding the time are described in the following chapters.

The earliest known description of an astrolabe reduced to a quadrant with no moving parts was in 1288 by Jacob ben Mahir ibn Tibbon (1236-1304), more widely known by his Latin name of Prophatius Judaeus or Profeit Tibbon. Tibbon's treatise was quickly improved by Peter Nightingale, whose account received wide distribution. The instrument was quickly named the *quadrans novus* (new quadrant) to differentiate it from the *quadrans vetus* (old quadrant).

The *quadrans vetus* used the scale commonly found on the back of old astrolabes to find the unequal hour based on the altitude of the Sun. The major disadvantage of the *quadrans vetus* was it could be used only to find the time in unequal hours The *quadrans novus* could be used to find the time and solve many other problems, and could generally be used at many places.

The basic idea behind the idea of the *quadrans novus* is the stereographic projection defining the components of a planispheric astrolabe is just as valid if the astrolabe parts are folded into a single quadrant. The result is an instrument capable of performing many of the functions of a standard astrolabe at much lower cost, but without the intuitive representation of the sky provided by the rotating rete.

It is not clear how popular the astrolabe quadrant became as only seven examples survive. There were, however, many treatises on the instrument so there is reason to believe many were made, perhaps of cardboard or wood. It is possible brass quadrants were not as highly prized as true astrolabes due to their simplicity and were recycled into other instruments. There are more treatises than instruments today. A variation of the astrolabe quadrant to be discussed in the next chapter was more popular in the Ottoman Empire from the 17th century until the early 20th century.

In Europe, use of the astrolabe quadrant and other time telling quadrants died out in the early 18th century along with the astrolabe as interest in astrology waned and accurate clocks become readily available.

Introduction to the Astrolabe Quadrant

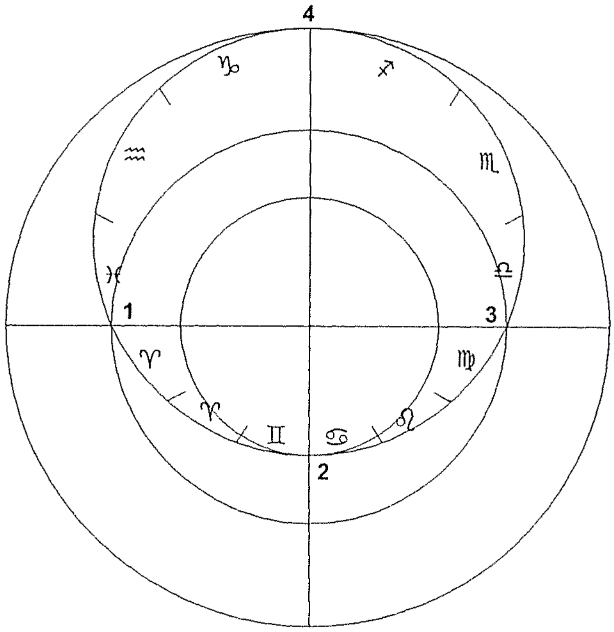


Figure 17-1. Normal Astrolabe

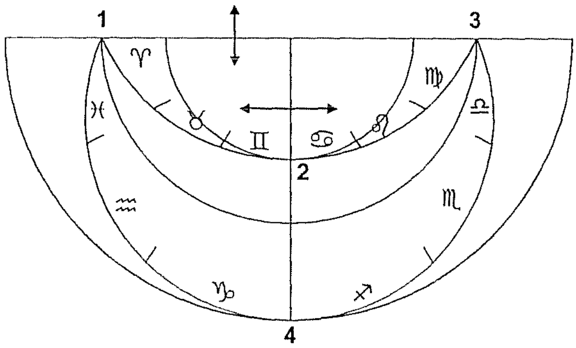


Figure 17-2. Astrolabe folded once

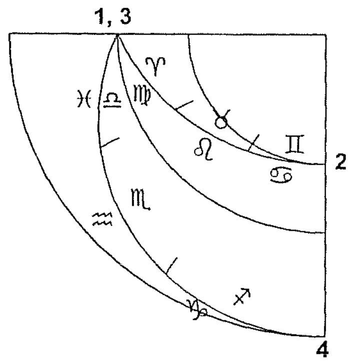


Figure 17-3. Second fold

The astrolabe quadrant is based on folding the rete and plate of an astrolabe twice. The rete is fixed with the vernal equinox on the eastern horizon (Figure 17-1). The astrolabe face is first folded from top to bottom (Figure 17-2). The resulting figure is then folded around the meridian to complete the process of reducing the astrolabe to a quadrant (Figure 16-3).

It is easy to see the tropics and the shape of the ecliptic are preserved with a little study of the folds. The entire ecliptic circle can be visualized by following the Sun as it starts at the vernal equinox (1) (the point where the ecliptic meets the equator) and proceeds through the first three signs (Aries, Taurus, Gemini) to the point where the ecliptic meets the Tropic of Cancer (2). The Sun then reverses direction on the ecliptic projection, and proceeds through the same three ecliptic divisions which now represent Cancer, Leo and Virgo, until it again meets the equator at the autumnal equinox (3). The Sun then proceeds along the larger ecliptic arc toward the Tropic of Capricorn through Libra, Scorpio, Sagittarius to the winter solstice (4). The Sun then reverses direction again and goes back to the vernal equinox through, Capricorn, Aquarius and Pisces (1).

The projection of the tropics is not affected by the rotation since the circles representing the tropics are concentric with the center of the instrument.

The horizon is also folded twice, but is often rotated 90° to make it is easier to differentiate the horizon arc from the ecliptic.

The function of the rotating rete is replaced by a weighted thread attached to the point on the quadrant corresponding to the center of the astrolabe. The thread swings freely and can be moved to various angles to simulate the rotation of the rete. The thread on an astrolabe quadrant is equipped with a sliding bead (often a pearl on fine old instruments) to allow points on the plate to be noted.

The *quadrans novus* can be used to solve most of the same problems that can be solved on an ordinary astrolabe but without the intuitive appeal afforded by the rotating rete. Also, the fundamental use of the plane astrolabe is to find the time, which can be done directly on the astrolabe, requires some calculations on the *quadrans novus*.

There are few elements that can be considered common between astrolabe quadrants from different periods and makers other than the basic layout. Strictly speaking, an astrolabe quadrant must use the stereographic projection. All astrolabe quadrants include the ecliptic projection, a degree scale for measuring altitudes, sights and the weighted thread for taking measurements. Other scales were added at the maker's discretion to allow for specific applications. Some instruments were made for a specific latitude and had horizon and altitude arcs for the location (the original description by Profeit Tibbon was of this type). Quadrants of this type, described in the next chapter, were used in the Ottoman Empire up to the beginning of the twentieth century. Other types of astrolabe quadrant included multiple horizons and could be used for certain problems for multiple latitudes.

A variation of the astrolabe quadrant was invented by Edmund Gunter (1581-1626) in 1623 that made solving problems related to the Sun's position, including telling time, quite simple and the Gunter's quadrant enjoyed considerable popularity. Another variation invented by Thomas Hardy in the mid-17th century was popularized by the noted engraver, Henry Sutton, and was generally called the "Sutton Quadrant". Both the Gunter and "Sutton" instruments were considerably easier to use than the Medieval *quadrans novus* and were included in technical discussions throughout the 18th century, long after interest in the plane astrolabe had virtually disappeared. Both of these quadrants are described later.

Other quadrants were developed for telling time using projections other than the stereographic projection.

The history of the astrolabe quadrant spans 500 years, which is a testimony to its ingenious construction.

The "quadrans novus"

This example is not a reproduction of a specific instrument, but does include elements that can be considered typical for a 14th or 15th century instrument of this type. The scales and layout of the instrument will be discusssed in this section. Most applications will become obvious during the discussion and solved sample problems are presented in the following section.

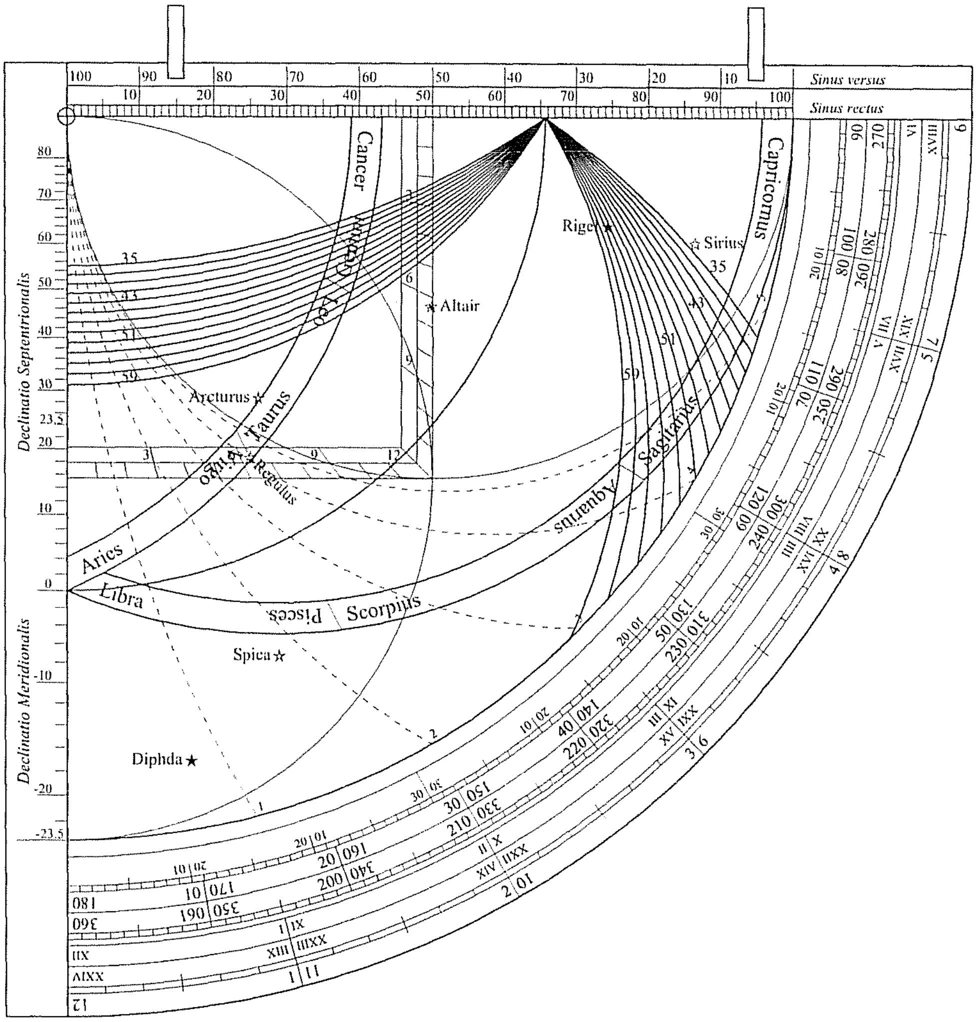


Figure 17-4. ***quadrans novus***

The front of this astrolabe quadrant contains the projection of the ecliptic, horizons for each 2° from 35° to 59°, the Tropic of Capricorn and Equator and several stars. Also included is an unequal hour scale and a shadow square. A number of scales for measuring altitudes and determining times, declination, right ascensions, longitudes, and for solving time-related problems are around the instrument margin. At first glance, the astrolabe quadrant may appear to be rather congested and obscure so each of the lines and scales is discussed individually.

The "quadrans novus" arcs and scales

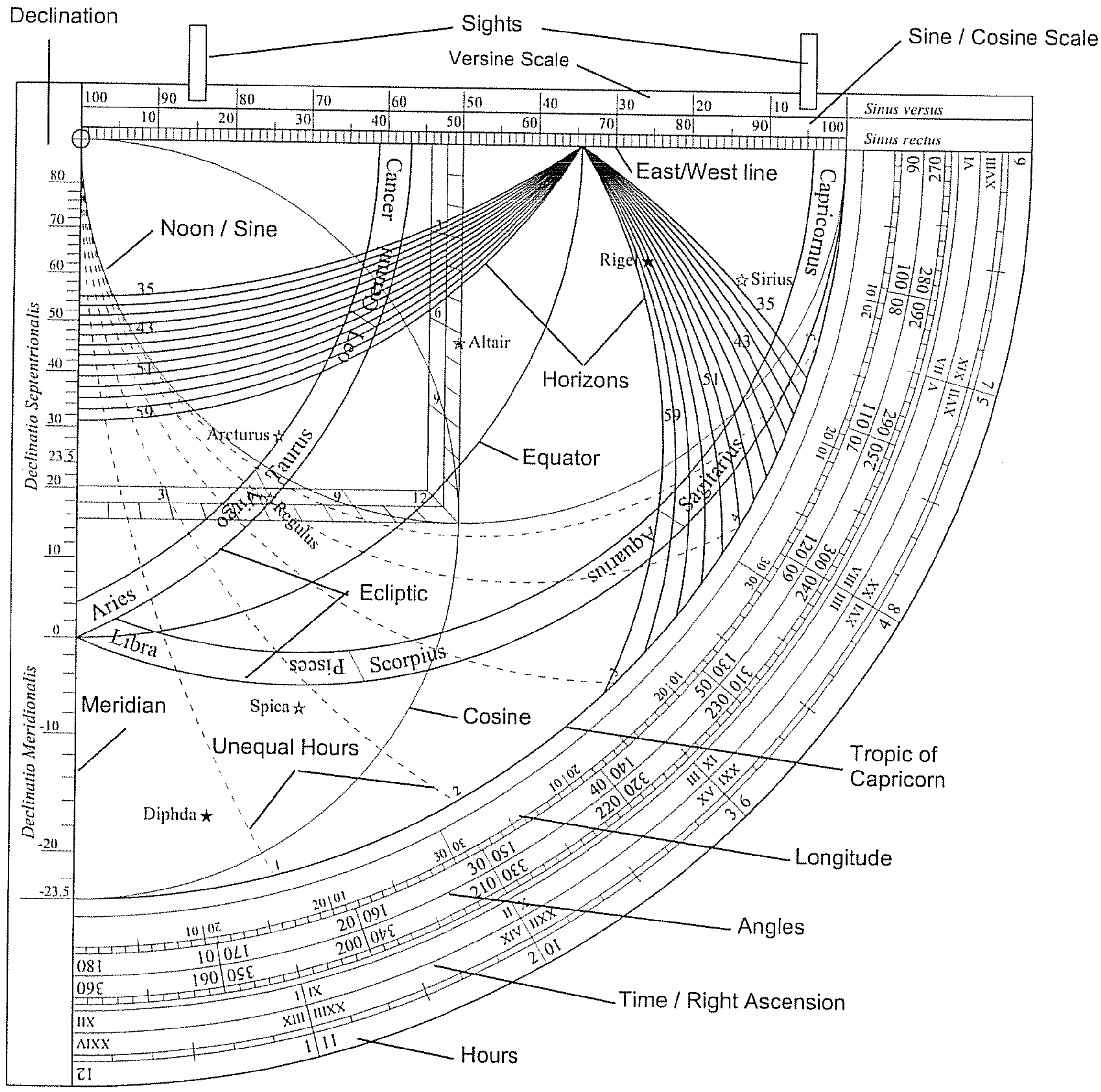


Figure 17-5. ***quadrans novus*** **Arcs and Scales**

The *quadrans novus* Interior

The following discussion assumes a good background in astrolabe principles.

The arcs and lines in the interior of the astrolabe quadrant generally reproduce the lines and circles on the rete and plate of a conventional planispheric astrolabe and have the same meaning.

The top line of the interior represents the line connecting East and West through your location, just as on a plane astrolabe. The left edge of the interior represents your local meridian.

The horizon arcs are particularly easy to visualize, since they are the result of a single fold of the astrolabe plate around the East/West line. The horizon section to the left of the equator is the part of the horizon below the East/West astrolabe line, and the horizon section to the right of the equator is the part of the horizon above the East/West line on an astrolabe. It is particularly important to become comfortable with the fact that the section of the horizon arc north of the equator is folded over the East-West line, because this particular fold determines which of the time scales to use.

An awkward point to some students is that the ecliptic on the astrolabe quadrant is rotated 90° from the orientation of the horizon arcs. It is not difficult to get used to the astrolabe quadrant form of the ecliptic with a little practice.

The section of the ecliptic arc to use for a specific problem depends on the Sun's longitude. As described earlier, the two ecliptic arcs represent the complete ecliptic circle by starting at Aries 0°, proceeding along the upper ecliptic arc to Cancer 0°, and reversing direction back to Libra 0°. The lower ecliptic arc is then followed from Libra 0° to Capricorn 0° and back to Aries 0°. Since each arc represents two sections of the ecliptic, a double scale of longitude degrees would be required to label the ecliptic. The ecliptic arcs are not graduated in degrees to keep the already complex interior of the quadrant from becoming even more congested. The longitude scale in the margin fills the need for finding the Sun's ecliptic position and its use is described below. The only use of the longitude scale is to read or set ecliptic divisions. It is graduated in degrees of the zodiac, but the spacing is different from the degrees scale below it. The spacing of the longitude degree tics look even at first glance, but it isn't, as can be seen with close examination.

The longitude scale shows each degree of solar longitude, with longer tics each five degrees and labels for each ten degrees. Notice the longitude scale is labeled in 10° sections in both directions, with one set of numerals "upside down" when the astrolabe quadrant is oriented as in Figure 17-5. It is easy to figure out which direction to use on the longitude scale. Note the names of the zodiac signs are also "right side up" and "upside down" depending on the direction of increasing longitude. The orientation of the zodiac sign label is the same as the orientation of the longitude scale labels for each sign. For example, the Pisces label on the ecliptic is "upside down". Use the "upside down" labels on the longitude scale for Pisces.

The front of the astrolabe quadrant includes scales found on the back of a plane astrolabe. In particular, the shadow square, only half of which is included, is identical to the shadow scale on an astrolabe. The unequal hour scale, while in a different place on the instrument, performs the same function as on the astrolabe or horary quadrant.

The sine and cosine scales are unique to the astrolabe quadrant (scales for sines and cosines are sometimes found on Islamic astrolabes in a different form and have a different use). These scales are needed to solve time-related problems unique to the *quadrans novus*. The sine or cosine of an angle is found by aligning the thread with an angle between 0° and 90° and setting the bead on the appropriate curve. The thread is then rotated to the scale along the top of the quadrant, and the value is read from the scale labeled *Sinus rectus*. The value is read in parts of 100, so the value of, say, the cosine of 30° is read as 86.6, which you must mentally convert to 0.866 by dividing by 100. Note the semicircle used to find the sine of an angle does double duty as the noon arc for finding unequal hours.

The theory behind the sine and cosine curves is quite simple. See Figure 17-6 and 17-7.

There is a well known theorem from plane geometry that a triangle inscribed in a circle is a right triangle with a diameter as the hypotenuse. Taking the diameter to be a length of 1, then the length of the side of the triangle defined by an angle a with the diameter is equal to cos a. The interior edge of the astrolabe quadrant defines the circle diameter and the thread is the chord defined by the angle measured on the degree scale. Therefore, the length of the chord defined by the bead is equal to cos a. The ratio of the length of the chord to the diameter, the cosine, is read from the *sinus rectus* scale.

The sine scale is defined by similar logic and is shown in Figure 17-7.

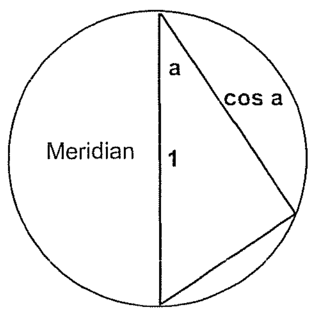


Figure 17-6. Cosine curve

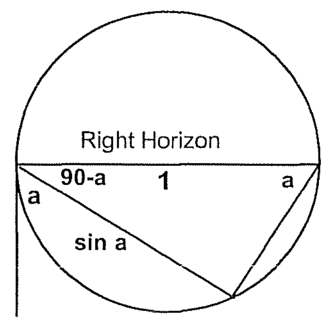


Figure 17-7. Sine curve

Scales

The longitude and *sinus rectus* scales have already been discussed. The scale along the top of the quadrant labeled *Sinus versus* is the 'versed sine' or versin, and is an old trigonometric construction = (1 – cos a). Its use for finding the time will be described later.

The degree scale shows angles from 0° to 360° as a quadruple scale related to the corresponding quadrant of the complete astrolabe. The most used and useful degree division is the scale from 0° to 90°, which is used to measure altitudes and to set angles for finding sines and cosines. The other sections of the degree scale can be used to convert between times (hour angles) and right ascension and degrees. The degree scale is also used to determine the angle from sunrise to noon or noon to sunset (the semi-diurnal arc).

There are two time scales. The hour scale with Roman numerals has graduations for a full 24 hours, and is used for finding right ascensions, and can be used to find times. The hour scale on the outer margin is used to tell time. The time scales are divided into ten minute intervals.

The scale on the left vertical edge of the quadrant is used to find declinations. In use, the thread is stretched over the point of interest, such as a point on the ecliptic or a star, and the bead is positioned over the point. The thread is then rotated to the declination scale, and the declination is read from the scale. *Declinatio Septentrionalis* means "northern declination" and *Declinatio Meridionalis* means "southern declination".

You should be able to read the degree scale to the nearest degree and the time scales to within about a minute or two, but it is hard to position the thread accurately and set the bead with only two hands. You may find this level of precision to be crude, but astrolabes and astrolabe quadrants were considered a marvel of technology and accuracy in the Middle Ages.

Stars

The example astrolabe quadrant includes the positions of the seven stars listed below. The astrolabe in the figure uses different star symbols to indicate which section of the right ascension scale to use. ★ indicates the star's right ascension is between 0h and 6h (Rigel and Diphda), ☆ shows the star's right ascension is between 6h and 12h (Sirius and Regulus), is the symbol for stars between 12h and 18h right ascension (Spica and Arcturus) is filled star with a white dot and right ascension between 18h and 24h (Altair) is an outline star with a black dot. The stars shown on the astrolabe quadrant are shown in Table 15-1.

Name	Right Ascension			Declination			Constellation
	Hours	Min	Sec	Degrees	Min	Sec	
Diphda	0	43	35.3	-17	59	12	Cetus
Rigel	5	14	32.2	-8	12	6	Orion
Sirius	6	45	8.9	-16	42	58	Canis Major
Regulus	10	8	22.2	11	58	2	Leo
Spica	13	25	11.5	-11	9	41	Virgo
Arcturus	14	15	39.6	19	10	57	Boötes
Altair	19	50	46.9	8	52	6	Aquila

Table 15-1. ***quadrans novus*** **Stars**

The width of the bead prevents the reading of declinations to closer than about a degree or so. You can try to get more accurate readings by using a pin, but even though the scales are accurately drawn, it is difficult to read them to more than one degree precision.

The *quadrans novus* Back

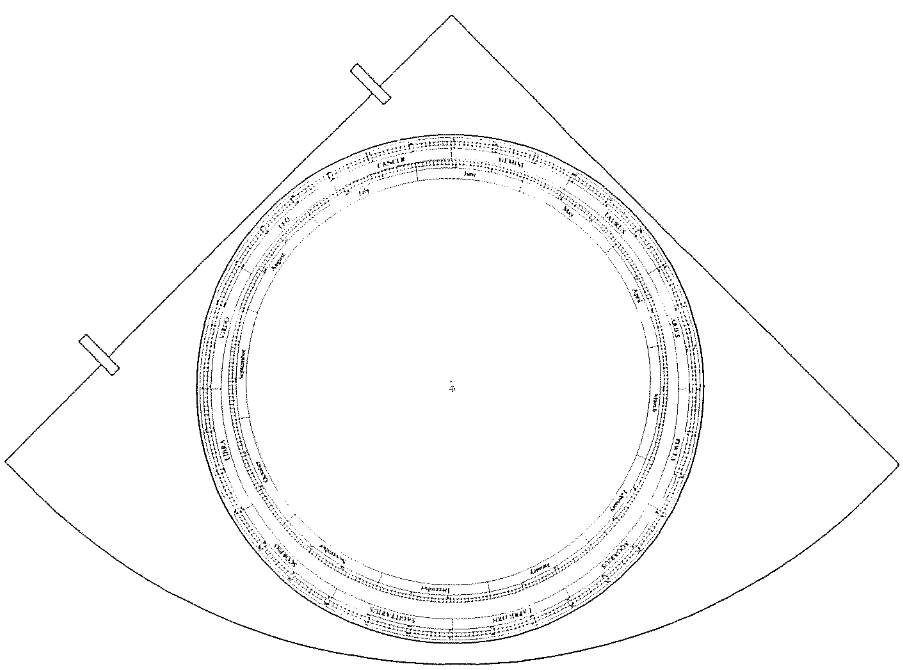


Figure 17-8. A *quadrans novus* Back

There are no standards for what might be engraved on the back of an astrolabe quadrant. Some surviving instruments had calendar information engraved, and the back was blank on at least one instrument. The back of an astrolabe quadrant should include a scale for finding the Sun's position in the zodiac for a date. Figure 17-8 is an example of the calendear/zodiac scales from the back of a planispheric astrolabe that might be used. You need a small prick to locate the center of the zodiac scale by marking the intersection of a line connecting Aries 0° and Libra 0° with a line connecting Cancer 0° and Capricorn 0°. The use of the scale is identical to the astrolabe, except you use the thread to select a date and read the associated longitude instead of the alidade.

Using the "quadrans novus"

This section contains sample problems illustrating the use of the astrolabe quadrant. You will have no trouble finding the solutions to other problems once these examples are mastered.

Position Related Problems

Solving problems related to the positions of celestial objects using the astrolabe quadrant is very similar to solving the same problems on a plane astrolabe.

1. Find the altitude of the Sun or a star.

The basic use of all quadrants was to measure the altitude of the Sun or a star above the horizon. An associated use is to measure the altitude of an object such as a tree or building to determine its height with the shadow square. The principle is illustrated in Figure 17-9.

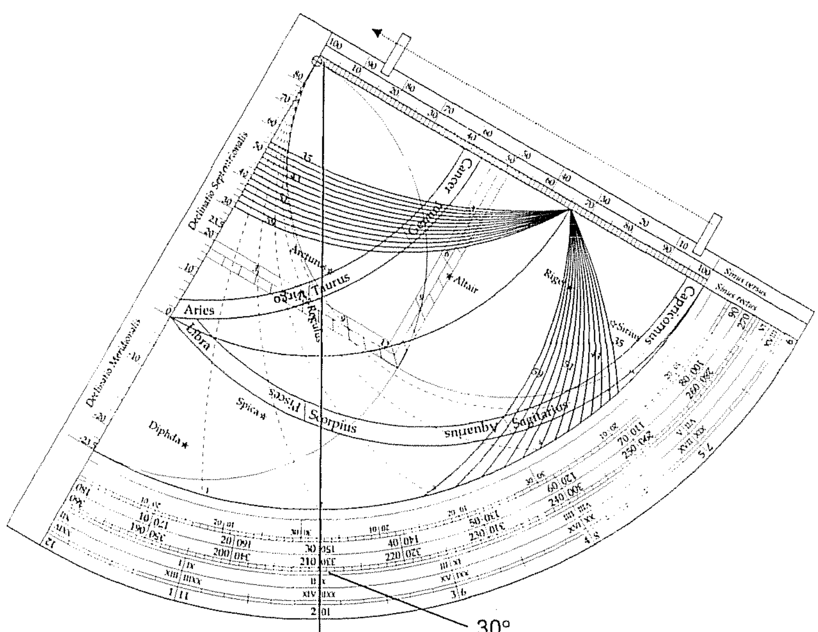


Figure 17-9. Reading the Sun's altitude as 30°

The principle of making an altitude reading is simple in principle. Simply raise the quadrant until you can view the star through the sights, hold the thread against the degree scale and then lower the quadrant and read the point where the thread cuts the scale. A Sun sight is similar except you allow the Sun to shine through the sights and you mark the thread position when a ray of sunlight goes directly through the sights. Never look directly at the Sun through the sights. Permanent eye damage is almost certain if you do.

Taking an accurate altitude reading with a quadrant requires a good bit of practice. The instrument has to be held very steady with one hand while the thread is positioned with the other hand. The orientation of the instrument virtually requires the quadrant be held with the right hand for making the sight while the thread is positioned with the left hand. You will have to experiment to find a comfortable position for the right hand that allows you to hold the quadrant steady while the left hand steadies the thread and holds it firmly on the scale as you lower the instrument to read the value.

2. Find the Sun's longitude and declination for a date.

Finding the Sun's longitude for a date on the astrolabe quadrant is identical to the procedure on the plane astrolabe except there is no alidade. Simply read the Sun's position in the zodiac (i.e. the longitude) using the scale on the back of the astrolabe quadrant.

Turn the astrolabe quadrant over and position the thread to the value found in the previous step.

Move the bead to intercept the ecliptic in the correct sign being careful to use the correct section of the longitude scale.

The Sun's declination can then be found by moving the thread to the declination scale and reading the value.

Example: Find the Sun's longitude and declination for November 15.

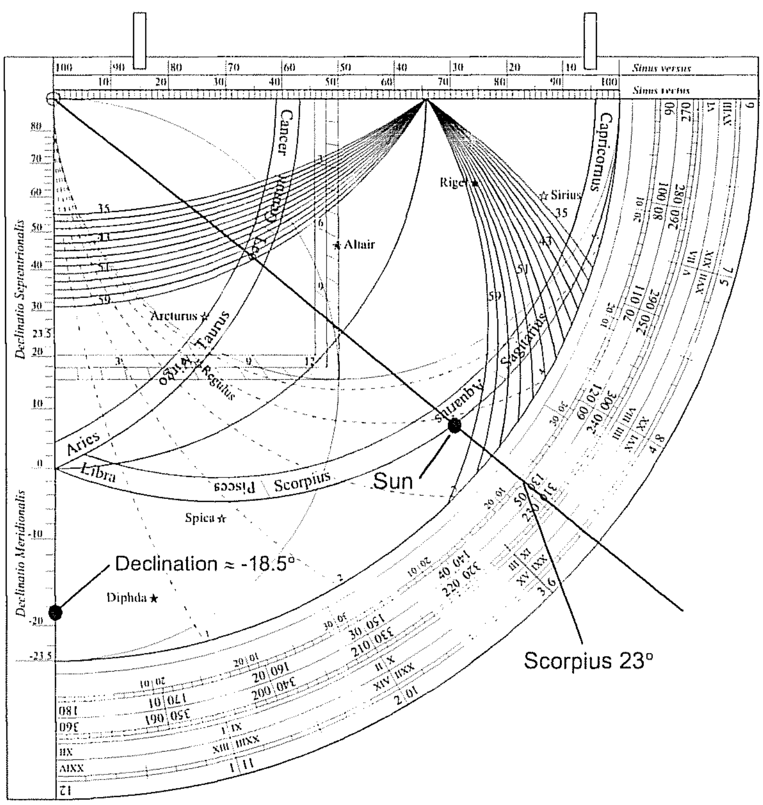


Figure 17-10. The Sun's position

The Sun's position in the zodiac for November 15 is found to be Scorpius 23° from the back of the astrolabe quadrant. Set the thread to 23° in Scorpius using the 'right side up' labels and set the bead on the ecliptic in Scorpius. Rotate the thread to the declination scale and read the Sun's declination as approximately –18.5°on the declination scale.

3. Find the declination and right ascension of a star.

Finding the declination of a star is very easy. Simply position the bead over the star and rotate the thread to the declination scale and read the declination.

Finding a star's right ascension requires knowledge of which quadrant of the complete astrolabe contains the star. Use Table 15-1 to associate the stars on the astrolabe quadrant with the correct right ascension scale. Stretch the thread across the star and read the right ascension from the scale. Check your answer with Table 15-1.

Example: Find the declination and right ascension of Altair.

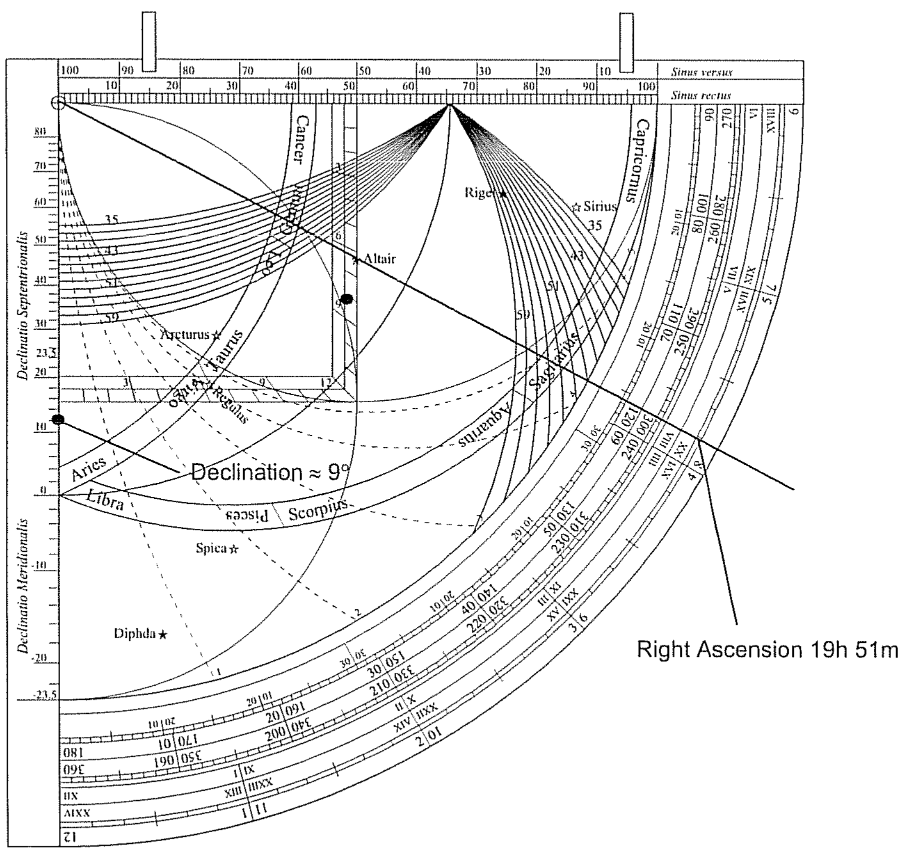


Figure 17-11. Star's coordinates

Stretch the thread across Altair and position the bead over the star symbol. Note the symbol in the figure indicates the right ascension is in the fourth quadrant (18h to 24h). Read the right ascension of Altair from the hour scale as approximately 19hr 51min.

Rotate the thread to the meridian and read the declination of Altair as approximately 9° North.

Time related problems.

Solving time related problems on the *quadrans novus* can be more complex than on the plane astrolabe. The following problems illustrate the procedures.

4. Find the time of sunrise and sunset.

The time of sunrise and sunset can be read directly from the astrolabe quadrant.

First, find the Sun's longitude and declination using the procedures of Problem 2. Set the bead to the Sun's declination. Rotate the thread until the bead is over the horizon arc closest to your latitude. The thread will cut the hour scale at the time of sunrise and sunset. However, some thought is needed to ensure the correct hour scale is used.

There are separate cases for whether you are finding the time of sunrise or sunset and whether the Sun's declination is north or south. If the Sun's declination is north, the horizon arcs to the left of the equator will be cut by the bead. In this case, sunrise is before 6 AM. The outermost hour scale ending at 6 shows the hours since midnight and, thus, the time of sunrise. sunset is after 6 PM so the hour scale for 6 to 12 shows the number of hours since noon, giving the time of sunset.

If the Sun's declination is south, the horizon arcs to the right of the equator will be cut by the bead. Sunrise will be after 6 AM. Use the outermost scale for 6 to 12 to find the number of hours since midnight to find the time of sunrise and the scale ending with 6 for the time of sunset, which is before 6 PM.

Example: Find the time of sunrise and sunset for November 15, for a latitude of 41° N.

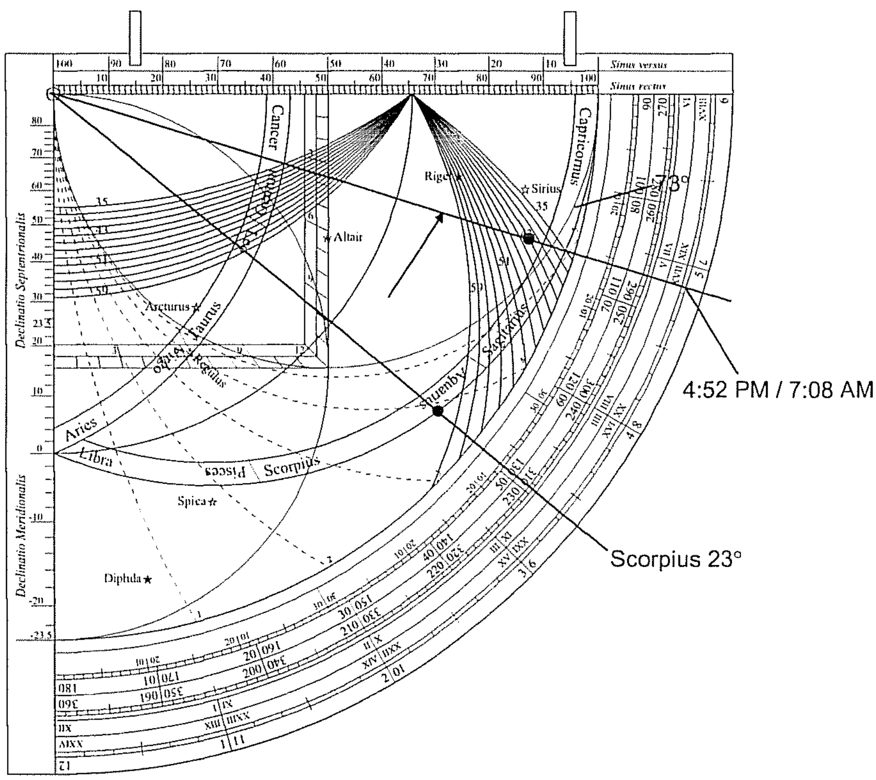


Figure 17-12. Time of Sunrise/Sunset

Using the result from Problem 2, the Sun's longitude on November 15, is Scorpius 23° and the Sun's declination is -18.5°. Set the thread over Scorpius 23° and set the bead to the ecliptic.

Rotate the thread until the bead is over the 41° horizon. Note the Sun's declination is south in November so sunset will be before 6 PM. Therefore, the time of sunset is read from the hour scale ending in 6. Read the time of sunset as approximately 4:51 PM. The time of sunrise is read from the 6 to 12 scale with the thread in same position. Read the time of sunrise as approximately 7:08 AM. The thread also shows the angle the Sun travels from noon to sunset, the semi-diurnal arc (73°).

5. Find the time from the altitude of the Sun.

Finding the time from the measured altitude of the Sun is more complicated on the astrolabe quadrant than on a complete astrolabe as it requires the solution of a simple equation. It is, however, still much simpler than finding the time from calculations alone as the astrolabe quadrant includes all of the required scales for the solution without reference to any books. With some practice, you can find the time to an accuracy of a minute or two after two or three minutes of calculation.

You do not find the current time directly on the astrolabe quadrant, but solve for the Sun's *hour angle.* The hour angle of a celestial object is the angle from the meridian (noon) to the object measured on the equator. The hour angle is converted to time by dividing the hour angle by 15 (there are 15° in an hour). The result is subtracted from 12 if the time is in the morning. For example, if the Sun's hour angle is 37.5°, the time is 37.5 / 15 = 2 ½ hours from noon or 9:30 AM if in the morning or 2:30 PM if in the afternoon.

The time is found by solving the following equation which was provided by Profat Tibbon:

$$\text{versin } H = \text{versin } A\ (1 - \sin h / \sin a)$$

H = Sun's hour angle
A = Length of the day's semi-diurnal arc
h = Sun's observed altitude
a = Sun's noon altitude

The angle A is found using the procedure in Problem 4 to find the time of sunrise/sunset.

h is measured using the astrolabe quadrant sights as in Problem 1.

a is found from the result of solving Problem 2 for the Sun's declination, δ, for the day and calculating the Sun's meridian or noon altitude from $a = (90° - \varphi) + \delta$, where φ is the latitude of the place. The term $(90 - \varphi)$ is called the *colatitude* and is the angle from the north pole to the zenith. For example, if the Sun's declination is -10° then the Sun's noon altitude at 41° N latitude will be:

$$(90° - 41°) + (-10°) = 49° - 10° = 39°.$$

Example: Find the time when the Sun's altitude is 24° in the morning on November 15, at a latitude of 41° N.

The Sun's declination, δ, on November 15, was found to be -18.5° in Problem 2. Therefore, the Sun's noon altitude on November 15, is (90° - 41°) + (-18.5°) = 49° - 18.5° = 30.5°.

The length of the Sun's diurnal arc, A, for November 15, in was found in Problem 4 to be 73°.

The equation for the Sun's hour angle can now be solved. The numbers in parentheses are calculated as a check on the results. You will not be able to read the values read from the astrolabe quadrant with the this level of precision.

1. Place the thread on the Sun's observed altitude (24°) and put the bead on the Noon/Sine curve. Rotate the thread to the *sinus rectus* scale and read the value of sin h as .41 (0.40674).
2. Place the thread on the Sun's noon altitude (30.5°) and put the bead on the Noon/Sine curve. Rotate the thread to the *sinus rectus* scale and read the value of sin a as .51 (0.50754).
3. Divide sin h by sin a. sin h / sin a = .41 / .51 = .80 (0.40674/0.50754 = 0.80139).
4. Subtract this value from 1. 1 - .80 = .20 (1 - 0.80139 = 0.19861).
5. Place the thread on the value of the Sun's diurnal arc for November 15 (73°). Place the bead on the cosine scale. Rotate the thread to the *sinus versus* scale and read the value of versin A as .71 (.70763).
6. Multiply this value and the result from step 4; 0.71 × 0.20 = 0.142 (0.14054).
7. Move the thread to the *sinus versus* scale and set the bead over 0.14. Rotate the thread until the bead is over the cosine scale. Read the hour angle, H, as 31° (30.74°).
8. Convert the hour angle to time by dividing by 15. 31°/15 = 2 hr. 4 min.. (30.74°/15 = 2 hr 2 min 58 sec.)
9. Since the observation is in the morning, the Sun is 2 hr. 4 min. before noon or 9:56 AM (9:57:02).

Therefore, the apparent solar time at 41° North when the Sun is 24° in altitude on the morning of November 15, as found on the astrolabe quadrant is 9:56 AM. The actual exact time is 9:57:46. Some precision is lost in rounding by using only two decimal places in the calculations using an estimated solar declination and using an approximation for the length of the diurnal arc. This level of accuracy was more than adequate in the Middle Ages and is acceptable for modern use. All things considered, the astrolabe quadrant gives a very good approximation of the time.

This time would have to be corrected for longitude and the equation of time to be converted to zone time.

Making the quadrans novus

Making an astrolabe quadrant is a non-trivial undertaking. There are a large number of elements to include and, consequently, a large number of computations. Fortunately, there are no new theoretical considerations and the same techniques used for the plane astrolabe apply exactly to the astrolabe quadrant.

The following steps outline making an astrolabe quadrant like the one in the figures. This instrument can be considered typical, but there are many possible variations. This discussion should, however, provide the essential elements for any astrolabe quadrant variation you might want to build.

The dimensions of the example quadrant are 5.85 inches from the center to the limb edge with a margin at the top of 0.35 inches and a side margin of 0.40 inches. This size is similar to surviving instruments.

The limb margin for the scales is 1.5 inches. This dimension defines the Tropic of Capricorn, which defines all of the other dimensions. The equator and ecliptic radii are found from the radius of the Tropic of Capricorn as on the astrolabe.

The sine and cosine curves are semi-circles with a diameter equal to the radius of the Tropic of Capricorn. The unequal hour scale is constructed as described in the previous chapter.

The sine and versine curves in the upper margin simply divide the upper edge into 100 parts. They are labeled and drawn as shown in the example quadrant.

The declination scale in the left margin uses the standard equation: $\mathbf{r = R_{eq} \tan [(90-\delta)/2]}$, where r is the distance from the equator.

Horizon centers and radii are calculated as on the astrolabe.

The limb scales require a lot of effort to draw due to the many labels, but there is no theoretical problem. The degree scale simply divides the limb in into 90° and the time scale is divided at 15° per hour.

The longitude scale is divided using the same technique as the astrolabe ecliptic.

Stars are positioned as on the astrolabe rete.

Chapter 18 - The Prophatius Quadrant

The quadrant originally described by Prophatius was more like the astrolabe than the *quadrans novus* in several respects. It was made for a specific latitude and included the horizon, almucantars and azimuth arcs found on the astrolabe plate. This type of quadrant was popular in the Ottoman Empire and was regularly used until the early 20th century.

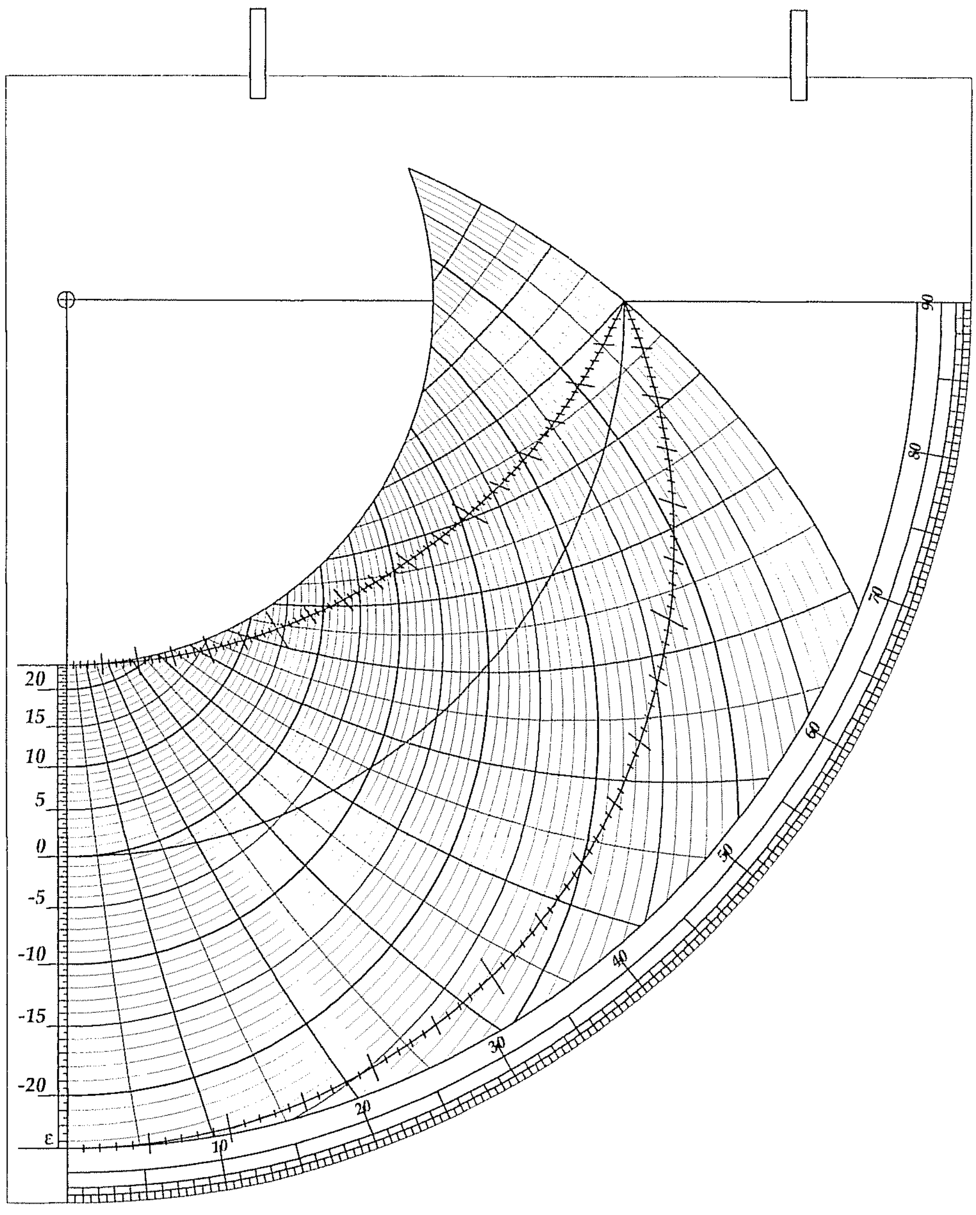


Figure 18-1. Prophatius Quadrant

The Prophatius quadrant is a direct implementation of the folded astrolabe plate discussed in the previous chapter. The only significant difference between the folding operation and the implementation is in the wedge of arcs above the right horizon. This wedge is the part of the plate above the horizon, outside the Tropic and Cancer and below the right horizon that has been folded up to allow the complete set of arcs to be included.

There is a modest amount of inconsistency, and even a low level of controversy, about the correct name to use for the various types of quadrants. The *quadrans novus* is often referred to as an "astrolabe quadrant" even though it is does not include all astrolabe elements. Others prefer to restrict the term "astrolabe quadrant" to the type shown in this chapter and use *quadrans novus* as the name of the specific instrument described in the previous chapter. The term "Profatius quadrant" is used here for this form of astrolabe quadrant to avoid the naming issue.

Figure 18-1 shows a representation of the scales included on a Prophatius style quadrant for 40°, the latitude of Istanbul. Almucantars are shown for each degree and the limb is divided to ½ degree. Azimuths for each 5° are shown.

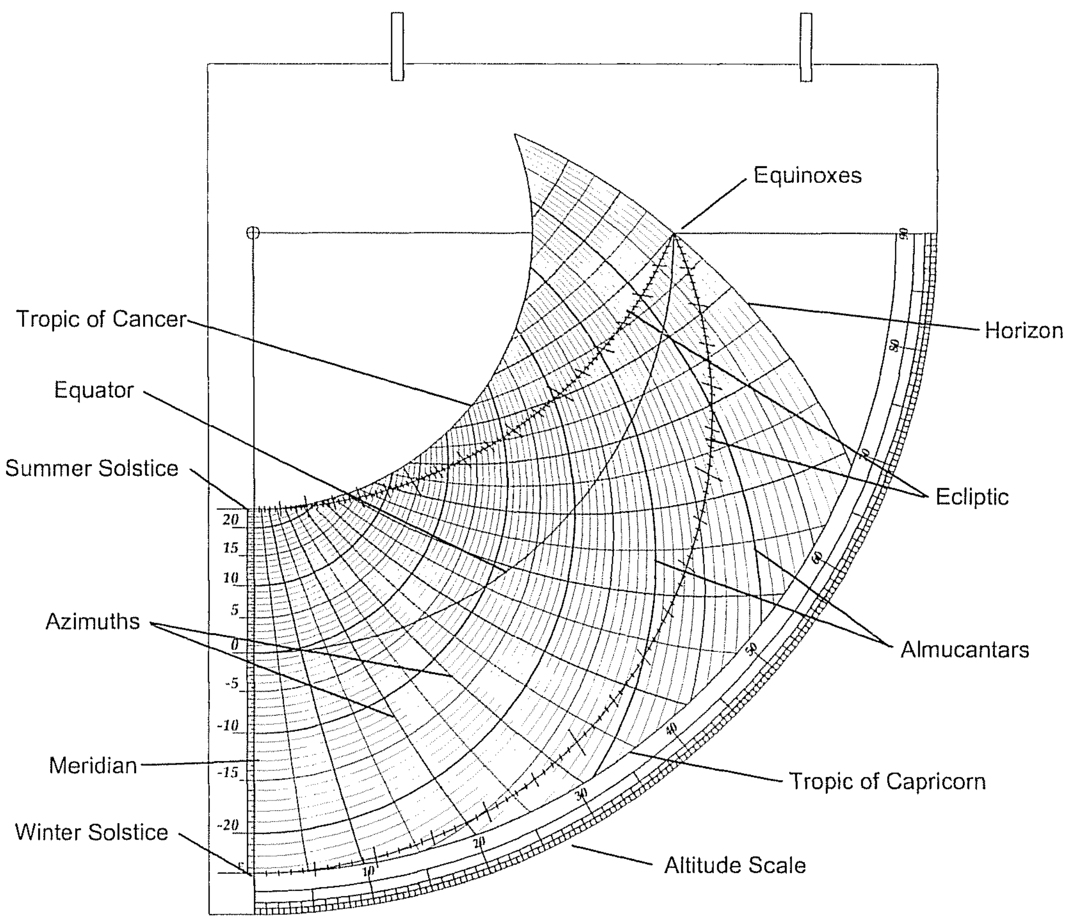


Figure 18-2. Prophatius Quadrant Scales

Turkish quadrants of this type were made of paper, varnished to a wood base. The base often had a recess for storage of the plumb and bob, and a plush storage bag for storing the instrument.

The quadrant in the figure is not complete. Turkish quadrants had a variety of scales that are not shown. Many instruments had a time scale along the limb and some had a calendar above the Tropic of Cancer. A variety of markings and annotation related to use and prayer times were often included.

There should be no mystery about the scales and use of this instrument if the *quadrans novus* is understood.

The ecliptic arcs each serve for half the year. The vernal equinox is where the ecliptic arcs meet at the top of the instrument. The shorter arc is used from the vernal equinox to the summer solstice and then reverses until the autumnal equinox. The longer arc is then used until the winter solstice and reverses to return to the vernal equinox.

The ecliptic arcs each serve for half the year. The vernal equinox is where the ecliptic arcs meet at the top of the instrument. The shorter arc is used from the vernal equinox to the summer solstice and then reverses until the autumnal equinox. The longer arc is then used until the winter solstice and reverses to return to the vernal equinox.

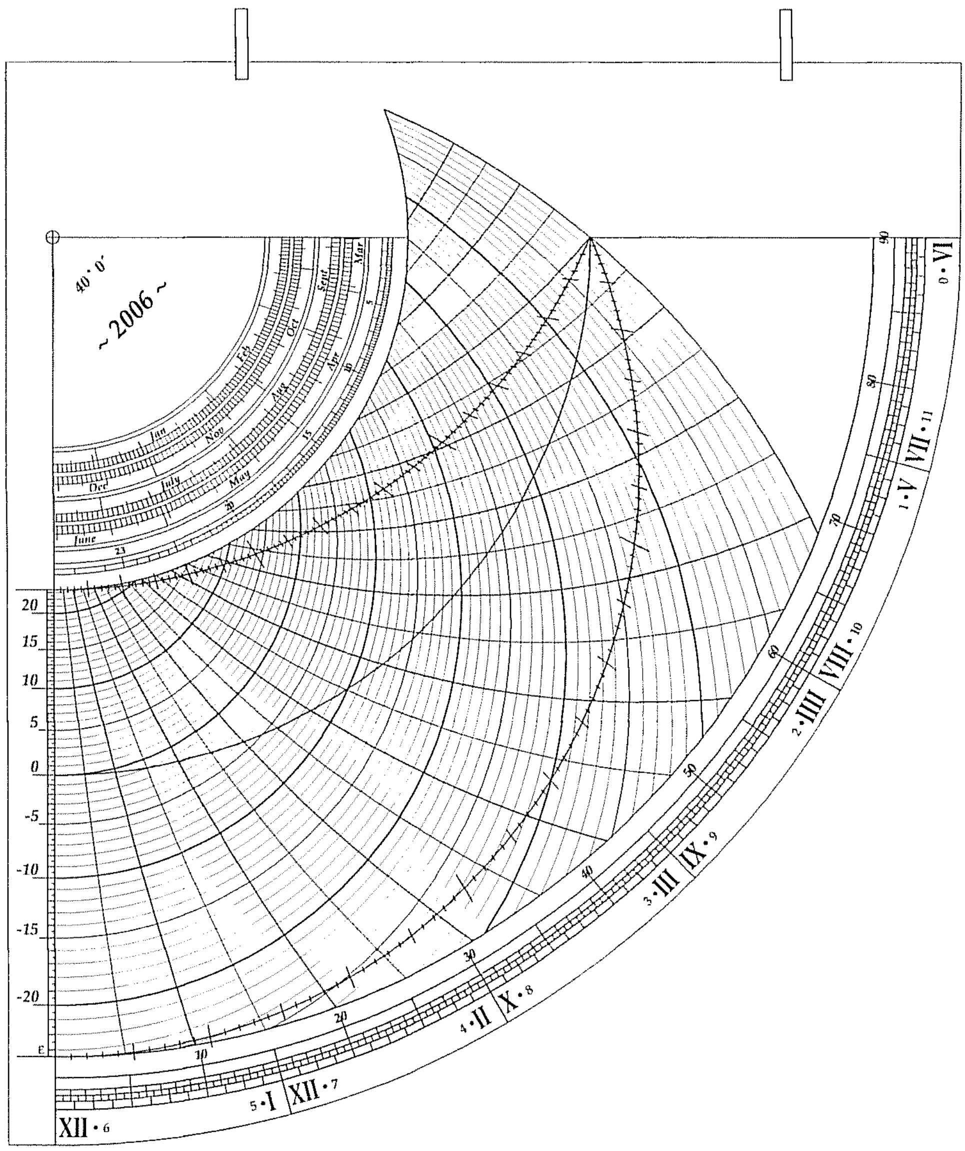


Figure 18-3. Prophatius Quadrant with Calendar and Time Scale

Figure 18-3 shows a westernized rendition of a Prophatius quadrant with a calendar and time scale. The addition of these scales makes the instrument much easier to use and adds the ability to find the Sun's right ascension. The quadrant in the figure should not be confused with an actual quadrant as used in the Ottoman Empire as such instruments would have different annotation and Turkish labels. The scales are comparable.

Using the Prophatius Quadrant

The use of this type of quadrant is quite similar to the plane astrolabe. The string and bob replace the rete and the bead is used to note declinations and altitudes.

None of the examples examined in preparing this chapter had labels for the ecliptic, azimuths or almucantars.

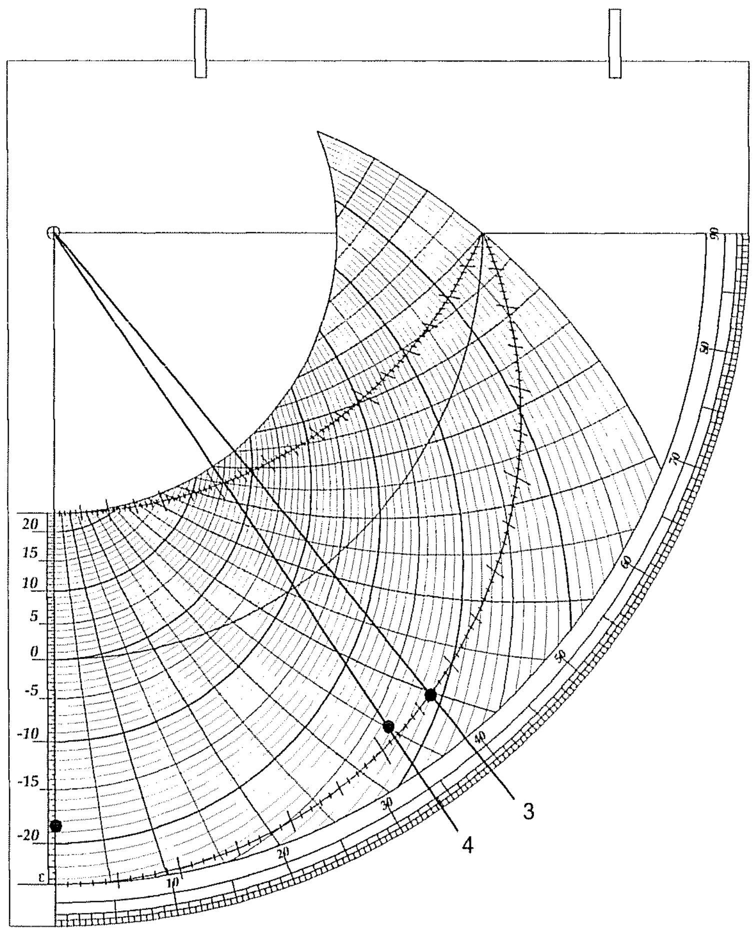


Figure 18-4. Prophatius Quadrant Example

Finding the time from the Sun's altitude (the steps are shown in Figure 18-4):

1. Find the Sun's longitude or declination from some source. Some quadrants included a calendar to make this easy and others included a table. A calendar scale above the Tropic of Cancer would be the easier option.

2. Say it is November 15, in the mid-afternoon. The Sun's The Sun's declination is -18° 31′ and the Sun's longitude is Scorpio 23°. The Sun's altitude is measured as about 23.5°.
3. Lay the string across the face of the astrolabe and set the bead at at -18.5° on the declination scale or Scorpio 23° on the ecliptic.
4. Move the string until the bead falls on the 23.5° almucantar.
5. Read the angle on the limb as 33.75°. The azimuth is 35° west.
6. Convert the angle reading to time at 15° per hour to be 2:15 PM apparent time.

Many quadrants of this type had a time scale on the limb to eliminate the final step.

The calendar scale allows you to find the Sun's declination and right ascension in a single step. The declination for the day is found by stretching the thread over a date tic and setting the bead to the ecliptic. The thread also points to the Sun's right ascension, which is read from the time scale using the Arabic numbers, and adding 12 hours for the second half of the year. The Sun's right ascension in the previous problem was 15h 24m.

Making the Prophatius Quadrant

Old quadrants should be studied to choose which scales to include.

Making a Prophatius quadrant problem presents no new challenges. The ecliptic, horizon, alumcantars and azimuth arcs are all calculated and drawn exactly as on a plane astrolabe.

The declination scale in the margin is divided like the astrolabe rule.

The time scale is simply the degree scale with divisions for each four degrees. Annotation can be applied to taste.

Labeling of the various arcs appears to be optional. The only use of the ecliptic is to set the Sun's declination, so the labels would be irrelevant to many users. Labeling the almucantars and azimuth arcs would make it a bit easier to use.

See page 276 for instructions on drawing the calendar scale. There is some variation in the declination for a date from year to year, so it is a good idea to make a Prophatius quadrant for a year halfway between leap years to average the error.

Chapter 19 - Gunter's Quadrant

Introduction

In 1623, Edmond Gunter (1581-1626) published a description of a quadrant in *The Description and Use of the Sector, Cross-Staff and other Instruments*, that is much easier to use than *quadrans novus* for finding the time. Gunter's quadrant is not a folded astrolabe although it is partly based on the stereographic projection, and many of its functions are familiar to astrolabe users, particularly those familiar with the *quadrans novus*.

It is interesting to speculate on whether Gunter's quadrant would have been developed if the Prophatius quadrant had been used in Europe, since it is no easier to use and does not provide any additional information.

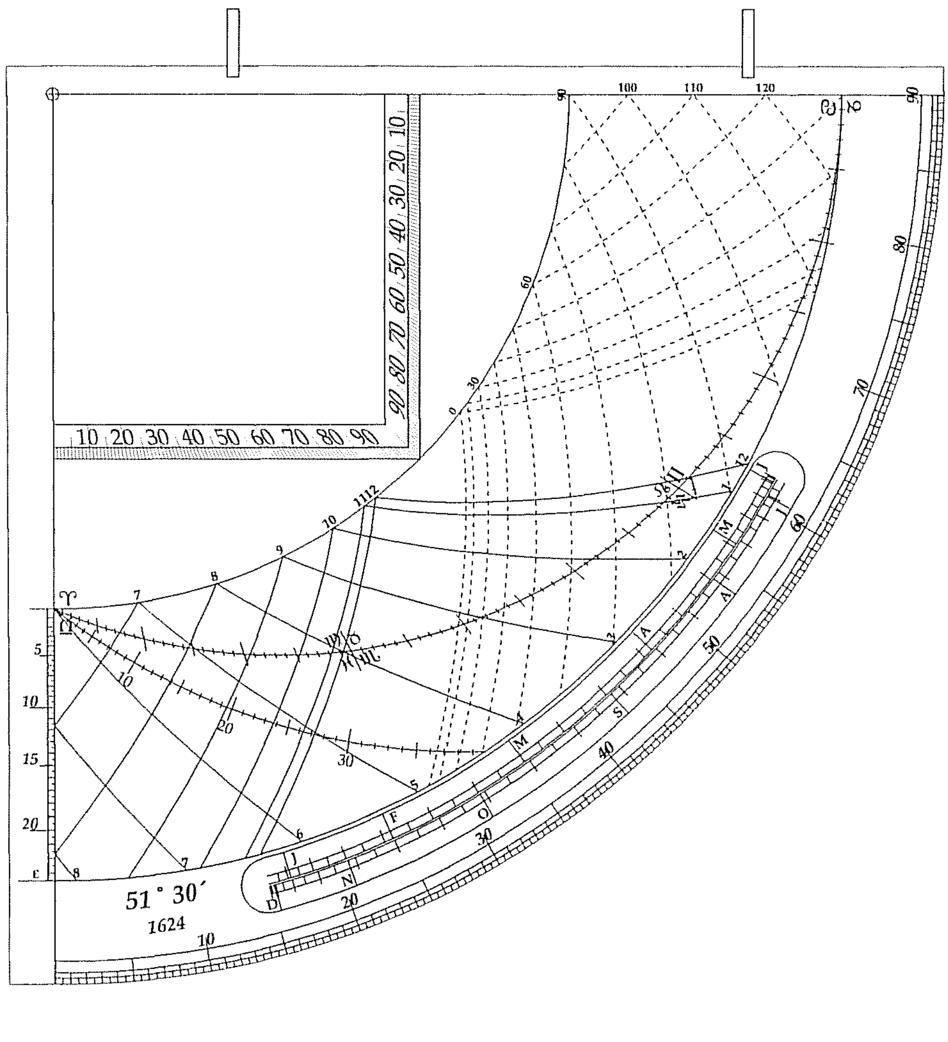


Figure 19-1. Gunter's Quadrant for London, 1624

Gunter was a mathematics teacher who in 1619 was named as the third Professor of Astronomy at Gresham College,London, a position he held for the rest of his life. Gresham College was founded in 1597 by Sir Thomas Gresham, founder of the London stock exchange and whose father had been Lord Mayor, to provide free public lectures to the citizens of London. The lecturers were paid (100/ pa) from rents from shops around the Royal Exchange bequeathed for this purpose. The Royal Society was founded at Gresham College in 1660. Gresham was unique in that lectures were given in both English and Latin, and were intended for practical men. Other Gresham lecturers include Christopher Wren and Robert Hooke. It is still in operation.

Gunter's treatise on the sector was originally written in Latin and circulated among his acquaintances privately. He is also remembered for "Gunter's Line", a logarithmic rule that anticipated the slide rule, contributions to surveying, and he was the first to use the terms cosine, cotangent, etc. to describe trigonometric operations on complements of angles (i.e. 90 degrees - angle).

There are few sources for information about Gunter's quadrant. Two are Gunter's own description of the instrument and the English translation of Nicolas Bion's *The Construction and Principal Uses of Mathematical Instruments* published by Edmund Stone in 1758. Both sources are a bit obscure for the modern reader.

Following is a paragraph from Gunter:

> *"Thus in our latitude of 51 gr. 30.Northward, the Sunne hauing 23 gr.30m.of North declination, if it ſhall be required to finde the altitude of the Sunne for ſeuen in the morning,; here becauſe the latitude and declination are both alike to the Northward, and the houre propoſed falleth betweene noone and ſix, you may take 23 gr.30m.the arke of the declination out of 78 gr.22m.the fourth arke belonging to the ſift houre from the meridian, ſo there will remaine 54 gr.52m.for your ſift arke. Then working according to the Canon, you ſhall find, ..."*

The same subject from Stone is:

> *"But if the Sun be not in the Equator, you muſt say, As the Co-ſine of the Hour from the Meridian is to the Radius, So is theTangent of the Latitude to the Tangent of a 4th Arc. Then conſider the Sun's Declination, and the Hour propoſed; if the Latitude and Declination be both alike, and the Hour fall between Noon and Six, ſubtract the Declination from the aforeſaid 4th Arc, and the Remainder will be a 5th arc."*

Suffice it to say it takes a measure of dedication to extract the essence of the instrument from these sources.

Description

Gunter's quadrant contains the essential scales for finding the time with a simple Sun sight, and can be used to solve a number of problems related to time and the position of the Sun. A few stars may be included for use at night. Some of the arcs and scales are based on a stereographic projection onto the equator. These arcs are drawn with the projection from both the south celestial pole and north celestial pole. They serve two functions depending on the projection origin. Like an astrolabe plate, the Gunter quadrant must be made for a specific latitude. As with other quadrants, Gunter's quadrant is equipped with sights and a weighted thread with a sliding bead.

An example of a representative Gunter's quadrant is shown in Figure 19-1, which implements the quadrant in Gunter's treatise for the latitude of London for 1624. This figure provides a reference if it is desired to reproduce the instrument described by Gunter. The quadrant scales are shown in Figure 19-2. The quadrant in Figure 19-3 is for the latitude of Washington, DC and modern time. Note Gunter's example included five stars that are not shown on this sample.

The degree scale on the limb is the normal quadrant scale of altitudes. This scale is also used to show right ascension. It is divided by half degrees.

The arc inside the degree scale is the stereographic projection of the Tropic of Cancer or the Tropic of Capricorn. The interpretation depends on the half of the year under consideration. This arc represents the Tropic of Capricorn when the Sun's declination is negative in the autumn and winter and the Tropic of Cancer when the Sun's declination is positive in the spring and summer. This arc represents a quadrant of the Tropic of Capricorn when the projection origin is at the south celestial pole and the Tropic of Cancer when the origin is at the north celestial pole.

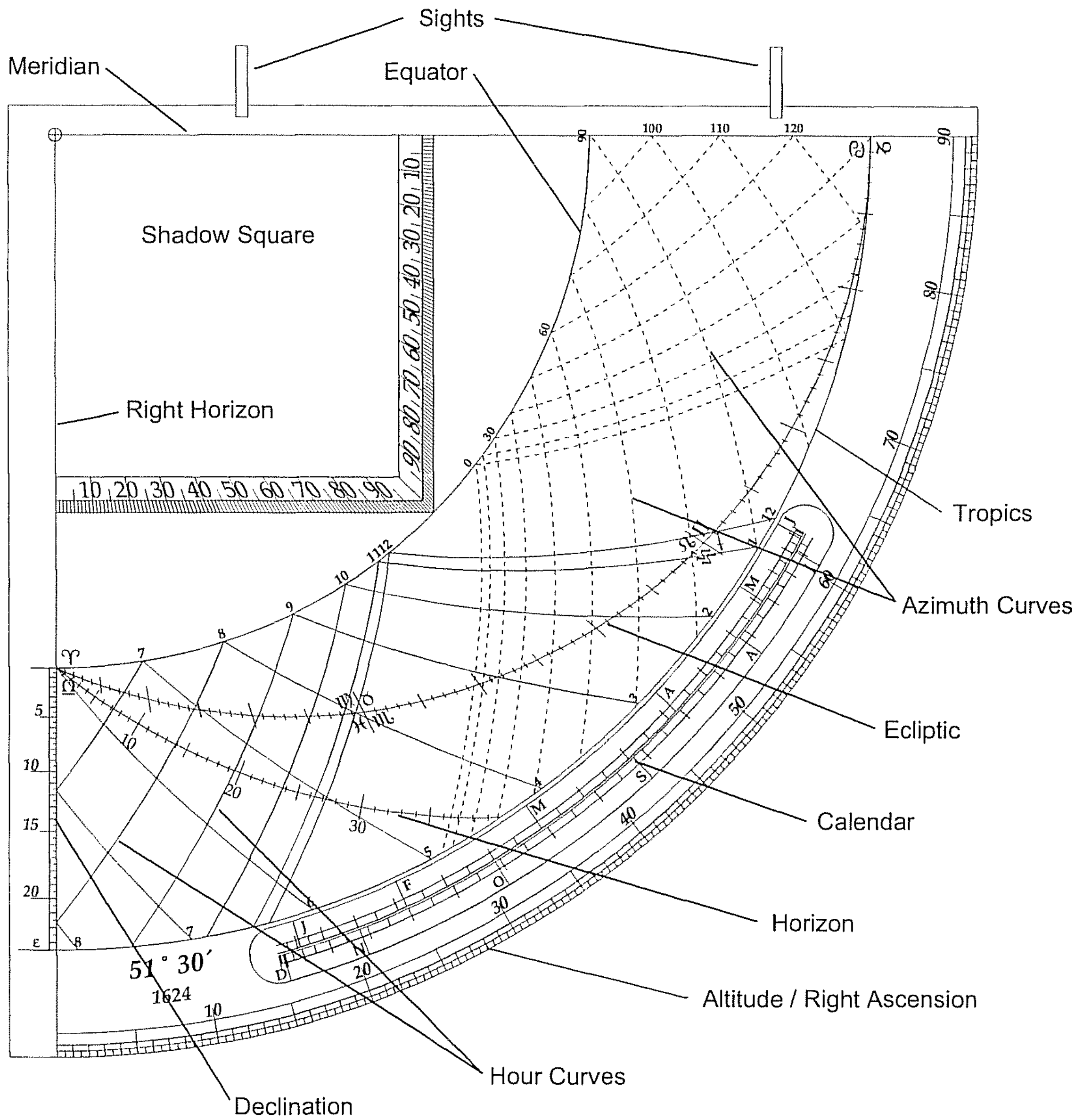


Figure 19-2. Gunter Quadrant Scales

The inner arc is the stereographic projection of the equator. The equator projection is the same regardless of the projection origin.

A scale of declination is in the left margin. The sign of the declination must be inferred from the time of year with positive declination in the spring and summer and negative declination in the autumn and winter.

The calendar scale just inside the degree scale on the limb associates the Sun's maximum altitude with each day of the year. When the thread is stretched over a date it shows the Sun's meridian altitude for the date on the limb scale and vice versa. Note the tic marks on the calendar scale are for noon of the date to simplify reading the Sun's altitude.

The portion of the horizon lying between the tropics is shown. This is the only part of the horizon needed to solve problems related to sunrise/sunset. The horizon scale is divided in degrees of azimuth from east or west.

The ecliptic is drawn between the tropics and is divided by the zodiac. The Sun moves over the scale twice in a year, once from the equator to the Tropic of Cancer then back to the equator, then from the equator to the Tropic of Capricorn and back to the equator. The zodiac signs shown indicate which direction the Sun is moving by the position of the sign. The degrees of longitude are generally not labeled since each tic represents four longitudes. The value assigned to a particular tic mark depends on the Sun's current sign and the direction the Sun is moving along the arc.

The hour curves represent equal hours. Each hour curve has two sections originating at the equator. The section used depends on the time of year. The section that arcs to the left is used in the autumn and winter, and the section to the right is used in the spring and summer. Each curve represents two times depending on whether it is morning or afternoon. The hour labels along the equator are used in the morning, and the hour labels along the Tropic arc are used in the afternoon. The hour curves are not stereographically projected and are not arcs of circles, but are calculated from the latitude, declination and Sun's hour angle.

Notice a line drawn from the apex through the point where the noon curve intersects the equator points to the latitude of the instrument on the limb degree scale.

The azimuth curves show the Sun's azimuth from south. Each azimuth angle is represented by two curves. Azimuth curves that angle to the left from the equator are used in the spring and summer and the curves angling to the right are used in the autumn and winter. The azimuth arcs are not stereographically projected and are not arcs of circles, but are calculated from latitude and declination. The Sun's altitude corresponding to a given azimuth is represented as co-altitude (90° - altitude.) on the limb degree scale to reduce the congestion on the left side of the instrument. Note the quadrant does not measure azimuth using the modern astronomical convention of the angle from north, increasing to the east. East and West on the quadrant are shown as 90°. East is 90° azimuth and west is 270° azimuth using the astronomical convention. An obsolete convention measured azimuth from south, increasing to the west.

A shadow scale of 100 divisions is normally included. This scale provides the tangent or cotangent of an angle. Tangents and cotangents greater than one must be inferred from the angle being measured (i.e., tangents of angles greater than 45° are greater than one).

Gunter included five stars on his example, apparently in an attempt to make the quadrant more useful at night. It is highly questionable whether any use was made of the stars since they are rather difficult to use and there are so few of them that it is problematical any of them would be in view when needed. Stars are not included in the examples shown. Gunter also suggested including a nocturnal on the back to make the total instrument usable at night, so he apparently did not have much confidence in using the stars to find the time.

A Gunter's quadrant for Washington, DC for 2003 is shown in Figure 19-3. Close comparison of this figure with Figure 19-1 will illustrate the effect of latitude on the quadrant. Washington is quite a bit south of London, so the Sun's altitude over the course of the year will be much higher in the sky and the horizon is much flatter. The azimuth curves are not included in Figure 19-3, because they would overlap the hour curves making the quadrant too congested. The calculations for this figure use modern values. Most significantly, the obliquity of the ecliptic is about 1/15 of a degree less than it was in 1624. This may appear to be a small value, but it affects the entire instrument.

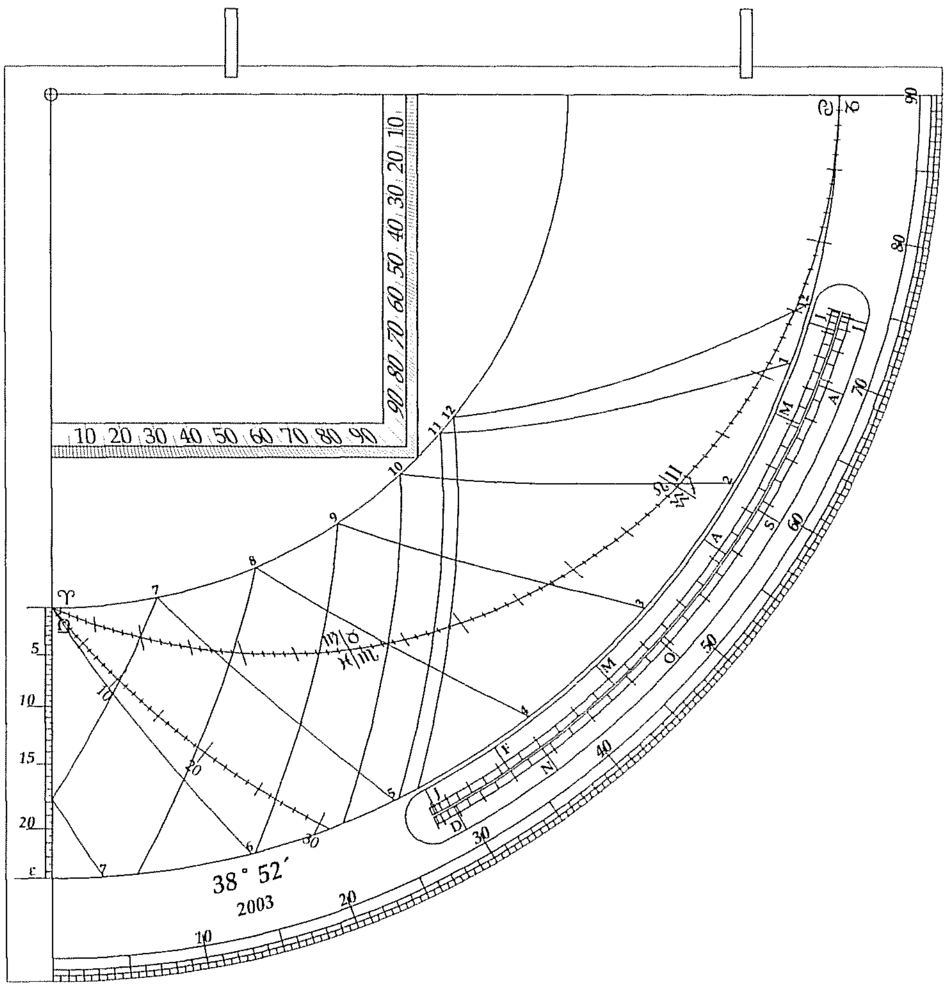


Figure 19-3. Gunter Quadrant for Washington, DC, 2003

The functions of the ordinary astrolabe and *quadrans novus* derive directly from the stereographic projection of the celestial sphere and timekeeping definitions. In a sense, the Gunter quadrant is not as pure an instrument as the astrolabe because the scales are not geometric results of the projection but are artificially calculated to provide the desired function. In one sense, it is a rather clever instrument combining the classic projection with calculations quite advanced for the time. One the other hand, it does not have the geometric appeal of the older instruments. It is, however, much easier to use and requires almost no astronomical knowledge to operate.

Using Gunter's Quadrant

The following examples use the example quadrant for 51° 30′, 1624 (Figure 19-4). The numbers in parentheses after an example are the true values. In general, it is not possible to read the quadrant to this level of precision.

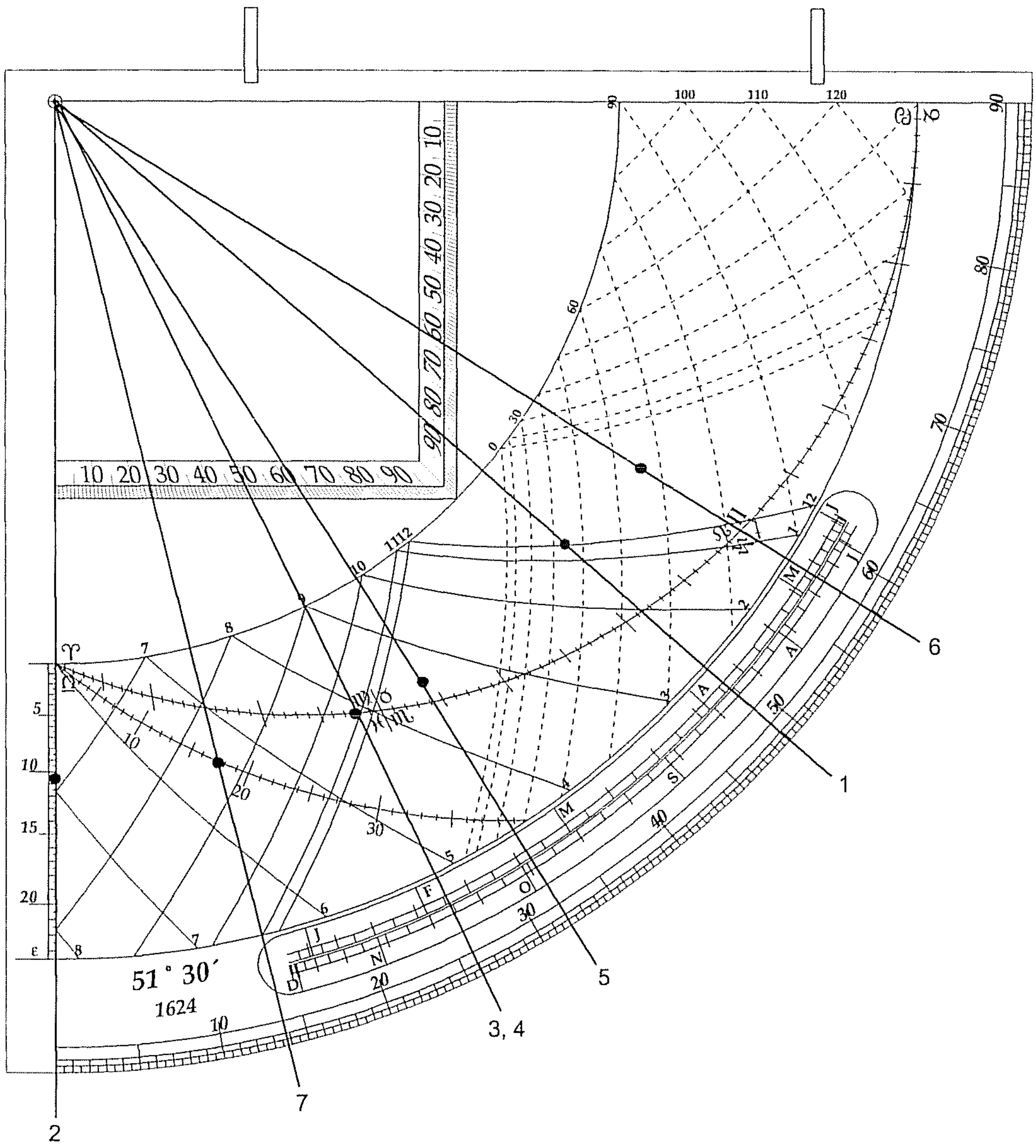


Figure 19-4. Sample Problems

Note the Julian calendar was still in use in England in 1624 even though most of continental Europe switched to the Gregorian calendar in 1582. The date of the vernal equinox in England in 1624 was March 10, and it was March 20 elsewhere in Europe. The use of the Julian calendar will make some of the results seem at odds with experience. This type of consideration is not unusual when working with old instruments and treatises. The quadrant showing the thread and bead for each sample problem is shown in Figure 19-4. The number next to each line is the number of the problem.

1. Find the Sun's maximum altitude for a date.

Lay the thread over the date on the calendar scale and read the Sun's noon altitude from the degree scale on the limb.

If the date is August 15, lay the thread over this point on the calendar scale and read 49° as the Sun's noon altitude on the limb degree scale (49.096°).

2. Find the Sun's declination for a date.

Lay the thread over the date on the calendar scale. Move the bead to the 12-hour curve. Rotate the thread to the left margin and read the Sun's declination from the declination scale.

Continuing with August 15, the Sun's declination is found to be about 10.5° (10° 36′).

3. Find the Sun's position in the zodiac for a date.

Lay the thread over the date on the calendar scale. Move the bead to the 12-hour curve. Rotate the thread until the bead is on the zodiac scale. Read the Sun's longitude from the zodiac scale considering the season.

For example, for August 15, the Sun's longitude is Virgo 2° (152.5°). The section of the zodiac arc to use is inferred from the date.

This problem can be worked in reverse to find the date corresponding to a given solar longitude.

4. Find the Sun's right ascension for a date.

Position the thread at the Sun's longitude in the zodiac and read the right ascension from the degree scale on the limb.

Continuing the example for August 15, set the thread on Virgo 2° and read 26° on the limb. Some interpretation of this result is needed. The Sun has moved from the vernal equinox where the right ascension is zero, through the summer solstice (90°) and is moving from right to left on the ecliptic. So, the Sun has moved through 90° + (90° - 26°) = 154°. Converting to time by dividing by 15 to get 10.27 hrs or 10 h. 16 m. (10h 18m).

5. Find the time of day.

Set the bead on the thread to the day's declination as in Problem 2 above. Measure the Sun's current altitude by letting the Sun shine through the holes in the sights and orienting the quadrant directly at the Sun. Read the Sun's altitude from the degree scale on the limb. Read the time from the hour curves being careful to use the correct section of the hour curves.

For August 15, the bead is set for a declination of 10.5°. If the Sun's altitude in the afternoon of August 15 is measured as 32°, the bead falls not quite halfway between the 3 and 4 hour curves using the afternoon labels at the bottom of the curves (3:24PM).

6. Find the Sun's azimuth from the meridian.

This problem is very similar to the previous problem except the azimuth curves are used instead of the hour curves. The difference is the position on the azimuth is found using the co-altitude (90° - altitude) on the degree scale.

For example, in example 5, the Sun's altitude was measured as 32°. We lay the thread on the limb angle of 90° - 32° = 58°. The bead will fall part way between the 60° and 70° curves at about 64° (63.8°). The azimuth is to the west since it is afternoon.

7. Find the time of sunrise (sunset).

The horizon arc is used to find the time of sunrise or sunset. There are two steps. First, the hour angle from east or west of sunrise/sunset is found and converted to time. Consider sunrise. Set the bead to the declination for the day and move the thread to the horizon arc. Read the hour angle from east from the degree scale on the limb (Gunter calls this the *ascensional distance*). Convert the hour angle to time by dividing by 15.

For example, on August 15, set the bead to the declination of 10.5° and move the thread so the bead is on the horizon. The thread falls on about 13.75° on the limb. Multiply 13.75° by 4 min./deg. to get 55 minutes. It is summer, so sunrise is before 6AM. Therefore, sunrise is 55 min. before 6AM or 5:05AM. (5:05AM). Gunter acknowledges this method is only approximate since it is difficult to read the hour angle with great precision.

The azimuth of sunrise from east is read from the tic marks on the horizon arc (17.3°).

Making Gunter's Quadrant

A different instrument is required for every latitude. It is prudent to calculate the astronomical constants such as the obliquity of the ecliptic for the year of the instrument before beginning the layout.

- **Basic design parameters.** Determine the size of the quadrant. The side dimension of most old Gunter quadrants was about six inches (15 cm), which gives a very useable instrument. A slightly larger instrument of say, eight inches (20 cm) would be a bit more accurate. The example quadrant has an overall size of 7.75 inches (19.7 cm).
- **Margins.** Margins outside the quadrant proper are needed for the declination scale and azimuth labels. About 1/2 inch (13 mm) is sufficient. The two margins need not be equal and it may be necessary for the margin on the side with the sights to be a bit larger to keep the sights from intruding on the scales.

  The intersection of the margins defines the center of the limb and tropic arcs and the hole for the thread. Note the limb, equator and tropics are drawn from this center. The radius of the limb is less than the length of the sides. Therefore, there must be a straight section the width of the margin from the tangent to the limb at the margin to the edge of the quadrant.
- **Limb division.** Divide the limb by degrees. It is possible to show half degrees on most quadrants. It is customary to box the degree scale for this type of instrument. Almost all 17th and 18th century instruments used boxed scales.

- **Tropics.** The radius of the tropics (**R_{cap}**) will be defined next, since this dimension determines all of the other scales. Room must be left for the calendar scale between the tropic and the limb. The example quadrant leaves 0.9 inch (2.3 cm) for the limb and calendar scales.

- **Equator.** The equations for drawing the stereographically projected elements are the same as for the astrolabe. Draw the equator arc: $R_{eq} = R_{cap} \tan [(90 - \varepsilon)/2]$. Set the compass to this radius and draw the arc from the right horizon to the meridian.

- **Declination scale.** The declination scale is needed for the rest of the construction so it needs to be drawn as accurately as possible. It should be divided into half-degree segments. The divisions need to be calculated for negative declinations from 0 to -ε. Your instrument will not be accurate if you do not use the exact obliquity for the year of the quadrant. The position of each declination point is:

$$R_\delta = R_{eq} \tan [(90 - \delta)/2]$$

- **Ecliptic.** The radius of the ecliptic arc can be calculated from:

$$R_{ec} = R_{cap} \cos^2[(90 - \varepsilon)/2] / 2 = R_{cap} / (1 + \cos 2\varepsilon).$$

The position of the ecliptic center is

$$y_{ec} = R_{eq} \tan \varepsilon.$$

Measure down the meridian y_{ec} units and draw an arc of a circle from the intersection of the equator with the right horizon to the intersection of the tropic with the meridian.

The ecliptic arc is divided in the same way as the ecliptic on an astrolabe and the same graphical method can be used. It is, however, simpler to calculate the Sun's declination for each longitude and mark the longitude using an arc from the declination scale. The Sun's longitude, λ, is found from spherical astronomy as, $\sin \lambda = \sin \delta / \sin \varepsilon$. Solve this equation for δ for each required longitude. Note that "only" 88 longitudes need to be calculated. Each calculated point is marked at the point where δ intersects the ecliptic.

- **Horizon**. The horizon radius is calculated from:

$$R_h = R_{eq} / \sin \varphi$$

The horizon center is:

$$y_h = R_{eq} / \tan \varphi$$

Draw the horizon circle from the intersection of the equator and the right horizon until it intersects the tropic arc.

Divide the horizon by degrees of azimuth from east/west. We know how to calculate the azimuth of sunrise/sunset, A, from spherical astronomy: $\cos A = \sin \delta / \cos \varphi$. Here, the azimuth angle is measured from east/west so we need $\cos(90 - A) = \sin A$. We need to mark specific azimuths so, to draw the tic marks, solve for the declination from $\sin \delta = \cos A \cos \varphi$. Calculate the declination for each tic, measure down the declination scale to the value and mark where the arc crosses the horizon. It is customary to mark each degree. The maximum azimuth shown on the horizon is the azimuth at the solstices and is calculated from $\sin A_{max} = \sin \varepsilon / \cos \varphi$.

- **Calendar scale**. This scale ties the Sun's meridian altitude to the date. It should be divided as finely as possible, but five days is sufficient for most purposes. Gunter suggests finding the Sun's declination for each day of the year from an almanac and calculating the Sun's meridian altitude from: $h = 90 - \varphi + \delta$. This is fine if you have an almanac for the year of the instrument, which is not always possible.

The Sun's declination for each day of the year can be calculated fairly easily on a computer. The Sun's declination for a date is $\sin \delta = \sin \varepsilon \sin \lambda$, where λ is the Sun's true geocentric longitude. The Sun's true longitude is calculated from:

True Longitude = Mean Longitude + True Anomaly - Mean Anomaly.

See the chapter on astronomical calculations for details on calculating these parameters. The meridian altitude is calculated using the equation above. Make sure the longitude is calculated at noon for each day. The Sun's declination changes fairly rapidly during parts of the year and there is enough difference in half a day to affect the accuracy of the scale.

If possible, you might consider drawing the calendar, hour curves and azimuth curves in color with one color representing spring /summer and afternoon (red) and a contrasting color (blue) for autumn/winter and morning.

The calendar scale in the examples is calculated for noon, UT. The calculations would be somewhat more precise if corrected for the time difference from your location to UT to account for the change in solar longitude due to the time difference. For example, the west coast of North America is a third of a day from the Greenwich meridian and there is a sensible change in longitude in that time in parts of the year. This correction may not seem to be required given the inherent lack of precision in the instrument, but there is some personal gratification in making it as accurate as possible.

- **Hour curves**. The hour curves are calculated from the standard equation from spherical astronomy that relates altitude to latitude, declination and hour angle. These curves are not arcs of circles so it is necessary to calculate a number of points and connect the points with a smooth curve. It is adequate to calculate the Sun's altitude for each degree of declination and connect the points with straight lines.

  The equation for calculating the Sun's altitude is:

  $$\sin h = \sin \varphi \sin \delta + \cos \varphi \cos \delta \cos H$$

  where H is the hour angle of the time. H = time from noon × 15°, e.g. the hour angle of 10AM/2PM = 30°. The calculations must be done for $-\varepsilon \leq \delta \leq \varepsilon$.

  Each point calculated is plotted on the quadrant at the polar coordinates (δ, h) if plotting manually; set the bead to δ and rotate the thread to h and locate the point.

  The point's rectangular coordinates can also be calculated from the declination arc for δ and the altitude, h. Let d be the distance inside the tropic arc for the day's declination:

  $c = R \tan[(90-\delta)/2]$. The x, y coordinates of the point to plot is then:

  $x = c \cos(90-h) = c \sin h$ and $y = c \sin(90-h) = c \cos h$

  Plot using (90-h) because we are drawing the quadrant in QIV.

  A considerable number of calculations are required to define the hour curves. The calculations can easily be done with a simple computer program, on a programmable calculator or with a spread sheet program.

- **Azimuth curves**. The azimuth curves show the Sun's azimuth for a given declination and altitude. The azimuth curves will overlap the hour curves for latitudes less than 45° so you may not want to include them for an instrument made for most of the US. The loss is not great as the azimuth curves are not used as often. They are, however, useful for some applications and may be desired. You may need to include only those curves that do not overlap the hour curves or you may be able to find a way to make the instrument useable with the overlapping curves.

The Sun's altitude given the azimuth, declination and latitude are need to locate the points on the curves. It is not possible to calculate the Sun's altitude directly, but Gunter solved the problem in a clever way (see below for an outline of the proof).

For a given azimuth, calculate the Sun's altitude when the declination is zero; i.e. the Sun is on the equator, from:

$$\tan h_0 = \cos A / \tan \varphi$$

Then calculate an auxiliary angle, x from:

$$\sin x = \cos h_0 \sin \delta / \sin \varphi$$

Calculate the Sun's altitude, h, for the azimuth and declination:

If $A < 90°$, $h = x + h_0$

If $A > 90°$, $h = x - h_0$

Once again, a significant number of calculations are required but this is not difficult with a spreadsheet program or a simple computer program. The curves are drawn using the same technique as the hour curves, remembering to plot h on the (90-h) division on the altitude scale.

Each point calculated is plotted on the quadrant at the polar coordinates (90-h, δ) if plotting manually; i.e., set the bead to δ and rotate the thread to 90-h and locate the point.

The point's rectangular coordinates can also be calculated from the declination arc for δ and the altitude, h. Let d be the distance inside the tropic arc for the day's declination: c = R tan[(90-δ)/2]. The x, y coordinates of the point to plot are then:

$$x = c \cos h, \; y = c \sin h$$

Draw the shadow square if desired. The square can be positioned according to taste. Each side of the square is divided into 100 equal divisions with each 10 divisions labeled. The labels should be 'sheared' so the slope of the characters matches the slope of the division.

You may or may not want to include stars on your quadrant. If so, the star's position is defined by its declination and right ascension, considering the appropriate quadrant. Be sure to precess the stars to the date of the quadrant. You will need to include a list of the stars included and their coordinates, perhaps on the back or inside the shadow square, in order for them to be useful..

Notes:

1. Gunter (Bion) outlines a method for the hour curves similar to the method shown above for the azimuth curves. They apparently did not realize the curves could be calculated directly without resorting to a trigonometric trick.

2. It would certainly be possible to make a Gunter's quadrant with hour curves reflecting modern time zones. It would only be necessary to adjust the hour angle of the hour curves to include the longitude correction of your location from the center of the time zone. Note, however, it would also be necessary to include a time zone adjustment when calculating the calendar scale to account for the Sun's declination change from UT to the local time zone. It would be very difficult to include the equation of time in the hour curves since each curve section is used for both positive and negative declination.

3. If you refer to Gunter's or Bion's treatise, you need to work each example carefully. There are many numerical errors. It is not clear whether the errors are computational or typesetting. One is, however, left with a sense of admiration for the huge amount of hand calculation required in the 17^{th} century to make an instrument of this type, particularly since the available trigonometry tables also had many errors. It is possible Gunter's quadrants were made with circular hour and azimuth arcs located by three points. This method would certainly reduce the effort required to make the instrument, but the result would be incorrect.

4. A proof of Gunther's method for finding the Sun's altitude for a given azimuth and declination is outlined below.

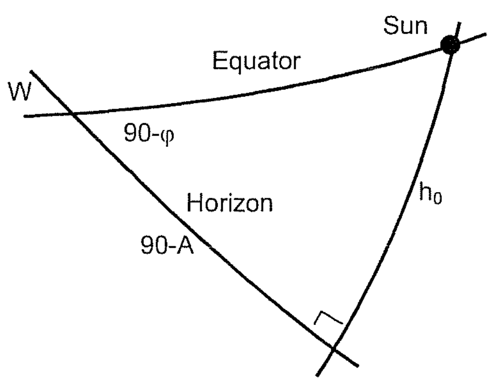
   The problem to be solved is to calculate h given δ, φ and A.

   There is a standard equation from spherical trigonometry that at first glance appears to solve the problem:

   $$\sin \delta = \sin \varphi \sin h - \cos \varphi \cos h \cos A$$

   However, this equation cannot be solved directly for h, although it can be solved by successive approximations.

   Gunter needed a way to solve directly for h, and he solved the problem by breaking it into two parts that can be solved. First, the Sun's altitude when it's declination is zero is solved. Then the additional altitude of the Sun when the declination is not zero is found, and the two components are added. The Sun's altitude when it is on the equator can be found using the construction below:



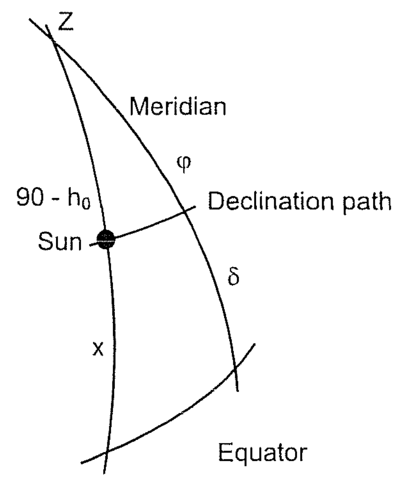
The construction is for the Sun when it is above the horizon, on the equator with an azimuth less than 90°.

This is a right spherical triangle than can be solved using Napier's rules as:

$$\sin (90 - A) = \tan h_0 \tan \varphi \quad \text{or} \quad \cos A = \tan h_0 \tan \varphi$$

The second part finds the additional altitude of the Sun when the declination is non-zero. The construction below is for the case of a positive declination. The case for negative declination is similar.

Call x the Sun's altitude above its altitude when the declination was zero. The construction uses two spherical triangles, both of which have the zenith as the apex, one side on the meridian and the other side the Sun's hour circle. The interior angle at the apex the Sun's azimuth. The base of the larger triangle is the equator, and the sides are φ on the meridian and $(90\text{-}h_0)$ on the Sun's hour circle. The base of the smaller triangle is the declination path of the Sun. The angular distance from the equator to the declination path on the meridian is the Sun's declination. The distance from the equator to the Sun on the Sun's hour circle is the unknown, x.



The two triangles are similar since the declination path and the equator are parallel and the triangles share a common apex. Therefore, $x : (90\text{-}h_0) :: \delta : \varphi$. And,

$$\sin x / \sin(90 - h_0) = \sin x / \cos h_0 = \sin \delta / \sin \varphi \quad \text{or} \quad \sin x = \cos h_0 \sin \delta / \sin \varphi$$

Chapter 20 - Sutton's Quadrant

There are only a few variations possible with the stereographic projection, and all of them have been used for astronomical purposes. The projection plane is the equator for most astrolabe-related instruments, but instruments have been made using the horizon as the projection plane with the nadir as the projection origin. A projection plane tangent to the celestial sphere with the projection origin opposite the point of tangency is popular for high quality star maps.

The usual astrolabe projection can be reversed to use the north celestial pole as the origin. This reversal of the projection origin is called the *southern astrolabe projection*,because it results in a normal-looking astrolabe for southern latitudes. The result of a southern projection for a northern latitude is shown in Figure 20-1. This projection style was popular with monumental astrolabe clocks and will be discussed again in the chapter on that subject. It was only a matter of time until someone used the southern astrolabe projection as the basis for an astrolabe quadrant[97].

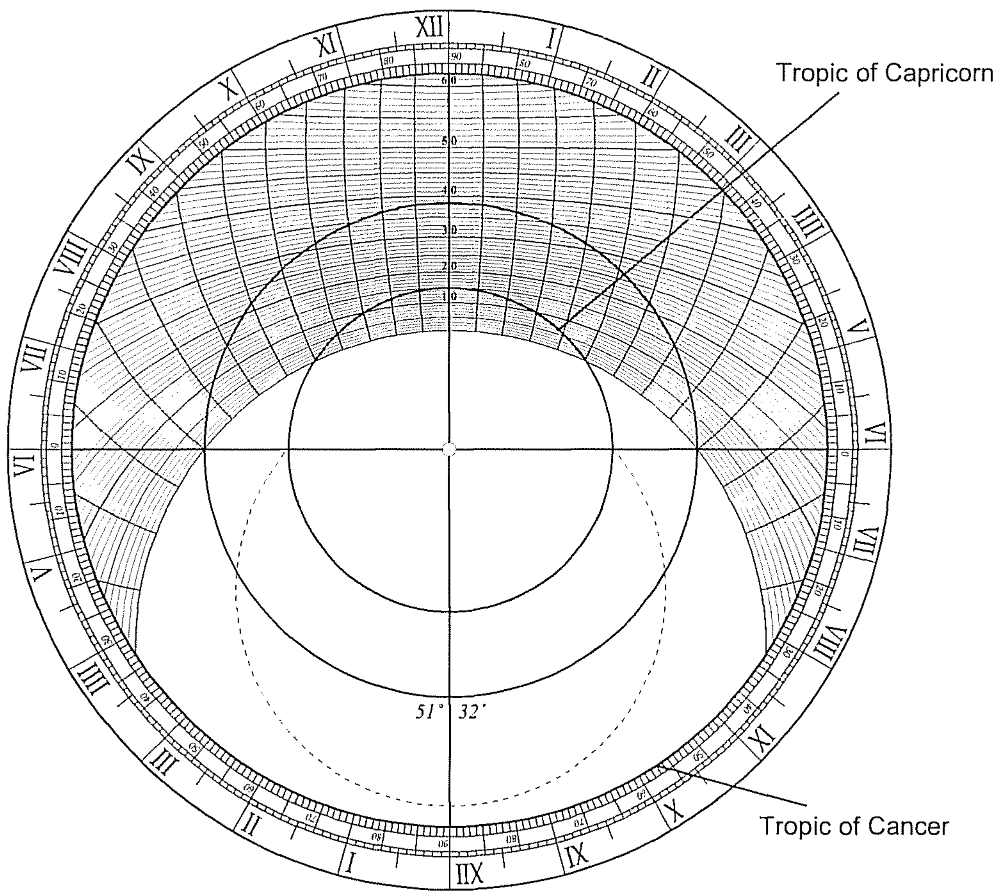


Figure 20-1. Astrolabe Plate from Southern Projection

The first European known to have applied the southern projection to an astrolabe quadrant was

[97] Islamic quadrants based on the southern projection were made before the 13th century.

Thomas Harvey[98] of London in the middle of the 17th century. Almost nothing is known about Harvey, but his quadrant design was adopted by Henry Sutton, a prominent printer involved in printing very accurate forms and grids. Sutton enjoyed some success with nicely printed paper versions of the quadrant varnished to a wood base and it became associated with Sutton's name rather than Harvey's. It was described in good detail by John Collins in, *The Sector on a Quadrant...*[99], published in 1659. Harvey, Collins and Sutton were all friends.

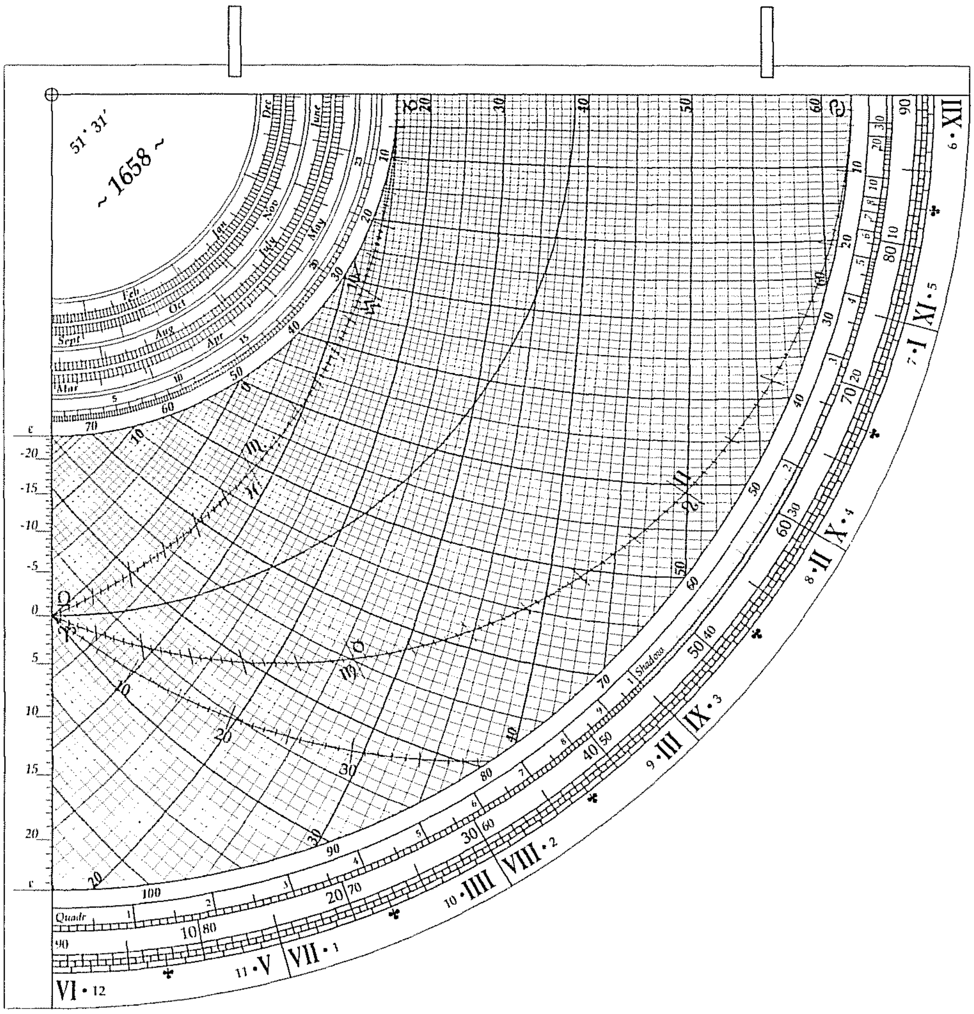


Figure 20-2. Sutton's Quadrant

Sutton produced the quadrant in several formats. The most common type of the surviving examples will be considered. He also produced a quadrant based on the horizontal projection.

The resulting quadrant is the upper right quarter of the southern astrolabe projection. This fourth of the astrolabe plate serves for both the morning and afternoon, since the curves are symmetrical about the meridian. The arcs reproduced on the quadrant are those above the right

[98] Collins spells his name as both 'Harvie' and 'Harvey'.
[99] The complete title is, *The Sector on a Quadrant or A treatise containing the description of the use of four several quadrants; two small and two great ones, each rendred many wayes, both general and particular. Each of them accommodated for dyalling; for the resolving of all proportions instrumentally, and for the ready finding of the hour and azimuth universally on the equal limb. Of great use to seamen and practioners of the mathematicks. Written by John Collins, Accountant Philomath.*

horizon and to the right of the meridian. Only the arcs between the tropics are shown, as the quadrant is intended to be used for the Sun.

The projected altitude and azimuth arcs below the right horizon form a rounded wedge that cannot be included on the quadrant. There is also a blank section below the horizon between Capricorn and the equator. The blank section is filled in with the missing curves. The altitude arcs will represent negative values in this area, but the curves are consistent because negative altitudes have the same relationship to Capricorn as the missing positive altitude arcs have to Cancer. Collins calls this area the "reverted tail."

Figure 20-2 is a representation of the quadrant known as Sutton's quadrant. It is, perhaps, unfair that this instrument is identified only with Henry Sutton, as it was apparently a product of teamwork, but after examining originals it is clear Sutton was a very skilled engraver, and one has to admire both his technical ability and artistic skill. Collins mentions that Sutton figured out the projection without any help, so he was obviously a capable mathematician also. Of all the instruments in this book, this is by far the most difficult to make, considering the number of scales and arcs to be drawn and the number of calculations required. The instrument in the figure recreates most of the elements found on the most common surviving version of the quadrant, which is also the implementation described by Collins.

In some respects, Sutton's quadrants are something of a virtuoso performance with scales included that appear to be of little practical value to the normal user and merely serve to fill up space and make the quadrant look more impressive. On the other hand, this quadrant style is one of the easiest to use for finding the time and has the capability of solving a number of astronomical problems for the knowledgeable user.

Description

See Figure 20-3. This figure is very similar to Sutton's product; scale content, positions and labels were measured from a photograph of a surviving instrument. Note, however, Sutton's quadrants include some scales not shown in the figure. These omitted scales were intended to be used for sundial design and do not relate to the primary function of the instrument. Also, Sutton included several stars on the face of the quadrant with their coordinates on the back. The stars are not included in our figures, as their utility is questionable. Other differences with Sutton's instruments are pointed out in the following discussion.

A cursory examination of the quadrant makes one positive aspect of quadrants immediately apparent. The quadrant in the figure is based on an instrument of about seven inches (16 cm.) on a side, or about the size of many astrolabes. However, on a quadrant of this size it is possible to show every degree of altitude and azimuth and divide the limb by half degrees without clutter. The quadrant form is potentially rather more accurate than an astrolabe of the same size. It would be interesting to know the precision requirements and achievements of actual users.

The upper edge of the quadrant represents the meridian. The left edge is the right horizon.
The outer arc of the quadrant interior is the Tropic of Cancer, and the inner limit is the Tropic of Capricorn. The equator is shown in the figure but was not included on Sutton's instruments.

As with all quadrants, it is equipped with a thread and sliding bead. Collins suggests using two beads to protect against getting a bad reading by stretching the thread. One bead is set to a fixed scale, such as Cancer and the other bead is set to the area of interest. The thread is stretched only until the reference bead is on the correct point to ensure the working bead is not displaced by pulling too hard on the thread.

Quadrant Interior

Altitude arcs

The altitude arcs are perpendicular to the meridian and curve either right or left depending on the altitude. Each degree is shown. Altitude arcs are labeled inside the quadrant at each end of the ten-degree arcs.

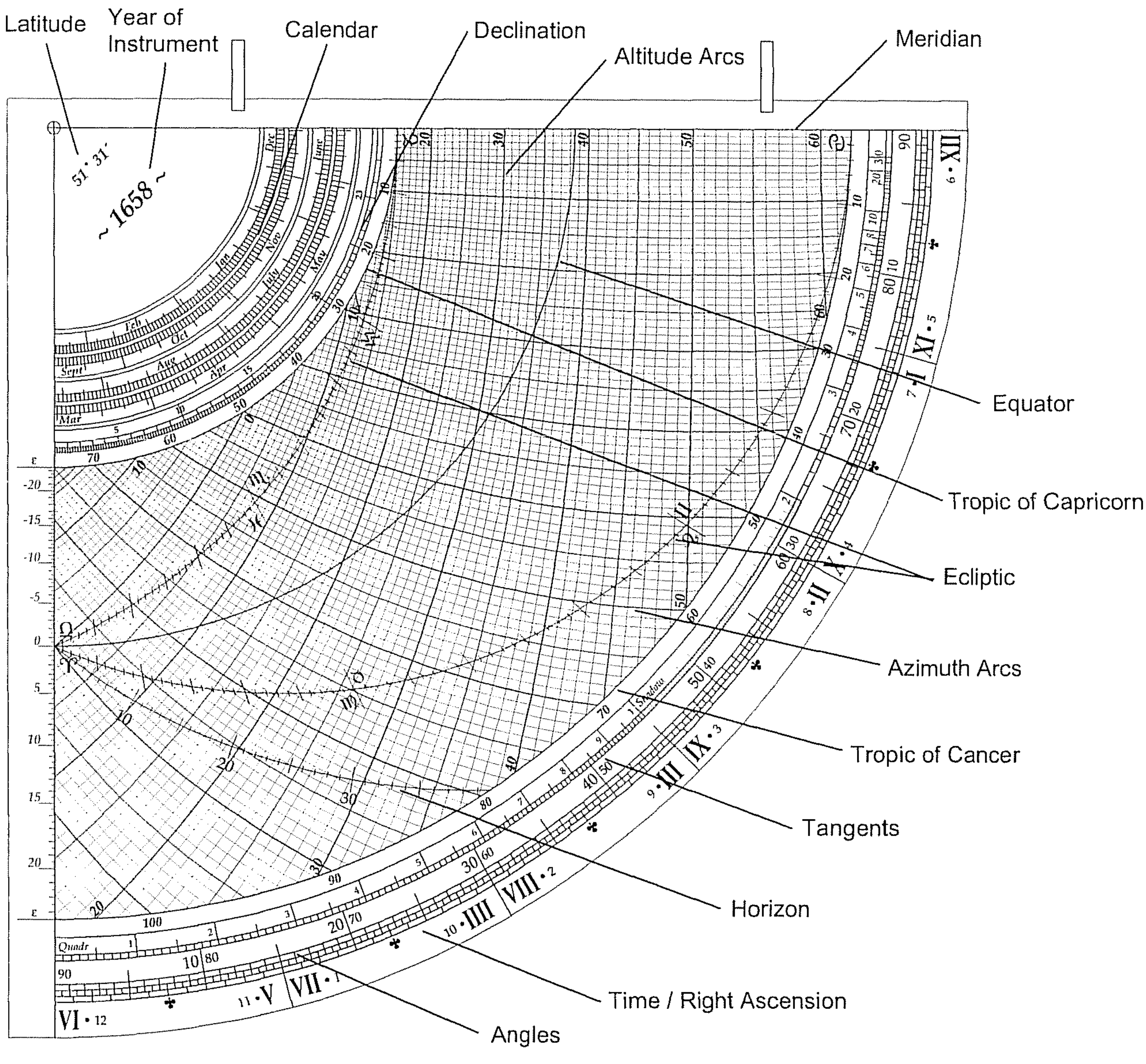


Figure 20-3. Sutton's Quadrant Scales

The main use of the quadrant is to find the time and to solve problems related to the Sun. The altitude arcs originating on the meridian between the tropics show the Sun's maximum altitude for the day, and the range of altitudes represented by the arcs between the tropics constitutes the annual range of meridian solar altitudes.

All of the altitude arcs that will fit on the quadrant are drawn, even though some of them may not be useful with the Sun, depending on the quadrant's latitude. Also, useful altitude arcs, such as twilight solar altitudes, may not be available for lower latitudes.

Azimuth arcs

Azimuths from the meridian are shown for all degrees that will fit on the quadrant. The azimuth arcs are labeled outside Cancer and inside Capricorn. Not all valid azimuth arcs will fit and it is not possible to read solar azimuths when the Sun is close to Cancer and near sunrise or sunset.

Horizon

The horizon is, as expected, the 0° altitude curve. However, this curve is truncated at the right horizon and is useful only in the winter. Therefore, an additional horizon curve that completes the scope of the horizon for the entire year is added. This horizon arc is identical to the horizon on Gunter's quadrant and is divided by azimuth angles.

Ecliptic

Unlike Gunter's quadrant, the entire ecliptic is shown. The zodiac signs are included on the figure and are not shown on Sutton's instruments. The ecliptic is folded as on the Prophatius quadrant.

Limb Scales

Angles

The angle scale is divided for half degrees from 0 - 90° and labeled for each 10° in both directions for use with the tangent scale, with larger numbers in the counterclockwise direction. The primary use of the angle scale is to mark observed altitudes but it can also be used as the angular element of problems to be solved. The angle scale can also represent right ascension measured in degrees.

Time / Right Ascension

The main use of this scale is to indicate the time of day. It doubles as a scale of right ascension. The scale is divided into five-minute sections. Roman numeral labels proceed from six to twelve for the morning hours and then from one to six for the afternoon hours in the reverse direction. Arabic numbers are used for right ascension. Twelve hours must be added mentally for right ascensions greater than 12 hours.

Tangent Scale

The tangent scale serves the same purpose as the shadow square on the astrolabe. The tangent or cotangent of any angle from 0-90° can be read from the scale when the thread is aligned with an angle. The half of the scale from 0 to 1 represents tangents of angles less than 45°, which are less than 1 in value. The scale value is divided by 10 to get the tangent. For example, the tangent of 31° is about 0.6. The 6 on the scale represents .6. The value of tangents of angles greater than 45° are shown directly on the scale.

The scale shows cotangents when using the clockwise angle scale labels.

The scale can be read to at least two places and to three places in some areas, which is fairly accurate for an analog instrument.

Upper Scales

The latitude and year for the quadrant is shown in the upper part of the quadrant.

Calendar Scale

The calendar scale is divided into the four sections of the year between equinoxes and solstices. The tics in the figure align with the solar right ascension scale on the limb for midnight of each date. The tic for December 31 is midnight of that day. The tic for January 1 is midnight, etc. It is not known how Sutton aligned the tics, but it could have been for noon of the day.

The calendar in the figure is for London in 1658. England used the Julian calendar until 1752, so the date of the vernal equinox is March 10. The vernal equinox was at about 10AM on March 10, 1658.

Declination Scale

The declination scale in the upper quadrant is aligned with the calendar scale. When the thread is laid across a date, it crosses this scale at the Sun's declination for the date. The sign of the declination must be supplied mentally based on the half of the year under consideration.

An additional declination scale is included in the figure on the left margin. This scale was not on Sutton's instruments but would be useful when working with celestial objects other than the Sun.

Quadrants for latitudes of 30° and 60° are shown in the figures below to illustrate the affect of latitude on the appearance of the instrument.

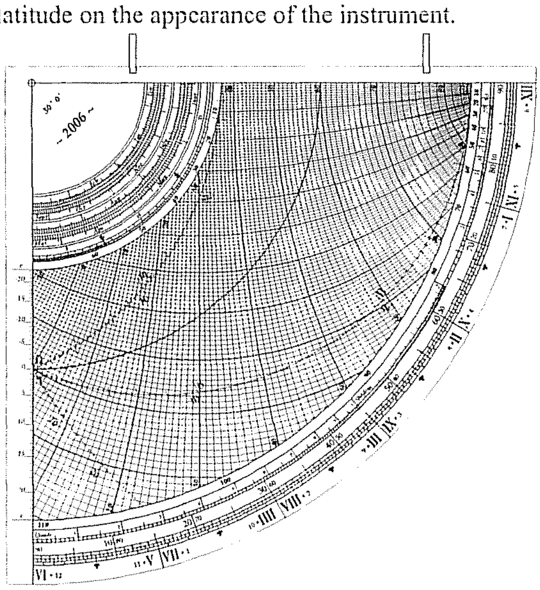


Figure 20-5. Sutton's Quadrant for 30°

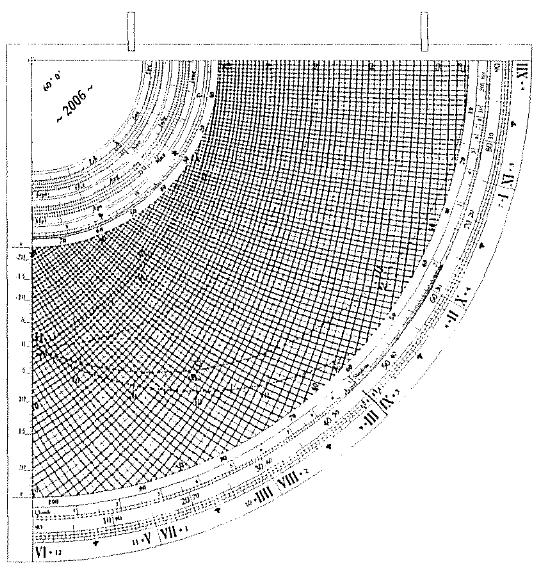


Figure 20-5. Sutton's Quadrant for 60°

Using Sutton's Quadrant

Following are several problems illustrating the operational simplicity of this instrument. The quadrant for the examples is for 40° latitude and modern time (Figure 20-6). The lines on the figure show the thread positions and are numbered to correspond to the example. The values in parentheses are the actual results.

1. **Find the time when the Sun's altitude is 38° in the afternoon on August 20.**

   Lay the thread over August 20 and set the bead on the lower ecliptic arc.

   Move the thread and bead to the 38° altitude arc.

   Read approximately 3:21 PM on the time scale.

   A contemporary user would need to correct for longitude, DST and the equation of time.

2. **Find the Sun's declination and right ascension on August 20.**

   Lay the thread over August 20. Recall the tic for a date is midnight, so the thread is positioned about half a tic before the long tic for August 20. Note the declination of approximately 12.4° (12° 21′) on the circular declination scale.

   Read the angle on the limb scale as about 59.6°. The right ascension as 9 hr. 56 min from the Arabic numerals on the time scale .

3. **What is the time and azimuth of sunrise on August 20?**

   Move the thread to August 20 and set the bead on the lower ecliptic arc. Move the thread and bead to the horizon arc and read the azimuth of sunrise as about over 16.5° on the scale (16.5°). Read the time of sunrise on the limb time scale as about 5:16 AM.

Other problems and their solutions will occur to the interested user.

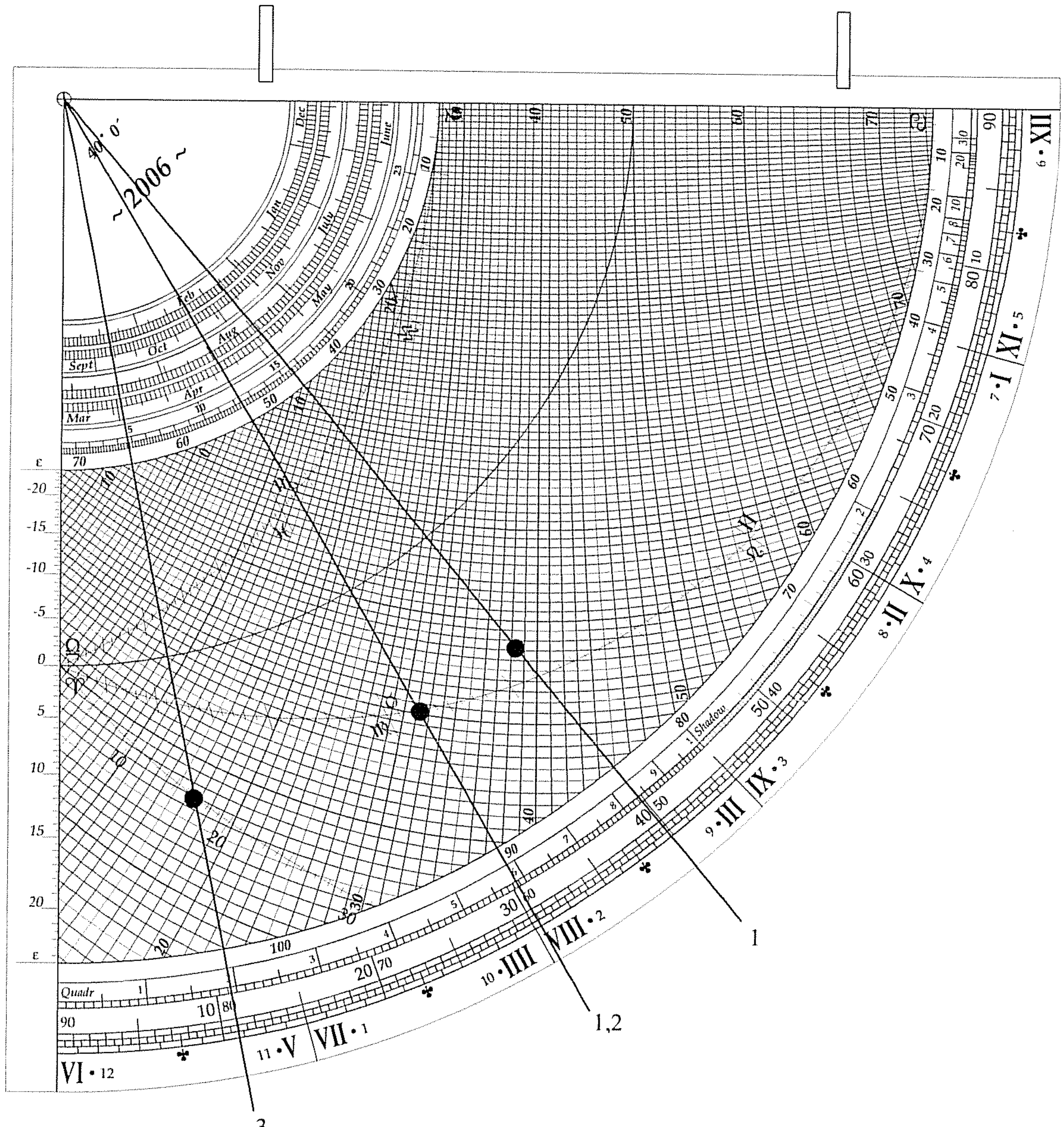


Figure 20-6. Sutton's Quadrant for 40°

Making Sutton's Quadrant

Drawing this instrument by hand as Sutton did (and in reverse since he was engraving printing plates!) is a meaningful effort. Drawing it with a computer program is scarcely easier due to the number of scales and labels that must be applied.

The equations used for the southern projection are essentially the same as those for the northern projection, but there are some special considerations that will be pointed out at the appropriate time.

A different instrument is required for every latitude. It is prudent to calculate the astronomical constants such as the obliquity of the ecliptic for the year of the instrument before beginning the layout.

Basic design parameters.

Determine the size of the quadrant. Collins implies a five-inch quadrant, which would be fairly easy to carry around and still be large enough for good accuracy. A slightly larger instrument is a bit easier to use accurately. The quadrant represented in the figures has a size of just over seven inches from the center to the round edge with a half-inch margin above the meridian to accommodate the sights. There is also a half- inch margin on each side.

- **Margins.** Margins outside the quadrant proper are needed for the declination scale and azimuth labels.. About 1/2 inch (13 mm) is sufficient. The two margins need not be equal and it may be necessary for the margin on the side with the sights to be a bit larger to keep the sights from intruding on the scales. Additional space in the margins will be needed if you are making a copy of Sutton's instrument.

  The intersection of the margins defines the center of the limb and tropic arcs and the hole for the thread. Note the limb, equator and tropics are drawn from this center. The radius of the limb is less than the length of the sides. Therefore, there must be a straight section the width of the margin from the tangent to the limb at the margin to the edge of the quadrant.

- **Tropics.** The radius of the Tropic of Cancer defines the entire instrument. The selection of the Cancer radius must allow sufficient space for the limb scales. The limb scale margin in the samples is 13% of the distance from the center to the limb edge. Once the Cancer radius is selected, the equator and Capricorn radii are calculated from the normal relations:

  $$R_{eq} = R_{can} \tan [(90 - \varepsilon)/2]$$

  $$R_{cap} = R_{eq} \tan [(90 - \varepsilon)/2]$$

Limb scales

The measurements for the scale circles can be taken from the figures or chosen to achieve a pleasing proportion. Seven circles are needed for all the scales and scale boxes.

- **Time / Right Ascension scale**. The scale is divided for each five minutes (1.25°) with longer tics each hour (15°). Sutton inserted a little decorative symbol each 30 minutes (7.5°). The time scale labels are shown on the sample figures.

- **Angle scale.** Divide the scale by degrees with each half-degree shown and longer tics for each five degree division and the ten degree tics connecting the bottom of the tangent scale. The counterclockwise labels are larger than the clockwise labels.

- **Tangent Scale.** The tics on the tangent scale are positioned at α = arctan a, where α is an angle on the limb. The scale in the examples is divided into several sections depending on the resolution. The 0 - 1 section represents tangents of 0 < arctan a < 45°. This section is labeled as 10 tan α; i.e. tan 35° = 0.7 and is labeled '7'. There are 10 divisions between each integer label, which gives fair accuracy when read to three significant digits.

  The 0 - 1 section has 100 divisions. The 2 - 5 section has 10 divisions per integer interval. The 5 - 8 section has four divisions for each integer interval and there are divisions for 9 - 20 and then single tics for 25 and 30. It doesn't make any sense to divide the scale higher than 30.

Upper Scales

There are 17 circular arcs defining the upper scales. The radii of these arcs for the examples were taken from a photograph of a surviving instrument. These radii as a fraction of the radius of the Tropic of Capricorn are, from the innermost to the outermost:

0.5970, 0.6096, 0.6448, 0.6675, 0.6801,
0.7028, 0.7380, 0.7506, 0.7859, 0.8086,
0.8212, 0.8438, 0.8791, 0.8917, 0.9269,
0.9421, 0.9572

- **Declination scale.** The declination scale associates the Sun's declination with the Sun's right ascension on the limb. The scale is divided for each 0.2 degrees of declination and labeled for each five degrees up to 20° and a final label for 23°.

  The scale is calculated by taking a declination, δ, and calculating the associated right ascension from:

  a) Calculate the Sun's longitude for declination from: $\sin \lambda = \sin \delta / \sin \varepsilon$
  b) Calculate the Sun's right ascension from: $\tan \alpha = \tan \lambda \cos \varepsilon$ or $\tan \alpha = (\sin \lambda \cos \varepsilon) / \cos \lambda$. (The second form is used with atan functions that return $-\pi \leq 0 \leq \pi$ and are called as atan2(y,x).)
  c) Draw the tic on the declination arc at angle α on the limb scale.

- **Calendar scale.** The calendar scale associates a date with a right ascension on the limb. Sutton made this scale by simply looking up the Sun's right ascension for each date and drawing the date tic (Sutton calculated his own table of solar declinations and right ascensions). This method is, of course, still valid using values from the Astronomical Almanac or other authoritative source, but very tedious.

  The Sun's daily right ascension can be calculated using the methods outlined in the chapter on astronomical calculations. In summary,

  Sun's true longitude: λ = **Sun's mean longitude + Sun's true anomaly - Sun's mean anomaly**

  Sun's right ascension, $\tan \alpha = \tan \lambda \cos \varepsilon$

  The right ascension must be calculated for $0° \leq \lambda \leq 360°$, which requires a polar arctangent. Then, it must be scaled to the correct quadrant to find the limb angle for drawing the tic. For example, if the value of $\alpha = 150°$, it will be drawn at $180° - \alpha = 30°$, which makes sense since 150° is 60° greater than 90° and is therefore 60° to the left of the end of the scale.

Quadrant Interior

The interior of the quadrant contains the altitude and azimuth arcs. The calculations of the center and radii of these arcs are very similar to those for a normal astrolabe, but with some additional considerations.

- **Altitude arcs.** Collins suggests making a "line of semi-tangents" scaled to the instrument for taking the radii of the arcs. This scale is $R_{eq} \tan (90-a)/2$ but labeled for a. The scale can be used with a set of dividers to get the radii, which are then transferred to the quadrant. The technique is valid, but very tedious and requires superior drafting skills.

The centers and radii are calculated from first priniciples.

There is one significant difference between the southern and northern projections. The northern projection always produces arcs for positive altitudes that curve in the same direction. On the southern projection, arcs for altitudes < latitude will produce arcs curving in the opposite direction of arcs for altitudes > latitude. The arc for altitude = latitude is a straight line.

Because of this factor, it is easiest to calculate the altitude arcs by determining the left and right intersection points on the meridian and using them to define the radius and center distance. All dimensions are on the meridian and distances from the center are x's.

a) Altitude (a) < latitude (φ): Arcs curve to left

Left intersection: $x_L = - R_{eq} \tan [(\varphi + a) / 2]$

Right intersection: $x_R = R_{eq} \cot [(\varphi - a) / 2]$

Center: $x_C = (x_L + x_R) / 2$

Radius: $r_a = x_C - x_L$

Note some of the arcs may be clipped by Capricorn, and the arcs will terminate at either Cancer or the right horizon. All of these arcs will be clipped by Capricorn for latitudes ≤ 33.25°.

b) Altitude = latitude

A straight line intersecting the meridian at: $x = R_{eq} \tan \varphi$

The line will be clipped by Capricorn if $\varphi < 33.25°$. It always terminates at Cancer.

c) Altitude > latitude.

Left intersection: $x_L = R_{eq} \tan [(\varphi + a) / 2]$

Right intersection: $x_R = R_{eq} \cot [(a - \varphi) / 2]$

Center: $x_C = (x_L + x_R) / 2$

Radius: $r_a = x_R - x_C$

These arcs will be clipped by Capricorn if $x_L < R_{cap}$. They will terminate at Cancer.

The altitude arcs are labeled at each end inside the quadrant.

These relations are derived using logic identical to that used to derive the planispheric astrolabe almucantars.

- **Azimuth arcs.** The azimuth arcs are identical to the astrolabe azimuth except the zenith and nadir are interchanged. The azimuth curves are calculated until it no longer fits inside the quadrant which occurs when the azimuth radius is greater than the distance from the azimuth center to the lower left corner of the quadrant.

The relations for calculating the azimuth arcs are:

Zenith: $x_Z = R_{eq} \tan[(90 + \varphi) / 2]$

Nadir: $x_N = -R_{eq} \tan[(90 - \varphi) / 2]$

Line of azimuth centers: $x_C = | x_Z + x_N | / 2$

Center line to zenith: $y_Z = x_Z - x_C$

Azimuth center: $x_A = -y_z \tan(A)$

Radius: $r_A = y_z / \cos(A)$

For A = 90°, $x_C = 0$ and radius = y_Z.

For A > 90°, calculate the arc for (A -90) and set the center below the meridian.

Azimuth arcs start at either Capricorn or the right horizon and end at Cancer.

The azimuth arcs are labeled in the margin just outside Cancer and just inside Capricorn.

Sutton's "Small Quadrant"

Collins also gives a general description of what he calls a "universal small pocket quadrant." This quadrant has only altitude arcs as it is intended to be used only to find the time from the Sun's altitude.

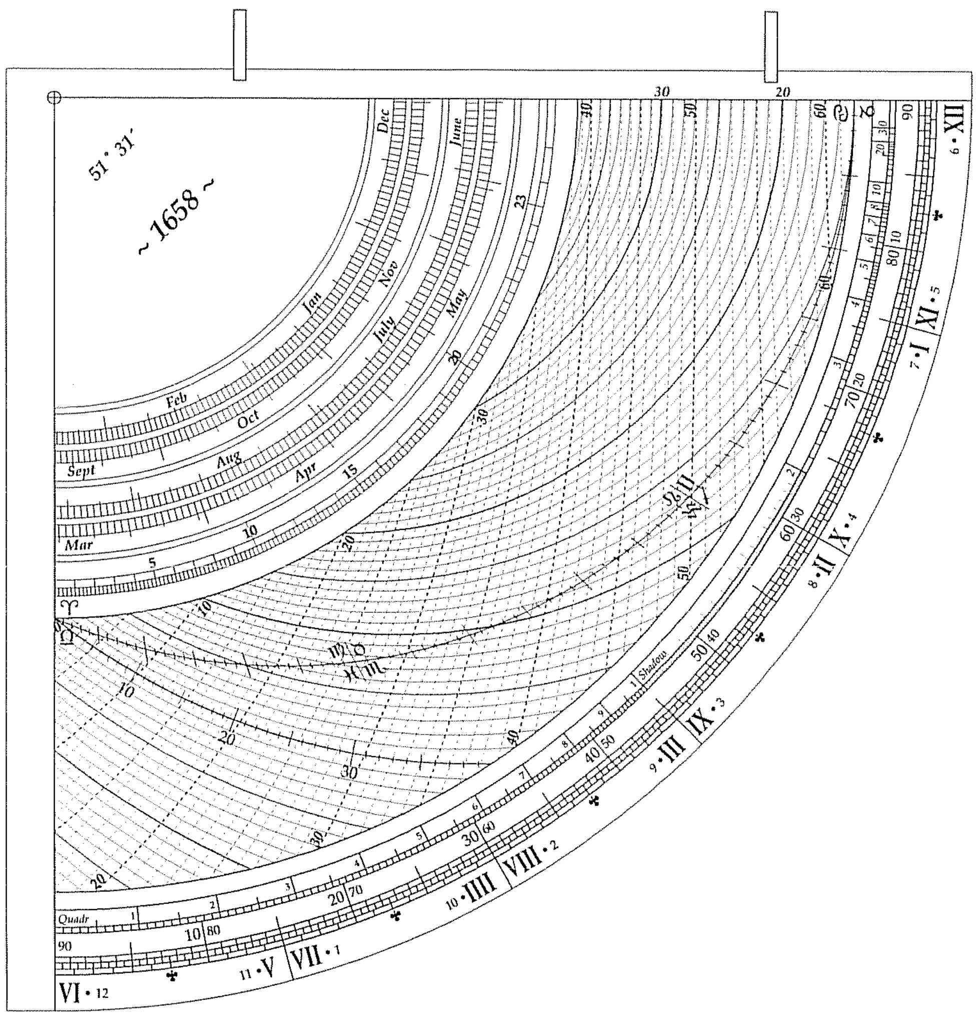


Figure 20-7. Sutton's "Small Quadrant"

Figure 20-7 shows a representation of this quadrant style. It is unique in that it is the only hybrid quadrant I have encountered[100]. The altitude arcs are from both the northern and southern stereographic projections.

The upper edge is the meridian and the vertical edge is the right horizon. Collins implies the edge of the quadrant is four inches.

The interior of the small quadrant is similar to Gunter's quadrant in that the outer arc represents either the Tropic of Cancer or the Tropic of Capricorn depending on which half of the year is under consideration. The ecliptic extends from the equator to the appropriate tropic and is

[100] But see Charette [2003] p. 86 for a discussion of the *musattar* quadrant.

labeled with the signs for the entire year. The horizon is divided by the azimuth of sunrise/sunset as on Gunter's quadrant and Sutton's quadrant.

There are two sets of altitude arcs. One set is used from the vernal equinox to the autumnal equinox, and the other set is used for the other half of the year. The outer arc represents the Tropic of Capricorn in the fall and winter, and represents the Tropic of Cancer in the spring and summer. The figure uses solid and dashed lines in an effort to make it easier to find the set of arcs to use.

The altitude arcs used in the fall and winter are the arcs from a normal astrolabe between the equator and the Tropic of Capricorn. These arcs originate at either the meridian or the outer arc (the Tropic of Capricorn in this case) and curve sharply to the left toward the equator. They are solid lines in the figure.

The spring and summer altitude arcs are from a southern projection astrolabe between the equator and the Tropic of Cancer. They start at the meridian or the equator and curve gradually to the left or right depending on the altitude, and are dashed in the figure.

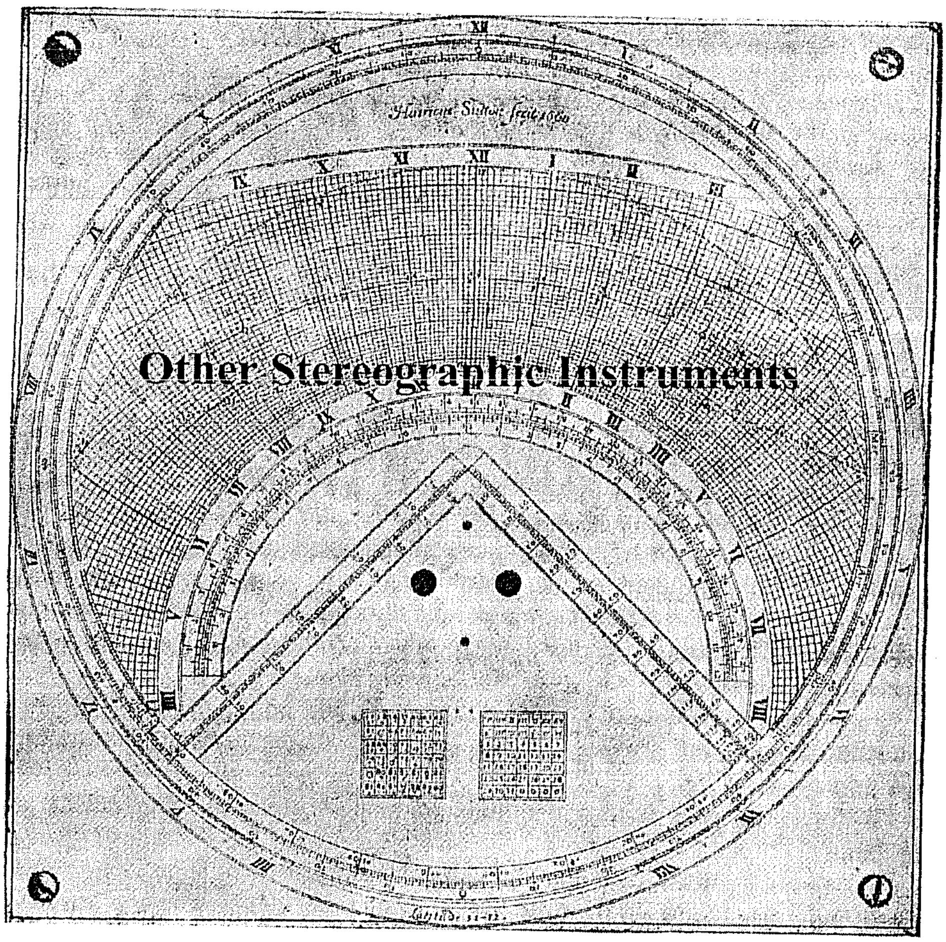
The original design provides scales that can be used to find the azimuth after some calculations. The calculations are very complicated, and it is highly unlikely anyone other than the makers ever mastered them.

The use of this quadrant to find the time is very simple once one gets used to which set of altitude arcs to use for a given date. Simply set the thread on the date and place the bead on the ecliptic. Then rotate the thread until the bead is on the correct altitude arc and read the time on the limb.

The example shown does not include all of the arcs included by Sutton as the scales along the edges are intended to be used to calculate the Sun's azimuth and are rather obscure. Collins gives a fragmented but complete description of the scales and their uses.

Making the small quadrant requires no additional description beyond what has already been covered.

Other Stereographic Instruments



Chapter 21 - Horizontal Instruments

Introduction

Several instrument variations employ the stereographic projection with the horizon as the projection plane. Such instruments are called "horizontal" to denote the horizon as the projection plane in the same manner that instruments with the equator as the projection plane are sometimes referred to as "equatorial".

Such instruments can be used to solve many of the problems related to time and the position of the Sun common on the plane astrolabe. It lacks, however, the ability to solve problems related to stars and is not nearly as versatile as the astrolabe. It is, however, rather pretty.

The horizontal stereographic instrument is derived from the projection of the celestial sphere on the local horizon with the nadir of the place as the projection origin (Figure 21-1). As such, it is very similar to an astrolabe plate with arcs of declination replacing the almucantars, and arcs of hour angle replacing azimuths. Only the portion of the sphere between the tropics is shown, and, like a sundial, hour arcs are included only for the length of daylight at the chosen latitude since it deals only with the Sun.

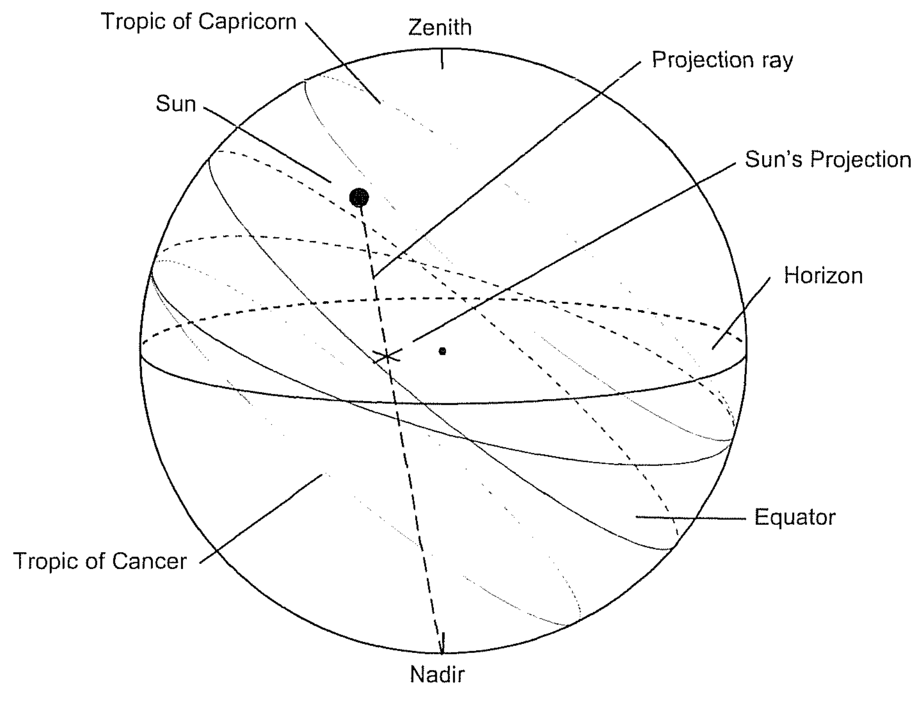


Figure 21-1. Horizontal Stereographic Projection

Several types of instruments using the stereographic projection with the horizon as the projection plane have been developed, all with the objective of finding the time from the Sun's altitude. Two implementations will be discussed: a portable horizontal instrument described by Georg Hartmann in 1527 and a horizontal projection integrated into a sundial by William Oughtred a hundred years later. Similar instruments, called the *musātira*, were described in Arabic treatises from the 10th and 13th centuries. For a more complete summary of the history of this instrument see Charette [2003] and Turner[101].

Description

The arcs and labels on the horizontal instrument are shown in Figure 21-2, which is drawn for latitude of 38° 42′, the author's home location. The instrument in the figure is somewhat generic and does not represent a specific implementation. Numerous variations are possible but the principles are illustrated in the example. Like other stereographic instruments, a different horizontal instrument is required for each latitude.

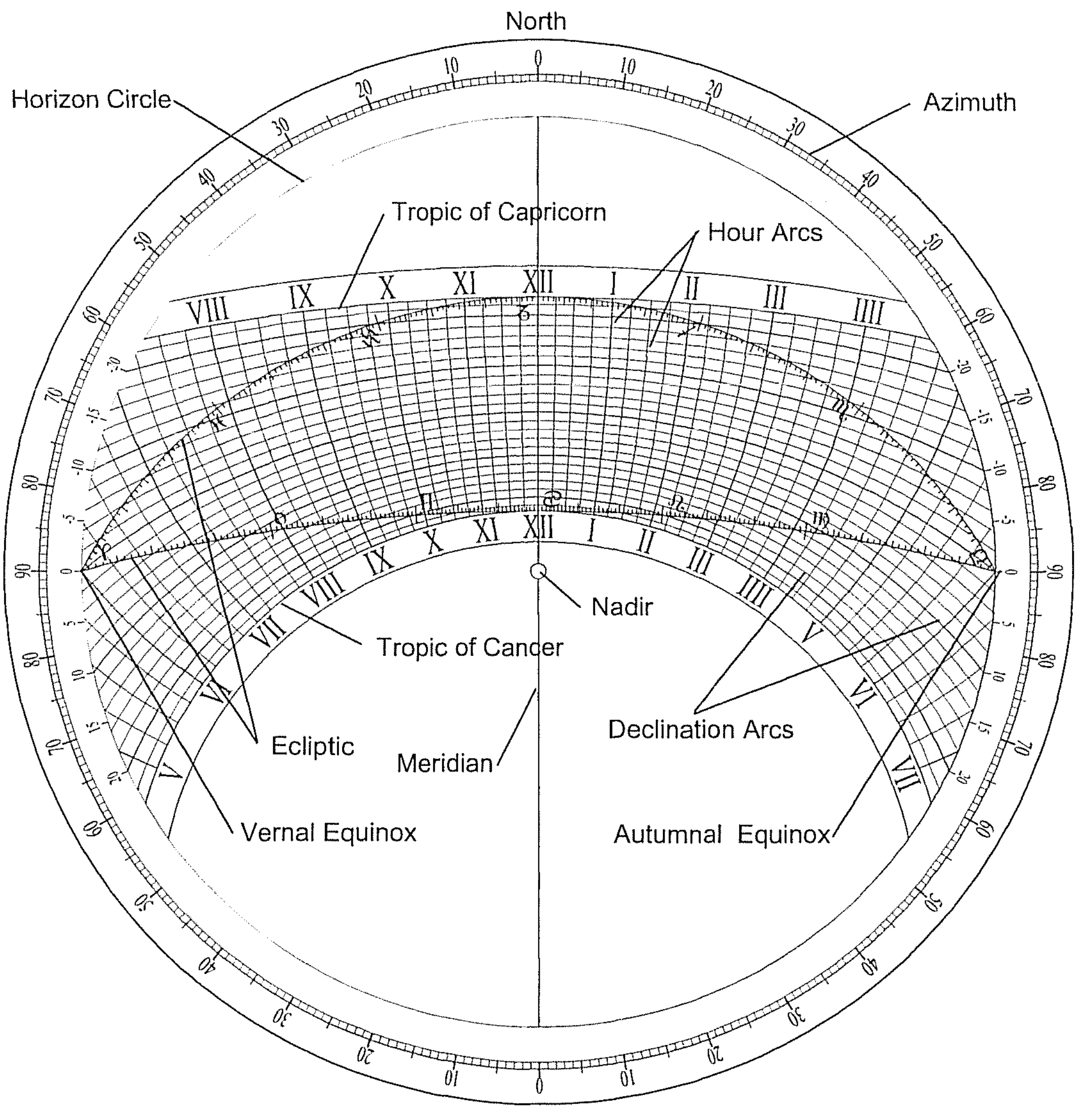


Figure 21-2. Horizontal instrument (38° 42′)

[101] Turner, A.J., *The Time Museum; Astrolabes and Astrolabe Related Instruments*, The Time Museum, 1985.

The use of the horizon instead of the equator should not be confusing when compared to an astrolabe since identical principles are used for both instruments. On the astrolabe, the declination circles are concentric around the center of the instrument, and hour angles are straight lines originating in the center. On the horizontal instrument, altitude circles are concentric around the instrument center, and straight lines represent azimuth angles from the center. Arcs of declination are equivalent to altitude arcs on the astrolabe, and hour angles are equivalent to the astrolabe's azimuth arcs. In short, the representation of declinations and altitudes are exchanged, as are hour angles and azimuth angles.

The instrument is intended to be laid flat with the top to the *north*, reverse of the astrolabe.

The limb is divided by degrees and represents angles of azimuth. The azimuth angles in the example are labeled as increasing from the meridian, which is not the modern astronomical convention (but is common in celestial navigation).

The circle inside the limb represents the horizon with west to the left, north at the top and east to the right. The vertical line through the center of the instrument indicates the meridian. The center of the plate is the zenith.

The arc near the top of the instrument is the Tropic of Capricorn. The arc close to the center is the Tropic of Cancer.

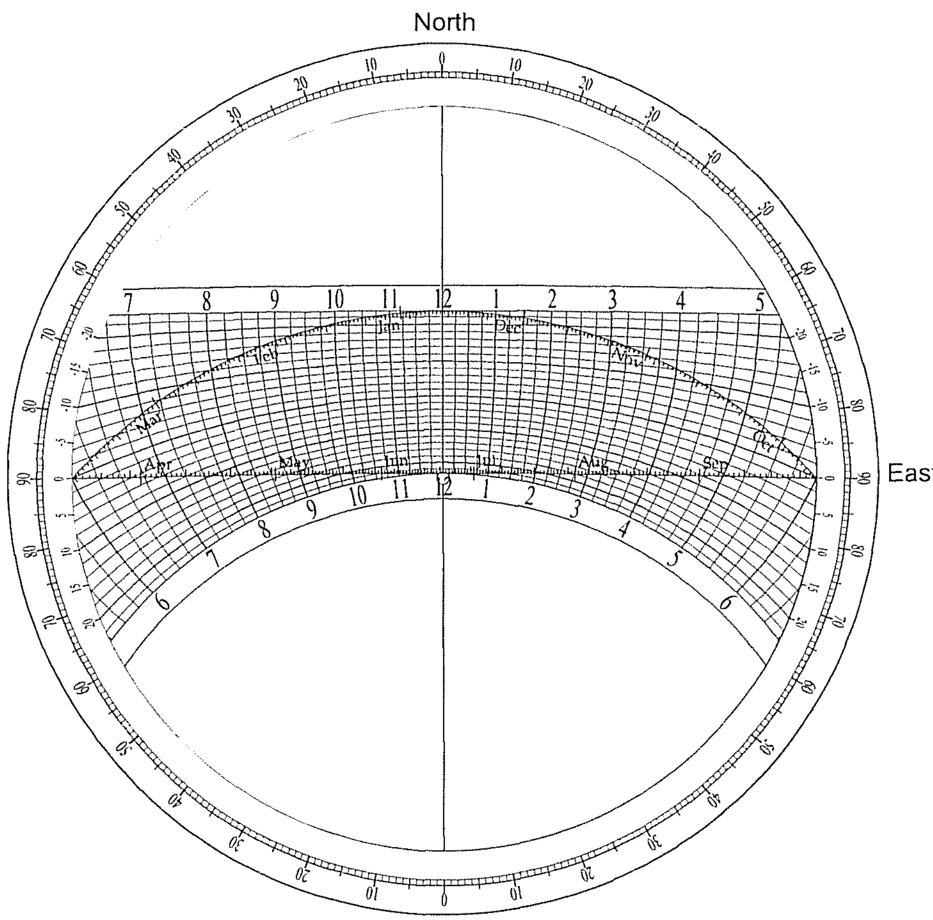


Figure 21-3. Horizontal instrument with the ecliptic divided by the calendar (25°)

The radial arcs represent the hour angle and are labeled as hours of time. The transverse arcs represent declination and are labeled outside the horizon in the example.

The ecliptic is drawn as two arcs. The lower arc, starting at the equator in the west, represents the vernal equinox to the autumnal equinox through the spring and summer. The upper arc, starting at the equator in the west, represents the autumnal equinox to the vernal equinox through the autumn and winter. The ecliptic is divided by the zodiac in Figure 21-2. The labeling of the ecliptic divisions is arbitrary and can originate at either side. The labeling in the figure has the solar longitude increasing to the east, which conforms to the definition of celestial longitude, but could just as easily be reversed without affecting the instrument's uses in any way.

It is easy to visualize the representation of the Sun on this instrument. On a given day, the Sun rises at the point where the appropriate declination curve meets the horizon circle in the east. The Sun then follows the declination curve over the course of the day and sets in the west.

A number of variations in labeling the horizontal instrument are possible.

Oughtred suggests dividing the ecliptic by the calendar as in Figure 21-3. The instrument in Figure 21-3 uses Arabic numerals to illustrate this style element, and is for 25° latitude to illustrate the effect of latitude on the scales.

Hartmann's "Compast"

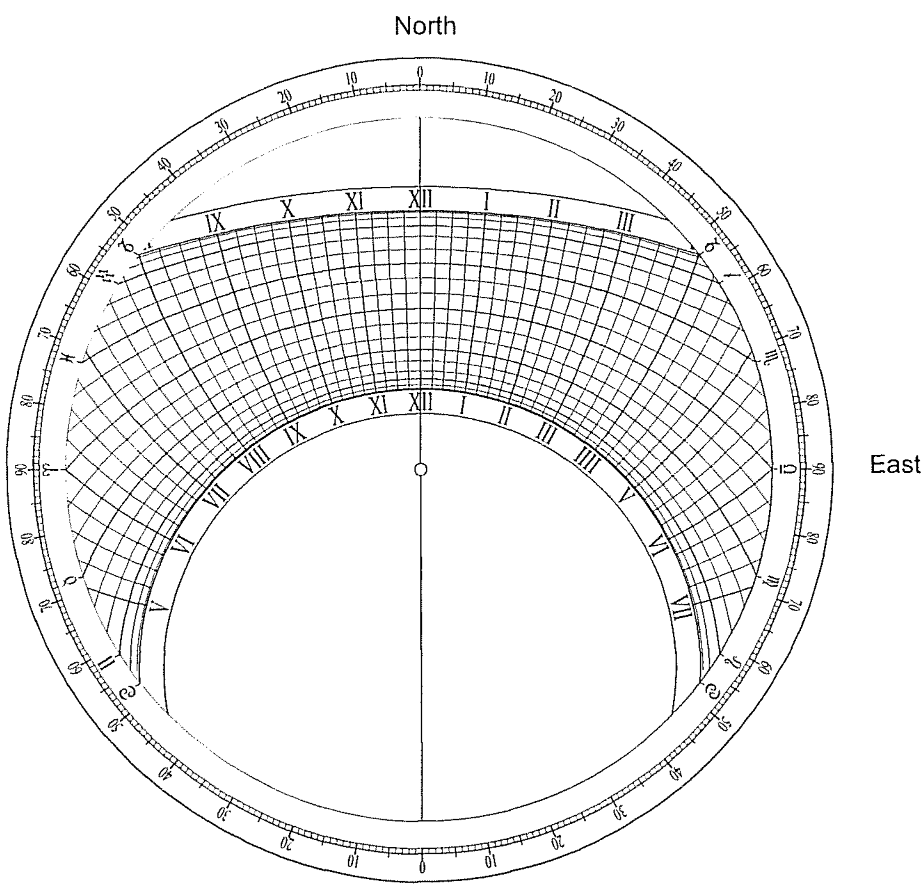


Figure 21-4. Georg Hartmann's 'Compast' (49°)

Georg Hartmann (1489-1564) of Nuremberg described a horizontal instrument in 1527, which he called the *Compast*[102]. Hartmann's implementation was intended to be portable and included a magnetic compass.

Hartmann's "Compast" (Figure 21-4) includes the declination curves corresponding to the zodiac and does not include the ecliptic. Hartmann's instrument also included "Nuremberg Hours" (not shown in the figure), which are equal hours counted 1, 2, ... beginning at sunrise. Nighttime hours are also numbered from 1, beginning at sunset[103]. Hartmann's instrument included a magnetic compass, with the appropriate magnetic deviation considered, for orienting the instrument to the meridian. No provision was made for ensuring it is horizontal. The instrument in Figure 21-4 is for Nuremburg at 49° latitude.

Time Measurement with the Horizontal Instrument

The measurement of the current time using the horizontal instrument is a bit problematical. The length, s, of the shadow from a vertical gnomon of height h for a given altitude a of the Sun is:

$$s = h \cot a = h \tan(90 - a)$$

The distance, d, of a stereographically projected point from the center of the instrument is:

$$d = r \tan [(90\text{-}a)/2]$$

where r is the radius of the horizon circle.

Clearly, the distances are not the same and this is why the horizontal stereographic projection is not normally used as a sundial. However, one desired use of such an instrument is to find the time from the Sun's altitude. There are several methods that can be used to make a horizontal instrument usable in this way. One option is to fit the back of the instrument with an alidade and scale for measuring the Sun's current altitude. The front of the instrument is fitted with a scale of stereographically projected altitudes. The scale may be drawn on a rule connected to the plate and free to rotate around the center of the instrument plate or engraved on the front and measured with a pair of dividers. Note is made of the measured altitude and the corresponding point located on the rotating rule or engraved scale. The rule is rotated until the altitude is on the day's declination arc and the time read from the hour arc.

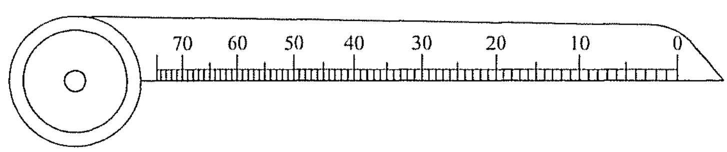


Figure 21-5. Stereographic altitude rule for 38° 42′

The same rule could also be used to solve any problem relating time and the Sun's altitude.

It would also be possible to equip a portable instrument with a vertical gnomon and a calibrated scale of solar altitudes for determining the Sun's current altitude. In this case, the instrument is suspended vertically since it would be difficult to confirm it is exactly horizontal. The time is

[102] Lamprey, John, Hartmann's Practika. A Manual for Making Sundials and Astrolabes with the Compass and Rule, John Lamprey, Bellvue, CO, 2002.
[103] There are always 24 Nuremburg hours in a day but they are divided into two sets of 12 hours only on the equinoxes. There will be fewer than 12 daylight hours in the autumn and winter and more than 12 in the summer.

found by aligning the instrument plate with the Sun, rotating the rule to the tip of the gnomon's shadow to find the altitude and then moving the rule to the day's declination arc as above.

Note a horizontal instrument can be used as a sundial if it is permanently mounted and carefully aligned to the meridian. In this case, the time is found from the point where the gnomon's shadow crosses the day's declination arc.

Oughtred's Double Dial

The most widely known horizontal instrument is a form of double sundial described by English mathematician William Oughtred (1574-1660). Oughtred described this type of instrument to Elias Allen, a London instrument maker in about 1627. Allen and his successors made a number of examples that have survived. Although the instruments made by Allen were in the form of a sundial, this was not required and the instrument can be made in several portable forms or engraved on an astrolabe. There is an interesting discussion in the literature[104] about Oughtred's dispute with Richard Delamain about who invented this type of instrument. In fact, neither did but that does not make the argument any less entertaining. It is, however, likely Oughtred was the first to combine the horizontal instrument with a sundial.

It must be stressed that the horizontal stereographic instrument is not a sundial in the normal sense. On most sundials the shadow from the gnomon or style falls across a number or scale to indicate the current time. The shadow from a vertical gnomon erected from the center of the horizontal instrument crosses many lines and the time cannot be read at a glance, although the time can be determined with some additional effort if the instrument is horizontal and aligned with the meridian.

Oughtred's double dial combines a standard horizontal sundial with the horizontal projection that makes it easy to read the time with the added capabilities of the horizontal stereographic instrument. This dial has the virtue of being self-aligning. When the dial is oriented so the sundial time matches the time from the horizontal projection scales, the meridian line is in the correct position. Once in place, the double dial allows the current time to be read from the sundial and common problems to be solved using the stereographically projected scales.

Elias Allen and his successors made a number of dials in this style characterized by a very high level of craftsmanship. Figure 21-6 is a double horizontal dial in the spirit of Allen's for a latitude of 51° 30′. Elias Allen also produced a quadrant based on this projection.

The double dial in the figure differs from Allen's in a number of details. See Michel for an example. For example, Allen's dial had the calendar scale along the side of the horizontal scales where the dial in the figure has the value of the Sun's declination. The azimuth degree scale was very small on Allen's instruments; he probably considered azimuths to be rather technical for the common user. Allen's dial had a time scale in minutes around the outer circle, and the altitude scale was graduated from 0 - 90°. The altitude scale in the figure is divided between 0° and the maximum solar altitude for the latitude. Allen's products also had additional ornamentation.

The tic toward the bottom of the meridian is the location of the tip of the gnomon.

[104] Turner, A.J., "William Oughtred, Richard Delamain and the Horizontal Instrument in Seventeenth Century England, *Annali dell'Istituto e Museo di Storia della Scienza di Firenze*, vi, 1981, 99-125.

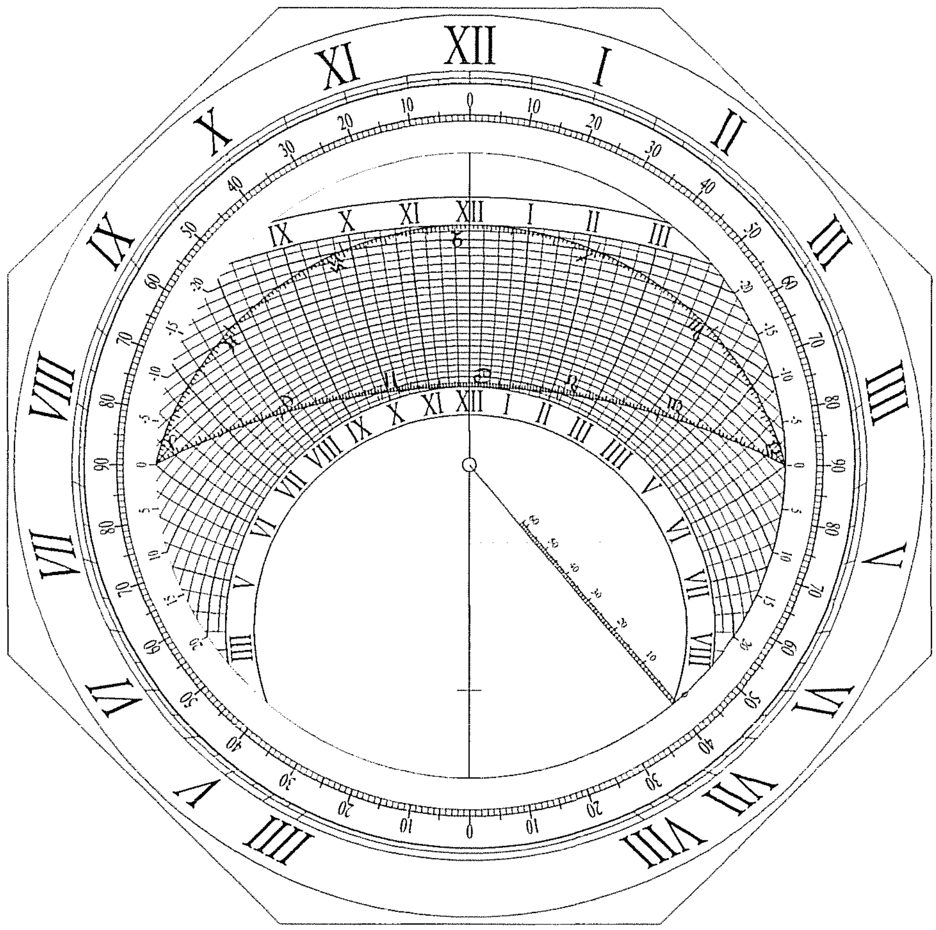


Figure 21-6. Double Horizontal Dial (51° 30′)

The gnomon[105] for the double dial is in the form of Figure 21-7. The angled style is the classic form for a horizontal sundial. The vertical section, which is aligned with the center of the plate, is used with the stereographic scales. Note the edges of both the sundial style and the vertical section must be ground to a sharp edge. It is not possible to use a thick gnomon on the double dial.

The shadow from the gnomon style indicates the time on the sundial time scale. The shadow of the vertical section crosses the stereographic scale at the angle of the Sun's azimuth and crosses the declination and hour lines. The hour line under the shadow of the vertical section passing through the declination arc for the current day is the current time. Finding the time on the horizontal projection part of the double dial is illustrated below.

[105] In keeping with standard sundial nomenclature, the device that casts a shadow on a sundial is called the gnomon. The edge that creates the shadow edge is called the *style*.

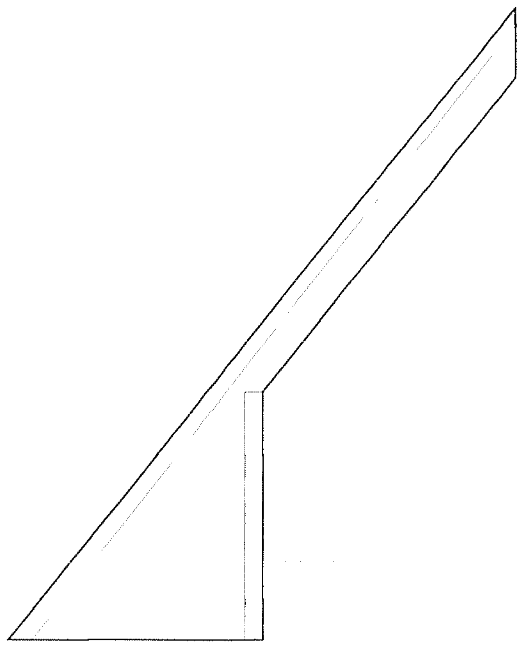


Figure 21-7. Double sundial gnomon for 51° 30′

Using the Horizontal Instrument

Several types of problems can be approached using the horizontal instrument. The first three problems can be solved by simply referring to the scales. Problem 4 concerns the current time. The remaining problems concern the double sundial.

Most uses of the horizontal instrument require only looking at it and, possibly, noting some positions with a finger. The instrument is assumed to have a rotating altitude rule.

1. **Find the Sun's declination for a day.** Locate the declination arc for the day from the ecliptic divisions. The Sun's declination can be read directly if the declination arcs are labeled. Otherwise, count the divisions from the Tropic of Cancer or Tropic of Capricorn.

   For example, referring to Figure 21-3, the declination for August 8 is about 16°.

2. **Find the time of sunrise and sunset.** Locate the declination arc for the day from the ecliptic divisions. Follow the day's declination arc to the end of the arc. Read the time of sunrise or sunset from the nearest hour arc.

   Continuing the example above, by following the 16° declination arc to the eastern horizon, the time of sunrise is found to be a few minutes before 5:30 AM.

3. **Find the azimuth of sunrise and sunset.** Locate the point of sunrise or sunset as in Problem 2. Rotate the rule to this point and read the azimuth from the outer degree scale.

   Continuing the same example, the azimuth of sunrise is about 72°.

4. **Find the time from the Sun's altitude.** Measure the Sun's altitude by some means. Locate this altitude on the rotating rule. Rotate the rule until the measured altitude is on the day's declination arc. Read the time from the hour arcs.

   On August 8, measure the Sun's altitude in the afternoon as about 48°. Locate 48° on the rotating rule and rotate the rule until 48° is on the 16° declination arc. Read the time as about 3PM.

5. **Align a double dial to the meridian.** This procedure is best done in the mid-morning or mid-afternoon when the Sun's altitude is about halfway between the horizon and the maximum altitude for the day. Mount the double dial perfectly horizontally. Rotate the dial until the meridian is approximately north and south. Locate the declination arc for the day. Read the time from the sundial and note the location where the shadow from the vertical section of the gnomon crosses the declination. Adjust the dial's position until the sundial time matches the hour curve that crosses the day's declination curve. A number of checks will be needed before you are confident the dial is perfectly oriented at which time it can be permanently mounted.

Using the Double Dial

Figure 21-8 is an example of the double dial showing the time. This example incorporates the ecliptic divided by the calendar and Arabic numerals for the hours on the stereographic section with the idea that this configuration is a bit easier for the modern user. The dial is for 51° 30'.

The date is March 7. The shadow from the sundial style shows it is 10:30 AM local apparent time.

The shadow from the vertical gnomon shows the Sun's azimuth is a bit over 26° (26.18°). The declination on March 7, is -5° 21'. Following this declination circle to the vertical gnomon shadow, it intersects the shadow at the 10:30 AM hour arc.

Therefore, the dial is correctly oriented to the meridian. If you measure the distance from the center to the intersection of the declination and vertical gnomon shadow and then measure the same distance on the altitude scale, you find the Sun's current altitude is 30°

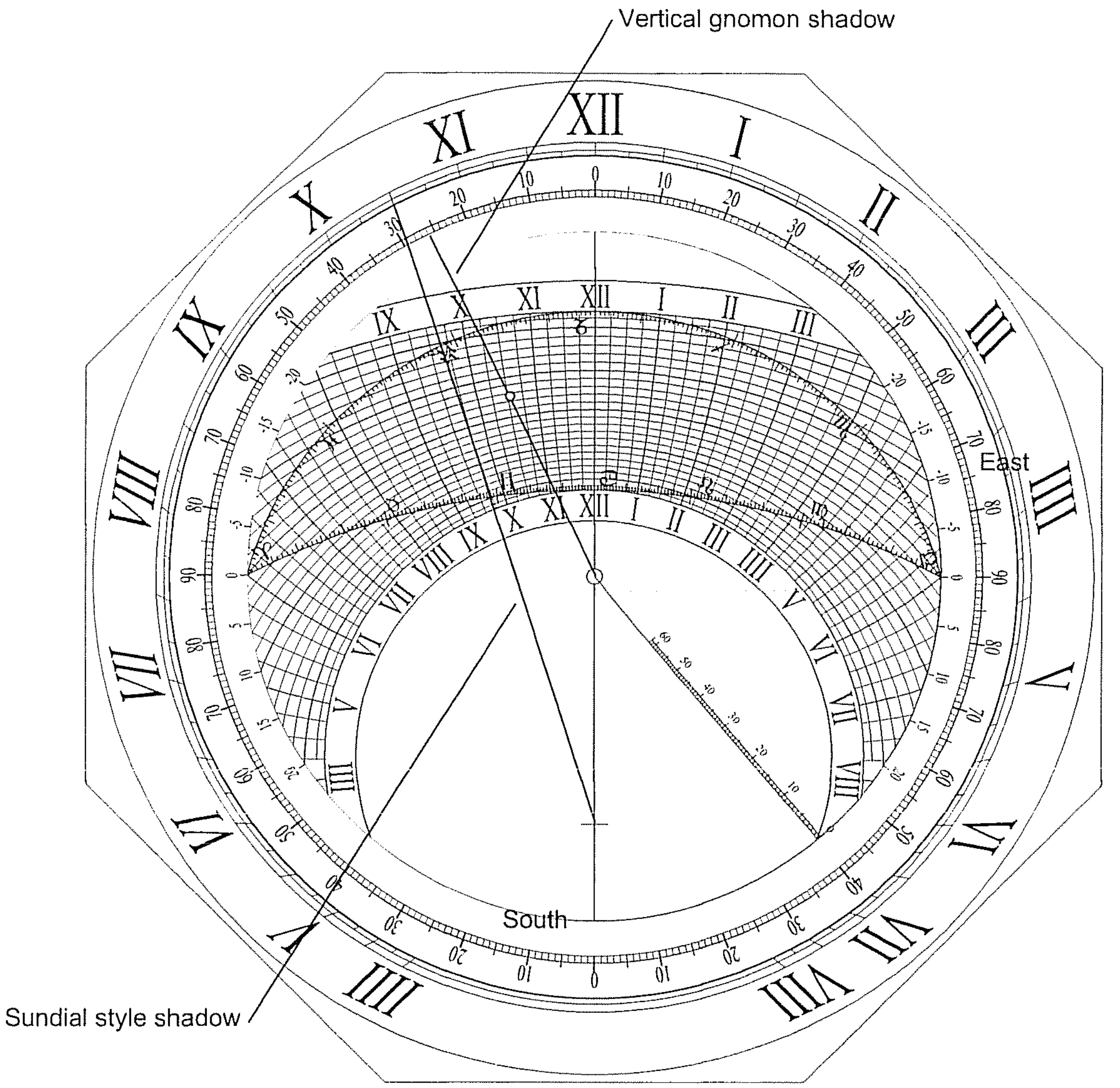


Figure 21-8. Double dial example (51° 30′)

Making the Horizontal Instrument

A different instrument is required for each latitude. Before beginning the layout and design it is necessary to decide which elements will be included. There are many options, whether to include the ecliptic, how the ecliptic will be divided if at all, which declination arcs to include and what they represent, and the character style for the hours. Roman numerals are traditional for classic style instruments, but Arabic numerals might be more appropriate for a modern interpretation.

1. **Basic design parameters.** You must decide on which elements you will include before beginning any layout. The overall size of the double dials made by Elias Allen and his successors was about 14 inches. Among the decisions required are whether or not to include a sundial, what scales will be included and their sizes, labels, etc. A horizontal instrument without a sundial can be any shape, but a round one seems traditional. A

double dial needs a place outside the dial proper to drill mounting holes. Old instruments were both square and octagonal, but the octagonal shape is more elegant. You will want to calculate the obliquity of the ecliptic for the year of the instrument.

2. **The concentric circles.** Locate the instrument center and draw the concentric circles for the azimuth scale and horizon. Some care should be taken to select the radius of the horizon to be an even value and measured accurately, because all of the arc calculations use this value. Let the radius of the horizon be **R**.

3. **The declination arcs.** The declination arcs use the same equations as the altitude arcs on the astrolabe. The number of declination curves to include depends on the size of the instrument. Allen's double dials had an arc for every degree of declination, but his dials were quite large. The examples have a declination arc for every two degrees. The arcs for the tropics should be drawn as carefully as possible.

   Let φ = latitude and δ = declination. The radius of the declination arc is:

   $$r = R \cos \delta / (\sin \varphi + \sin \delta)$$

   The location of the declination circle center below the center of the instrument is:

   $$y = R \cos \varphi / (\sin \varphi + \sin \delta)$$

   Draw the declination circle within the horizon.

4. **The hour arcs.** The hour arcs use the same principles as the azimuth arcs on an astrolabe and are constructed using identical logic. Each hour arc passes through the north and south pole. The centers of the hour arcs are on a line halfway between the poles. The angle of the hour arc at the pole is equal to the hour angle. You can include hour arcs for each 15 minutes or for each 10 minutes depending on the size of the instrument. Any greater resolution gets too congested. The examples have hour arcs for each 15 minutes.

   The hours to include can be calculated from the hour angle of sunrise at the tropics. The maximum hour on the Tropic of Cancer will be the integer part of:

   $$H_{Can} = \arccos(\tan \varphi \tan \varepsilon) / 15$$

   The maximum hour to include is the hour angle of sunset at the summer solstice converted to time.

   Similarly, the maximum hour on the Tropic of Capricorn will be:

   $$H_{Cap} = \arccos(\tan \varphi \tan -\varepsilon) / 15$$

   First, calculate the location of the line of centers halfway between the projection of the poles. The projection of the north pole is at:

   $$N = R \tan [(90 - \varphi) / 2]$$

   And the south pole is projected to:

   $$S = -R \tan [(90 + \varphi) / 2]$$

   The line of centers is halfway between at:

   $$C = (N + S) / 2$$

   And the distance from the south pole to the center line is:

   $$y_{ha} = S - C$$

The centers of each hour arc will lie on the line of centers.

Let H be the hour angle (i.e., the time in hours × 15°). The distance of the center of an hour arc from the meridian on the line of centers is:

$$x = y_{ha} \tan (90 - H)$$

(90 – H) is used because the angle is measured from the meridian.

The radius of the hour arc is:

$$r_{ha} = yha / \cos (90 - H)$$

Note the hour arc for hour angle of 6 hours requires special handling to avoid dividing by 0. For the 6 o'clock line, x = 0 and rha = yha.

The hour arcs need be calculated for only one side of the instrument. The other side can be drawn by symmetry.

The hour arcs can be labeled with either Roman numerals or Arabic numerals depending on the style of the instrument.

Sawyer[106] derives different equations for the declination and hour arcs. The form used here is consistent with astrolabe usage.

5. **The ecliptic.** If your instrument includes the ecliptic, and most will, you will find drawing and dividing the ecliptic is somewhat easier on the horizontal instrument than on an astrolabe.

   There are two ways to find the center and radius of the ecliptic arcs.

   Each ecliptic arc passes through the point where the 0° declination arc intersects the horizon and the point where the appropriate tropic intersects the meridian. The ecliptic arcs can be drawn by constructing arcs of circles passing through these three points.

   Alternatively, the radius and center of the ecliptic arcs can be calculated directly:

$$y_{ec} = - R / \tan (\varphi + \varepsilon)$$

$$r_{ec} = R / \sin (\varphi + \varepsilon)$$

   where ε has the appropriate sign for the tropic.

   Dividing the ecliptic is also quite straightforward. Each ecliptic division is defined by the declination of the point to be divided. The declination of the zodiac signs can be looked up in an almanac or calculated directly from the solar longitude:

$$\sin \delta = \sin \varepsilon \sin \lambda$$

   The ecliptic division point is where the ecliptic curve meets the calculated declination curve. The zodiac division of the ecliptic is symmetrical. The calendar divisions must be calculated for each day of the year.

   Calculating the declination for a given day of the year is discussed in the appendix on astronomical calculation.

[106] Sawyer, Fred, "William Oughtred's Double Horizontal Dial", *The Compendium of the North American Sundial Society*, 4, 1, March, 1997, 1-12.

6. **The double sundial.** If you wish to make a double sundial, the calculations are simply the standard horizontal sundial equations. You first must decide on the point where the tip of the gnomon will lie. This is mainly an artistic decision. The dial in Figure 21-6, places the tip of the gnomon at half the radius of the outer dial circle. Other locations are possible.

   For each hour angle included, the angle of the hour line from the meridian is:

   $$\tan D = \tan H \sin \varphi$$

   You need only calculate the hour lines for one side of the instrument and then draw the other side by symmetry.

   The sundial hour lines are drawn to point toward the tip of the gnomon. Allen's dials used 'sheared' numerals angled toward the gnomon tip. This is not strictly required but it does present an elegant look.

   The angle of the style on the gnomon is equal to the latitude. The style and the vertical gnomon must be beveled to a straight edge, which is very hard to do accurately.

The Horizontal Projection Quadrant

It is, of course, possible to implement a quadrant using the horizontal projection. In fact, the result works very nicely and is comparable in ease-of-use to the Gunter or Sutton quadrants. This quadrant style is actually a more practical implementation than the Compast since it includes the ability to measure the Sun's altitude directly.

Figure 21-9 shows a horizontal projection quadrant similar to the style suggested by Phillip Apian in *de utilitate trientis* in 1586[107]. The quadrant interior contains the horizontal projection required for telling time from the Sun and includes the hour arcs needed for a latitude. The limb scale is the standard altitude measurement scale. The scale on the top of the quadrant is the stereographic projection of the altitude and includes only possible solar altitudes for the latitude in this sample. The quadrant is equipped with a thread and bead connected at the hole in the upper left corner of the projection.

In use, the Sun's altitude is measured in the normal way using the thread. The thread is rotated to the stereographic altitude scale and the bead is set to the measured altitude. The thread is then rotated to the point where the bead is on the declination arc for the day and the time is read from the hour arcs. For example, referring to Figure 21-9, on April 6 (Aries 17°), the Sun's altitude in the afternoon is 38°(1). Set the bead on the thread to 38° (2), rotate the thread and bead to the Aries 17° declination arc and read the apparent solar time as 2:15 PM (3).

There is considerable opportunity for enhancement to the basic form. Apain showed options that included unequal hours and Babylonin and Italian hours.

Figure Figure 21-10 shows a horizontal projection quadrant in a more modern style using solar declination in degrees, graduated for each degree of declination and each five minutes of time. The Sun's declination for a day would be found from a scale on the back of the quadrant.

This quadrant style is drawn using the same methods used for the double dial. The only new consideration is the angle of the top of the quadrant which is found from the intersection point of the Tropic of Cancer and the circle defining the outer limit of the projection.

[107] Janin [1979]

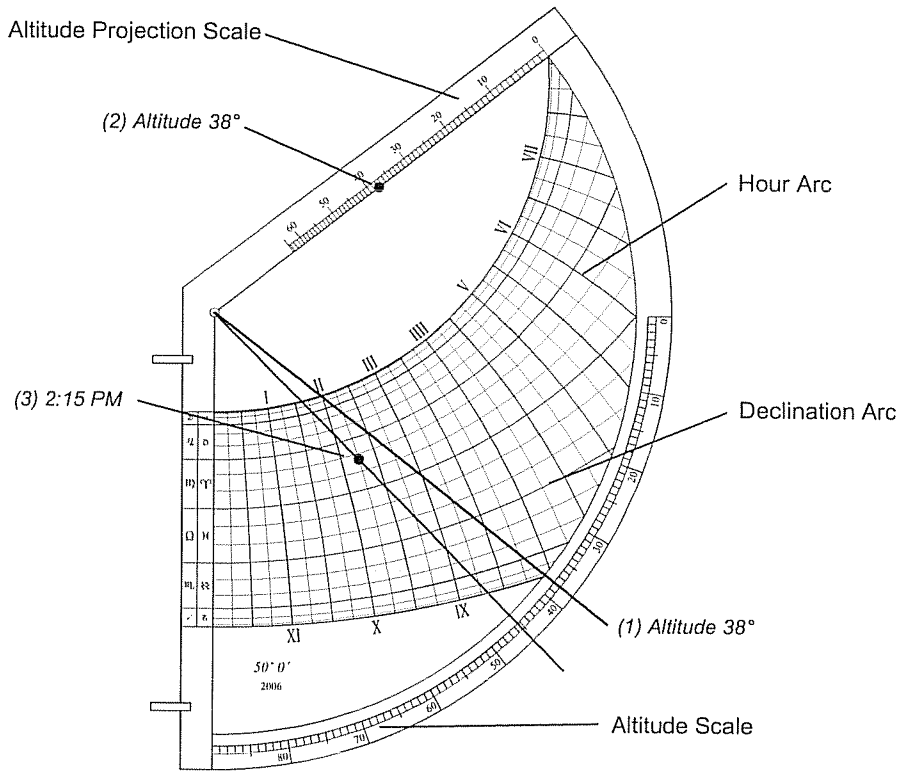


Figure 21-9. Horizontal Projection Quadrant

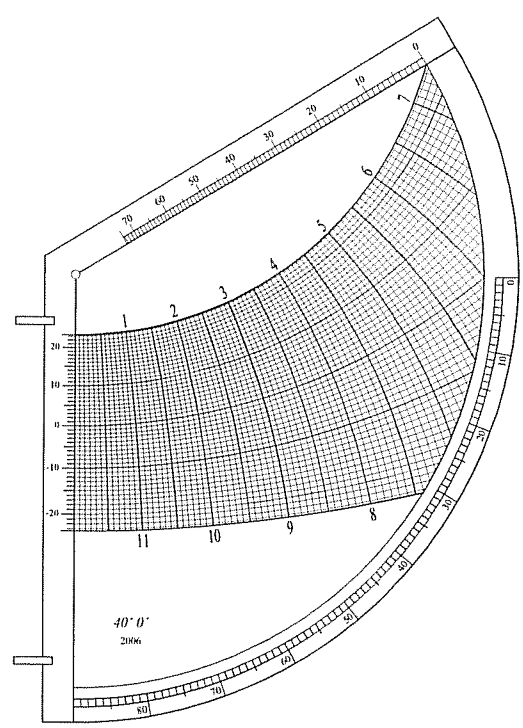


Figure 21-10. Modernized Horizontal Quadrant

Chapter 22 - Astrolabe Variations

Several variations on the astrolabe were suggested over the centuries in which the astrolabe was admired and used by highly educated and respected scholars. For a variety of reasons, none of these variations ever gained much popularity but they are interesting nonetheless and illustrate innovation was not as stagnant as some historical overviews would have you believe.

These astrolabes that are substantially different from the basic planispheric astrolabe although they work on identical principles.

van Maelcote's Astrolabe

In about 1600, the Belgian Jesuit Odo van Maelcote[108] (Malcotius) (1572-1614) suggested a form of the planispheric astrolabe offering the potential for greater accuracy for a given instrument size and accommodating both a north and south latitude. Similar Islamic astrolabe variations date from the the 10[th] century at the latest.

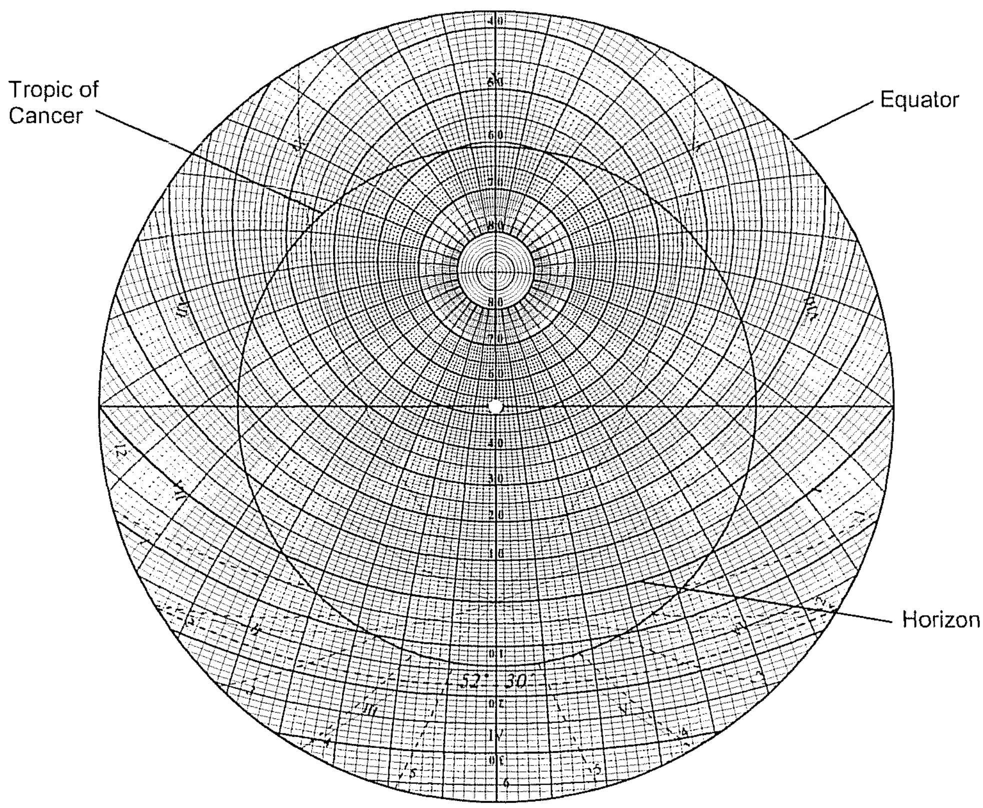


Figure 22-1. Plate for van Maelcote's astrolabe

Apparently, only one instrument of this type was ever made, by Lambert Damery[109], and it was described in only two treatises. van Maelcote was apparently not the first to describe such an

[108] Van Maelcote studied under Christorpher Calvius, who was instrumental in the Gregrorian caldendar reform, at the Jesuit Collegio Romano.
[109] Turner [1985], n. 233, p. 83.

instrument, which was included in a manuscript from about 1400 by Henri Arnault, a student of Jean Fusoris. It is unlikely van Maelcote knew of the earlier work and his name is associated with this type of instrument since it is the only version ever published with any meaningful distribution. It is also called the *Aequinoctial Astrolabe.*

Van Maelcote's astrolabe incorporates two ideas that, in combination, accomplish his objectives. First, the circle defining the circumference of the plate represents the equator. For this reason, Damray calls this instrument, *Astrolabium Aequinoctiale Odonis Malcotij.* The smaller interior circle represents either the Tropic of Cancer, for positive declinations, or the Tropic of Capricorn, for negative declinations. The plate in Figure 22-1, which is for 52° 30′ north latitude, is in the position used for positive declinations so the tropic represents the Tropic of Cancer.

The unequal hour and Houses of Heaven curves are exactly as drawn on a normal astrolabe plate, except they are limited to the area inside the equator.

The plate in the figure is divided with a resolution of 1° for the almucantars and 2° for the azimuths. Since the perimeter of the plate represents the equator, there is more room for scales than on a normal astrolabe of the same dimensions. There is ample room on a full sized plate for 1° resolution, hence the potential for higher accuracy.

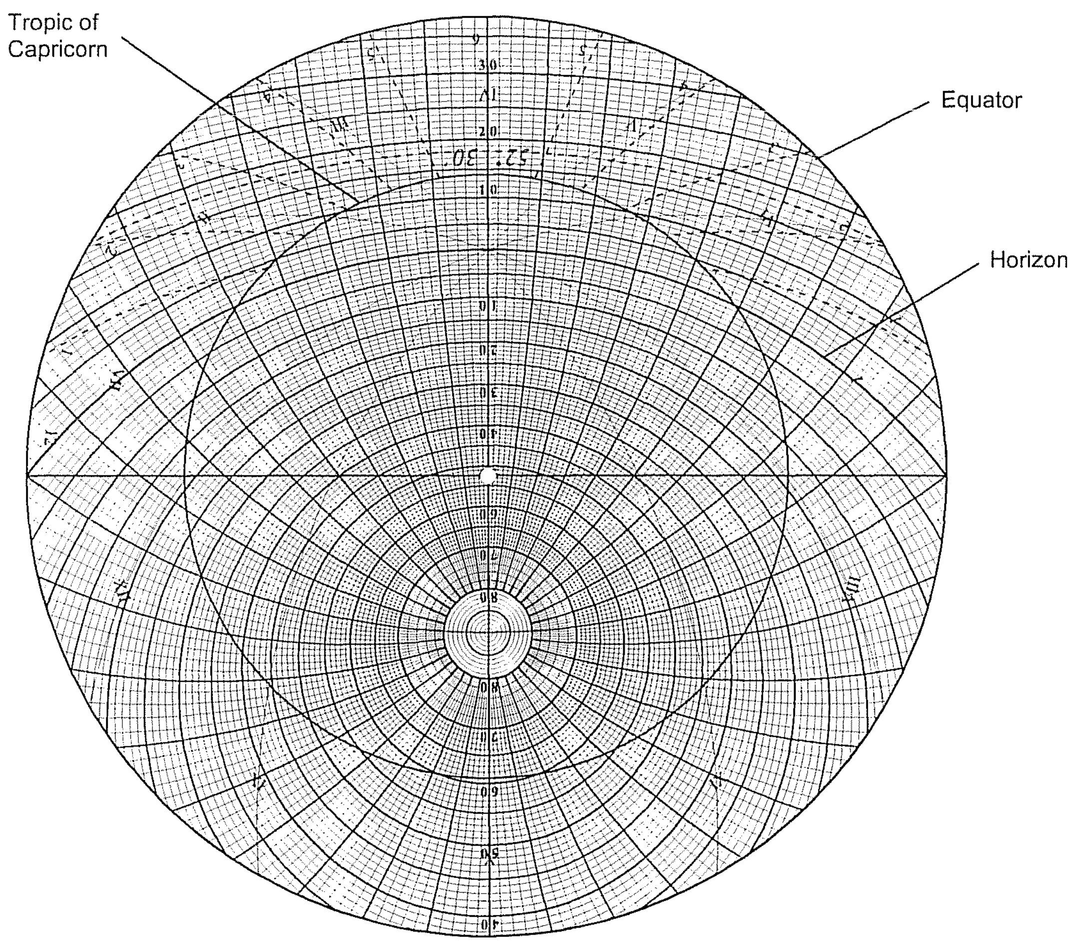


Figure 22-2. van Maelcote's plate for southern declinations

The second element of van Maelcote's astrolabe, which makes it usable for any declination, is based on the fact that inverting a plate for a northern astrolabe makes it a plate for a southern astrolabe. You have probably noticed that all the almucantars for negative altitudes that will fit are drawn on the plate. The negative altitudes for the northern declinations become the positive altitudes for the southern declinations.

The plate in Figure 22-2 shows the plate oriented for working with southern declinations. The plate is simply reversed and is now a plate created from a southern projection. Notice the almucantars for negative altitudes are now positive altitudes for a southern projection astrolabe plate for the same latitude.

The rete design participates in this configuration of the Tropics, as shown in Figure 22-3, using a transparent rete. The portion of the ecliptic used for positive declinations is from Aries to Libra and is the same as this portion of the ecliptic from a normal astrolabe rete. The solid stars on the rete (★) are those with northern declinations and right side up labels. The outline stars (☆) have southern declinations and have upside down labels in this view. The portion of the ecliptic from Libra through Pisces is the same as this section of the ecliptic from a southern projection astrolabe.

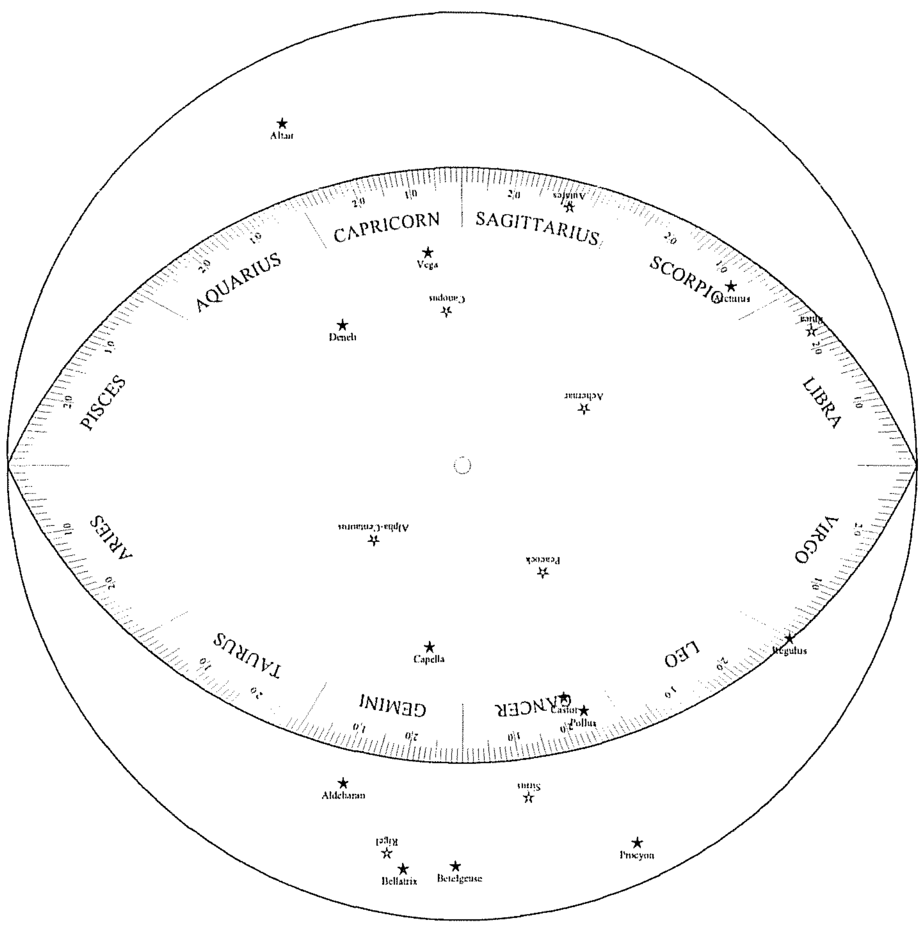


Figure 22-3. Rete for van Maelcote's astrolabe

It has already been pointed out that van Maelcote was not the first to envision this form astrolabe. In fact, a rete of this form was mentioned in old Arabic astrolabe treatises, and a similar form was called *al-āsī* (myrtle leaf). This rete format was imposed on a normal astrolabe with the Tropic of Capricorn as the plate circumference, but was identical otherwise. At least one instrument in this form survives. Islamic astrolabe variations in this form are discussed in the next section of this chapter.

The rule for van Maelcote's astrolabe is identical to the rule from a normal astrolabe except it is divided only to 0° declination. The sign of the declination must be inferred from the current use of the astrolabe.

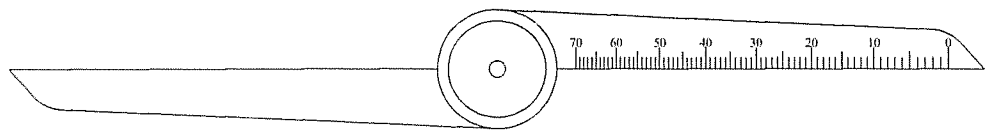


Figure 22-4. Rule for van Maelcote's astrolabe

Using van Maelcote's astrolabe

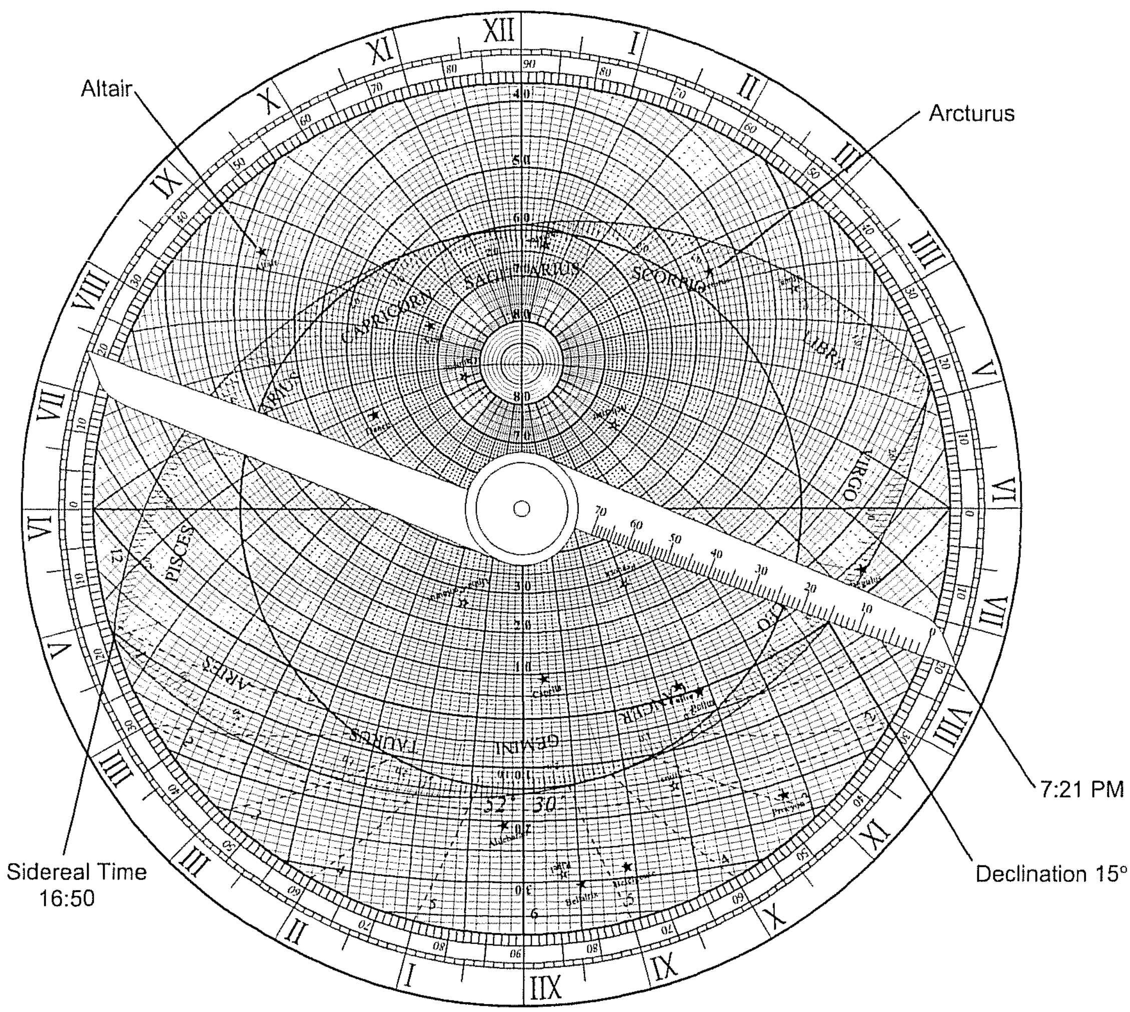


Figure 22-5. van Maelcote's astrolabe set for August 12

Using van Maelcote's astrolabe is almost identical to using a normal astrolabe for positive declinations. Figure 22-5 shows an instrument set for sunset on August 12. The Sun's declination is positive in August, so the plate is used in its normal position.

As on a normal astrolabe, set the rule to the Sun's longitude for the day (Leo 20°) and move the rule and rete until their intersection is on the western horizon. The figure shows sunset is at about 7:21 PM, and the Sun's declination is about 15° (14° 51'). Arcturus is at an altitude of about 45° in the southwest and Altair is at an altitude of 33° in the southeast and the sidereal time is about 16:50. Any problem involving a star with positive declination is solved in exactly the same way as on a normal astrolabe.

Problems with negative declinations require a bit of orientation, but are not substantially different. For example, to find the time of sunrise on November 15, reverse the orientation of the plate and use the Libra – Pisces section of the ecliptic.

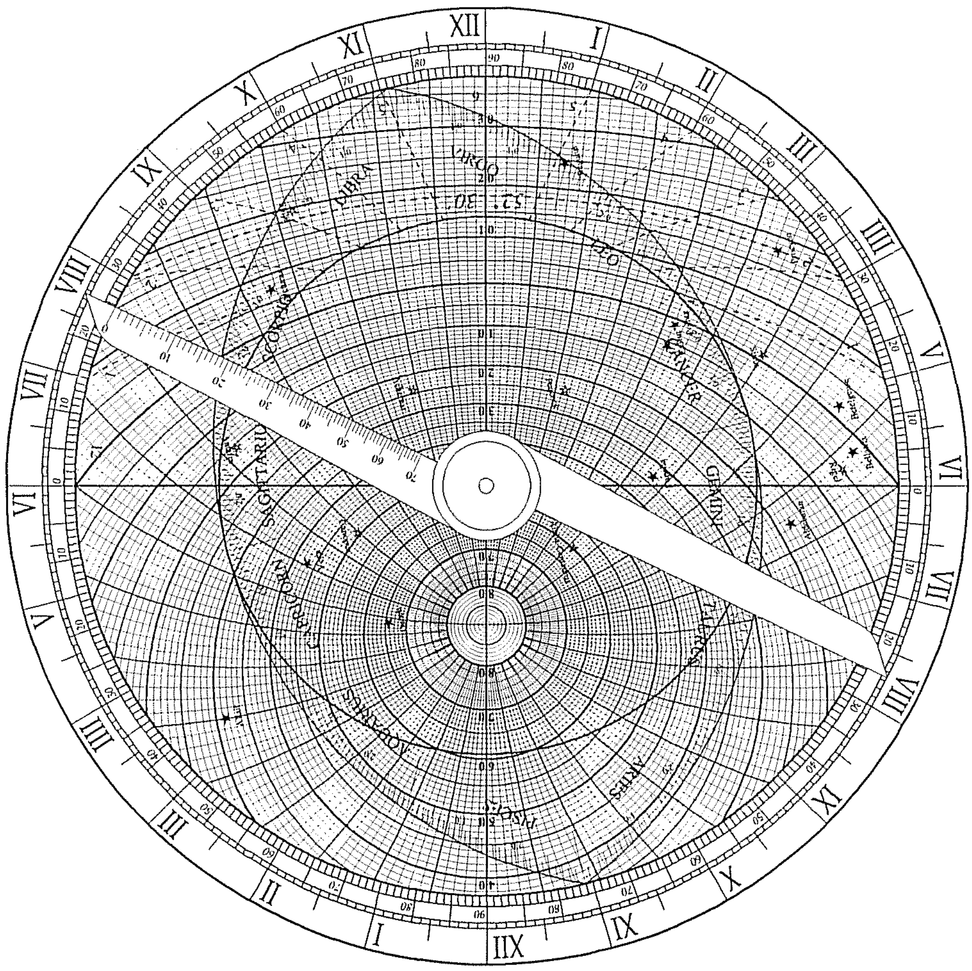


Figure 22-6. Maelcote astrolabe — Sunrise November 15

The astrolabe is set in the usual way for the Sun's longitude (Scorpio 23°). Once set we see the Sun's declination is about -18.5° (-18° 28') and the time of sunrise is about 7:43 AM. The sidereal time is about 11 hr 5 min. We also see Spica is at an altitude of about 20° in the east and Sirius is just setting. The answers are not labeled in the figure to give you the opportunity to confirm the results.

This astrolabe form can also be used for southern latitudes by exchanging the orientation of the plate from the northern latitude positions. In this way, the southern circumpolar stars are above the horizon. Operation is otherwise identical.

van Maelcote's astrolabe was defined at a time when the Earth's southern hemisphere was beginning to be explored in detail. It is easy to understand enthusiasm on the part of the developer for an instrument that could be used all over the world. However, serious navigators and explorers required more specialized instruments and the astrolabe was no longer widely used by the time European settlements were firmly established in most of the souther hemisphere. This variation, however potentially flexible, was simply too late to be widely used.

Making van Maelcote's Astrolabe

No new methods are required to make an astrolabe of this type. The only tricky part concerns the negative altitude arcs, which curve in the opposite direction for altitudes less than -φ as shown in the following figure. The radii of the lower altitude arcs get very large and the almucantar for -φ is a straight line.

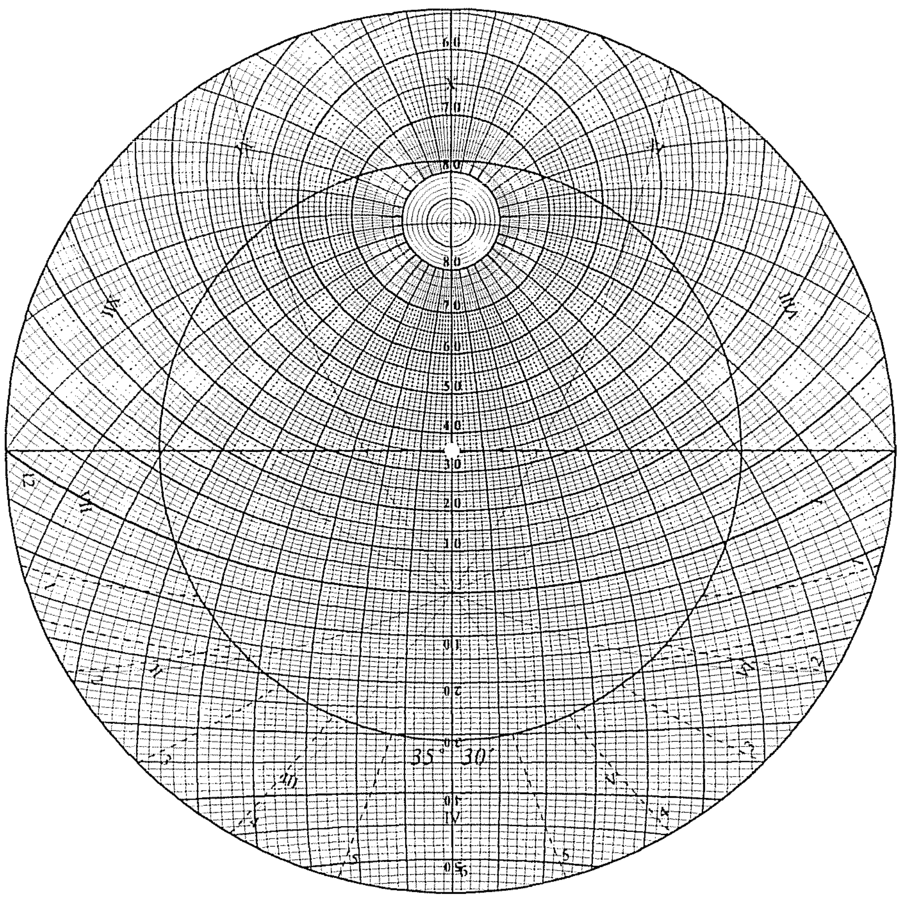


Figure 22-7. Aequinoctial Astrolabe Plate for 35° 30′

The Linear Astrolabe of al-Ṭūsī

Perhaps the mechanically simplest device ever described that would perform some astrolabe functions is the linear or "stick" astrolabe developed by Sharaf al-Dīn al-Ṭūsī [110] (ca. 1135 - 1213). Little has been written about this instrument. This treatment is based on Michel., with some changes and additions. The theory and operation of the plane astrolabe must be thoroughly mastered before tackling the linear astrolabe.

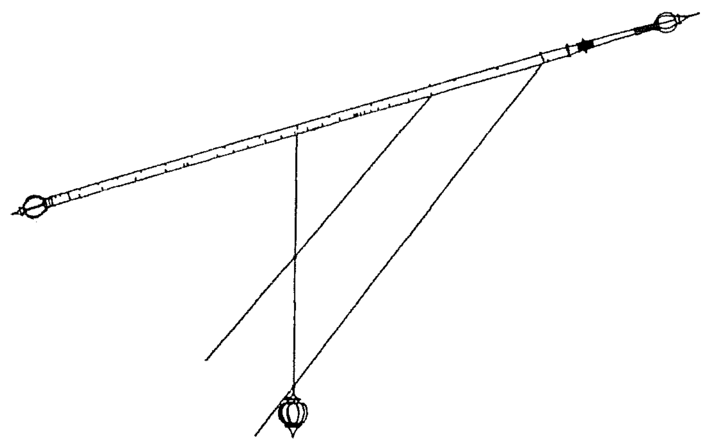


Figure 22-8. The Linear Astrolabe of al-Tūsī

al-Ṭūsī's linear astrolabe is a stick with some scales along its length and three threads or strings. The thread with the weight is a plumb line to give a horizon reference. The thread at the right end of the stick in the figure is used with the plumb line to take measurements and we refer to it as the "measuring thread". The third string is moveable and is used to set the instrument for specific uses. Note, the measuring thread can be on either end of the stick and our figures show it on the other end.

The linear astrolabe can be used to measure the altitude of the Sun or a star and to find the approximate time from the Sun's altitude, along with other functions. Users testify it is more accurate than you might think for such a simple device.

In use, you point the stick at something in the sky, usually the Sun, and use the measuring thread and a scale on the stick to measure its altitude. Once the altitude is known, the moveable thread can be used to find the length of the day or to find the approximate time in either equal or unequal hours, the hour angle of the Sun or a star, the time of sunrise/sunset or estimate the declination of the Sun or a star. You can also find the approximate time from the altitude of a star of known declination with some simple calculations. The scales suggested by al-Ṭūsī will be described, but you may be able to think of other scales that could be added to increase its utility. It is capable of a lot considering its simplicity.

Despite its apparent simplicity, the linear astrolabe is not particularly easy to use. You have to understand the instrument totally to make use of its capabilities and making accurate altitude measurements is very hard for one person to do alone, particularly if there is any wind.

[110] Full name: Sharaf al-Din Al-Muzaffar ibn Muhammad ibn Al-Muzaffar al-Tusi. Not to be confused with Nasir al-Dīn al-Ṭūsī, the famous astronomer who founded the observatory of Maragha.

It is, however, an interesting approach that encourages experimentation at almost no cost.

Basic linear astrolabe functions

Consider measuring celestial altitude angles with just a stick and a string. There's no place to put a curved altitude scale on a straight stick, so another way must be found. See Figure 22-9.

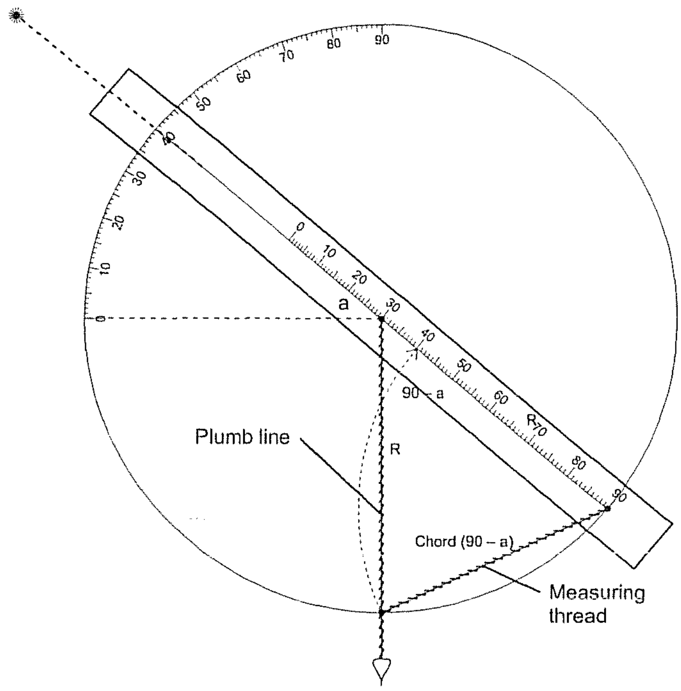


Figure 22-9. Measuring altitudes with a stick

We start with a straight rod or tube about two feet long and two somewhat shorter strings. One of the strings will be a plumb line, weighted at one end. The other string will be used for measurements. Pick a point near the center of the stick and suspend the plumb bob from that point. Select a convenient distance from the center and connect the measuring thread. The distance from the plumb line at the center to the measuring thread attachment point is R. A knot is tied in the plumb line at a distance R from the plumb line's attachment point. You may want to have some sort of sights on the ends of the stick, but this is not really required. A scale described below will be glued to the stick.

When the stick is pointed toward an object in the sky with altitude a, the plumb line will make an angle (90-a) with the stick. A triangle is defined by placing the measuring line at the knot in the plumb line. The measuring line defines a chord for the angle (90-a). A scale of chords on the stick itself shows the measured altitude. In the figure, the measured altitude is 40°. The measuring thread is stretched to the knot in the plumb line. The intersection point on the measuring thread is rotated to intersect the scale at 40°, the required altitude in the example.

The chord length scale in the figure is actually the chord subtended by the angle (90-a), labeled for the measured altitude, a. The length of a chord subtending an angle a in a circle of radius R is 2 R sin a/2, so a scale of 2 R sin (90-a/)2 divided by the chord length and labeled for a is needed. The scale can also be labeled for 90-a, in which case the value read from the scale is the zenith distance. This form of the scale is used later.

The modest accomplishment or measuring the altitude has a number of uses. For example, knowing the noon altitude of the Sun or the meridian altitude of a star of known declination allows our latitude to be calculated. Also, with the addition of a scale of sines and a little arithmetic, the unequal hour can be estimated using the equation on page 226.

It would also be possible to add a scale showing the length of a shadow cast by a gnomon of a given length for the Sun's altitude, thus duplicating the astrolabe's shadow square.

Theoretically, this is a quite accurate way to measure altitudes since the stick can be made rather long and the scale can be finely drawn. In practice, it is hard for one person to sight an object in the sky and move the measuring thread to the knot and mark the intersection length. However, it is quite easy for two people to do and surprisingly accurate.

Now, let's consider what else be done with a stick and string, this time using an astrolabe as a starting point. Think about what is involved in finding the length of the day with an astrolabe; The Sun is placed on the horizon and the time of sunrise or sunset read from the limb. We are actually reading the Sun's hour angle at sunrise or sunset converted to time. The measured angle is half the length of the day.

The Sun or a star follows the circle of declination on the astrolabe plate over a day. The diurnal arc for a given declination is the angle measured on the declination circle above the horizon. Put another way, the Sun's position is used to define a declination circle. The semi-diurnal arc is the angle on the declination circle from the horizon to the meridian.

On an astrolabe, a given declination is represented by a circle with its center at the center of the plate. The horizon is represented by a circle with its center on the meridian. The points where a given declination circle intersect the horizon circle represent the points of rising and setting. The semi-diurnal arc of the Sun can be found from the angle from the intersection of the declination circle and horizon to the meridian. Figure 22-10 shows the diurnal arc for a declination of 10° and a latitude of 38.7°.

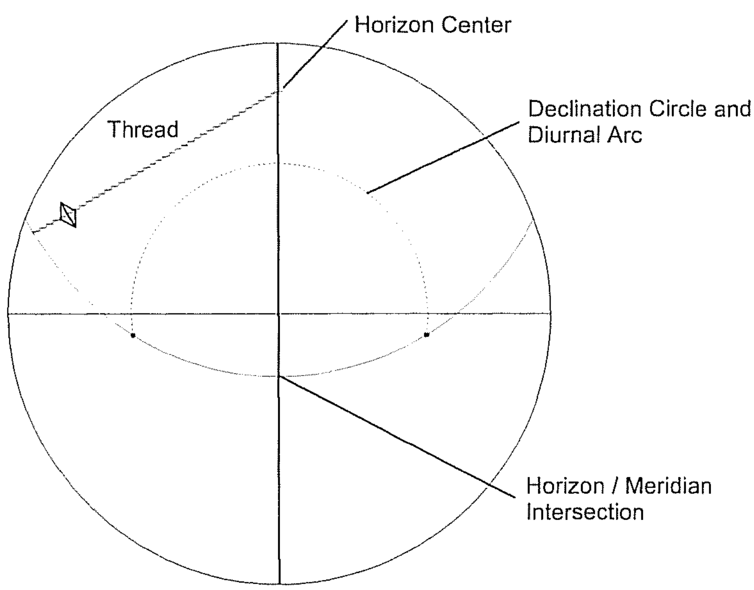


Figure 22-10. Diurnal arc on astrolabe plate

If the location of the center of the horizon circle on the meridian and its radius are known, a thread attached to the stick astrolabe can be attached at the horizon center. The point on the string at the horizon radius will trace out the horizon circle when it is rotated. The plumb thread and measuring thread are used to find the angle from the rising point on the horizon to the meridian (the semi-diurnal arc).

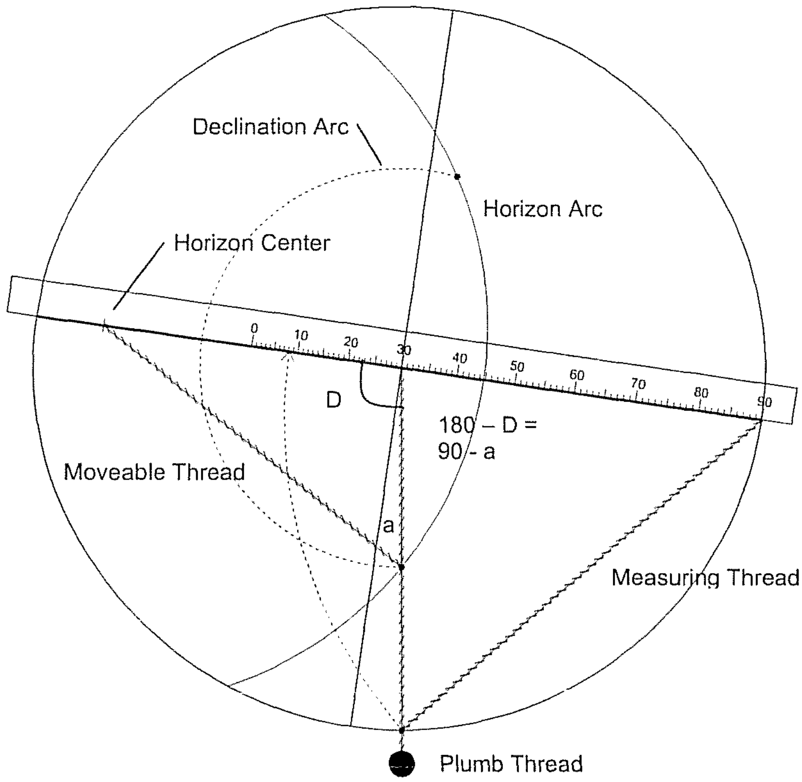


Figure 22-11. Measuring the semi-diurnal arc

See Figure 22-11. The stick or rod represents the meridian. The length of the semi-diurnal arc can be found when the astrolabe stick and threads are set up as shown in Figure 22-11. Scales for finding the center and radius of the horizon and the radius of the declination circle are included on the stick (Figure 22-12). One end of the moveable thread is attached to the horizon center and the horizon radius is noted. The point on the plumb line intersecting the declination circle is found. The point on the moveable thread at the horizon radius is tied to the plumb thread at the declination point. The entire arrangement is held up and tilted until the threads are taut and the plumb line falls vertically. The point on the measuring thread intersecting the plumb thread at the point equal to the radius, the Tropic of Capricorn, is found and this point is rotated to the scale and the value read (a little over 8° in the figure).

The plumb line intersects the horizon at the rising point on the declination circle. D is the semi-diurnal arc, the angle from the plumb line (the rising point) to the meridian. Here the angle a in the figure is the difference in the diurnal arc from the equinoxes and 90 – a = 180 - D. Thus, the chord of 90 – a represents the semi-diurnal arc. The measuring thread is used to find the angle a using the scale of chords. Depending on how our chord scale is divided, we can either read the size of the arc directly or by adding 90°. The length of the day in hours is the (semi-diurnal arc × 2)/15.

Numerically, for latitude = 38.7° and declination = +10° (April 15), the hour angle of sunset (the semi-diurnal arc) is 98.1° or 6 hr 32 min. a is the excess over the equinox diurnal arc = 8.1°.

Figure 22-12 shows the full scale suggested by al-Ṭūsī. This scale is intended to be carved into the rod or written on paper and glued to the rod. It includes four elements.

1. A scale of chords of 90-a on the left side.
2. A scale of almucantar centers (the horizon is almucantar 0).
3. A scale of almucantar lower intersections with the meridian.
4. A scale of solar declinations at entry to the zodiac signs.

The scales of chords and declinations are the same for all latitudes. The almucantar centers and intersections depend on the latitude. The scale in the figure is calculated for 45° latitude.

The scale of chords is used with the measuring thread to find angles. When labeled as in the figure, the altitude is read as the zenith distance. The measuring thread can be attached to either end of the stick and the scale can be reversed if you find it more convenient to attach the measuring thread at the end opposite the eye.

Our examples use the chord scale divided by the altitude angle rather than the complement of the altitude (zenith distance). There are advantages and disadvantages for either method of labeling and, in fact, it might be useful to include both sets of labels.

The scale of centers and meridian intersections are used to set the moveable thread.

The scale of declinations is used to define declination circles on the plumb line. This scale could easily be divided by the calendar. This scale is fairly dense and it is difficult to set declinations accurately.

al-Ṭūsī suggests an additional double scale of the Sun's right ascension and the right ascensions of bright stars to be used find the time with the equation:

$$t = H_c + (\alpha_c - \alpha_s)$$

where H_c is the hour angle of the Sun or a star which is added to the difference in the right ascensions of the star and the Sun.

In practice, it is useful for the scale of centers to be marked by pegs, grooves or small holes in the rod to allow the end of the thread to be anchored. The moveable thread is attached to the plumb line using a running knot that can be tightened at the correct radius.

We will now discuss how to use the threads and scales to find the hour angle of the Sun from noon, from which the current time is easily calculated. It is interesting that the time in either equal or unequal hours can be found using the same procedure.

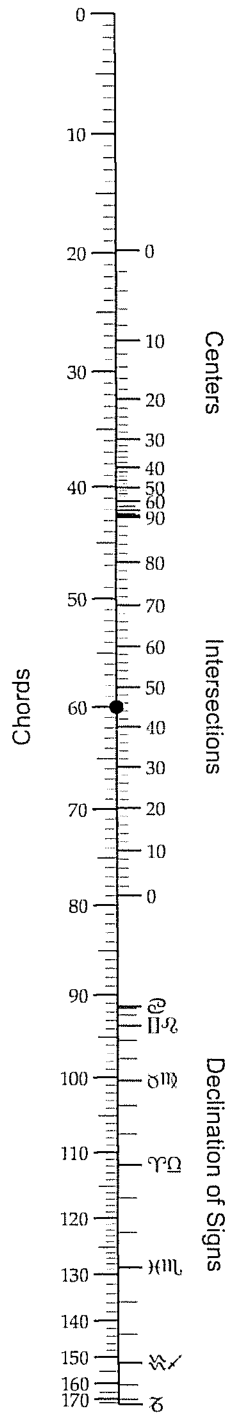


Figure 22-12. al-Tūsī's Scale

The procedure is almost identical to finding the length of the day, but uses the almucantar for the Sun's current altitude, which we assume is 50°.

The linear astrolabe is set up in exactly the same way as for finding the diurnal arc, except now the moveable thread is attached to the center of the 50° almucantar. Locate the almucantar radius and attach it to the plumb line at the point for the radius of the declination circle.

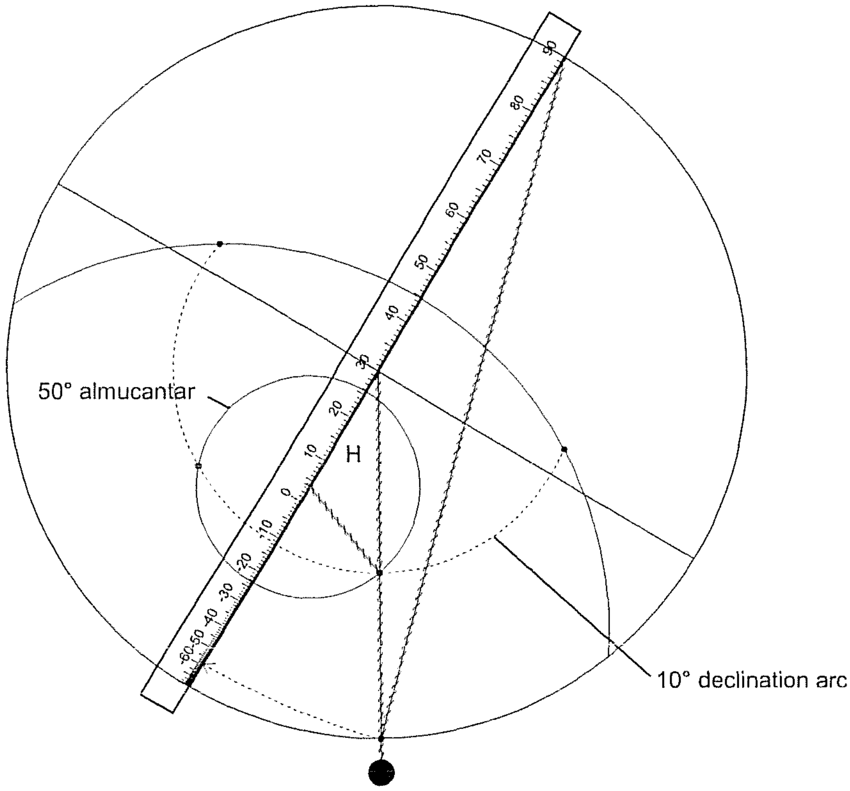


Figure 22-13. Finding the time with the linear astrolabe

Here angle H in the figure is the hour angle of the Sun or star. The measuring thread is used as above to find H and then the angle is converted to time. In this case, H = 90 – (the value read on the altitude scale). Read 59° on the altitude scale giving H = 90° – 59° = 31°. The values used in the figure are: $\varphi = 38.7°$, $\delta = 10°$ (April 15, ♈ 25.5°), altitude = 50°, hour angle = 31° = 2:04, apparent time = 12:00 – 2:04 = 9:56 AM.

Time from noon can be found as either equal hours, by dividing a by 15, or found in unequal hours by finding the number of degrees per unequal hour by dividing the semi-diurnal arc found earlier by 6 to find the number of degrees per hour and then dividing a by this value.

This example illustrates why this device never became more popular. The user must understand both the plane astrolabe and how the full astrolabe transfers to the linear astrolabe to use it effectively and there are still details that take significant practice to master. It is much easier to follow the theory from the figures in the book than it is to work the instrument as a few minutes dealing with sticks, strings and knots will demonstrate. I assume it can be mastered with perseverance, but I have never met anyone who achieved this level of expertise.

Making al-Ṭūsī's Linear Astrolabe

There are many options available when making a linear astrolabe. You can vary the size and the scales depending on your objective. It is fun to make one just to see how it works, which is what

I did. You can also make a museum quality piece using fine woods and careful construction, such as the one made by Roderick Webster for the Adler Planetarium. The following discussion is rather general since much of the fun associated with making this sort of device is in figuring out the details for your instrument.

I made my sample using materials already at hand; a ½" dowel rod, sewing thread and clear tape. Rod Webster's instrument is made of mahogany with brass sights and a carefully crafted scale. The choice on the approach is yours alone.

I just printed the scale, cut it into a strip and taped it to the dowel rod with transparent packing tape. I located the threads using push pins and used a fishing weight for the plumb line. It occurs to me that a nice linear astrolabe could be made from a piece of round or square tubing with holes drilled at the appropriate locations, but I have not yet made one this way.

The scales are easy to lay out.

The chord scale has already been covered. The chord scale needs to extend over most of the length of the rod. The chord scale labeling requires some thought. I prefer to label it by altitude since measuring altitude it the most demanding application. Labeling the scale by zenith distance is more traditional.

The almucantar centers and radii are calculated using the standard equations (page 167).

The declination scale is calculated using $\sin \delta = \sin \varepsilon \sin \lambda$. The distance of the declination tics from the center of the rod is $R_{eq} \tan (90 - \delta)/2$. You might consider dividing the declination scale by the calendar (see page 361 for an overview of the calculations).

You might find it interesting to construct additional scales for other uses as suggested above. Considering the simplicity of the device, experimentation can be done at almost no expense and modest effort.

It has been suggested that the linear astrolabe might be easier to use if rods were used instead of threads. The measuring rod could be divided with a scale of chords to reduce congestion on the rod. The end of the moveable rod could be set with a thumb screw and the attachment of the measuring rod to the plumb line could be accomplished with a sliding device with a screw to tighten it in position. A metal strip, such as rule could be substituted for the rod. Such a device would be fun to make and might be much easier to use.

Islamic Astrolabe Variations

Experiments with the basic astrolabe form were undertaken early in the astrolabe's history. Astrolabe variations date from the 9th or 10th century and are mentioned in by Ibrāhīm ibn Sinān (907-946)[111]. Many variations were described, none of which ever gained wide popularity. One of the great appeals of the classic astrolabe is the intuitive astronomical representation of the sky. All of the astrolabe varieties compromised this advantage is some way without offering offsetting advantages, thus limiting acceptance. All of the astrolabe styles worked in the sense of allowing solutions to the same problems as the classic astrolabe, but with somewhat more complexity. That is, you have to think about what you are doing, which has never been popular.

[111] King [2003].

Apparently, the first modifications explored involved astrolabes that use both the northern and southern projection on the same instrument[112]. The initial result was an instrument very similar to the aequinoctial astrolabe discussed in the previous section, but with the outer limit of the astrolabe plate at the Tropic of Capricorn, as on the traditional instrument. An example is shown in Figure 22-14, which combines a northern projection for Aries through Virgo and a southern projection for Libra through Pisces.

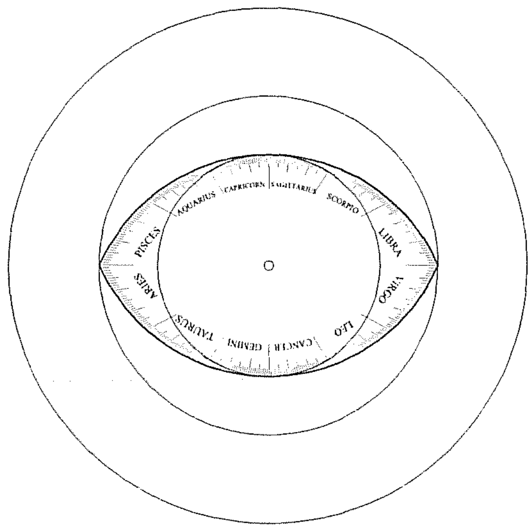


Figure 22-14. Myrtle Astrolabe Rete

This form is described in almost all medieval Islamic instrument treatises. We might describe the ecliptic shape as a convex lens, but it apparently reminded Islamic astronomers of the shape of leaf from a myrtle tree and it is called *al-āsī* (myrtle-leaf).

This form requires a plate incorporating a full range of almucantars for either the northern or southern projection. The almucantars for altitudes below the horizon in the northern projection represent altitudes above the horizon in the southern projection. The plate orientation is reversed depending on the season.

The plate for the myrtle astrolabe is virtually identical to the aequinoctial astrolabe in the previous section except that the outer limit is the Tropic of Capricorn. Note it is not necessary for the unequal hour arcs to extend past the equator for the myrtle, but the hour arcs from the equator to Capricorn are used in other formats. A sample mixed projection plate for Cairo is shown in Figure 22-15. The aequinoctial astrolabe was anticipated by Ibn al-Sarraj in the 14th century.

[112] This section makes extensive use of Charette [2003], pp.66-78.

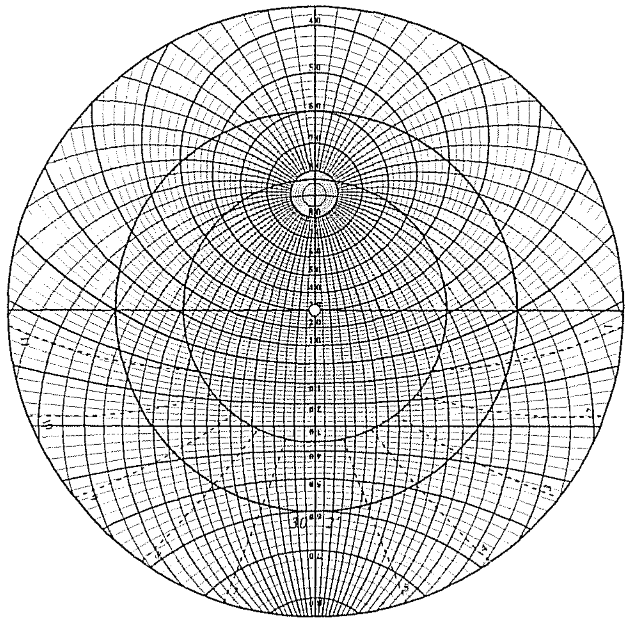


Figure 22-15. Mixed Projection Plate

The opposite orientation occurs to you immediately once you have jumped the intellectual hurdle of mixing projections on a single instrument. Reversing the projection for each zodiac section from the myrtle rete yields the rete in Figure 22-16. The hour glass shape of this ecliptic representation reminded the makers of the Arabic drum called *tabl*. Hence the name *al-muṭabbal*, "like a *tabl*". The Drum rete uses the same plate as the Myrtle rete. The Drum rete is actually rather good in that it makes the ecliptic divisions more coarse, and hence more readable, for the entire ecliptic.

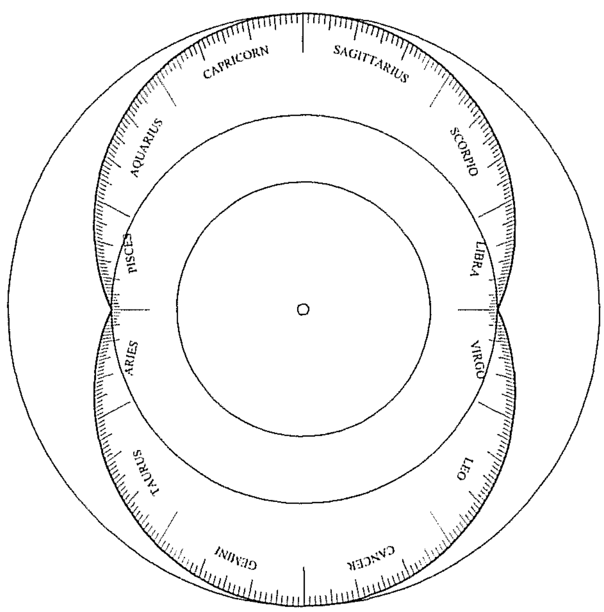


Figure 22-16. Drum Astrolabe Rete

It is not a great intellectual leap to realize, if the ecliptic can be split in half by opposite projections, why not project smaller sections or even single signs with different projections? Some rather interesting and fanciful rete patterns emerge. We will adopt the clear notation used by Charette to describe these rete patterns. Each 30° sign is represented by an "N" or an "S", signifying whether this sign uses a northern or southern projection. Thus, the Myrtle rete is described by NNN | NNN | SSS | SSS and the Drum rete is SSS | SSS | NNN | NNN.

There are 2^{12} (4,096) possible combinations if every possible configuration is considered valid. However, if there is a requirement that the rete be symmetrical about one or both axes, the number of valid combinations is reduced to 118[113], not counting the two basic cases. Only a few of these have been discussed in the literature.

The following mixed rete figures represent eight formats discussed in medieval literature:

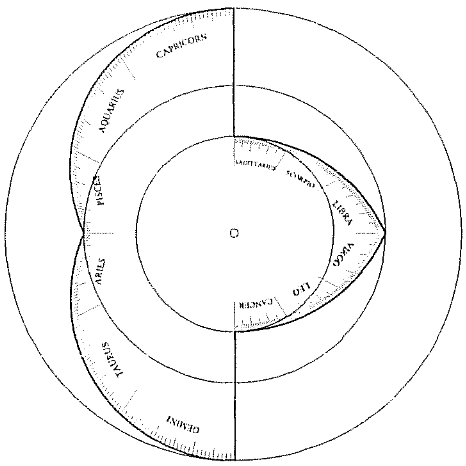


Figure 22-17. Crab (*musarṭan*)

SSS | NNN | SSS | NNN

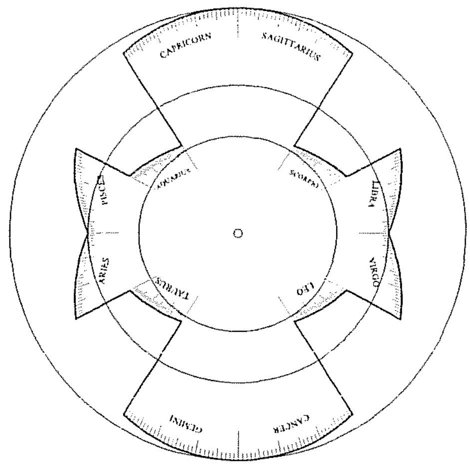


Figure 22-18. Conch (*sadafī*)

SNS | SNS | NSN | NSN

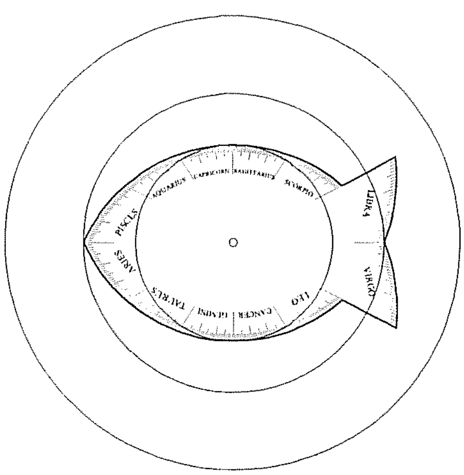


Figure 22-19. Fish (*samakī*)

NNN | NNS | NSS | SSS

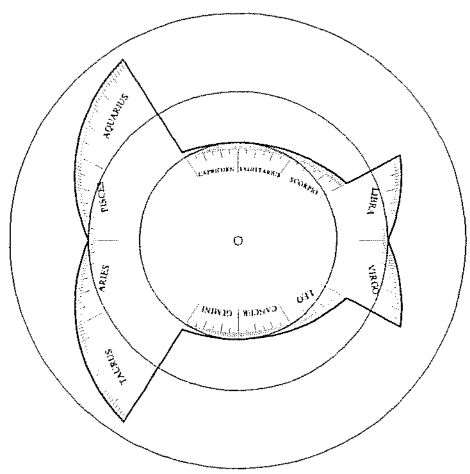


Figure 22-20. Jug (*bāṭī*)

SSN | NNS | NSS | SNN

[113] Charette [2003], n. 76, p. 70.

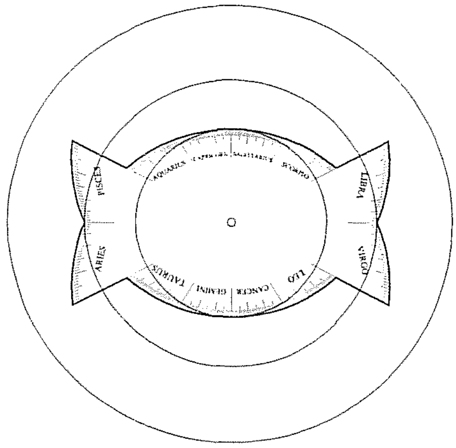


Figure 22-21. Jar (*narjisdānī*)

SNN | NNS | NSS | SSN

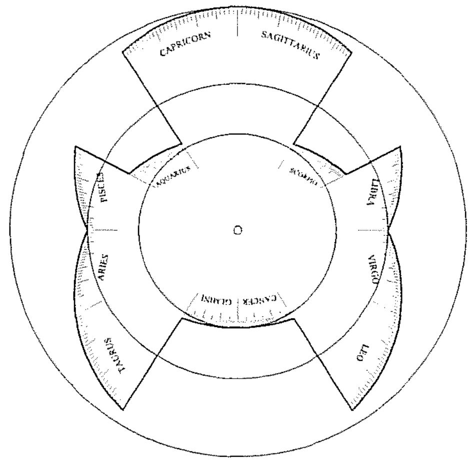


Figure 22-22. Bull (*thawrī*)

SSN | NSS | NSN | NSN

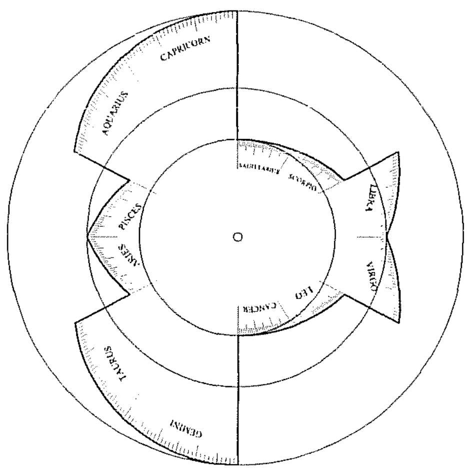


Figure 22-23. Buffalo (*jāmūsī*)

NSS | NNS | NSS | NNS

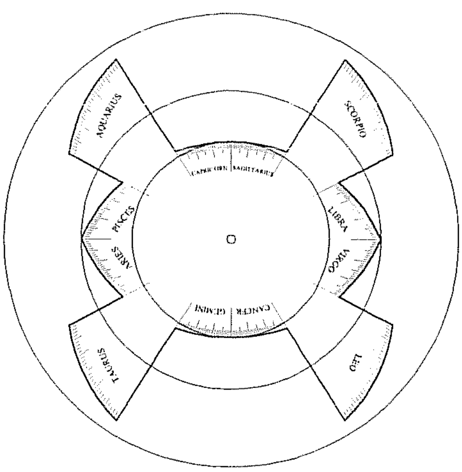


Figure 22-24. Tortoise (*sulaḥfī*)

NSN | NSN | SNS | SNS

As mentioned above, a great many other configurations are possible. The names applied to the various configurations is not completely consistent in such literature as there is on this subject.

Somewhat surprisingly, the mixed retes are not difficult to draw. Simply calculate the rete divisions for the northern projection. Invert the rete and use the sign 180° away for the sections that use the southern projection.

The mixed projection plate in Figure 22-15 can be used with any of the configurations. However, the plate orientation must be selected depending on the projection for each sign. For example, on the Crab rete (Figure 22-17), when the Sun is in Aries through Gemini, the plate is oriented for the southern projection. When the Sun is in Cancer through Virgo, the plate is oriented for the northern projection, etc. Once the plate is correctly oriented, the location of the

Sun in the ecliptic can be set to an almucantar and the problem solution continues in the normal way. Finding the unequal hour requires the additional step of noting the almucantar and reading the unequal hour from the equivalent almucantars on the opposite projection since the nadir on the ecliptic cannot be used for this function. Medieval instrument makers were not satisfied with this relatively simple solution and apparently continued their virtuoso performance by defining complex plates customized for each configuration.

Consider the Crab rete in Figure 22-17. The rete requires both a northern and southern projection over the entire plate area. One way to provide this is shown in Figure 22-25. This plate includes the northern projection of the celestial sphere above the horizon upright and inverted on the left side and the same for the southern projection on the right side. The entire sky for both projections is represented on the plate by rotating the plate 180°.

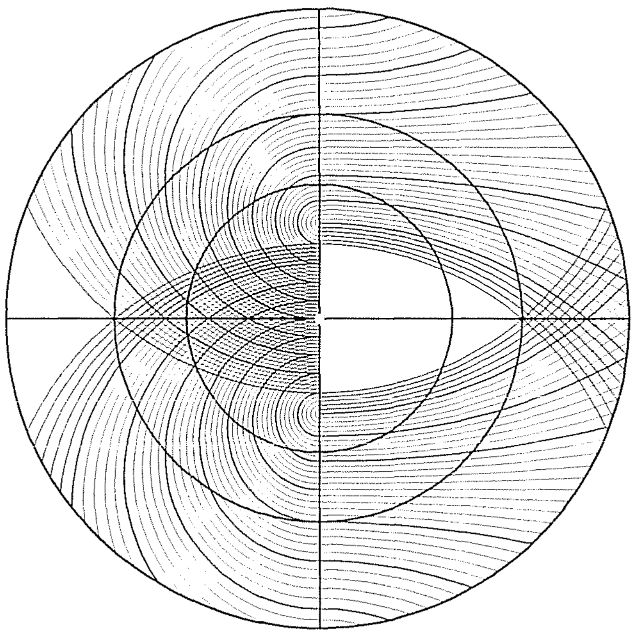


Figure 22-25. Mixed Projection Plate for Crab Rete (40°)

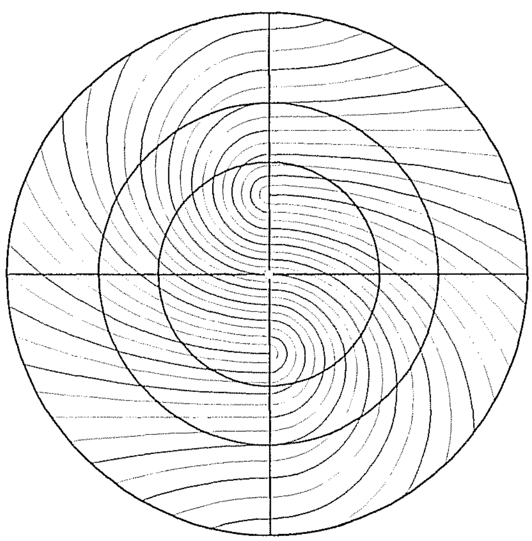


Figure 22-26. Mixed Projection Plate

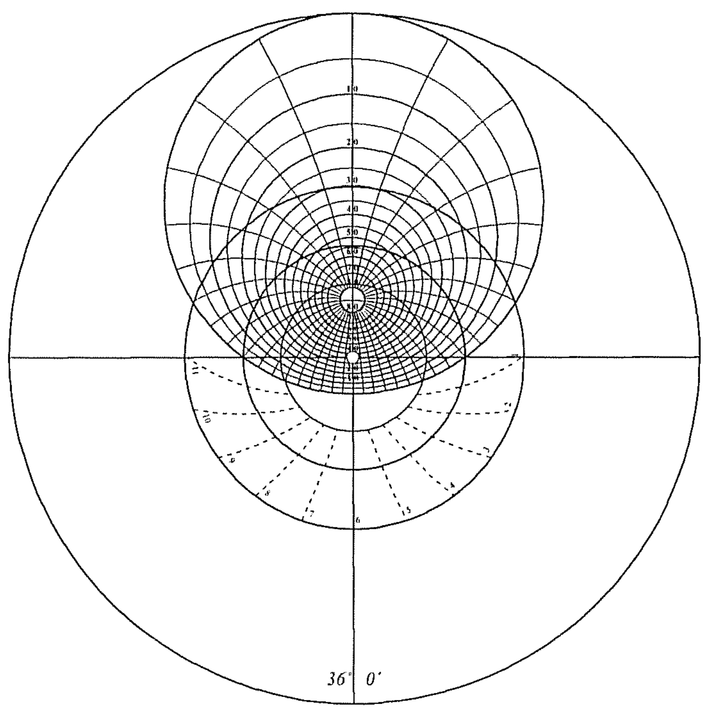
A number of these plate variations were described in treatises, some of them quite intricate and may include different projections inside and outside the equator. One that I rather like is shown in Figure 22-26.

There is no question some astrolabes were made using these configurations. But I tend to agree with Michel that they are more of a "*coquetterie géométrique*" (geometric affectation) intended more to amuse the reader of a treatise or to impress with the virtuosity of the writer than a serious attempt at improving the astrolabe.

Even more complexity can be introduced. al-Sijzī introduced an additional complication with two different projection scales using four circles for the tropics; an additional "tropic" is drawn inside the Tropic of Cancer treating the northern projection Tropic of Cancer as a second equator. Thus, each sign can be shown in four ways. It is not clear to me why anyone would invest much effort on this extreme approach.

The Myrtle and Drum astrolabes are both practical. The Drum gives greater resolution for the first half of the ecliptic and the Myrtle offers a less congested rete. One practical variation of the Myrtle rete reduces the ecliptic congestion even more. The entire ecliptic can be reduced to a single arc representing Aries – Virgo using the northern projection and reversing and inverting the southern projection for Libra – Pisces. King [2003] discusses a surviving example of such an implementation from the 14th century.

There were some Islamic astrolabe variations with realistic uses. The *kāmil* astrolabe extends the outer diameter of the plate to include the entire horizon (Figure 22-27). This approach is very instinctive since it actually increases the didactive uses of the astrolabe by showing the entire horizon. The unequal hour arcs are used only within the annual limits of the Sun's motion and should extend only to the tropics. This format is such an obvious extension of the classic astrolabe that virtually every student I have ever had has been drawn to experiment with this approach without coaching.



igure 22-27. *Kāmil* Astrolabe Plate for 36°

Kāmil means "complete", a name that was applied to a variety of instruments by different authors. The *kāmil* plate becomes unwieldy for low latitudes but works well for latitudes above about 35°. Note that a normal plate includes the entire horizon for latitudes above the Tropic of Cancer.

Other modifications were suggested, apparently intended to reduce instrument cost. The rete can be practically eliminated by substituting a thread with a bead that can be set to a scale of longitudes or an ecliptic circle drawn on the plate. The thread is then set to the almucantar to show the time. Stars are drawn directly on the plate to provide a coordinate reference. Similarly, the longitude scale can be engraved on a rotating rule.

Many other astrolabe and quadrant variations were defined by Islamic masters, some of them quite clever, including instruments that did not use the stereographic projection. The interested reader is referred to Charette [2003], who also includes a complete bibliography. However, you should be warned that these instrument variations can be as addictive to modern practitioners as they were to 13th century *mīqāti*.

Planisphere

The planisphere is a star finder. The planisphere is not directly related to the astrolabe but the two devices are sometimes confused by students so it seems useful to discuss the planisphere to the extent the differences are understood.

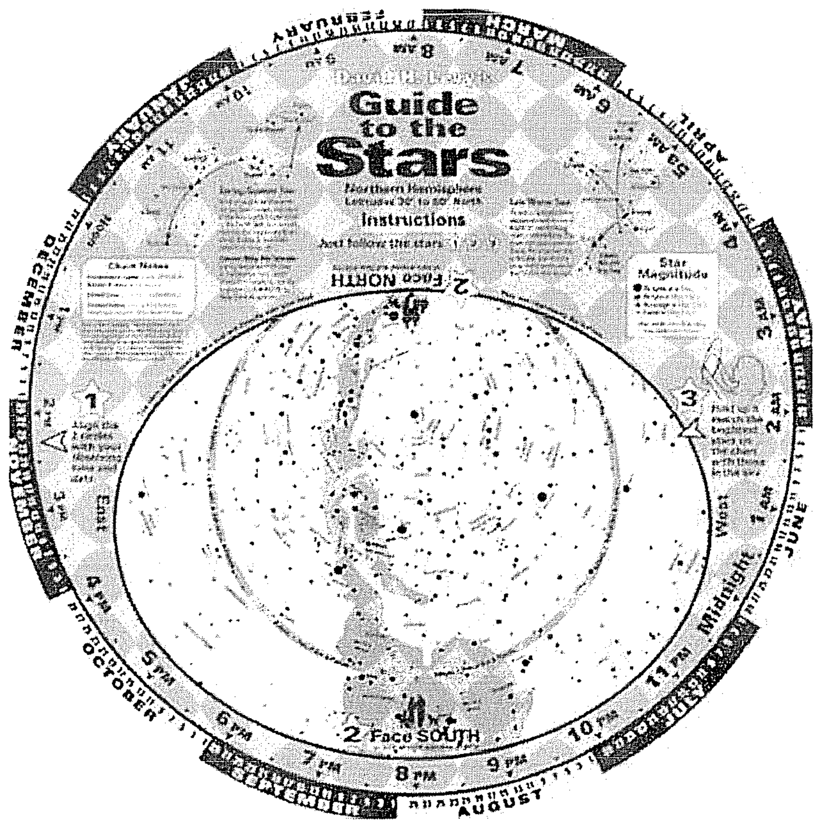


Figure 22-28. Planisphere

A typical planisphere star finder is shown in Figure 22-28. It consists of a circular star map centered on the north celestial pole with a calendar scale around the outside. The star map is beneath a rotating sheet, usually of plastic, with a cutout for the local horizon and a time scale around the periphery. The top sheet is rotated so the current time of day is near a date on the star map. The stars within the horizon cutout are the ones visible at that time. In use in the field, the planisphere is held over the head, oriented to the north.

While planispheres are accurately made, they are not precision instruments and are intended only to allow you to find approximately which stars are visible. They are useful to beginning astronomy students as an aid for learning the constellations and can be used by more advanced astronomers to make rough plans for an observing session.

It would be possible to make the planisphere star map using the stereographic projection but none are made in that way today. All planispheres use the polar aspect of the Azimuthal Equidistant projection.

The term, "Azimuthal Equidistant" sounds a lot more complicated than it is. *Azimuthal* means the projection plane is tangent to the sphere. *Equidistant* means the scale is chosen so equal intervals of declination project to equal distances on the star map. The projection is significantly simpler with the polar aspect. The projection is not conformal and there is meaningful shape distortion away from the pole. However, the view of the relative distance between stars looks close to what you actually see, which makes constellation identification easy.

Figure 22-29 shows a planisphere star map using the same stars on the astrolabe rete shown earlier. Notice circles of declination are evenly spaced. As on the astrolabe plate, right ascensions plot as radial lines. Unlike the stereographic projection as used on the astrolabe, the planisphere star map is as seen from the inside of the celestial sphere.

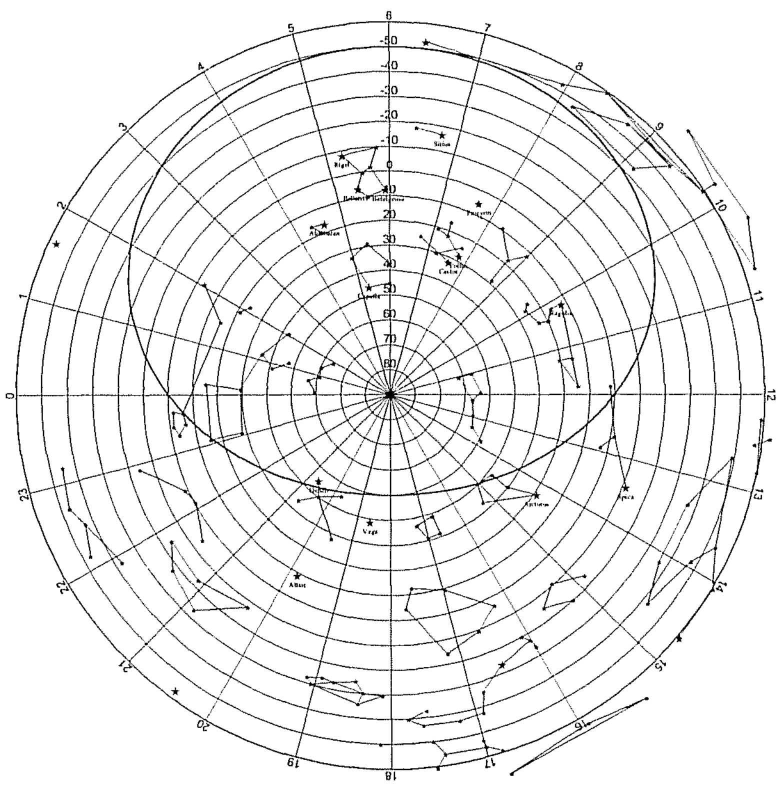


Figure 22-29. Planisphere Star Map and Horizon

The horizon cutout is shown as the oblate circle outlined with a heavy line. The horizon in the figure is for 40° latitude. The planisphere in the figure above is oriented to a local sidereal time of six hours as shown by the label on the radial, which occurs at about midnight on December 22, 8 PM on February 21, etc.

A variation of the planisphere used by experienced celestial navigators is the 2102-D Star-Finder and its successor, the CP 300/U. The 2102-D was developed by Capt. Gilbert T. Rude (1881-1962), one time chief of the Division of Coastal Surveys for the U.S. Coast and Geodetic Survey.[114] Originally called, "The Mariner's Practical Star Finder and Identifier", it was adopted by the U.S. Navy and furnished to all naval vessels. The NOAA survey vessel 'Rude' (pronounced 'Rudy') is named after him. The CP 330/U was developed by the U.S. Air Force in the mid-1950's. The two versions are almost identical. The CP 300/U has more stars (66 vs. 57), some constellation asterisms, and an arm for fine adjustments to the altitude/azimuth grid. The British version, the N.P. 323, is very similar but is made of cardboard.

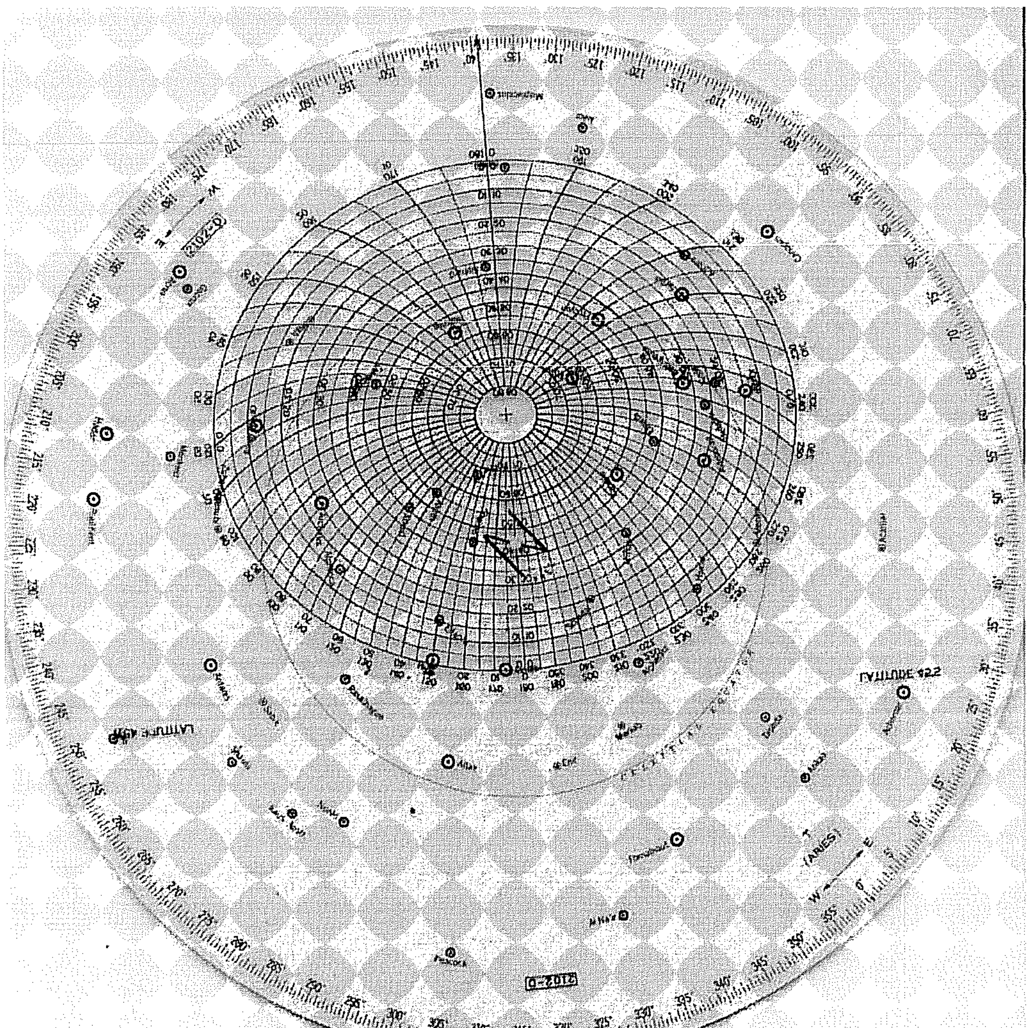


Figure 22-30. 2102-D Star Finder

[114] Maine Coastal News, **12-7**, 15-31 May, 1999.

The primary purpose of the 2102-D is to find the navigational stars used by celestial navigators. This is a critical navigation function and the 2102-D is a professional instrument. The main uses of the 2102-D are to find a visible navigation star when the sky is partly obscured and to pre-plan fixes for efficiency and improved accuracy.

Unlike the simple planisphere, the 2102-D horizon (Figure 22-30) contains scales of altitude and azimuth which allows the 2102-D to be used much more precisely than a planisphere and provides other uses related to the altitude and azimuth..

You look down on a 2103-D, like an astrolabe, and you are looking at the outside of the celestial sphere. In principle, the 2102-D star map is the same as the simple planisphere.

To use the 2102-D you install a transparent disk containing the altitude/azimuch scales for a latitude on the fixed disk which has a projection of the navigation stars. The scale around the perimeter is right ascension in degrees, but is used in the sense of LHA ♈.You estimate the LHA ♈ for your meridian and time and set the tranparent rotating disk to that value. You can then see which stars are visible and read their approximate altitude and azimuth from the grid. An astrolabe rete for the same function is discussed on page 109. The 2102-D is also useful for planning sights, particularly of the moon and planets.

Making a Planisphere

The star map

The theory behind a planisphere star map is rather simpler than for an astrolabe rete. Like the astrolabe rete, star positions are derived from the star's declination (δ) and right ascension (α). The angle of the star is the right ascension converted to degrees and the radial position of the star is the declination scaled linearly to the size of the map.

Let ρ be the radial distance of a star from the center of the map, R be the radius of the map and δ_{min} be the least declination to be shown (-60° on the sample above).

$$\rho = R\frac{90-\delta}{90-\delta_{min}}$$

The coordinates of the center of the star are:

$$x = \rho \cos(180 - \alpha)$$
$$y = \rho \sin(180 - \alpha)$$

The supplement of the right ascension is used to make the sense of right ascension go in the direction used on the planisphere. Star coordinates should be processed to the date of the map.

The horizon cutout

The limits of the horizon cutout are found from the declination of each point on the horizon.

The equation for converting from equatorial to horizontal coordinates is:

$$\sin h = \sin\varphi \sin\delta + \cos\varphi \cos\delta \cos\alpha$$

At the horizon, h = 0. Collecting terms we find:

$$\tan\delta = \frac{\cos\alpha}{\tan\varphi}$$

The coordinates of each point of the horizon are found as above. The horizon in Figure 22-29 was drawn by connecting the points on the horizon for each degree of right ascension with line segments.

Most planispheres use symbols of various sizes to represent the brightness of stars and show constellations as the common asterisms. There are several computer versions of star catalogs available, but the star positions will have to be precessed to the current epoch. From an implementation point-of-view, the hardest part of creating a planisphere star map is labeling all of the elements.

Other Astrolabe Related Devices

The power of the projections used for astrolabes, particularly universal astrolabes, has been recognized for centuries, and a large number of related devices have been developed to capitalize on their strengths.

For example, the device in Figure 22-31, which is a modern implementation of the Rojas astrolabe, was offered by the United States Power Squadron as an easy way to convert between right ascension/declination to altitude/azimuth.

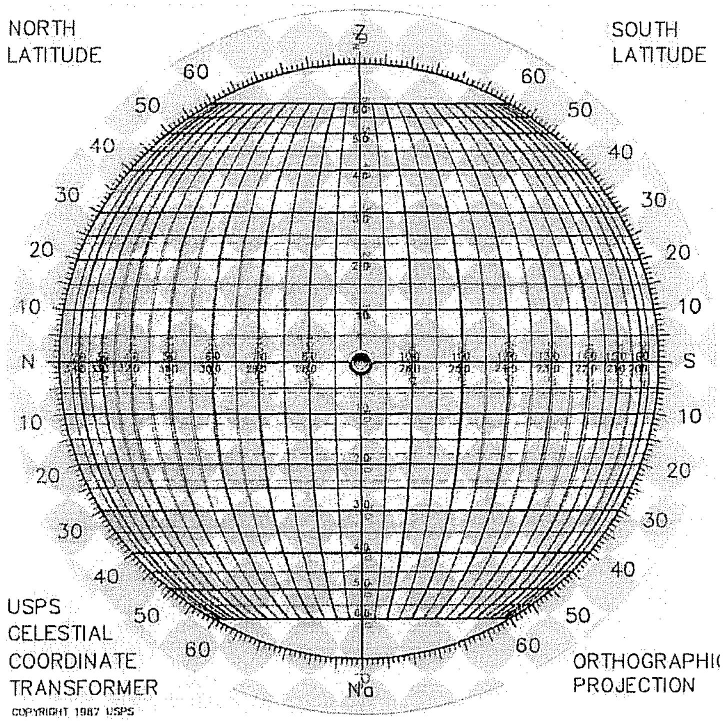


Figure 22-31. USPS Coodinate Converter

The device in Figure 22-32, was a nicely packaged instrument using the saphea projection for a variety of navigation application.

The pelorus is a shipboard instrument for locating bearings. The pelorus is sometimes equipped with a horizontal stereographic projection for determining altitudes and azimuths.

There have been literally hundreds of such devices developed, most for celestial navigation applications. They seem to come and go and, with the advent of GPS, have become rare collectors items.

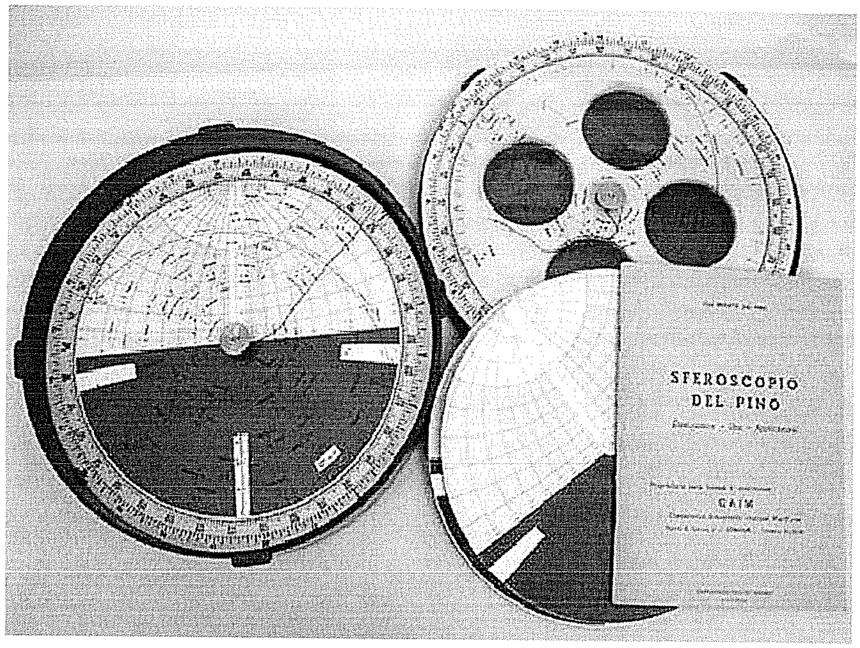


Figure 22-32. Saphea Based Navigation Tool

Chapter 23 - Astrolabe Clocks

For some reason, people love complex mechanical devices and the more complicated the better. The literature is full of learned expositions postulating deep philosophical theories about why geared machines have been so popular. In the final analysis, they are just neat.

The history of machines demonstrating celestial movement is as old as the history of machines. Some of the earliest documented mechanical devices are astronomical in nature. Archimedes is said to have made mechanical globes around 212 BC. The "Antikythera Machine"[115] from ca 80 BC is a very complex astronomical analog computer. The Tower of the Winds apparently contained several celestial machines. Price[116] goes so far as to state that astronomical machines came first, with time-telling as a byproduct.

It is only natural the astrolabe would be considered as a good candidate for a geared mechanism. The earliest documented geared astrolabe was described by al-Bīrūnī and used a simple gear train to relate the age of the moon with solar time[117]. Such an implementation would be useful for calendar problems for the lunar Islamic calendar. A similar geared astrolabe of Persian origin made in 1221/2 is in the collection of the Museum of the History of Science at Oxford. It is reasonable to conclude the earliest clocks with astrolabe dials originated in medieval Islam. Such clocks were powered by water flowing at a controlled rate and did not include gears, but used ropes and pulleys to derive the required ratios. Such a clock still exits at Qarawīyīn University at Fez, Morocco[118].

The earliest European time mechanisms were simple geared devices regulated by the slow fall of a weight controlled by water flowing from a cistern. The first such devices would cause a lever to strike a bell to signal some time of interest, such as a prayer time. They gradually become more sophisticated and easier to tend.

Simple clocks began to be used to signal the hours in towns by the end of the 12th century. The escapement was developed and applied to clocks in the late 13th century, which made it possible for the first time to have an unattended machine signaling the time at a distance. A dial with a single hour hand was added somewhat later.

It was inevitable astronomical functions would be added as knowledge about the verge and foliot escapement dispersed and the fabrication of gear trains became more common. Monumental clocks with astronomical functions became popular for churches and public buildings in the 15th century.

Floor-standing and table clocks of all kinds, particularly those with astronomical or animated figures, became popular during the Reformation. Centers for clock production developed in Blois and Paris in France, Augsburg and Nuremberg in Germany, Geneva and London[119]. Most such elaborate clocks were commissioned by the nobility or wealthy, and the industry suffered greatly after the Peace of Westphalia (1648) ended the Thirty Years War, which led to the fragmentation of central Europe into many small principalities that could no longer afford such

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[115] Price, D. J. deSolla, "Gears from the Greeks. The Antikythera mechanism – a calendar computer from ca. 80 B.C.", Transactions of the American Philosphical Society, new ser., 64, pt. 7:5-70.

[116] Price, Derek de Solla, "On the Origin of Clockwork, Perpetual Motion Devices and the Compass, *Contributions from the Museum of History and Technology*,Washington, D.C., Smithsonian Institution, 1959, Bull. 218, Paper 6), pp. 81-112.

[117] King, Henry C., "Geared to the Stars", University of Toronto Press, 1978, p. 15.

[118] *ibid.* p.16.

[119] Bedini, Silvio A., "The Mechanical Clock and the Scientific Revolution", from *The Clockwork Universe. German Clocks and Automata 1550-1650*, Neale Watson Academic Publications, Inc., New York, 1980. p. 20.

luxuries. It is tempting to speculate that complex clocks were so popular at that turbulent time because they were a metaphor for order and logic, where a central, invisible authority causes the world to operate in mystical ways.

Monumental Astrolabe Clocks

A number of European cities invested heavily in monumental clocks as a point of civic pride and to impress the populace with the erudition of their leaders. Many of these monumental clocks had astronomical elements, the most popular of which was an astrolabe dial. The astrolabe dial is ideal for community clocks, since it links time with the Sun and sky. Such clocks usually included a hand representing the Sun rotating once in a day, an astrolabe rete rotating once in a sidereal day, and a hand with a moon figure showing the phase and approximate position of the moon.

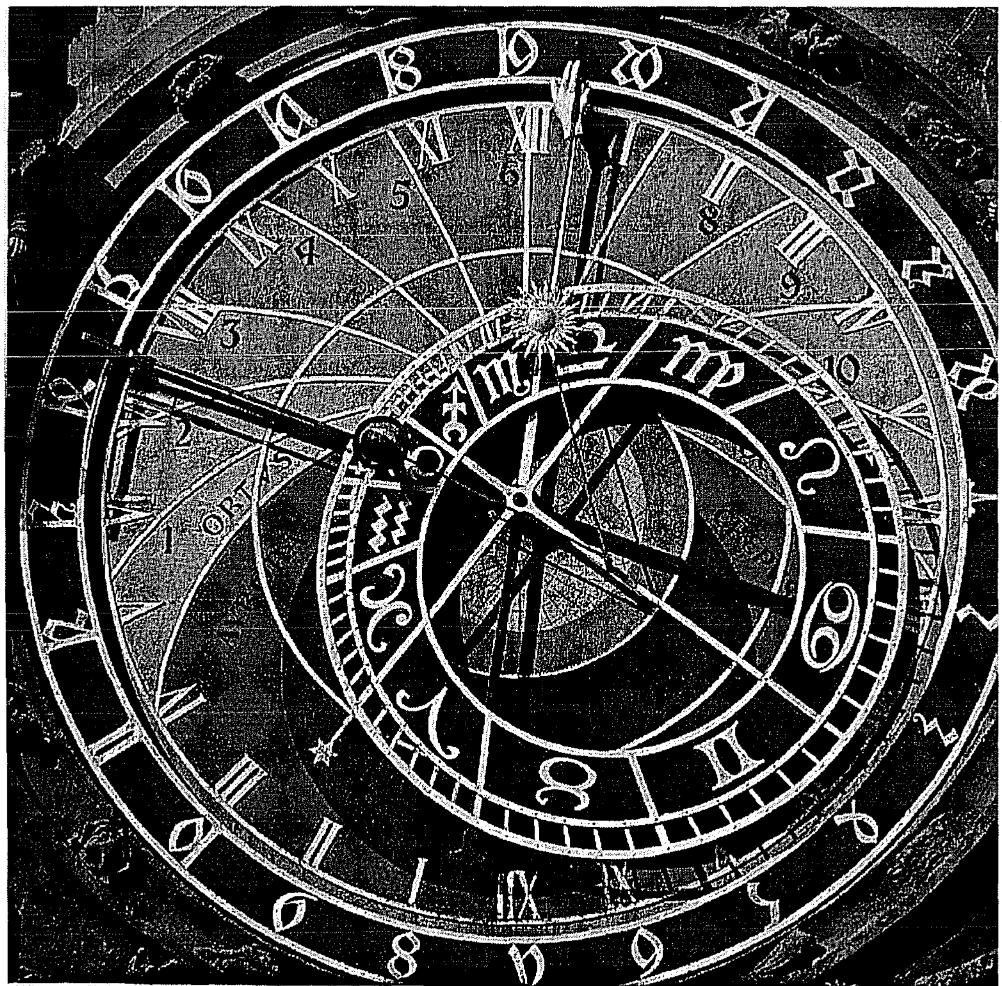


Figure 23-1. Prague Clock Astrolabe Dial

Anyone looking at the clock would be able to see the time, how high the Sun is in the sky, and the Sun's position in the zodiac at a glance. Most such clocks also had a hand showing the moon, and some of the more elaborate ones had mechanical figures performing on the hour.

The plate used on most monumental astrolabe clocks was somewhat different from the plate of an ordinary astrolabe in that it used a projection from the north celestial pole. This projection causes the horizon to curve in the opposite direction of the normal astrolabe horizon, but gives a more intuitive view of the Sun's current position.

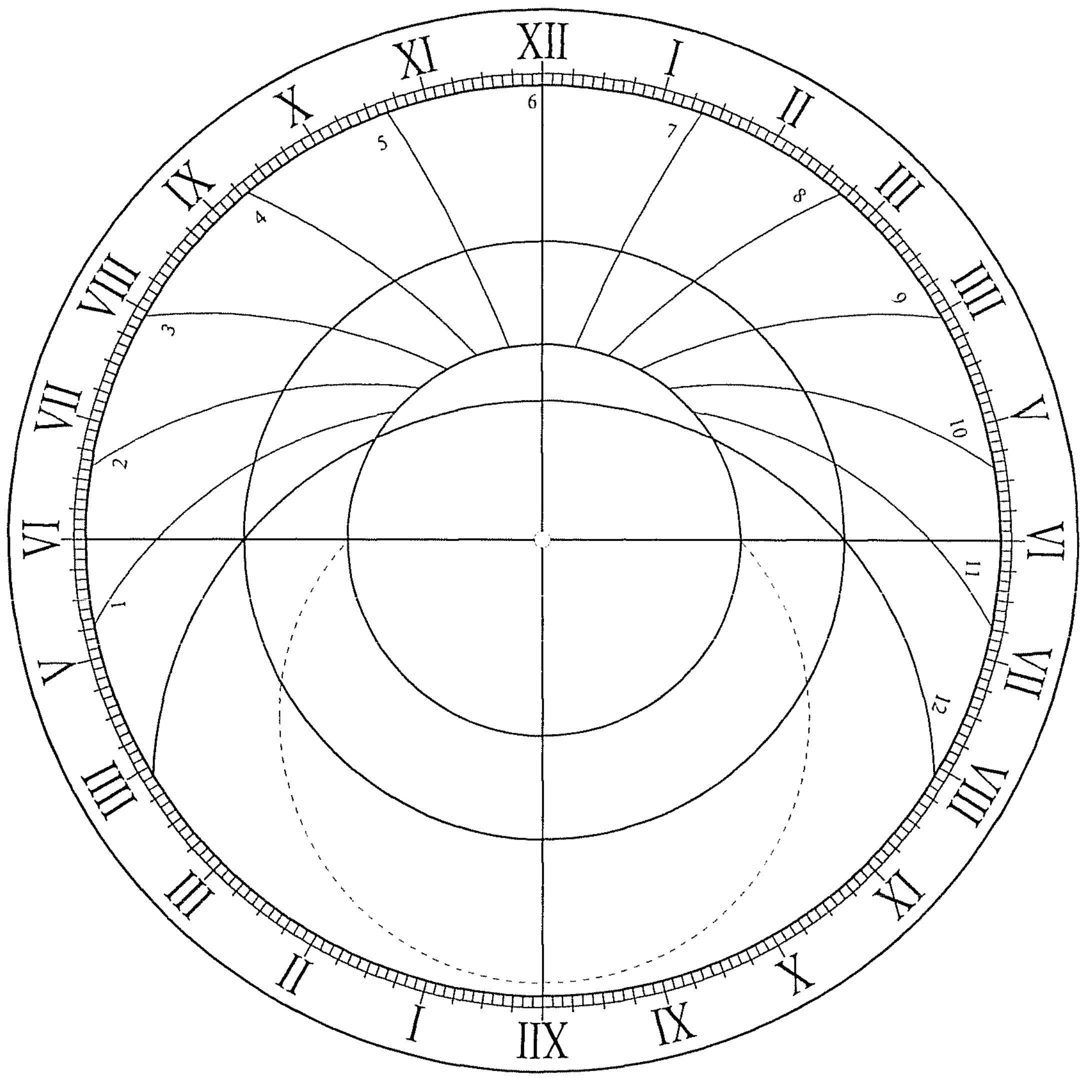


Figure 23-2. Monumental Astrolabe Clock Dial

Figure 23-2 is a representation of the features found on the dials of monumental astrolabe clocks. The dial in the figure is for the same latitude as the Prague clock, 50° 5′. The convex curvature of the horizon arc makes it easy for anyone with no knowledge of astrolabes to visualize the height of the Sun above the horizon. Altitude arcs are not included since the intention is to give only a general idea of the Sun's position, not an actual measurement. The dashed circle below the horizon is the normal 18° crepuscular arc. The fact that it is a full circle vividly demonstrates it never gets completely dark in the middle of summer at this relatively far northern latitude.

Most people counted time in unequal hours when these large clocks were built, but the equal hours shown by clocks were gradually coming into acceptance. The fact minutes were not indicated on the dial is both an acknowledgement of the rather irregular timekeeping quality of old clocks, and a testimonial to the casual attitude about the time of day before time ruled our lives. The unequal hours are shown on the clock dial above the horizon to give an immediate sense of the unequal hour during the day. It is unlikely the clock would be visible at night, since there was little or no community illumination at that time.

The Prague clock is an excellent example of this type of clock. The Prague astrolabe clock was originally constructed in about 1410 and has undergone numerous restorations and modifications. The clock is geared so the rete, which is divided by the zodiac into five-degree sections, rotates in a sidereal day. A hand with a Sun image riding on the ecliptic rotates in a mean solar day. The time, position of the Sun in the zodiac, sidereal time and the Sun's approximate altitude above the horizon can be seen at a glance. Another hand shows the phase and position of the moon.

Notice the zodiac is labeled in the reverse direction on the southern projection ecliptic as a result of the reversal of the projection origin.

The original clock probably had only an astrolabe dial with a concentric ring that was adjusted to Bohemian hours each day at sunset (i.e., 24 equal hours with the day beginning at sunset). Modifications were made in about 1490 to add a manually adjusted calendar. The calendar was given its own drive mechanism in 1566, and the limb was divided into the normal two 12-hour sections (the calendar is on a separate dial below the clock). A new striking mechanism and the moon hand were added in the mid-17th century. Moving figures of the Apostles were added sometime after 1659.

A major renovation was undertaken in 1865, when a new striking mechanism was installed, the calendar was repainted and a new escapement was added, replacing the old verge and foliot. The clock was seriously damaged during street fighting on May 8, 1945, but has been carefully restored. The diameter of the dial is 3.1 m, and the ecliptic diameter is 2.8 m.

Careful examination of the picture shows the time to be about 12:30 PM with the Sun in the beginning of Scorpio and the moon at about Capricorn 15°. We can conjecture the picture was taken at about 12:30 PM on Thursday, October 25, 1990. Sunset in Prague on October 25, is 5:07 PM.

Clock makers have struggled with methods of representing celestial movements with gears since the first clocks were made. It is a difficult problem and one not admiting to a simple solution. Basically, the problem is that we want the celestial sphere, the rete on an astrolabe, to rotate once in a sidereal day, and the clock hands to rotate in a mean solar day. A sidereal day is about 23 hours 56 minutes 4.09054 seconds, or about 3 minutes 56 seconds shorter than a mean solar day. The ratio of a sidereal day to a mean solar day has been approached with varying degrees of accuracy. The most accurate mechanical solution is likely on Jens Olsen's clock in Copenhagen. The old clocks were able to approximate the ratio well enough to satisfy most people.

The classic approximate ratio of a sidereal day to a solar day is 366/365. This means there would be exactly one more sidereal day than solar days in a 365-day year. This works out nicely on a clock, since the Sun would appear to go around the ecliptic in exactly one year of 365 days. The mechanism would need to be adjusted in leap years or when the relationship between mean time and the zodiac was a half-day in error.

The maker of the Prague clock used fairly simple gearing to rotate the Sun hand and the astrolabe rete at near the correct rates (Figure 23-3). The hands are mounted on tubes that are free to rotate around a shaft. The motion work moving the hands is driven by a lantern pinion of 24 teeth that rotates 15 ¼ (61/4) times per day. The pinion drives two wheels[120]. A wheel of 366

[120] Clock people call gears, 'wheels'.

teeth rotates once in 24 hours and is connected to the hour hand (the 24 tooth pinion rotates 15 ¼ times in a day = 366 teeth per day). The same pinion drives a wheel of 365 teeth connected to the rete. Therefore, the rete wheel rotates 366 teeth per day, so it gains one tooth per day and the Sun moves around the rete in a year. The rete gear will gain 365 teeth over the course of a 365-day year, so the wheels will be in the same orientation every 365 days. Note this gearing is a bit tricky to make. The Prague clock used teeth on the wheels filed to a triangular shape. Clearly, this mechanism had a lot of "slop" in the train that would not be acceptable to modern technicians. The fact that there are two concentric wheels with different counts driven by a single pinion presents practical problems of tooth pitch and depth.

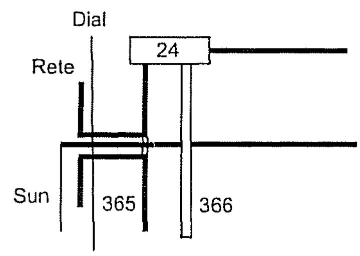


Figure 23-3. Prague Clock Gearing

Other existing clocks with astrolabe dials are at Lund Cathedral, Sweden (1380), Berne, Switzerland (1530), Münster Cathedral (1540) and Strasbourg Cathedral, France (first clock 1354). King discusses a number of other clocks that no longer exist.

Personal Clocks with Astrolabe Dials

Smaller clocks with astrolabe dials usually incorporate a normal astrolabe plate and rete with gearing that produces more-or-less correct rotational rates. It is beyond the scope of this work to attempt to cover all of the methods used to produce the solar/sidereal rate clock gearing. Instead, we will cover one popular method that worked rather well.

One of the earliest smaller clocks, which was a floor standing unit incorporating six astronomical dials, was made by Giovanni de 'Dondi of Padua ca. 1380. King discusses the epicyclic gearing of the sidereal rate mechanism.

Augsburg in Bavaria became a center of complex mechanical device manufacture in the 17th and 18th centuries and many beautiful table clocks with astrolabe dials were produced. The gearing used in the Augsburg clocks used differential-epicyclic gearing. It is possible to produce gear ratios with this type of gearing that would require a large number of wheels in a normal gear train.

A representation of the Augsburg gearing is shown in Figure 23-4. Only the part of the train used for the astrolabe rete and Sun hand are shown. Differential-epicyclic gearing uses the idea that a wheel mounted on and moving with a wheel can turn at a rate controlled by the difference in the rates of two wheels. This allows very small differences in rotational speed to be amplified and used to control other wheels. Differential-epicyclic gearing can produce gear ratios that would require many wheels with conventional gearing. The oldest such gearing was found the Antikythera machine. Production in complex clocks and machines declined in Augsburg due to the Thiry Years War and this type of gearing was not revived until the mid-18th century.

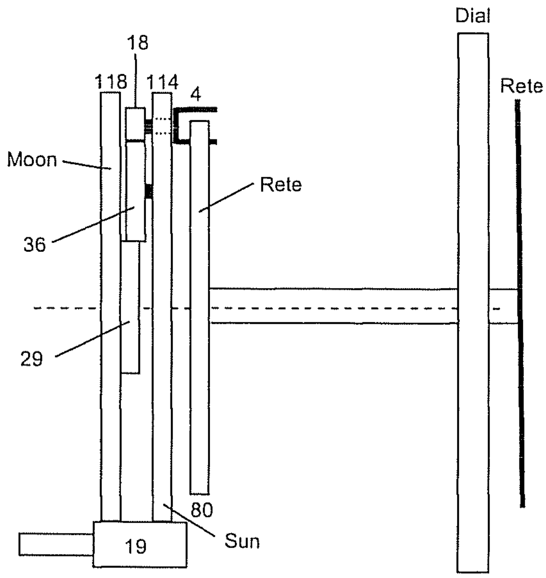


Figure 23-4. Augsburg Differential-Epicyclic Gear Train

The Augsburg train works as follows:

1. The train has two driven wheels of 118 and 114 teeth driven by a pinion of 19 rotating six times a day. The wheel of 118 is connected to a hand showing the moon and will not be discussed further. The wheel of 114 rotates in 24 hours and is connected to the Sun indicator.
2. Integrated into or attached coaxially to the moon wheel of 118 is a wheel of 29. This wheel engages an eccentric wheel of 36 attached to the body of the Sun wheel of 114.
3. The eccentric wheel of 36 engages a wheel of 18 that has a pinion of four leaves. The arbor for the wheel of 18 passes through the Sun wheel. The pinion drives the rete wheel of 80 which turns freely on the central shaft. The rate of the rete wheel is the rate of solar wheel plus the rotational rate of the pinion.
4. The moon wheel (118) rotates slower than the solar wheel (114) by 118/114 × 24 hr giving the time of rotation of the moon indicator of 24h 50m 32s, almost. Therefore, the moon indicator moves slowly counterclockwise relative to the Sun; i.e. the moon will appear to rise a bit later each day (the true mean lunar day is 24h 50m 28s).
5. The wheel of 29 connected to the moon wheel causes the epicyclic idler wheel of 36 to turn at the rate of the difference in speed of the Sun and moon wheels. Each rotation of the Sun wheel causes wheel 29 to lose (118-114)/118 = 2/59 revolution.
6. This movement is imparted to idler 36 and to pinion 18+4 as 29/36 × 36/18 = 29/18. For each revolution of wheel 29, pinion 18+4 turns 29/18 revolution.
7. In a day (one rotation of the Sun wheel), pinion 18+4 turns 29/18 × 2/59 = 28/531 revolution.
8. Pinion 19+4 is pushing the rete wheel around plus the rotation of the pinion. Therefore, in one revolution of the Sun wheel, the rete wheel rotates 24 h + 4/80 × 29/531 = .00273 × 24 hr. This translates to 3m 55.9s or a total rate of 23h 56m 4.1s which is a good approximation of a sidereal day.

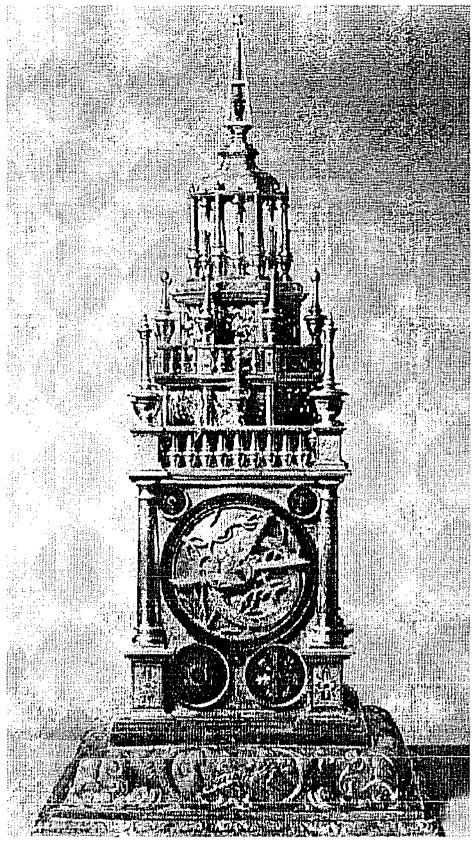


Figure 23-5. Augsburg Clock with Astrolabe Dial

The clock in the figure was made by Nikolaus Rugendas the Younger.

In summary, this gear train is capable of reproducing the solar/sidereal rates rather closely, assuming the rest of the clock operates at the required rate, which is not a good assumption on the old clocks but could be today using modern materials and methods.

The Ulysse-Nardin "Astrolabium Galileo Galilei"

One of the most beautiful and certainly the smallest astrolabe time-keeper is the astrolabe wrist watch called the "Astrolabium" from the prestigious Swiss watch manufacturer, Ulysse-Nardin.. This watch was described by a Ulysse-Nardin executive as "wearable art," a term that certainly applies.

The Astrolabium is a fully functional automated astrolabe that includes a rete rotating at the sidereal rate, a lunar position indicator, lunar nodes hand, and a day/date calendar as well as hands for normal timekeeping.

The prototype of the Astrolabium watch was developed by Dr. Ludwig Oechslin of Lucerne, Switzerland in 1983 under commission from Ulysse-Nardin[121]. It has been offered with no significant mechanical changes since it was introduced to the public two years later at the Basel watch fair.

[121] "The Trilogy of Time", Ulysse-Nardin, Le Locle, Switzerland, p.22

The self winding movement uses epicyclic-differential gearing to reproduce the astronomical rates and requires the use of space-age materials to satisfy the temperature stability and high strength with low weight requirements of a wrist watch. The astronomical train runs on tiny ball bearings to reduce friction sufficiently to allow such a complex movement to be driven by the going train. Ulysse-Nardin says the gearing is accurate to one day in 144,000 years, which is one hour in 6,000 years or one minute in 100 years[122].

The "Astrolabium" uses a normal northern stereographic projection for the plate and rete. The plate for your latitude is installed when you order the watch. The horizon, a crepuscular arc for -18°, almucantars for 30° and 60°, azimuth arcs for 30°, 60° and 90° (the prime vertical) and unequal hour arcs are included on the plate. The watch in Figure 23-6 is from a special platinum series and does not include the unequal hours. They are included on the normal gold case watch model.

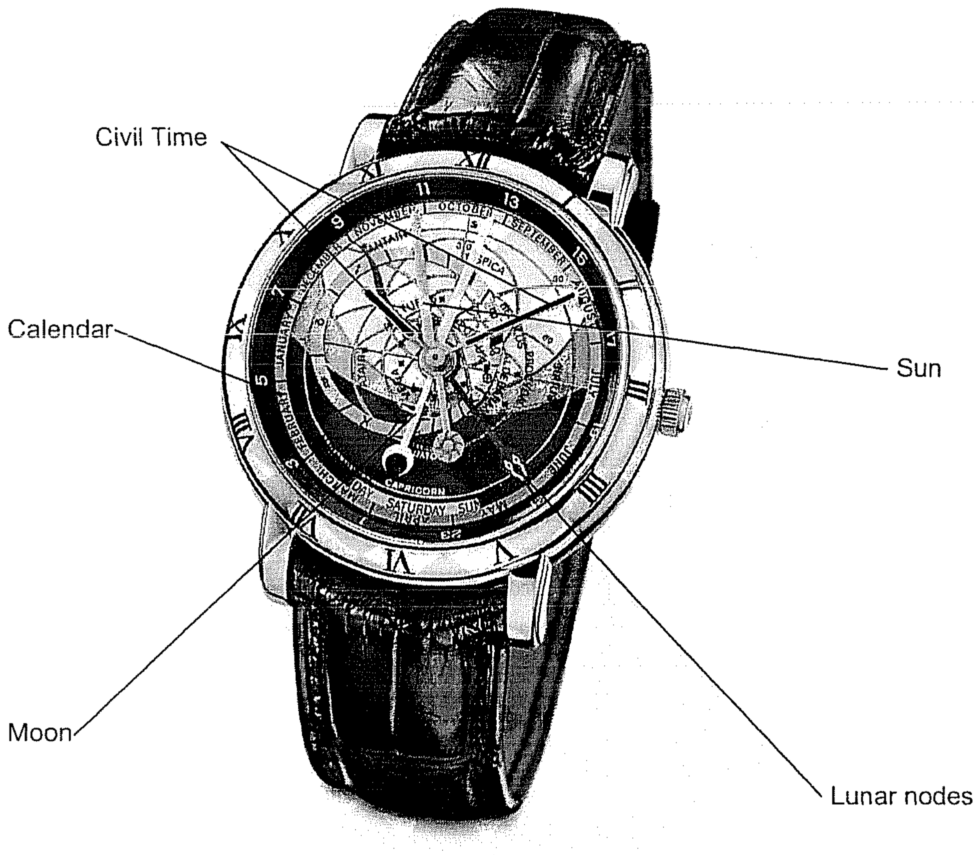


Figure 23-6. Ulysse-Nardin "Astrolabium"

The rete includes the ecliptic divided into the twelve zodiac sections and 16 stars.

Civil time is shown by normal watch hands.

122 *ibid*, p.28.

The Sun hand serves two purposes. It shows the Sun's position in the ecliptic and also points to the date on the calendar. The calendar ring rotates at the sidereal rate with the rete and the Sun hand rotates at the mean solar rate. The Sun hand serves the same purposes as the rule on an astrolabe including showing the unequal hour.

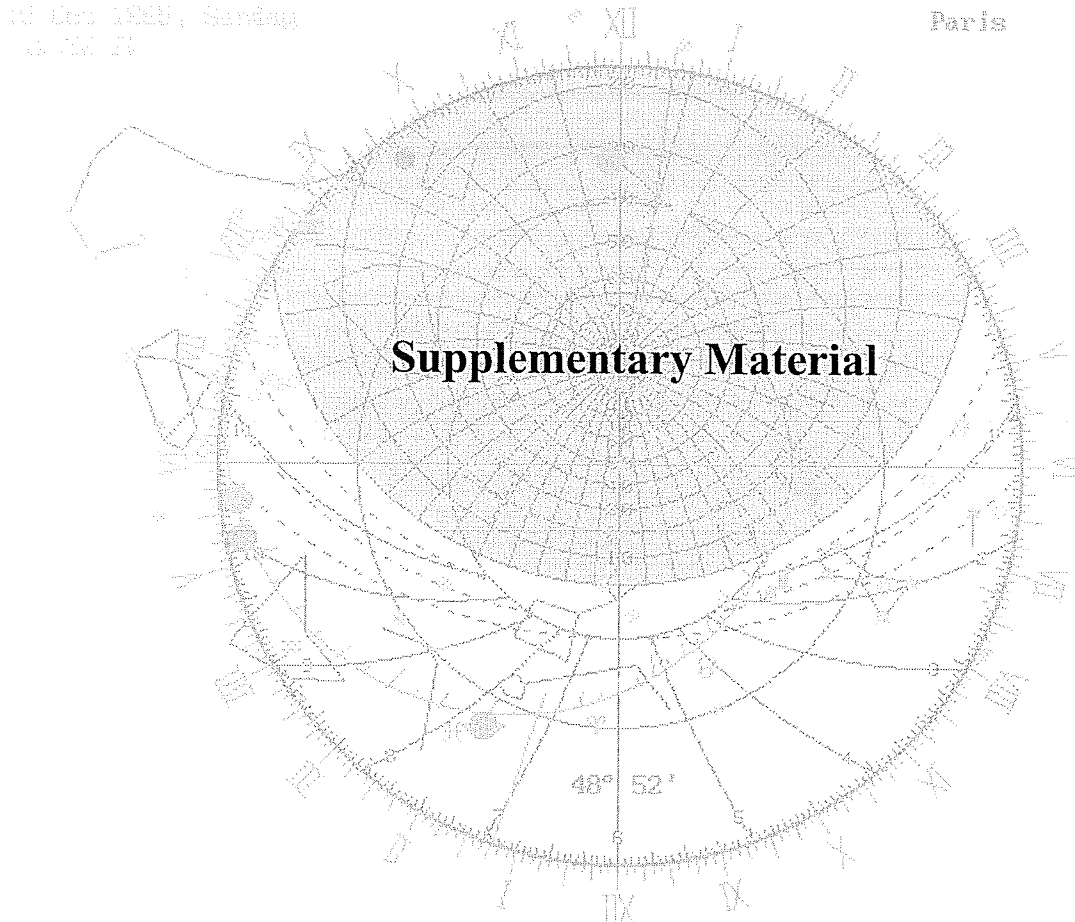
The moon hand rotates at the lunar mean synodic rate and can be used to estimate the moon's phase and it also indicates conditions when an eclipse might occur. You can also watch the moon rise, culminate and set.

The lunar nodes hand rotates once in a draconitic month and is used to estimate eclipse conditions. An eclipse can occur when the moon is new or full and the moon is near one of its nodes. A lunar eclipse occurs at full moon and a solar eclipse at new moon. An eclipse may occur when the Sun, moon and lunar node hands are all aligned. The moon's actual motion is very complex, and the Astrolabium can only show averages so the indications are an approximation.

The Astrolabium in the picture is set for about 10:12 AM on Saturday, November 2. The Sun hand shows the Sun is about halfway through Scorpio. The moon is a little west of the meridian at an altitude of about 45°. We can infer from the moon's position relative to the Sun that the moon will be a small waning crescent. With a little more effort we can determine the Astrolabium is set for November 2, 2002, from the moon's position relative to the Sun.

Procyon is setting, Altair is rising and Spica is just past culmination. The moon is not near a node so there will not be an eclipse this month.

The Astrolabium is set with a three-position crown. The first position is for manual winding, the second position adjusts the Sun hand and all other astronomical settings and the third position sets the current civil time.



Supplementary Material

Chapter 24 - Astronomical Background

The astrolabe is an astronomical instrument so there are some astronomy terms you need to understand before you can completely understand the astrolabe. Astronomers have a habit of using complicated sounding words for simple ideas. Don't let the words scare you. The ideas are very easy. The following discussion is very basic and concentrates on the historical elements of postional astronomy. A good astronomy text will expand and clarify the subject.

The Celestial Sphere

The Earth can be considered to be in the center of an enormous sphere of indeterminate but very large radius. The celestial sphere appears to rotate on an axis about the North Pole due to the Earth's rotation. There are three planes that define great circles on the celestial sphere from which most celestial positions are measured: the equator, the ecliptic and the local horizon. The celestial equator is perpendicular to the axis of the Earth's rotation and 90° south of the north celestial pole. For all practical purposes, the celestial equator can be considered to be the extension of the Earth's equator. The ecliptic is the path of the Sun on the celestial sphere. The local horizon is the plane perpendicular to the zenith for a specific location on the Earth. The coordinates of objects in the sky can be measured from the equator, the ecliptic or the horizon.

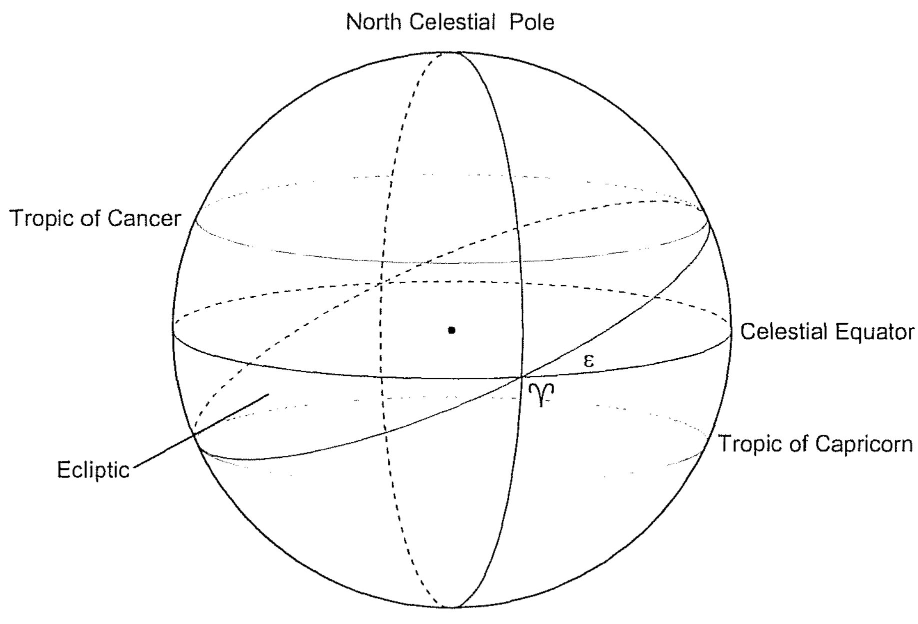


Figure 24-1. The Celestial Sphere

Figure 24-1 shows the principle circles on the celestial sphere. The celestial equator is, for our purposes, an extension of the Earth's equator. The ecliptic and celestial equator meet at the vernal equinox, which is also known as the 'First Point of Aries' and is associated with the ♈ symbol. Right ascensions and celestial longitudes are measured from the vernal equinox.

The angle between the equator and the ecliptic is the *obliquity* of the ecliptic and is shown by ε.

The Tropic of Cancer is a small circle on the celestial sphere, parallel to the celestial equator, that marks the northern limit of the Sun. Similarly, The Tropic of Capricorn marks the Sun's most southerly limit.

The basic measurement systems used in positional astronomy will now be discussed. Celestial postions can be specified with the Earth as the center (geocentric) or with the Sun at the center (heliocentric).

Equatorial Coordinates

Equatorial measurements use the celestial equator as the reference plane. The position of the Sun and stars are usually shown in equatorial coordinates. Note equatorial coordinates are always geocentric.

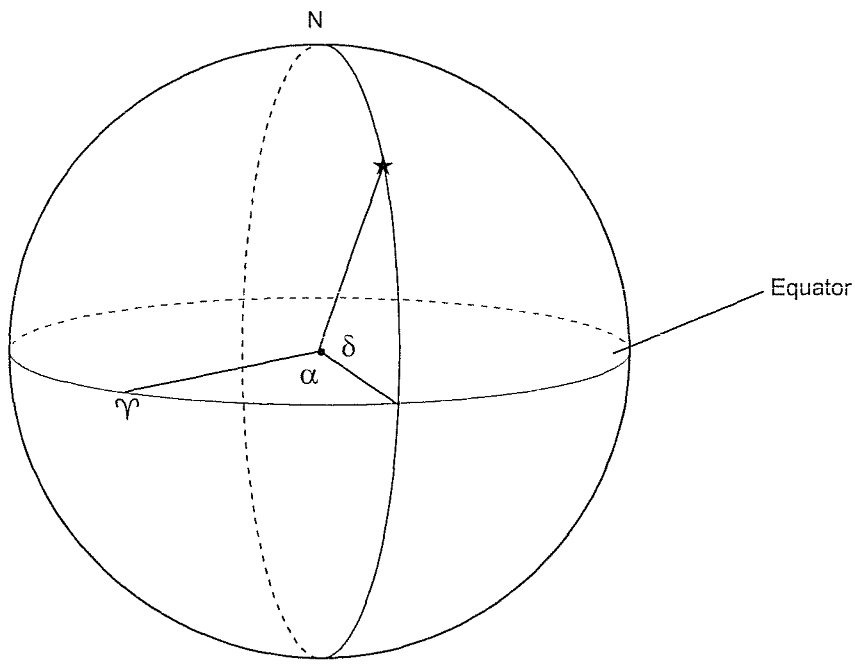


Figure 24-2. Equatorial Coordinates

Star positions are measured in ways very similar to the way latitude and longitude are measured on Earth. The latitude of a place on the Earth is the angle of the place from the equator measured from the center of the Earth. If you live in a place with latitude 39° north, a line drawn from your home to the center of the Earth would make an angle of 39° with the plane of the equator. The equator makes a convenient place from which to measure latitudes since it is easy to figure out where it is and the position of the equator does not change.

Longitude is more difficult to measure on the Earth, because there is not a natural place to call 0°. By international agreement, the world has agreed that 0° longitude is the meridian of the Old Royal Observatory in Greenwich, England, not far from London. There is nothing special about Greenwich making it ideal for being 0° in longitude, and it was chosen mainly because many of the navigation charts in use when the decision was made were based on that location. Other places have been used. In the 18th century, the French navy used navigation tables with Paris as 0° longitude. The marker of the zero point can still be seen on the Île de la Cité.

A star's position is measured just like latitude and longitude on Earth, but with a different name. The angle of a star above or below the celestial equator is called its declination (δ). Declination is measured just like terrestrial latitude except instead of saying so many degrees north or south, declination is positive or negative. Positive declinations mean the object is north of the celestial equator and negative declinations mean it is south of the equator. It is not called latitude because, in astronomy, latitude means something completely different that will covered later. The star Vega has a declination of 38° 47′. This means Vega is 38° 47′ north of the celestial equator.

A more formal definition of declination is the angle of a celestial body from the celestial equator with north taken as positive.

The problem of picking a zero point for measuring the astronomical equivalent to terrestrial longitude is easier than it was on Earth because there is a natural place to call the zero point. The Sun is exactly on the celestial equator twice a year, in the spring and fall. The days when the Sun is on the equator is called an equinox because the length of day and night are equal on these dates. The point where the Sun crosses the equator in the spring, the vernal equinox, is the reference point for a star's east-west location. This is a good place to start measuring star positions because it is a very well defined location and can always be found.

The east-west position of stars is called right ascension (α). The name "right ascension" derives from the most difficult problem in ancient mathematical astronomy, finding the length of the day as a function of the latitude of a place. This problem was called the "rising time problem" because the solution involved figuring out how long it takes a given number of ecliptic degrees to rise above the horizon. An important element of the problem was the rising times of the ecliptic when viewed from the equator (*sphaera recta*). The rising time for a section of the ecliptic is found directly at the equator. The "rising time at sphaera recta" in Latin is "ascensio recta" or right ascension. Because of its relation to rising times, right ascension is measured like time, in hours, minutes and seconds. Right ascension is measured on the equator and ranges from 0h at the vernal equinox to 24h, increasing to the east. 1h right ascension is 15° **east** along the equator. 2h is 30° east of the vernal equinox and so on. The right ascension of Vega is 18h 36m 56s which means Vega is a little over 279° east of the vernal equinox, measured on the celestial equator. Star coordinates are almost always given by the star's declination and right ascension.

Right ascension is formally defined by a great circle passing through the celestial poles and the object. The angle, measured on the equator, from the vernal equinox to the intersection of this great circle with the celestial equator is the right ascension. The angle is expressed in units of time by dividing by 15°/hr.

A measurement related to right ascension is the hour angle of a celestial object. The hour angle is the angle on the equator, of the great circle passing through the poles and the object, measured from the local meridian, with west taken as positive. Hour angle is expressed in units of time. Local sidereal time is the hour angle of the vernal equinox.

Ecliptic Coordinates

We know today that the Earth orbits the Sun in a slightly elliptical orbit. The plane defined by the Earth in its orbit around the Sun is called the **ecliptic** (because eclipses occur only when the moon is near this plane).

The rotational axis of the Earth is tilted slightly (currently about 23° 26′) to the plane of the ecliptic. The angle of the tilt of the Earth's axis is called the obliquity of the ecliptic (ε). The tilt of the Earth's axis causes the Sun to be higher in the sky in part of the year and lower in sky at other times. The equator and ecliptic meet at the equinoxes. The Sun's declination will be equal to plus or minus the obliquity of the ecliptic at the solstices. The maximum northerly declination of the Sun is called the Tropic of Cancer and the maximum negative (southern) solar declination is called the Tropic of Capricorn.

Celestial positions can also be measured on the ecliptic. See Figure 24-3. The angle of a celestial object above or below the ecliptic is called its latitude (β) with north taken as the positive direction. The angle of the intersection of the great circle on the celestial sphere passing through a celestial object and the ecliptic poles, measured on the ecliptic is the object's longitude (λ). Longitude is measured from 0° to 360°, increasing to the east from the vernal equinox. The positions of the Sun and planets are typically given by latitude and longitude although equatorial coordinates are also used. Ecliptic coordinates can be either heliocentric (Sun centered) or geocentric (Earth centered). The geocentric longitude of the Sun equals the heliocentric longitude of the Earth ± 180°.

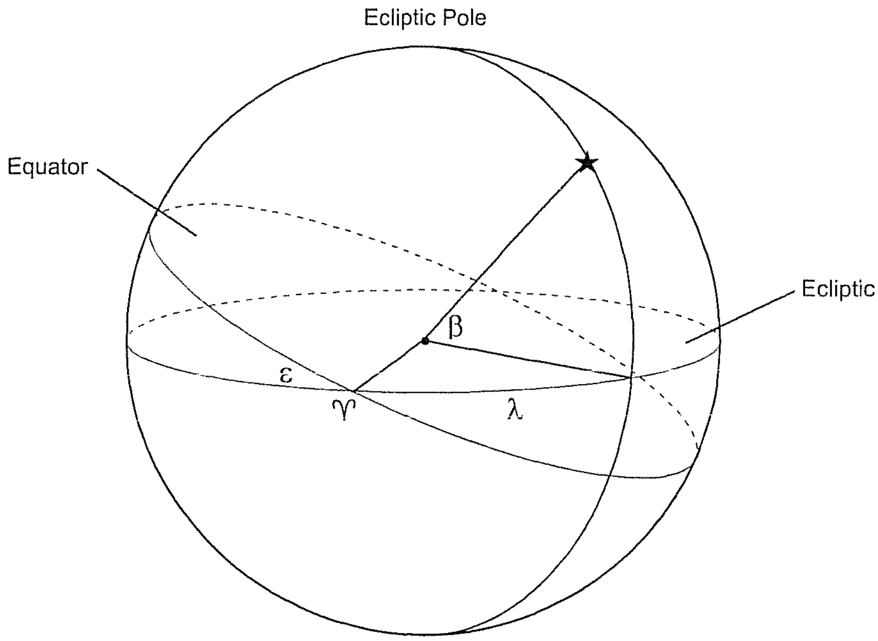


Figure 24-3. Ecliptic Coordinates

Mediation

Medieval and Renaissance astronomical tracts often use a hybrid coordinate system that mixes equatorial and ecliptic coordinates somewhat to describe celestial positions, probably for simplicity in laying out astrolabes. The coordinates used were declination and mediation (*coeli mediatio,* literally "measure of the sky"). The declination used is the normal definition. Mediation is the point on the ecliptic that crosses the meridian at the same time as a star; i.e. has the same right ascension. Longitude is measured on the ecliptic. Mediation is measured on the equator. Mediation was usually expressed in terms of the zodiac, the point on the zodiac with the same right ascension as a star, which can get a little confusing.

Figure 24-4, shows the mediation of Arcturus. Arcturus' right ascension is about 14 hr 16 min. Arcturus is on the meridian in the figure. The ecliptic degree on the meridian at the same time is about Scorpio 6°.

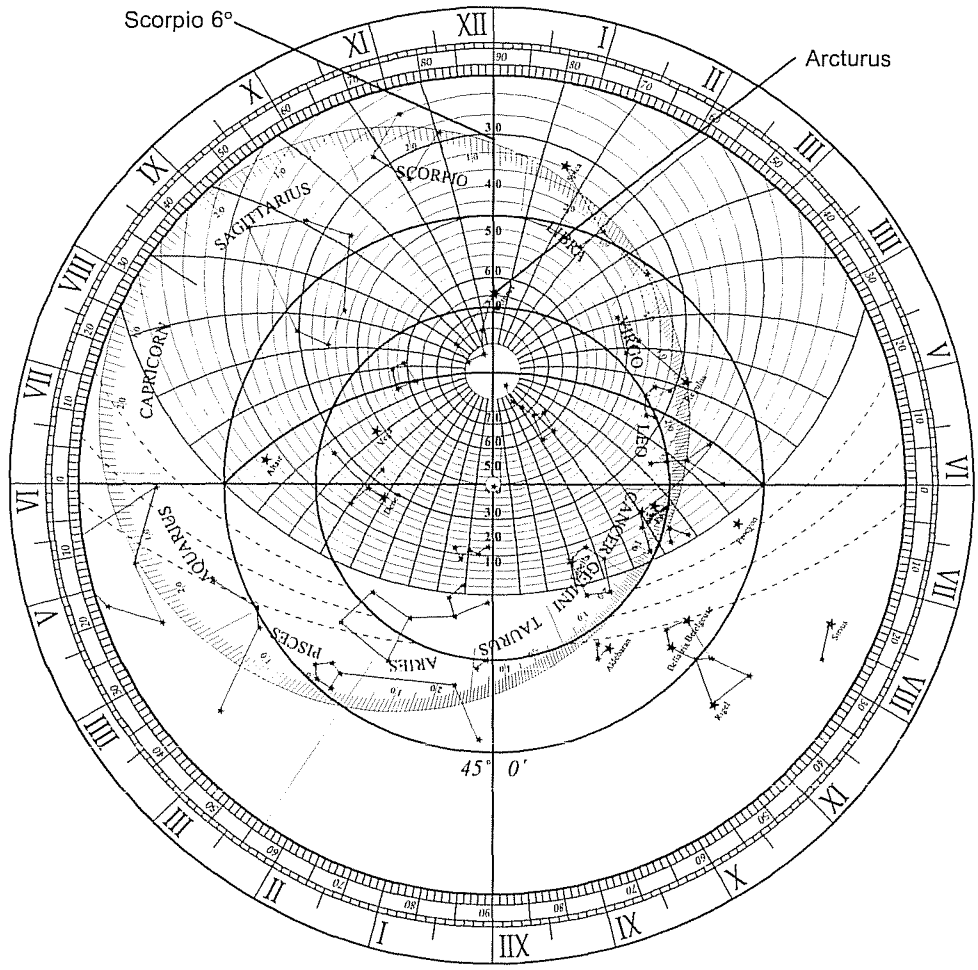


Figure 24-4. Mediation of Arcturus

A star's mediation is calculated by finding the ecliptic longitude, λ, corresponding to a star's right ascension, α : **tan λ = tan α / cos ε**. The result is then converted to a zodiac position.

Continuing the example above, take ε = 23° 26′ 25″ = 23.4403°. Arcturus' right ascension = α = 14 hr 15 min 39.6 sec.= 14.261 hr × 15 °/hr = 213.915°. Solving the equation above the mediation is 216.235° ≈ Scopio 6.2°.

The Sun's Position

Much astronomical thinking has been handed down over thousands of years and it helps us understand why things are measured as they are by knowing a little about how the ideas were developed long ago. Our ancient ancestors discovered the ecliptic by watching the Sun rise and set over a long period of time. Just before sunrise and just after sunset you can see the stars that are close to the Sun. With careful observation you notice the Sun moves among the stars a little each day but it always follows the same path and always returns to the same place in the stars after a full year. The path the Sun takes through the stars is called the ecliptic and was so named long before it was realized that the Earth orbits the Sun. With more careful observation you notice some "stars" also move and they are never far from the Sun's path. The "stars" that move are the planets (the word planet is based on the Greek word for "wanderer"). In ancient civilizations, they knew more about the ecliptic than they did about the equator.

It might be interesting to note that ancient civilizations were very good at geometry and were fascinated by regular geometrical figures such as the equilateral triangle. Each of the angles of an equilateral triangle is 60°, and six equilateral triangles will fit into a circle. Most ancient civilizations used base 60 numbers instead of our base 10 numbers, and our decimal 60 is 1,0 base 60. Six triangles with angles 1,0 in a circle has undeniable appeal and is probably why we say a circle has 360°.

Almost all ancient civilizations used a calendar based on the Moon. A lunar month starts when the first tiny crescent after a new moon can be seen just at sunset. Lunar months are 29 or 30 days long and there are twelve lunar months in a year. This is a natural, if irregular, way to define the calendar. The Hebrew and Islamic calendars in use today are lunar calendars.

About 550 BC, our ancestors divided the ecliptic into twelve sections to match the number of months in the year. These ecliptic divisions are each 30° in celestial longitude wide. The collection of the 12, 30° sections of the ecliptic is called the zodiac. The divisions were named for constellations near each of the divisions at that time. The names given to the twelve divisions (Aries, Taurus, Gemini, Cancer, Leo, Virgo, Libra, Scorpio, Sagittarius, Capricorn, Aquarius, Pisces) have stuck even though the constellations that originally gave them their names have moved to other parts of the sky (see precession below). It is very important to understand that the names of the sections of the zodiac have nothing to do with astrology and also have nothing to do with where the constellations of the same name are now. The zodiac is merely a convenient way to divide the ecliptic into equal sections of longitude. If you look at a star chart, you will see that the constellations that gave the zodiac signs their names are not very close to the section of the zodiac with the same name. For example, the constellation neatly centered in the section of the zodiac called Aquarius is the constellation Capricorn. The stars have precessed that far since the zodiac was named.

If you take no other fact from this book, please remember this one: the zodiac has nothing to do with astrology or the current positions of any constellation. It is only a convenient way of dividing the ecliptic by longitude. Astronomers (and astrologers) used the zodiac to specify the positions of celestial bodies. For example, saying Mars is at Leo 10° is the same thing as saying Mars' longitude is 130° and nothing more.

The Sun moves along the ecliptic through the year in a very predictable way. If you know the date you can figure out where the Sun is on the ecliptic and if you know the Sun's longitude, you can figure out the date. The astrolabe uses the position of the Sun in the ecliptic to tell time.

The backs of many astrolabes use a very old model of the Sun to define the calendar to solar longitude (zodiac) conversion scales. It can be shown that the position of the Sun in the sky can be equally well described by considering the Earth to orbit the Sun or by Sun orbiting the Earth. This is why the ancient Earth centered models of the Sun worked so well.

Observation of the daily change in the Sun's longitude reveals the Sun's longitude increases faster for part of the year and slower in the other parts. It was recognized sometime in the 3rd century BC that the variation in the speed of the Sun could be represented by eccentric motion. In this model (Figure 24-5), the observer is at T. The Sun, S, rotates at a constant rate equal to its average daily motion on a circle centered at O. The distance between the centers, ε, is the eccentricity expressed as the fraction of the radius of the eccentric. *Aphelion* (A) is the point where the Sun is farthest from the Earth. *Perihelion* (P) is the point where the Sun is closest to the Earth. The line connecting A and P is the *line of apsides*. To an observer at T, the Sun will appear to speed up and slow down over the course of the year in a manner very close to observations.

The basic notion that makes the eccentric motion model necessary is the ancient metaphysical belief that celestial objects **must** travel in circles at a constant velocity. Any other notion is counter to Aristotelian philosophical constructions that define the circle as the perfect plane figure. Since the universe must be perfect it follows that the Sun and planets must follow the perfect path. Since the circular model does not match observations it was necessary to construct a model using perfect movement but still "preserved the phenomena" (i.e. allowed one to calculate positions without regard to what is physically happening).

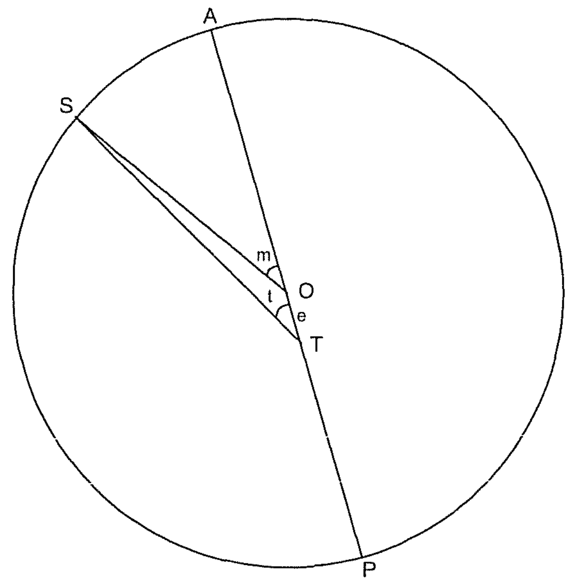


Figure 24-5. Eccentric model of Sun's motion.

Angles measured from the line connecting aphelion and perihelion (the line of apsides) are called *anomalies*. By convention, modern astronomers measure anomalies on elliptical orbits from perihelion. The ancients measured anomalies from aphelion. Either system is equally valid. m, the *mean anomaly*, is the angle of the Sun on the eccentric circle from the line of apsides and increases at a constant rate by the definition of the eccentric. This definition is the same for both elliptical and eccentric orbits. t is the *true anomaly* and is the angle of the Sun from the line of apsides as seen by an observer at T (the Earth). The difference between the mean and true anomalies (the angle at S) is called the *equation of the center*. It can be shown[123] that the true anomaly in this model is an excellent representation of reality for the Sun. For an elliptical orbit of eccentricity e′, the true anomaly can be expressed as:

True Anomaly (ν) = $M + 2e' \sin M + 5/4\, e'^2 \sin 2M + \ldots$

where **M** is the mean anomaly of the elliptical orbit.

The true anomaly of the eccentric model is:

True Anomaly (t) = $m + e \sin m + \frac{1}{2} e^2 \sin 2m + \ldots$

[123] Neugebauer, Otto, *A History of Ancient Mathematical Astronomy*, Springer-Verlag, New York (1978), pp. 1098-1102. Also see Smart, W.M., *Textbook on Spherical Astronomy*, Cambridge, New York (1977). pp. 119-120.

The two equations above give similar answers for small values of e if the eccentricity of the eccentric model is expressed as twice the value of the elliptical model (e = 2e'). The eccentricity of the Sun's orbit is only about 0.017 and the agreement is good for such a small eccentricity. The maximum error is ¾ e^2.

The eccentric model of the Sun's motion is the basis for the calendar/zodiac conversion scales on the back of the astrolabe and the ecliptic divisions on the rete. In practice, the zodiac scale is drawn and evenly divided into 360° with the vernal equinox at the 3 o'clock position.

The location of the line of apsides is defined by the longitude of aphelion on the zodiac.

The center of the calendar circle is offset from the center of the instrument by 2e in the direction of perihelion and the calendar circle is drawn[124].

The date of the vernal equinox is marked and the calendar circle evenly divided into 365 days from that point (analytically, the calendar is rotated about its center by an angle equal to the mean anomaly of January 0). The observer is at the center of the instrument so the Sun rotates around the calendar circle and the alidade indicates the Sun's true anomaly on the longitude (zodiac) scale. The *astrolabiste* took the values from tables or instruction books.

The values used for e and the longitude of aphelion were determined from a construction as in Figure 24-6[125]. This construction was used for centuries from Ptolemy to Copernicus. In the figure, M is the center of the Sun's circular orbit around the Earth. O is location of the observer on the Earth. O and M are offset by the eccentricity of the construction, OM. The Sun's longitude at any time is the angle from the vernal equinox. Aphelion is the point on the Sun's orbit when it is the greatest distance from O at A. The objective is to find the values of the longitude of aphelion (λ_A) and the eccentricity from the lengths of the seasons.

The procedure is to find the number of days from the vernal equinox (♈ 0°) to the summer solstice (♋ 0°) and the number of days from the summer solstice to the autumnal equinox (♎ 0°) which are converted to degrees (α_1 and α_2). The figure is then solved for e and λ_A. Ptolemy used values of 94½ days for the spring and 92½ days for the summer, Ptolemy found e to be 1/24 and $\lambda_A = 65.5°$.

The same problem can be solved using modern values for the lengths of the seasons to test its validity.

ΔHFM is isosceles with base angles δ_1 and apex angle 360 – (α_1+α_2), therefore $\delta_1 = [(\alpha_1+\alpha_2)-180]/2$. $\delta_2 = \alpha_1-90-\delta_1$.

OM' = R sin δ_2. MM' = R sin δ_1. OM =$\sqrt{(OM'^2 + MM'^2)}$. sin λ_A = MM' / OM and e = OM / R.
R was taken as 60.

Solving this figure for the year 2006 using values from a readily available astronomy computer program, the number of days of spring = 92.7505 and the days of summer = 93.6515. The mean daily motion is 0.98564 °/day, giving $\alpha_1 = 91.4184°$ and $\alpha_2 = 92.3064°$. $\delta_1 = 1.8624°$ and $\delta_2 = -0.4440°$. OM' = -0.4650, MM' = 1.9500 and e = 0.03341. sin λ_A = 1.9500/2.0046 = 0.97273 and λ_A = 76.588° or (180° - 76.588°) = 103.412°. The figure has changed in the nearly 2000 years since Ptolemy and α_2 is now greater than α_1. Therefore, M must be in the second quadrant relative to O. Hence, $\lambda_A > 90°$ and λ_A = 103.412°.

[124] The eccentricity, e, in this context is the eccentricity of the Earth's elliptical orbit.
[125] Neugebauer, *op cit.* pp. 56-59.

The eccentricity of the Ptolemaic model is twice the value for the elliptical orbit. The elliptical eccentricity derived = 0.03341/2 = 0.016705. The correct values are e = 0.016706 × 2 = 0.033412 and λ_A = 103.044°. The small differences are primarily due to the fact that the eccentric circular motion is not an exact model for the elliptical motion or consider perturbations in the Earth's orbit due to the gravitational influences from the other planets.

The perihelion and aphelion are precessing, advancing eastward along the ecliptic about 11 arcseconds per year in relation to the fixed stars. This is the difference in arcseconds between the sidereal year and the anomalistic year. At this rate, they will take about 114,000 years to make a complete revolution.

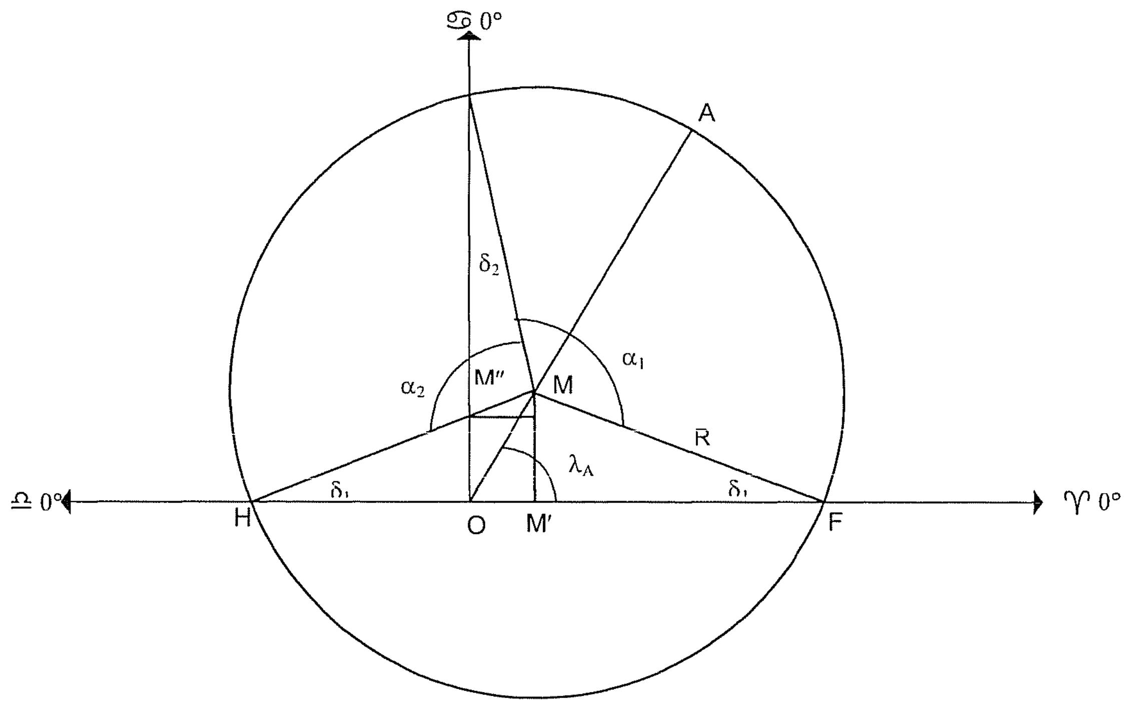


Figure 24-6. Determining eccentric model parameters.

Time

We take timekeeping for granted. All of us have known how to tell time since we were small children. If you want to know what time it is, just look at your watch or glance at any of the hundreds of clocks that surround us. If you know what time it is where you are, you can easily figure out what time it is just about anywhere in the world by adding or subtracting a few hours. It was not always this simple.

Through all of human history, until the last 150 years or so, time was told by the Sun. Noon was when the Sun was due south of wherever you happen to be. The path of the Sun for any given day traces out an arc in the sky and the highest point of the arc is when the Sun is due south. Noon has always been an important time because it signals time for lunch! The length of the day is measured from noon to noon because the Sun can be seen at noon and it can't be seen at midnight.

The north-south line through a place is very important in astronomy. It is called the *meridian*. When the Sun crosses your meridian it is called *meridian passage* or *culmination*. Culmination means the Sun reaches its maximum altitude for the day when it crosses the meridian.

The tricky part about telling time from the Sun is that the direction of south is different for different places. Every place with a different longitude has a different meridian. If one place is west of another place, its noon will be later because it takes the Sun a little more time to reach due south for the place farther west. As a result, different towns had different times. This was a minor inconvenience in olden days when people didn't move around too much but, once railroads started moving people quickly between cities, it became very inconvenient for every town to have a different time. Time zones were invented to give towns with similar longitudes the same clock time in order to make it easier to keep track of what time it is in different places, which was particularly important for train schedules[126].

The time it takes the Sun to go from noon on one day to noon the next day has been defined as 24 equal hours. Time zones are one hour wide. Since the Sun goes around a full circle in 24 hours it travels 360°/24 hours or 15° in one hour so the center of the time zones are located 15° apart in terrestrial longitude[127]. The time used for everyday life is called *civil time*. Every place in the same time zone has the same civil time[128]. The time you get from the Sun is called *apparent solar time*. "Apparent" means it is what you really see or is apparent to you. At any given instant, every place with a different longitude has a different apparent solar time.

The previous paragraph simplifies the real definition of the length of the day somewhat. The real Sun does not behave quite so nicely as will be discussed in a bit. The amount of time passing between meridian passages of the Sun is not constant. In fact, the length of the day is the <u>average</u> time between noons. This average is defined by a *fictitious mean Sun* that, for the sake of defining the standard, is assumed to orbit the Earth in the plane of the equator. The time between meridian passages of the fictitious mean Sun at Greenwich is one day of 24 hours. The hour angle of the fictitious mean Sun at Greenwich is called Greenwich Mean Time (GMT). Universal Time (UT), which is also starndardized for Greenwich, has a somewhat more complicated definition based on atomic clocks.

When you use the astrolabe to solve problems involving time, you will be working with apparent solar time. An astrolabe or sundial shows you apparent solar time. Converting apparent solar time to civil time (the time on your watch) involves two ideas. The first corrects for where you are in your time zone, and the second corrects for the fact that the Sun does not move at a constant speed.

Correcting for where you are in your time zone is very simple. Since the Sun moves 15° in an hour, it moves 1° in four minutes (60 minutes for 15° = 4 minutes per degree). Time zones are centered each 15° in terrestrial longitude. In the United States, EST is centered at 75° west longitude, CST at 90° W, MST at 105° W, and PST at 120° W. European Continental Time is centered at 15° East longitude. To calculate your longitude correction, *subtract* your longitude from the longitude of the center of your time zone. For example, if you live at 77° W longitude, your longitude correction is 75° - 77° = -2° or -8 minutes. Time corrections are always

[126] The first country to adopt a standard time was Scotland, in 1848. English rail and telegraph companies used Greenwich time from 1827 and the country was de facto standardized by 1855 but was not official until 1880. The USA was the first country to adopt time zones in 1870, primarily due to the efforts of Charles F. Dowd (1825-1904). Times zones were officially adopted by act of Congress in 1918. The Greenwich meridian was adopted as the starting point for longitude in 1884.

[127] Time zone boundaries are adjusted to match political borders and seldom match the longitude definitions in populated areas. For example, although most of Spain is west of the Greenwich meridian, Spain uses European Continental time which is centered at 15° East longitude. Similar situations exist in much of Alaska, Argentina and Greenland.

[128] There are a few places that do not observe standard time zones. For example, the time in Iran, India, Newfoundland and parts of Australia is offset by a half hour from the standard zone time. The time in Nepal is offset by three quarters of an hours. There are other places with irregular times.

subtracted from apparent solar time, so, to correct Sun time to zone time you subtract -8 minutes. If the apparent solar time at 77° W longitude is 10:00AM, the zone time is 10:00 - (-8 minutes) = 10:08. This makes sense because the Sun has had to travel a little bit farther to reach your location than it did to reach the center of the time zone.

During daylight savings time you have to add an hour to apparent solar time to get civil time.

The other correction from apparent solar time to civil time uses the equation of time. Everybody knows (or should know) the Earth orbits the Sun in a slightly elliptical orbit and it moves a little faster in its orbit when it is closer to the Sun than it does when it is farther away. The time between noons is a bit shorter when the Earth is closer to the Sun and moving faster and a bit longer when the Earth is further from the Sun and moving slower. The difference due to this factor is called **the equation of the center** and can be calculated with some effort. See Smart for a lucid discussion of the math.

As the Earth goes around the Sun it defines a plane called the ecliptic. The axis of the Earth is tilted about 23° 26′ from the ecliptic. The true Sun appears to follow the ecliptic, but the basis for timekeeping is the fictitious mean Sun obiting on the plane of the equator. There is a slight difference in the time shown by the true Sun and standard time due to this effect. It is called the **reduction to the equator.**

When you combine these two factors, it turns out the time for the Sun to return to the local meridian on successive days is usually not exactly 24 hours. The difference between mean time and apparent solar time is called the equation of time. The equation of time is not what we normally think of as an "equation": a formula that can be calculated (although it is possible to calculate the value of the equation of time). In this context the word "equation" is an archaic term that signifies a value to be added to get the right answer.

Let's do a little experiment. The purpose of the experiment is to make a simple sundial that will tell us exactly where the Sun is at noon. Take a straight stick and mount it on a flat board so it is exactly perpendicular to the board. Take the board outside and put it in sunlight and lay it so the board is perfectly horizontal. Set your watch to be exactly on time and wait until noon. Make a mark on the board at the tip of the shadow made by the stick exactly at the instant of noon. Repeat the measurement at noon every day for an entire year. What is the shape of the figure made by the marks on the board?

The figure made by the shadow of a vertical stick is shown in Figure 24-7. This shape is called an *analemma*. The little circle represents the stick and the vertical line shows the direction of north and south (south is at the top). The analemma shows the Sun is usually not due south at noon. If the Sun is a little to the west of south at noon, the shadow will be a little to the left of the north-south line. We say the Sun is *early* if it is west of south at noon. The Sun can be as much as 16 minutes early. If the Sun is east of south at noon, the shadow will fall to the right of the line. The Sun is *late* if it is east of south at noon. The Sun can be 14 minutes late. The difference in the time when the Sun is due south and noon is called the *equation of time*. If you want to tell time from the sun, and get the same time your watch says, you have to correct for the equation of time.

The dates when the equation of time is zero are April 15, June 13, September 1 and December 25. You can find these days on the analemma curve with a little thought. The Sun is low in the sky in the winter so the shadow cast by our stick will be long in the winter. In the summer, the shadow will be short because the Sun is high in the sky. Look at the analemma and find the dates when the value of the equation of time is zero.

The maximum value of the equation of time is when the Sun is 16 minutes 25 seconds early on November 3. The minimum value is on February 10 when the Sun is 14 minutes, 15 seconds late.

To correct apparent solar time for the equation of time, subtract the value of the equation of time from the apparent solar time. Continuing the example above, if the day is March 7, the equation of time is about -11 minutes. On March 7, the Sun reaches the meridian about 11 minutes later than on the average day. The civil time for 10:00 apparent solar time on March 7, for 77° W is 10:00 - (-8 min.) - (-11) minutes = 10:19.

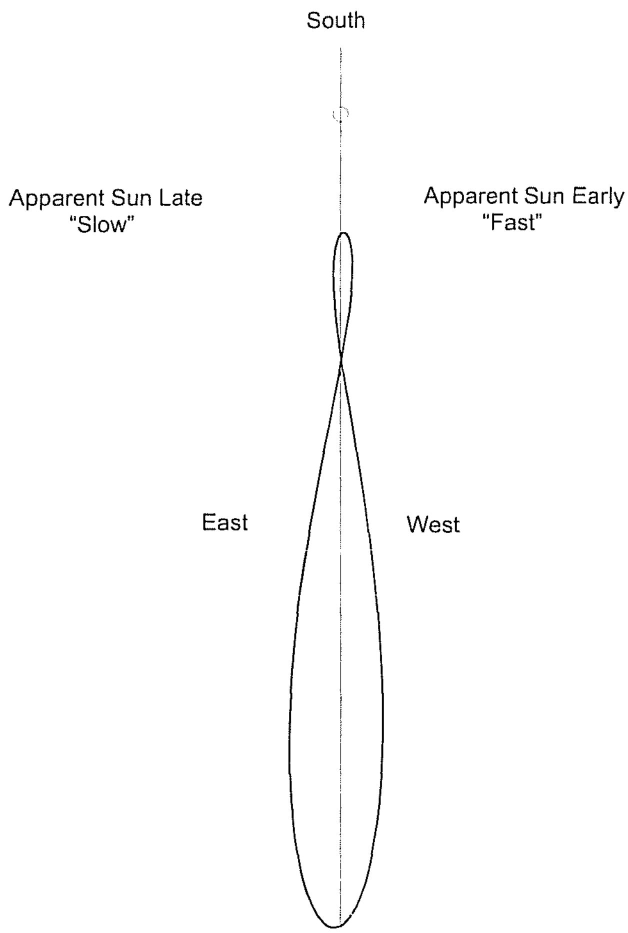


Figure 24-7. Analemma for 38° 58′ N.

The term, "Sun Late" means the apparent Sun crosses the meridian after 12:00 Noon, civil time. "Sun Early" means the apparent Sun crosses the meridian before 12:00 Noon, civil time.

The equation of time can be represented in several ways. The analemma representation is common on globes of the Earth. It is most commonly shown as a graph as in Figure 24-8. This graph show the equation of time in minutes for 2006. There is very little difference in the graph from year to year, but there is a significant difference for years in the distant past.

In summary,

Civil Time = Apparent Solar Time - Longitude Correction - Equation of Time (+ Daylight Savings Time)

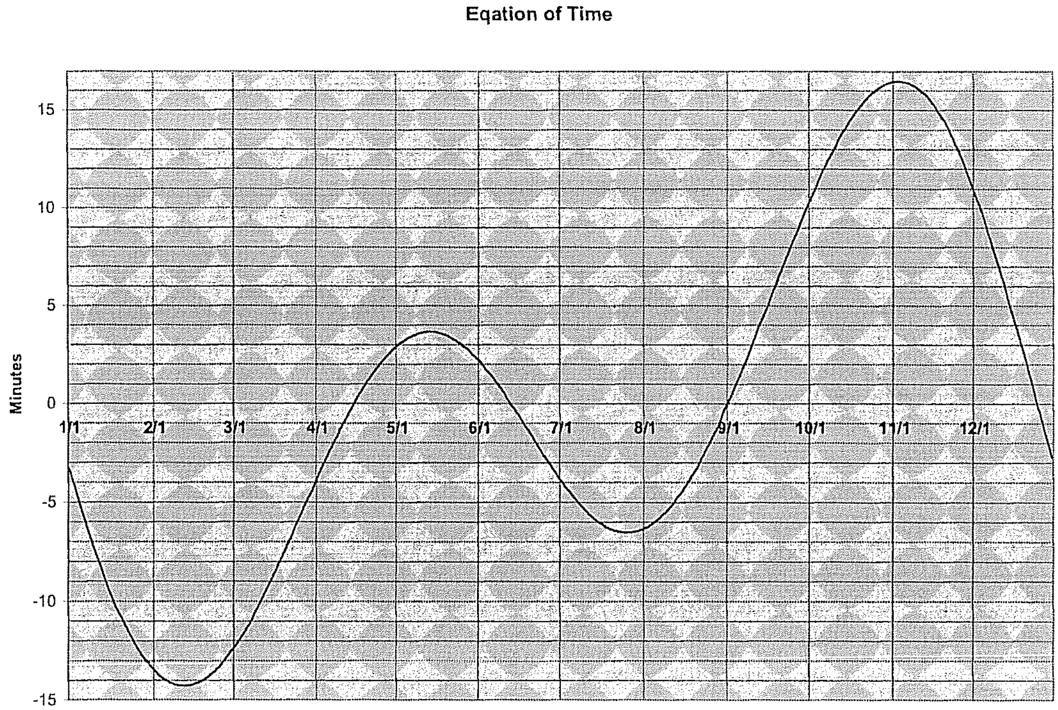


Figure 24-8. Equation of Time Graph

The analemma curve results if the value of the equation of time is plotted against the Sun's declination as shown in Figure 24-9.

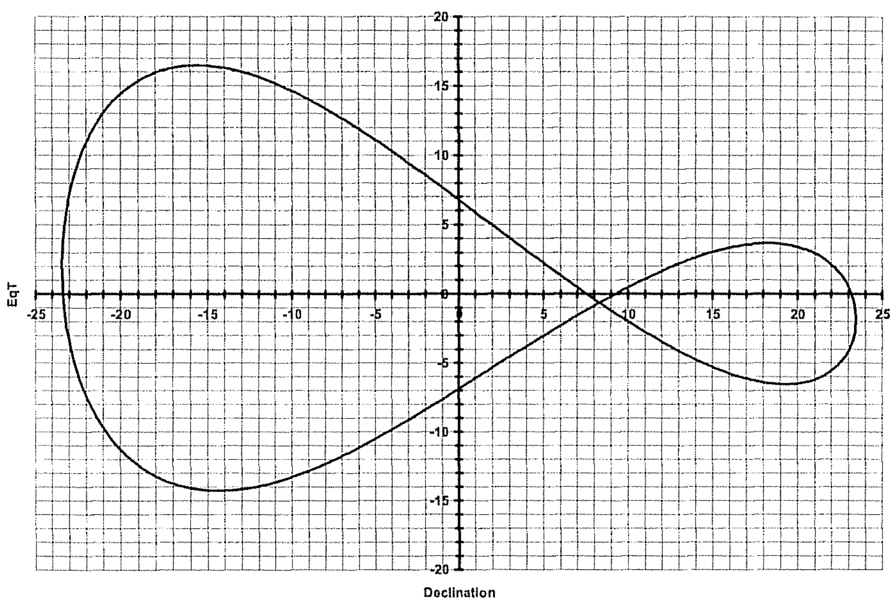


Figure 24-9. Equation of Time Analemma

The equation of time drawn in polar coordinates is shown on the modern astrolabe back on page 149, Figure 8-37.

Sidereal Time

The term *sidereal time* literally means "star time", that is, the right ascension of stars exactly south of a place at a specific time. Sidereal time is defined as the hour angle of the vernal equinox and thus, is the right ascension of all stars on the local meridian at a certain time. A sidereal day is the length of time it takes for the same star to be on the local meridian from day to day and is about four minutes shorter than a civil day. You know what stars are visible if you know the sidereal time.

Finding the sidereal time on an astrolabe is very simple. Simply read the hour on the limb pointed to by the First Point of Aries on the rete. Similarly, you can find the right ascension of any object shown on the rete by positioning it on the meridian and noting the sidereal time.

Chapter 25 - Astronomical Calculations

Some facility in performing basic astronomical calculations is required in order to design any astronomical instrument. This chapter contains some of the fundamental calculations required and an introduction on how to implement them on a computer. Sample programs are included as examples of how the calculations might be done.

Introduction

The study of calculating and finding the location of celestial bodies is called Positional Astronomy. This is, perhaps, the oldest real science and certainly one of the most precise. The literature on positional astronomy is huge, and many distinguished scientists have devoted their lives and reputations to the subject. Only some very basic results can be presented here. Smart is highly recommended as a good introduction to the basic theory. Meeus is indispensable for calculation details. *The Astronomical Almanac* and *Supplement to the Astronomical Almanac* are required for detailed definitions and precise values of astronomical values for testing the accuracy of your results.

A feeling that might occur to you as you pass through this section is how much easier it is to use an astrolabe to solve a positional astronomy problem than it is to calculate the answer.

Notation

The following symbols are used in this chapter:

φ	Terrestrial latitude
β	Ecliptic latitude
λ	Ecliptic longitude
δ	Declination
α	Right ascension
h	Altitude above the horizon
A	Azimuth, measured westward from south on the horizon
H	Hour angle. measured westward from south on the equator
ε	Obliquity of the ecliptic

Tranformation of Coordinates

It is often necessary to convert between coordinate system. For example, you might need to find the Sun's azimuth and altitude for a given day and time at a specified latitude given the declination. This is equivalent to finding the Sun's position in horizontal coordinates (altitude, azimuth) from its equatorial coordinates (declination, right ascension). The following equations are straightforward applications of basic spherical trigonometry and are found in all astronomy texts:

Equatorial to horizontal coordinates:

$$\sin h = \sin\varphi \sin\delta + \cos\varphi \cos\delta \cos H$$

$$\tan A = \frac{\sin H}{\cos H \sin \varphi - \tan \delta \cos \varphi}$$ [129]

Horizontal to equatorial coordinates:

$$\tan H = \frac{\sin A}{\cos A \sin \varphi - \cos \varphi \cos h \cos A}$$

$$\sin \delta = \sin \varphi \sin h - \cos \varphi \cos h \cos A$$

Ecliptic to equatorial coordinates:

$$\tan \alpha = \frac{\sin \lambda \cos \varepsilon - \tan \beta \sin \varepsilon}{\cos \lambda}$$

$$\sin \delta = \sin \beta \cos \varepsilon + \cos \beta \sin \varepsilon \sin \lambda$$

Equatorial to ecliptic coordinates:

$$\tan \lambda = \frac{\sin \alpha \cos \varepsilon + \tan \delta \sin \varepsilon}{\cos \alpha}$$

$$\sin \beta = \sin \delta \cos \varepsilon - \cos \delta \sin \varepsilon \sin$$

Time in astronomical calculations

Almost all astronomical positions change over time. Otherwise, a value would not have to be calculated. A problem that has plagued astronomers over the centuries is how to express the time of a specific celestial event. The first innovation was over 2,000 years ago when it became clear it did not make sense to describe celestial events using the unequal hour convention employed for everyday life. It is very difficult to compare the time of an eclipse observed in Rome to an eclipse in Alexandria, because they are at different latitudes and, therefore, the length of the hour was different, and the amount of the difference depended on the time of year. It was recognized very early it was much more convenient for astronomers to keep time in equal hours that do not change over the course of the year. Specifying dates of celestial events has also been a problem. We will discuss the evolution of some of these considerations.

Calendars and timekeeping have a very untidy history. Different cultures used a wide variety of calendars and methods for keeping time. For example, the Egyptian calendar used 12 months of 30 days each, with the left over days used for festivals at the end of the year. Most other ancient cultures used lunar calendars whereby the start of a month was determined by the first physical visibility of the crescent moon. Calendar dates were usually tied to a ruler's reign. The date of some event would often be described as occurring on a day of a lunar month in a given year of someone's reign. It is difficult at best to convert dates with such a description into our familiar calendar, and often impossible. It is hard enough to find when specific rulers were in power, but it may be impossible to determine the beginning dates of empirically determined months which

[129] The azimuth value calculated by this expression is the azimuth from south. The North American convention is to measure azimuth from north. The azimuth from north is A - 180° (+ 360° if negative).

may depend on local visibility and political conditions. Widespread use of the Julian calendar made things somewhat simpler in the western world for some time. Later, the Gregorian calendar was implemented at widely varying times in different countries ranging from 1582 to 1927. In some countries, different areas used different calendars and different days as the first day of the year. There was a time in France when neighboring villages would have the same date but in different years while a third nearby village would have a different date.

Timekeeping has a similarly sordid past. Most cultures counted days as beginning at sunrise but others counted them from sunset. An argument about which day to use for Jesus' crucifixion raged for centuries because the Jewish day started at sunset and the Roman day started at sunrise and the gospel of John seemed to give a different date than Luke, Matthew and Mark. A choice was made by the First Council of Nicea in 325, but the issue was not settled for many years. Further, timekeeping in the ancient and medieval world normally used hours that expressed the fraction of the day or night that had passed. The length of these unequal hours varied through the year by an amount depending on the latitude of a specific place. Astronomers adopted a system of equal hours very early, but translating from historical records to time of day can be very tricky since several methods of keeping time locally have been used. For example, Italian hours are equal hours beginning at sunset and Babylonian hours are equal hours beginning at sunrise. Counting unequal hours can be from either sunrise or sunset. One must be certain of the timekeeping technique used in a specific place at a certain time to place events at the proper time of day.

Astronomers and historians need a way to count time that is independent of calendars in order to position events from different eras. Astronomers in the ancient and medieval world tended to do calculations using the Egyptian calendar because of its consistency and relative ease for computing the number of days between events, but this was not a complete solution.

The *Julian Period* provides a sequence of days independent of any calendar and was defined to address this problem. The Julian Period is a continuous sequence of **days**. The Julian Period is expressed only as a day number. For example, the Julian day number for December 17, 1966, is 2439477. There is no day/month/year in the Julian Period. Years in the Julian Period are relevant only when determining the Julian day number from a given calendar date.

The Origin of the Julian Period

A method providing an unvarying sequence of days, independent of any calendar and undisturbed by liturgical or political considerations was developed by Joseph Justus Scaliger (1540-1609) of Leyden and introduced in *De emenatione temporum* in 1583. Scaliger wanted to define a long sequence of days with a definite starting point and included all of recorded history. To this end he introduced the concept of the Julian Period, so called because it contains exactly 7980 Julian years of 365.25 days[130]. A farside lunar crater of 84 km diameter at 21.1 degrees south latitude; 108.9 degrees east longitude was named for Joseph Justus Scaliger by the International Astronomical Union (IAU) in 1970.

The Julian Period is independent of any calendar system and counts days sequentially from January 1, 4713 BC. In the Julian calendar, dates are counted forward from January 1, AD 1 and backwards from December 31, 1 BC. There is no year 0. There is a year 0 in the Julian Period, which leads to describing dates BC as a negative number. Thus, the year 46 BC has the number -45 in the Julian Period. Also, the Julian Period can be extended backwards indefinitely. The Julian calendar began in 46 BC, but a date such as June 27, -431 (the beginning of the first Metonic cycle) is a perfectly valid date in the Julian Period even though it is nearly 200 years before the Julian calendar was defined[131]. Astronomers would record this date as Julian day 1563813, which is the number of days that had passed since January 1, -4712, and is totally

[130] Moyer, Gordon, "The Origin of the Julian Day System", Sky and Telescoope, April, 1981. pp 311-313.
[131] Historians would normally write this date as 27 June 432 BC.

unambiguous. Julian days can be converted to calendar dates in either the Julian or Gregorian calendars using relatively simple calculations.

Realistically, a period of sequential days could have been started from any arbitrary date. Scaliger used three date cycles that were used in his era. They are, the solar cycle of 28 years in which calendars repeat in the same order, the "Golden Number" which is the year in the current 19 year lunar cycle[132] in which lunar phases repeat on the same date (approximately) and the 15 year cycle of indiction introduced by Constantine I in AD 313 as a period for taxation. The periods of these three cycles are mutually prime (i.e., have no common divisor). Therefore, if the cycles all start on the same date there will be no other date on which all three cycles have the same number. The length of the Julian Period is $28 \times 19 \times 15 = 7980$ years.

In order to describe the various periods we need to use a mathematical term from the theory of numbers called *modulus* arithmetic. The concept is very simple. The modulus of a and b is the remainder of a/b [written a (mod b)]. For example, 235 (mod 19) = 7. That is, 235/19 = 12 with a remainder of 7. Similarly, 7 (mod 19) = 7. Two numbers x and y are said to be *congruent mod b* if x (mod b) = y (mod b). 45 and 235 are congruent mod 19 since 45 mod 19 = 7 and 235 mod 19 = 7. This is written as $235 \equiv 7 \pmod{19}$ and is said out loud as " 235 is congruent to 7 mod 19." 19 is called the *modulus* (or *modul*) and 7 is the *residue*.

The start of the Julian Period is defined by a year in which all three cycles began on the same date.

The 19 year soli/lunar cycle has been known since antiquity as a period in which 235 lunar months is very nearly 19 years of 365.25 days. The beginning of a 19 year cycle can be reliably chosen as AD 532, which is the first year of the cycle used by Dionysius Exiguus for calculating the date of Easter[133]. The number of a specific year in the cycle is called the **"Golden Number"** (*numerus aureus*), a term introduced in 1200 by Alexandre de Villedieu in his *Massa Compoti* on the grounds that, "this number excels all other lunar ratios as gold excels all other materials." As the first year in the cycle, the Golden Number for 532 is 1. 532 mod 19 = 0. Therefore, the Golden Number for year n = (n mod 19) + 1. The starting year, n, of a 19 year cycle, n, can be written $n \equiv 0$ mod 19.

The solar cycle (*ciclus solaris*) is a period of 28 years in which calendars repeat in the same order. A year of 365 days has 52 weeks plus one day. Therefore, if the first day of the year falls on a given day of the week, it will fall on the next day of the week in the next year and so on. All years could be described by seven calendars, each beginning a different day, if it were not for leap years. The insertion of leap years requires seven additional calendars. The period in which all of the possible calendars repeat *in the same order* is 28 years. Solar cycles start in a leap year. In the Julian calendar, all solar cycles begin in a leap year in which January 1, falls on Monday. 1560 was such a year[134]. Since the year of the solar cycle beginning in 1560 = 1, and 1541 mod 28 = 1, we can calculate the year in the solar cycle for any year in the Julian calendar as (Year - 19) mod 28. The first year of a solar cycle can be written as $n \equiv 20$ mod 28.

The solar cycle works as follows. Call Sunday the first day of the week. Number the normal year calendars 1, 2, ..., 7 for the day of the week that begins the year. Number the leap year calendars as 1L, 2L, ... , 7L. Lay out the years so the calendar starts on the correct day starting with a leap year. The solar cycles from 1924 to 2092 begin on a Tuesday, so we start the sequence with calendar 3L. The number of a year in the solar cycle is called the *concurrent*. Solar cycle lists sometimes identify leap years with a "B" for *bisextilis*. Solar cycles begin in 1979, 2008, etc.

[132] Called the Metonic cycle after Meton of Athens who is credited with its discovery or, in church literature, the Alexandrine cycle.
[133] Dionysius Exiguus also introduced the practice of numbering years *anno domini nostri Jesu Christi* or AD.
[134] The starting day of the week for solar cycles is affected by century years that are not leap years in the Gregorian calendar (1700, 1800, 1900, 2100...). Solar cycles currently start on Tuesdays.

Examination of this calendar sequence shows that common year calendars are used three times each, repeating in cycles of six years and eleven years. The leap year calendars are each used once. The day of the week represented by the calendar in year 1 changes from century to century in the Gregorian calendar due to the non-leap year century years. For example, the year in the solar cycle (the *concurrent)* for 2006 is 27. The calendar for year 27 is shown by 1 in the table which indicates that January 1, 2006 is on Sunday. Thus, the 2006 **dominical letter** is A.

Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Calendar	3L	5	6	7	1L	3	4	5	6L	1	2	3	4L	6

Year	15	16	17	18	19	20	21	22	23	24	25	26	27	28
Calendar	7	1	2L	4	5	6	7L	2	3	4	5L	7	1	2

Following is a table of the calendars used in the current solar cycle. Leap years are **bold**.

2008	**3**	**2012**	**1**	**2016**	**6**	**2020**	**4**	**2024**	**2**	**2028**	**7**	**2032**	**5**
2009	5	2013	3	2017	1	2021	6	2025	4	2029	2	2033	7
2010	6	2014	4	2018	2	2022	7	2026	5	2030	3	2034	1
2011	7	2015	5	2019	3	2023	1	2027	6	2031	4	2035	2

The 15 year cycle of indiction was introduced in AD 313 by Constantine I. Thus, the *indicitio* of 313 is 1. 301 mod 15 = 1. Therefore, the indictio of any year is (Year -12) mod 15 and the first year, n ≡13 mod 15.

The beginning of the Julian Period is a year in which the year's Golden Number = Solar Cycle = Indictio = 1. This year can be found by successive subtractions from the known starting dates of each of the cycles or analytically. If we count back 276, 19 year cycles from 532, 224, 28 year cycles from 1560 and 335, 15 year cycles from 313 we find all three cycles begin in -4712. Since all three cycles start on January 1, the beginning of the Julian Period is Monday, January 1, -4712.

The beginning of the Julian Period can also be found analytically using number theory.

Analytically, the beginning year, n_0, must satisfy the conditions:

$n_0 \equiv 0 \pmod{19}$, $n_0 \equiv 20 \pmod{28}$, $n_0 \equiv 13 \pmod{15}$

Following is one method of solving the congruence relations above:

First, we calculate the "magic numbers" from the product of the three periods: $15 \times 28 \times 19 = 7980$.

7980 / 19 = 420, 7980 / 28 = 285, 7980 / 15 = 532

We form three new congruence relations using the "magic numbers":

$420\,a \equiv 1 \pmod{19}$, $285\,b \equiv 1 \pmod{28}$, $532\,c \equiv 1 \pmod{15}$

We find a, b and c through trial and error. For example, 420 = 2 (mod 19), 420 × 2 = 4 (mod 19), ... and so on until we find a =10 gives 1 (mod 19). Similarly, b = 17 and c = 13.

We form the result from the original congruences, the "magic numbers" and a, b and c. We rewrite the original congruences using postive residues as $n_0 \equiv 19 \bmod 19$, $n_0 \equiv 20 \bmod 28$, $n_0 \equiv 13 \bmod 15$

$$n_0 \equiv (19\times420\times10) + (20\times285\times17) + (13\times532\times13) \pmod{7980}$$
$$n_0 \equiv 266608 \pmod{7980}$$
$$n_0 \equiv 3268 \pmod{7980}$$
$$n_0 \equiv -4712 \pmod{7980}$$

That is, Julian Periods start in years -4712 + k 7980. If k =0, the year is -4712.

Gauss suggests a similar method[135].

Given congruence relations $x \equiv a \pmod{A}$, $x \equiv b \pmod{B}$, $x \equiv c \pmod{C}$,..., A, B, C, ... relative primes.

Find $\alpha \equiv 1 \pmod{A}$ and $\equiv 0$ (mod BC..), $\beta \equiv 1 \pmod{B}$ and $\equiv 0$ (mod AC..), $\gamma \equiv 1 \pmod{C}$ and $\equiv 0$ (mod AB..)

Then, $x \equiv (a \times \alpha) + (b \times \beta) + (c \times \gamma) + \ldots$ (mod ABC..).

For A = 15, B = 28 and C = 19 we find $\alpha = 6916$, $\beta = 4200$ and $\gamma = 4845$.

Therefore,

$n_0 \equiv (15 \times 6916) + (28 \times 4200) + (19 \times 4845) \pmod{7980}$ which is the same as the relation above.

Note that α, β and γ can also be found directly. Since $\alpha \equiv 0$ (mod BC) then α = BCx and $\alpha \equiv 1$ (mod A) so α = Ay + 1. Therefore, BC x = Ay + 1 or BC x - Ay = 1. This form of an equation is known as a linear Diophantine equation[136] which can be solved directly. In the example above, x = 13, y = 461 so $\alpha = 13 \times 532 = 15 \times 461 + 1 = 6916$. β and γ can be found similarly.

Uses of the Julian Period

The Julian Period is primarily used by astronomers as a consistent way to keep track of time, but it is also used in business as a way to manage sequences of time. Astronomers usually observe at night, and it would be inconvenient for the date to change in the middle of an observation, so Julian days are defined as beginning at Greenwich noon. Also, astronomers need an accurate way to specify the time of day in addition to the date. Julian days are expressed as a day number with a fraction. The fraction is the part of the day that has passed in universal time (i.e., the time at Greenwich). A Julian day number such as 2430259.0 which has 0 as the fraction says the exact instant is noon, universal time. Similarly, 2430259.5 specifies midnight, universal time. Thus, Julian day 2440423.62240 corresponds to July 21, 1969 at 2:56:15 UTC, the moment Neil Armstrong first set foot on the moon. This date and time can easily be converted to local date and time for other places than Greenwich by allowing for time zones and situations where the date at Greenwich is different than the local date. In the example above, Houston is six hours earlier than Greenwich (five hours during daylight savings time) so the same instant at the Manned Space Flight Center was July 20, 1969 at 9:56:15 CDT.

Based on this refinement, the Julian Period began at noon on Monday, January 1, -4712.

Julian day numbers are used as an unambiguous method of recording the time of observations and as the time value when performing astronomical calculations. For example, the Sun's mean longitude can be calculated for a date by:

$$\mathbf{L = 280^\circ.46645 + 36000^\circ.76983\ T + 0^\circ.0003032\ T^2}$$

[135] C. F. Gauss, *Disquisitiones Arithmeticae*, article 36.
[136] A Diophantine equation is an equation with integer coefficients and integer solutions.

where T is the number of Julian centuries of 36,525 days before or after noon, January 1, 2000.

T = (JD - 2451545.0) / 36525.

2451545.0 is the Julian day number for January 1, 2000[137], the 0 decimal specifies noon UT. Note this is the definition of T defining an *epoch* for astronomical positions. The epoch is updated each 20 years and new base positions for stars and equations for astronomical values are published in advance of the epoch change. Astronomers typically specify Julian dates with at least five or six decimal places in order to specify to a tenth of a second or better precision.

The meaning of the Julian day number for use in calculations has been slightly modified in recent years to reflect *dyanimical time* instead of UTC in order to compensate for slight irregularities in the Earth's rotation. Dynamical time is based on time from atomic clocks and flows at a constant rate whereas civil time is based on the rotation of the Earth, which varies by small, unpredictable amounts. The difference between UTC and dynamical time is only about one minute in the early twenty-first century, but is several hours for dates in the past and days for dates in the far past..

Historians use Julian day numbers in order to convert dates between calendars. Times used in industry for date calculations are often called Julian days, but are actually in the form of the day of the century and were one of the major sources of Year 2000 computer program problems because many programs rely on being able to subtract 'Julian days' to find the number of days between events.

Calculating Julian days

There are many algorithms for calculating the Julian day number for a given calendar date. The following algorithm is valid for any positive Julian day number, but not for negative Julian days. INT(x) means the integer part of x.

The Julian day corresponding to a date in the Julian or Gregorian calendar can be calculated as[138]:

Let Y be the year, M the number of month (January =1, February = 2, ...) and D the day of the month.

1. If M = 1 or 2, Y = Y - 1, M = M + 12.

2. If M > 2, leave M and Y unchanged.

1 and 2 above mean January and February are treated as months 13 and 14 of the previous year. This allows leap years to be treated consistently by adding the leap day at the end of a year.

3. Set B = 0 if the date is in the Julian calendar. If the date is in the Gregorian calendar, calculate A = INT(Y / 100). Calculate B = 2 - A + INT(A / 4). The meaning of B is discussed below.

4. The Julian day is:

$$JD = INT(365.25 \times Y) + INT (30.6001 (M + 1)) + D + 1720994.5 + B$$

The fraction of the day is added as a decimal value.

[137] Also called J2000.0.
[138] Meeus, Jean, *Astronimical Algorithms*, Willmann-Bell, Richmond, VA (1991), pp.60-62.

B is a clever method of compensating for the differences between the Julian and Gregorian calendars. Recall B is calculated only when a date is in the Gregorian calendar. The Gregorian calendar was initially instituted on October 15, 1582. The day before was October 4, 1582. Thus, 10 days were dropped in order to bring the calendar in agreement with the seasons. Also, in the Gregorian calendar, century years not divisible by 400 are not leap years. Therefore, the calculation would count too many days for years after 1700 (1600 was a leap year) without a correction. The correction provided by the calculation of B corrects for both factors.

B = 2 - A + INT(A/4). Recall B is calculated only for the Gregorian calendar so A ≥ 15. The correction for A = 15 and 16 (i.e. in the 1500's and 1600's) is -10, the number of days dropped from the calendar when the Gregorian calendar was implemented. INT(A/4) is 3 for A = 15, 2 - A = -13 so B =-10. Similarly, B = -10 for A = 16. The year 1700 was not a leap year so an additional day must be dropped. B = -11 for A = 17, -12 for A = 18, -13 for A = 19 and 20, and so on. One cannot describe a calculation such as this by relating it to first principles. It is simply a convenient calculating technique that gives the right answer. The ancients loved this sort of thing and called it, "preserving the phenomena".

The day of the week can be found directly from the Julian day number. Recall the Julian Period began on a Monday (JD = 0). If we number the days of the week as Monday = 1, Tuesday = 2, etc. then the number of the day of the week is (JD + 1) mod 7. Using the example of the first lunar landing above, Julian day (2440423 + 1) / 7 = 348632 with no remainder. Since day 0 is Sunday, the first lunar landing was on Sunday.

A sample C routine to calculate the Julian day from the current date and time follows. Note there is a line commented out which allows you to compensate for when the Gregorian calendar was introduced.

```
// Julian day for date

double Julian (int mo, int day, int year, double ut)
{
    int jm, jy;
    double jday, wv, dayj;

    if (mo > 2)
          {jm = mo + 1; jy = year;}
    else
          {jm = mo + 13; jy = year - 1;}

    dayj = day + ut/24.0;
    jday = ipart(365.25 * jy) + ipart(30.6001 * jm) + dayj + 1720994.5;

    //Return Julian calendar jday if before 10/4/1582
    //if (jday <= 2299160.0)
    //Modification to return jday if before 9/2/1752 English Gregorian calendar
    if (jday <= 2361220)
     return (jday);               //Julian day in Julian calendar
    else
     wv = ipart(0.01*year);

    return (jday + 2 - wv + ipart(wv*0.25));     //Julian day in Gregorian calendar
}
```

Here is the same routine in BASIC:

```
' Calculate Julian Day for Date

Public Function JD(Mo As Integer, Day As Integer, Year As Integer, UTC As
Double) As Double
Dim JM As Integer, JY As Integer, WV As Integer

If Mo > 2 Then
    JM = Mo + 1
```

```
    JY = Year
Else: JM = Mo + 13
    JY = Year - 1
End If

JD = Int(365.25 * JY) + Int(30.6001 * JM) + Day + 1720994.5

If JY > 1582 Then
    WV = Int(JY / 100)
    JD = JD + 2 - WV + Int(WV / 4)
End If

JD = JD + UTC

End Function
```

Calculating the date from the Julian day number

The date corresponding to a Julian day number can be calculated from[139]:

1. Add 0.5 to the JD. Let Z be the integer part and F be the fractional part.

2. If $Z < 2299161$, take $A = Z$. (JD 2299161 is Oct. 5, 1582, any earlier day must be in the Julian calendar).

3. If $Z \geq 2299161$, calculate:

$$\alpha = \mathrm{INT}(\ (Z - 1867216.25) / 36524.25\ )$$

$$A = Z + 1 + \alpha - \mathrm{INT}\ (\alpha / 4\ )$$

4. Now, calculate:

$$B = A + 1524$$

$$C = \mathrm{INT}(\ (\ B - 122.1\ ) / 365.25\ )$$

$$D = \mathrm{INT}\ (\ 365.25\ C)$$

$$E = \mathrm{INT}\ (\ (\ B - D) / 30.6001\ )$$

The day of the month (with decimals) is:

$$\text{Day of Month} = B - D - \mathrm{INT}(\ 30.6001\ E\ ) + F$$

The month number, m is:

$$\text{Month} = E - 1 \text{ if } E < 14.$$

$$\text{Month} = E - 13 \text{ if } E \geq 14.$$

The year is:

$$\text{Year} = C - 4716 \text{ if } m > 2.$$

$$\text{Year} = C - 4715 \text{ if } m \leq 2.$$

[139] Meeus, op. cit., pp. 63-64.

The resulting date is at Greenwich and may need to be adjusted to the local time and date, which can get a bit tricky if the local date and the date at Greenwich are different.

Following is a BASIC program that implements this algorithm. G% is a switch to specify the use of the Gregorian calendar. This function gets a bit involved considering time zones, particularly when the date is different in your time zone from UT. The base algorithm gives the date at Greenwich, which has to be adjusted in some cases. The routine checks UDIF which is the time difference in hours from UT, to make this determination and adjusts if needed. JDAY# is a global containing the Julian day to convert. WV is a working variable. MO, DAY and YR are integers.

```
' Julian Day to Date

REDO: JZ=FIX(JDAY#+.5)
        JF=JDAY#+.5-JZ
        IF NOT G% THEN JA=JZ: GOTO JULIAN
        WV=FIX((JZ-1867216.25#)/36524.25)
        JA=JZ+1+WV-INT(WV/4)
JULIAN: JB=JA+1524
        JC=INT((JB-122.1)/365.25)
        JD=INT(365.25*JC)
        JE=INT((JB-JD)/30.6001)
        DJY=JB-JD-INT(30.6001*JE)+JF
        DAY=FIX(DJY-UDIF)
        IF DAY=0 THEN
                JSV#=JDAY#
                JDAY#=JDAY#-UDIF
                IDO=-1
                GOTO REDO
        IF IDO THEN
                IDO=0
                JDAY#=JSV#
        IF JE<13.5 THEN MO=JE-1 ELSE MO=JE-13
        IF MO>2.5 THEN YR=JC-4716 ELSE YR=JC-4715
        IF ND% THEN GOTO EXIT
        ELSE
                LOCATE 1,1:
                PRINT USING "##_/##_/####";MO,DAY,YR;:PRINT C$
EXIT: ND%=0:RETURN
```

Following is a similar routine in C that sets the date and UT corresponding to a Julian day:

```
// Date corresponding to Julian day
void jdate(double jd, int *mo, int *day, int *year, double *ut)
{
        double z, f, alpha, a, b, c, d, e, wv ;

        z = ipart(jd + 0.5) ;    f = frac(jd + 0.5) ;

        if ( z < 2299161.0) a = z ;
        else {
                alpha = ipart((z-1867216.25)/36524.25) ;
                a = z + 1 + alpha - ipart(alpha/4.0) ;
                }
        b = a + 1524 ;
        c = ipart((b-122.1)/365.25) ;
        d = ipart(365.25 * c) ;
        e = ipart((b-d) / 30.6001) ;

        wv = b - d - ipart(30.6001 * e) + f ;
        *day = ipart(wv) ;
        *ut = frac(wv) ;

        if (e < 14) *mo = e - 1 ;
        else *mo = e - 13 ;

        if (*mo > 2) *year = c - 4716.0 ;
        else *year = c - 4715.0 ;
```

```
        return ;
}
```

This routine is called as:

```
int mo, day, year ;
double jday, ut ;

jdate(jday, &mo, &day, &year, &ut) ;
```

The Sun's Position

Many of the scales on the instruments described in this book require accurate values for the Sun's orbit and position. A few fundamental orbital mechanics terms must be understood in order to completely understand the required calculations. Following is an introductory description of planetary orbits. Meeus is a highly recommended reference for details.
Figure 25-1 shows a planet's orbit in space. The only orbit we are really interested in is the Earth's, but the terms apply to all planetary orbits. The orbit in the figure is much more eccentric than any of the real planetary orbits in order to highlight the concepts.

You may recall from basic science that Johannes Kepler discovered the fundamental rules of planetary orbits. In particular, planets travel around the Sun in elliptical orbits with the Sun at one focus[140]. **f** in the figure is the focus of the elliptical orbit. **P** is perihelion, the point on the orbit where the planet is closest to the Sun. **A** is aphelion, the point on the orbit where the planet is farthest from the Sun.

The position of the planet in its orbit is described as the angle of the planet from the line of apsides, the line connecting perihelion and aphelion (the major axis of the ellipse). Angles measured from the line of apsides are called *anomalies*. The angle **v** is called the *true anomaly*, and is the angle from the orbit's major axis to the planet measured from the focus (i.e., the sun). Kepler used an auxiliary angle (**E**) called the *eccentric anomaly* measured from the center of the orbit ellipse to calculate the true anomaly.

However, none of the planetary orbits have the same major axis. Therefore, if we are to describe planetary positions in a consistent way it is necessary to measure the planet's position from a fixed point common to all the planetary orbits. Such a point is the vernal equinox.

As seen from the Earth, the vernal equinox is a fixed direction in space that is easily and consistently measured and, incidentally, falls on both the ecliptic plane and the plane of the equator, which makes conversion between ecliptic and equatorial coordinate systems rather simple. Planetary positions measured from the vernal equinox (♈) are called *longitudes*. The *true longitude* (λ) of a planet is the angle of the planet from the vernal equinox. The angle from the vernal equinox to perihelion is called the *longitude of perihelion* (π in the figure — ϖ is sometimes used for this value but the symbol is so close to ω we prefer π). Thus, the true longitude of a planet is $\theta = \pi + \nu$. Calculation of a planet's position in its orbit is performed using simple formulas developed by Kepler in the 17th century.

[140] The focus is actually at the center of gravity of the planet-Sun system, but the distinction is not significant for our purposes.

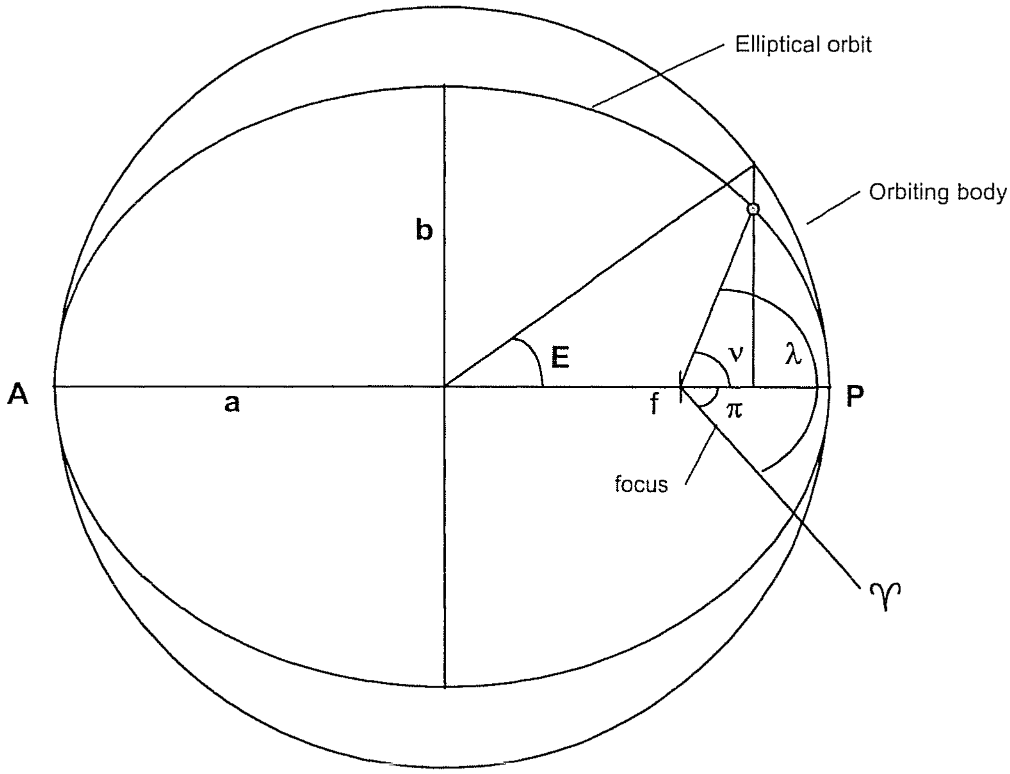


Figure 25-1. The Orbit in Space

Kepler used the same terminology and concepts as Ptolemy. Recall the eccentric model of the Sun uses the concept of an imaginary Sun moving at a constant rate on a circle. The angle of the fictitious Sun from the line of apsides is called the *mean anomaly*, **M**. Kepler's construction uses the same idea. The circle in Figure 25-1 is used in exactly the same way with the same meaning and terminology. Kepler derived an equation using the mean anomaly to calculate the *eccentric anomaly*, E. The eccentric anomaly is then used to calculate the *true anomaly*, ν. Once we know the true anomaly we can calculate the Sun's true longitude, λ, from:

$$\lambda = \nu + \pi$$

In practice it is easier to calculate the Sun's mean longitude (L) and mean anomaly (M). $\pi = L - M$.

Kepler's equation is:

$$E = M + e \sin E$$

e is the eccentricity of the Earth's orbit.

Once we have the eccentric anomaly, we solve for the true anomaly from:

$$\tan\frac{v}{2} = \sqrt{\frac{1+e}{1-e}}\tan\frac{E}{2}$$

We can then solve for the true longitude, from which the Sun's declination and right ascension is derived.

Kepler's equation cannot be solved directly for E because E is on both sides of the equal sign. This is called a *trancendental equation*. However, for small values of e it can be solved quite easily using successive approximations. We take a first estimate of E as M, solve the equation and then substitute the result as a better approximation of E, solve it again... until the change in E between two approximations is as small as desired. This value for E is then used to continue the calculation.

The following C subroutine to calculate the Sun's eccentric anomaly for a date and time (T) implements this approach:

```
// Sun's eccentric anomaly (radians)

double kepler (double T)
{
    double m, e0, e, ea ;
    m = radian(manom(T)) ;
    ea = m ;
    e = ecc(T) ;

    do
        {e0 = ea; ea = m + e * sin(e0) ; }
    while (fabs(ea - e0) > 1.0e-09) ;
    return (ea) ;
}
```

However, in order to solve Kepler's equation, the value of e and M for the instant for the calculation must be known. Recalling from the discussion above that T is the time in Julian centuries from J2000.0, M and e can be calculated from:

$$\mathbf{M = 357°.52910 + 35\,999°.050\,30\,T - 0°.000\,1559\,T^2 - 0°.000\,000\,48\,T^3}$$

$$\mathbf{e = 0.016\,708\,617 - 0.000\,042\,037\,T - 0.000\,000\,1236\,T^2}$$

The resulting value for M will need to be normalized to 0-360°. C routines to perform these calculations are listed below.

Many equations require the value of the obliquity of the Earth's orbit, which can be calculated approximately from:

$$\boldsymbol{\varepsilon = 23°.43929111 - 1.3004167 \times 10^{-2}\,T - 1.638889 \times 10^{-7}\,T^2 + 5.036111 \times 10^{-7}\,T^3}$$

To be perfectly precise, the value of the obliquity above is not the exact value. This value would have to be corrected for nutation to be absolutely correct, but the correction amounts to a few arc-seconds and is unnecessary for these instruments.

The Sun's geocentric true longitude is calculated from:

True Longitude = Mean Longitude + True Anomaly - Mean Anomaly.

To calculate the Sun's declination for a date and time, first calculate the Sun's geocentric true longitude and then, noting the Sun's latitude is 0, calculate the declination from:

$\sin\delta = \sin\varepsilon \sin\lambda$

The Sun's altitude and azimuth are calculated from the Sun's declination.

Putting it all together, the following C routines calculate the Sun's maximum altitude for a day:

```
//-------- Part of main program -----------------------------------

//Calculate jday for noon of date to get Sun's noon altitude

jday = Julian( mo, day ,YEAR, 12.0) ;           //Julian day for date

T = (jday - 2451545.0) / 36525.0 ;              //T for astronomy calc

//Declination for day

decl = asin(sin(radian(obl(T))*sin(radian(geolong(T))))) ;

//Sun's max altitude for day

alt = PI_2 - latr + decl ;
.
.
.
//-------- Called Subroutines ------------------------------------

// Obliquity of ecliptic (degrees)

double obl(double T)
{
  return (((5.036111e-07 * T - 1.638889e-07) * T - .013004167) * T +
              23.4392911) ;
}

// Eccentricity of Earth's orbit

double ecc(double T)
{
    return ((-1.236E-07 * T - 4.2037E-05) * T + .016708617) ;
}

// Sun's eccentric anomaly (radians)

double kepler (double T)
{
    double m, e0, e, ea ;
    m = radian(manom(T)) ;
    ea = m ;
    e = ecc(T) ;
    do
      { e0 = ea; ea = m + e * sin(e0); }
    while (fabs(ea - e0) > 1.0e-09);
    return (ea);
}

// True anomaly from eccentric anomaly (degrees)

double tanom(double T)
{
    double ea, e, ta ;

    ea = kepler (T) ;
    e = ecc(T) ;
    ta = atan(tan(ea / 2.0) * sqrt((1.0 + e) / (1.0 - e))) ;
    if ((ta < 0.0) || (ta > 0.0 && ea > PI)) ta = ta + PI ;
    ta = 2.0 * ta ;
    if ((ta < PI) && (ea >= 3.0 * PI_2)) ta = ta + PI ;
    return (degree(ta)) ;
}
```

```
// Sun's mean anomaly (degrees)

double manom(double T)
{
    return (normal(((-4.8e-07 * T - .0001559) * T + 35999.0503) * T + 357.5291)) ;
}

// Sun's mean longitude (degrees)

double mlong (double T)
{
    return (normal(((.000000021 * T) + .00030368 * T + 36000.7698231) * T +
280.466449)) ;
}

// Sun's geocentric longitude (degrees)

double geolong (double T)
{
    return (normal(tanom(T) + mlong(T) - manom(T))) ;
}

// normalize angle to 0 - 360

double normal (double x)
{
    while (x < 0) {x += 360.0;}
    return (frac(x/360.0) * 360.0) ;
}
// Hour angle of sunrise given latitude and declination. Args in radians.

double sunrise (double lat, double decl)
{
    return (degree(acos(- tan (lat) * tan (decl))));
}

// Sun's Declination in degrees (from true longitude)

double declination (double T)
{
    return (degree(asin(sin(radian(obl(T))) * sin(radian(geolong(T))))));
}

// Sun's right ascension from true longitude in degrees

double right_ascension (double T)
{
    double tlong ;
    tlong = radian(geolong(T)) ;
    return degree(normal(atan2((sin(tlong) * cos(radian(obl(T)))), cos(tlong)))) ;
}

//  Altitude from latitude, declination, hour angle. Arguments in radians

double altitude (double lat, double decl, double hr_ang)
{
    return (degree(asin(sin(lat)*sin(decl)+cos(lat)*cos(decl)*cos(hr_ang))));
}

//  Azimuth from latitude, declination and hour angle. Arguments in radians

double azimuth (double lat, double decl, double hr_ang)
{
    return (degree(atan2(sin(hr_ang),(cos(hr_ang)*sin(lat)-tan(decl)*cos(lat)))));
}
```

These routines can be used in many ways in many programs once they are coded and tested.

You may notice the use of parentheses in the above routines is not an exact reproduction of a series. This style of coding power series minimizes the number of multiplications required to evaluate the polynomial in T.

The following routine is useful for checking the Julian day of the vernal equinox for the back of the astrolabe[141]:

```
// Julian Date of Vernal Equinox for YEAR

double vernal (int YEAR)
{
   double Y;

   if (YEAR >= 1000) {
      Y = (YEAR - 2000) / 1000 ;          //Valid only from 1000 to 3000
      return (((-.00057*Y-.00411)*Y+.05169)*Y+365242.37404)*Y+2451623.80984 ;
      }
   else {
      Y = YEAR / 1000 ;                   //Year -1000 to 1000
      return (((-0.00071*Y-0.00111)*Y+.06134)*Y+365242.1374)*Y+1721139.29189 ;
      }
}
```

Stellar Precession

It is critical to the accuracy of your astrolabe that the star positions be precessed to the date of the instrument. There are several methods of varying accuracy for calculating precessed stellar coordinates. There is no reason not to use the best possible method given the speed of modern computers. We can ignore proper motion for this application.

The equations for rigorous precession are[142]:

$$\mathbf{A = \cos \delta_0 \sin (\alpha_0 + \zeta)}$$

$$\mathbf{B = \cos \theta \cos \delta_0 \cos (\alpha_0 + \zeta) - \sin \theta \sin \delta_0}$$

$$\mathbf{C = \sin \theta \cos \delta_0 \cos (\alpha_0 + \zeta) + \cos \theta \sin \delta_0}$$

$$\mathbf{Tan\ (\alpha - z) = A / B}$$

$$\mathbf{Sin\ \delta = C}$$

Where α_0 and δ_0 are the J2000.0 right ascension and declination and :

$$\mathbf{\zeta = 2306''.2181\ T + 0''.30188\ T^2 + 0''.017998\ T^3}$$

$$\mathbf{z = 2306''.2181\ T + 1''.09468\ T^2 + 0''.018203\ T^3}$$

$$\mathbf{\theta = 2004''.3109\ T - 0''.42665\ T^2 - 0''.041833\ T^3}$$

The following C subroutine calculates the coordinates of a star precessed from its J2000.0 position. The calling parameters are right ascension in decimal hours, and declination in decimal degrees. The J2000.0 right ascension must be converted to decimal values before calling the precession routine as in:

```
int rah, ram, ras ;                //RA hours, min sec
int decd, decm, decs ;             //Decl deg, min, sec
```

[141] Meeus ibid p. 166
[142] Meeus *ibid* p. 126.

```
double ra, decl ;                    //RA dec hrs, Decl dec deg
.
ra = rah + (ram/60.0) + (ras/36000.0) ;
decl = abs(decd) + fabs(decm/60.0) + (decs/3600.0) ;

//     Handle negative declination

if ((decd < 0) || (decm < 0)) decl = -decl ;

precess(ra, decl, &rap, &declp, T)

(more code)

//-------------------------------------------------------------

//      Precess star from J2000.0 coordinates
//     Sets rap to ra in dec hrs, decl in deg.

void precess (double ra, double decl, double *rap, double *dp, double T)
{
    double rar, dr, zeta, z, theta, a, b, c ;

    rar = radian(ra * 15.0) ;               //RA to degrees (radians)
    dr = radian(decl) ;                     //Declination to radians

    zeta = radian((((.017998 * T + .30188) * T + 2306.2181) * T) / 3600) ;
    z = radian((((.018203 * T + 1.09468) * T + 2306.2181) * T) / 3600) ;
    theta = radian((((-.041833 * T + -.42665) * T + 2004.3109) * T) / 3600) ;

    a = cos(dr) * sin(rar + zeta) ;
    b = cos(theta) * cos(dr) * cos(rar + zeta) - sin(theta) * sin(dr) ;
    c = sin(theta) * cos(dr) * cos(rar + zeta) + cos(theta) * sin(dr) ;

    *rap = normal(degree(z + atan2(a, b)))/15.0 ;
    *dp = degree(asin(c)) ;
}
```

Here is the same function in BASIC.

```
Public Sub PrecessStar(RAD As Double, Decld As Double, RAP As Double, DeclP As Double,
T As Double)
' RAH decimal hours, DECL in degrees.
' Returns RAP in decimal hours and DECLP in degrees precessed to T
Dim RA As Double, DECL As Double
Dim ZETA As Double, Z As Double, Theta As Double
Dim A As Double, B As Double, C As Double

RA = RADIAN(RAD * 15)              'RA in radians
DECL = RADIAN(Decld)               'Declination in radians
ZETA = RADIAN((((0.017998 * T + 0.30188) * T + 2306.2181) * T) / 3600)
Z = RADIAN((((0.018203 * T + 1.09468) * T + 2306.2181) * T) / 3600)
Theta = RADIAN((((-0.041833 * T + -0.42665) * T + 2004.3109) * T) / 3600)

A = Cos(DECL) * Sin(RA + ZETA)
B = Cos(Theta) * Cos(DECL) * Cos(RA + ZETA) - Sin(Theta) * Sin(DECL)
C = Sin(Theta) * Cos(DECL) * Cos(RA + ZETA) + Cos(Theta) * Sin(DECL)

RAP = Z + POLAR(A, B)
RAP = DEGREE(RAP) / 15#
DeclP = ARCSIN(C)
DeclP = DEGREE(DeclP)
End Sub
```

The Equation of Time

According to Smart, the equation of time can be calculated with sufficient accuracy for this type of application from:

$$E = y \sin 2L - 2 e \sin M + 4 e y \sin M \cos 2L - \tfrac{1}{2} y^2 \sin 4L - 5/4\, e^2 \sin 2M$$

Where:

- L = Sun's mean longitude
- e = eccentricity of the Earth's orbit
- M = Sun's mean anomaly
- $y = \tan^2 \varepsilon/2$ (ε = obliquity of ecliptic)

E is in radians. It can be converted to decimal minutes by converting to degrees and multiplying by four.

The following C routine implements this relationship using support routines previously discussed:

```
// Equation of time in decimal minutes (Smart pg. 149, eq. 28)

double eqt(double T)
{
        double obt, rmlong, rmanom, e, eqtl ;

        obt = tan(radian(obl(T) / 2.0)) ; obt*=obt ;
        rmlong = radian(mlong(T)) ;          // Solar Mean Longitude
        rmanom = radian(manom(T)) ;          //Mean Anomaly
        e = ecc(T);                          //Eccentricity of Earth's Orbit

        //      Calculate EQT in RADIANS

        eqtl = obt*sin(2.0*rmlong)-2.0*e*sin(rmanom)+4.0*e*obt*sin(rmanom)*cos(2.0*rmlong);
        eqtl = eqtl-0.5*obt*obt*sin(4.0*rmlong)-1.25*e*e *sin(2.0*rmanom) ;
        return (4.0 * degree(eqtl)) ;        //EqT in minutes
}
```

The equation of time can also be calculated from **Sun's mean longitude – Sun's right ascension**, which may be more accurate if you are calculating exact values for the Sun's right ascension.

Calculating Time

As promised, a procedure for calculating the approximate time from the altitude of a body of known declination and right ascension will be outlined. It is <u>much</u> easier to solve this problem on an astrolabe.

The problem is to calculate the current solar apparent time given h, δ, α, φ, λ, and the date. For a star just look up the J2000.0 δ and α and precess the star to the date. The Sun is bit problematical because we don't know the time at which to calculate δ and α, so we have to make do with a time within a few hours of the desired time. The error will not be as large as the error reading the astrolabe. This minor dilemma illustrates why this problem is a bit tricky. We can calculate celestial positions very accurately if we know the date and time. Here, we want to calculate the time, so the standard equations do not apply.

What we can calculate is the star or Sun's hour angle from: $\cos H = \dfrac{\sin h - \sin\varphi \sin\delta}{\cos\varphi \cos\delta}$. This is just the equation for converting from equatorial to horizontal coordinates solved for H.

Knowing the hour angle gives us a way to calculate the local sidereal time. Right ascension is the hour angle of the star from the vernal equinox. Therefore, the right ascension on the meridian at this instant, the local sidereal time (LST), is the star's right ascension plus its hour angle: LST = H + α.

With LST known, we can calculate the sidereal time at Greenwich (GST) from GST = LST + λ, with western longitudes taken as positive.

We can also calculate GST for 0 hr UT (GST0) in degrees for the date from:

$$GST0 = 100.46061837 + 36000.770053608\ T + 0.000387933\ T^2 - T^3 / 38710000$$

T is the interval in Julian centuries from J2000.0 as above. GST0 will have to be reduced to the interval 0-360° and can be converted to time by dividing by 15.

One civil hour = 1.00273790935 sidereal hour. Therefore, (GST-GST0)/ 1.00273790935 is the number of civil hours that have passed since midnight, and is the current UT.

Our local time is simply UT – our time zone, 5 hr. difference in this case..

We must then add the longitude correction and equation of time to get the local apparent solar time.

As we said, this is just an outline of the procedure. It is necessary to account for situations when the date at Greenwich is different from the local date and, of course, make sure times and angles are in the correct range. Following is the example on Page 19:

Date: 11/9/2006 φ = 38° 58′, λ = 77°.

Star: Altair h = 40°, δ = 8° 53′ 9″, α = 19h 51m 6s = 297.775° (precessed)

Solving equation above for hour angle: H = 44.739578°.

LST = H + α = 342.514578° = 22h 50m 3.5s.

GST = LST + λ = 59.514579° = 3h 58m 3.5s.

GST0 for Julian day 2454049.5 (T = 0.0685695) = 49.014491° = 3h 16m 3.5s.

UT = (GST-GST0)/1.00273790935 = 10.471459° = 0h 41m 53s.

Local zone time = UT – 5h = 19:41:53

Longitude correction = -8 min., Equation of time = 16m 9 s.

Local Apparent Time = 19:41:53 + 8m 9s = 19:50:2.2.

The current UT was found in the problem above. We can calculate any astronomical position we want for a given UT. First calculate the Julian day (2454049.52910) and then T (0.068570270). Then calculate the Sun's position in the following steps:

1. Calculate the obliquity of the ecliptic (ε). (23.43874°)
2. Calculate the Sun's true anomaly by solving Kepler's equation. (304.426°)
3. Calculate the Sun's mean anomaly and mean longitude. (305.994° and 229.049°)
4. Calculate the Sun's geocentric longitude (λ)= true anomaly + mean longitude – mean anomaly. (227.481°)
5. Calculate the Sun's declination from $\sin \delta = \sin \varepsilon \sin \lambda$. (-17° 3′)
6. Calculate the Sun's right ascension from $\tan \alpha = \cos \varepsilon \tan \lambda$. (15h 00m)
7. Calculate the Sun's altitude and azimuth using the equations at the beginning of this chapter. (-31.845° and 273.353°). This step requires the LST we found earlier in order to find the Sun's hour angle: H = (LST - α) × 15.

The only purpose of this discussion has been to point out how much easier it is to solve astronomy problems with an astrolabe than it is is calculate the result. On the other hand, you have to do the calculations to get precise answers. The accuracy of astrolabe results are generally adequate for personal use, particularly for finding the time. The low accuracy for problems involving celestial positions is why the astrolabe was abandoned.

Chapter 26 - Computers and Astrolabes

Introduction

In this chapter some topics on how computers can be used to design and make astrolabes will be introduced. You can skip this entire chapter if you are not interested in writing astrolabe programs. On the other hand, the techniques used to draw an astrolabe on a computer should be interesting background, and this material might be useful in the future if you decide to start down this rewarding and interesting road.

Included are some useful algorithms for drawing the astrolabe components, including source code in BASIC and C or C++. BASIC and C code is included because these are the two most popular programming languages. The included code can be translated to other languages quite easily. I started my astrolabe programming on a programmable calculator, moved to an obscure but very powerful language called APL, then to plotter programs and then to BASIC because it was available and, as an interpreted language, required the minimum amount of software to code and test. I later moved to C/C++ because, strangely, I find BASIC to be a bit wordy for my taste, and the Borland C development environment I use has spectacular editing and debugging facilities. The choice of a programming language is highly personal, and there is a strong sentiment among programmers of, "what I know is best". By all means, use whatever language you feel most comfortable with. If you are new to programming, I highly recommend BASIC as your learning vehicle. You can move to C or Visual BASIC over time as you gain confidence and can justify the cost of the development tools.

Virtually all astrolabes made today are laid out and drawn by computers. That is not to say there is not a lot of hand craftsmanship involved in making an astrolabe, but the arcs and scales are almost always computer generated for increased accuracy and fabrication convenience. Computers have virtually replaced hand drawing instruments. Hand engraving is rapidly becoming a lost art, and new manufacturing technologies are available that are much faster, cheaper and more accurate than is possible with handwork.

It is also no longer necessary to actually make an astrolabe instrument. An astrolabe drawn on the screen of a computer can be rather more flexible than a metal instrument. Since you can change the instrument's latitude or features with a few key strokes, the moon, planets and other celestial objects can be included if desired, and the astrolabe can be animated to illustrate astronomical principles in a very graphic way.

An astrolabe implemented on a computer embodies all of the educational potential of an instrument and, in fact, offers opportunities to exploit the highly intuitive astrolabe format to expand educational opportunities. In addition, a computerized astrolabe on a laptop is just as portable as an old astrolabe although it lacks the observational capability and charm of a working instrument.

That being said, there are many ways computers can be applied to astrolabes ranging from simplifying and speeding up the design process to producing working instruments. Following are some of the more common uses of computers for astrolabe related activities:

- Use of a spread sheet program to calculate the size and locations of the various astrolabe elements or basic astronomical calculations.
- Writing a program to calculate and print essential information used in the design of an astrolabe.

- Using a Computer Aided Design (CAD) program to design and lay out an instrument that will be made.
- Drawing an astrolabe on the screen of a computer to replace a static instrument and make it more flexible.
- Printing astrolabe components as masters for manufacturing or making a working instrument.

Using a spread sheet program to calculate the radius and center location of the almucantars is not very difficult and certainly within the ability of anyone with the interest. Some spreadsheet programs even have facilities for drawing pictures, which allows you to print the layout of a specific instrument. However, spreadsheet programs do not have the capability of producing a complete astrolabe. There are just too many details you can't control and certain astronomical calculations are awkward on a spreadsheet. However, spreadsheets are very useful for checking whether you have the math right on some tricky point. All of the sample values in the calculation summary section were created using a spreadsheet program. Spreadsheets get very awkward for the non-professional if a meaningful amount of logic is required to determine a value. For example, determining the correct quadrant for an arctangent can be very tricky. It is generally faster and easier to write a small program for this type of calculation. However, a spreadsheet duplicating the plate construction parameters such as those in old astrolabe treatises is easy.

Writing a simple program to print out basic design information is not difficult, even for the novice programmer. It is also a wonderful exercise to make sure you understand all of the details of some subject. It has been said you can't really understand an instrument until you have made one. While writing a program is not really a substitute for making an actual instrument, it does sharpen your understanding of each critical element.

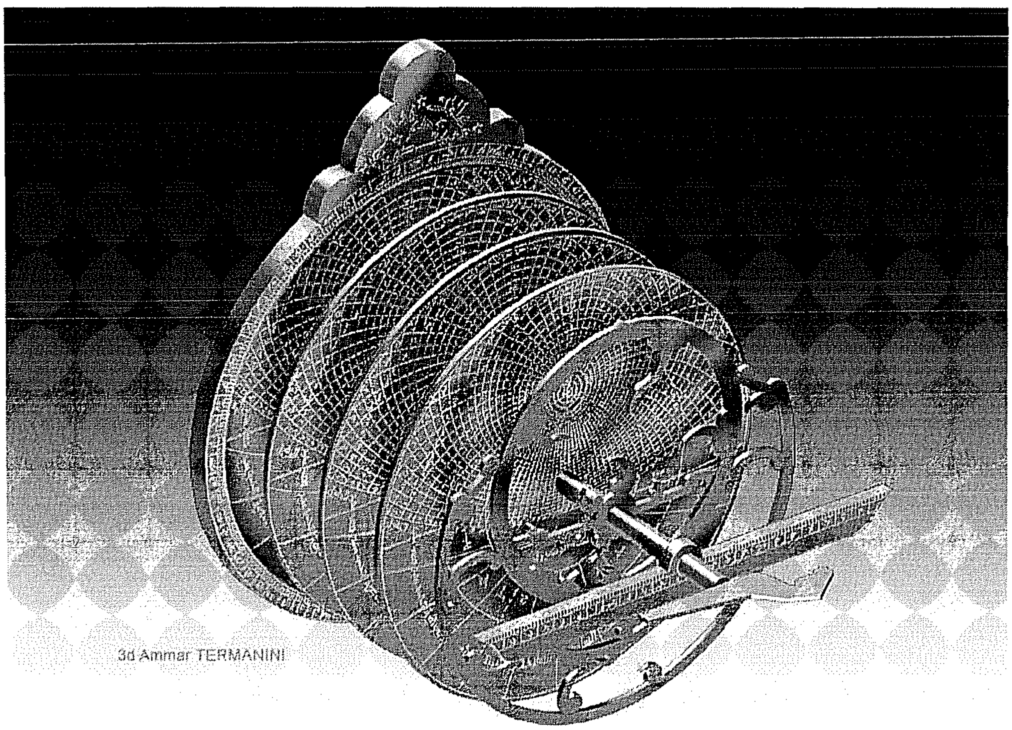


Figure 26-1. CAD Astrolabe

The most common method of making metal astrolabes today is chemical milling whereby the metal piece is coated with light sensitive photoresist, exposed to light and immersed in an acid bath. This process requires a mask of the component that can be drawn manually, but is usually computer produced. People skilled in chemical milling seem to also have skill in using complex Computer Aided Design (CAD) tools.

Figure 26-1 shows an AutoCAD display of an astrolabe made by Dr. Hasan Bilani of the University of Aleppo, Syria. Figure 26-2 is the front of the finished instrument.

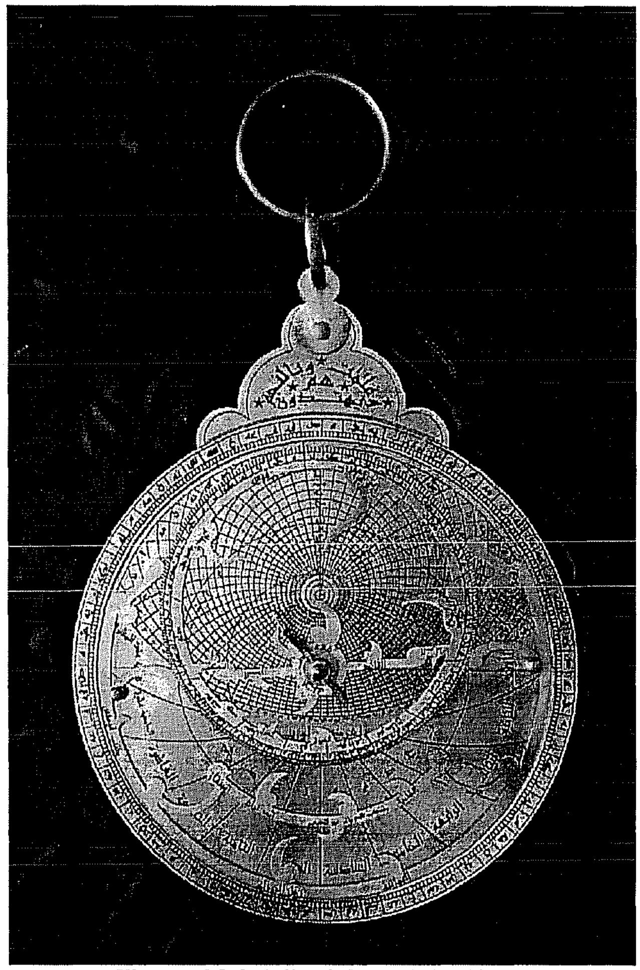


Figure 26-2. Bilani Astrolabe Front

Some beautiful astrolabe reproductions have been made by individuals using CAD. However, CAD programs can be quite expensive and all of them require a significant investment in time to learn the program and use it at the level required. We will not discuss this technique further since it is so specialized.

Drawing a computer generated astrolabe on the computer screen is a task for the experienced programmer, but it is fun and the results can be quite rewarding. However, the amount of effort required is high and dedication is required. In fact, it probably takes longer and is more work than hand engraving for an expert in either field. Also, drawing a computer generated astrolabe requires considerable background in astronomical calculations and computer graphics. On the other hand, it is also a terrific exercise in learning both disciplines. Your results can get more sophisticated as you gain experience in both areas. Computer graphics facilities are becoming much more common and easier to learn and use. One cannot expect to master the art of graphics programming in a few days, but it is not unreasonable to expect someone who is fairly well versed in programming in either C or BASIC could get good results after a few weeks effort.

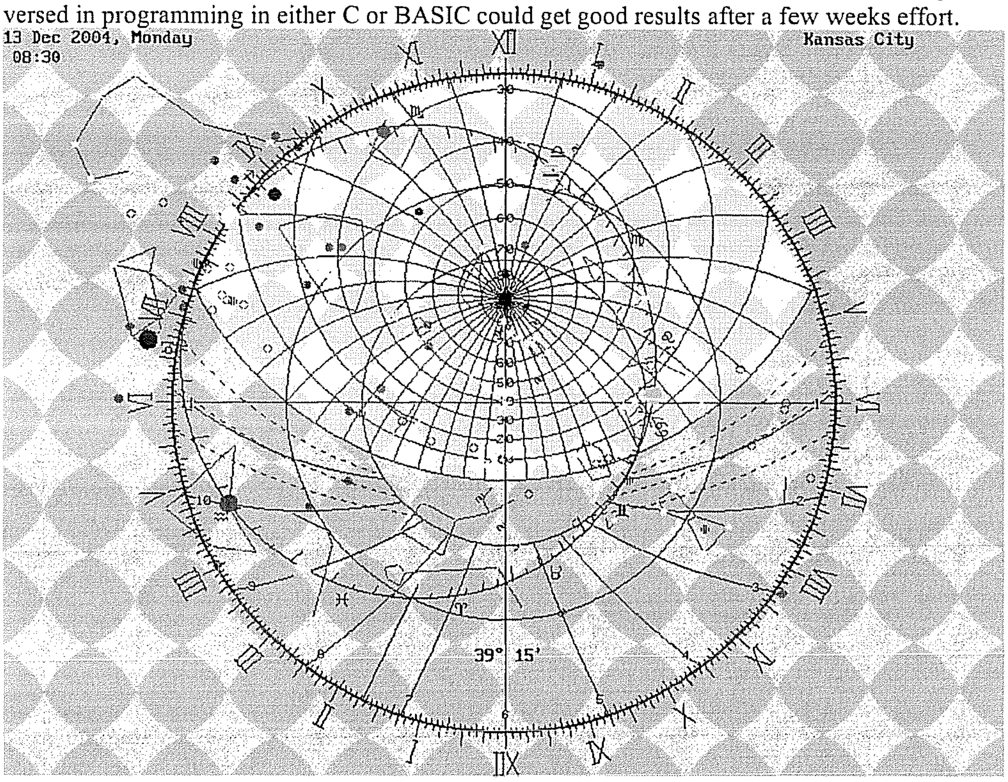


Figure 26-3. Computer Astrolabe

Figure 26-3 is a screen shot of a computerized astrolabe called *The Electric Astrolabe*[143] written by the author. This implementation can be set to any latitude, north or south and for any date with a valid Julian day number. It can show the astrolabe plate for either northern or southern projection. All astrolabe components can be selectively displayed. 150 stars in constellation asterisms are included, as is the moon, all planets out to Neptune, all Messier objects and other celestial objects selected by the user. The display can be animated at virtually any time increment one step at a time or continuously. The program can also show tabulated position data for displayed objects and an animated orrery.

Computers were much slower when The Electric Astrolabe was written in the early 1990's, so it was written entirely in assembler language and uses very low level graphic interfaces in order to achieve the required animation speed. It is a DOS program and, hopefully, someone will write a program of this type for Windows someday as it has proved to be a very useful educational tool for astrolabes and basic positional astronomy.

Printing a complete astrolabe that looks good is a bit easier. Probably the best facility for producing high quality printed output of graphics is PostScript. For example, PostScript allows fonts to be sized and rotated and it can produce almost any shape you can imagine. Almost all of the figures in this book were created with PostScript.

PostScript is a programming language that is executed in a PostScript capable printer. You either create a PostScript program with a text editor and send it to the printer, or you write a program that "writes" a PostScript program that runs when it is sent to the printer. Learning PostScript is challenge in itself because it is quite different from other computer languages. However, it is

[143] Morrison, J.E., "The Electronic Astrolabe", International Science Reviews, March, 1994

ideally suited for drawing old instruments using relatively basic functions, and many of the advanced PostScript capabilities are not required. A discussion of how to write PostScript programs is far beyond this book, but a brief introduction and sample programs are included later so you can see what it looks like.

Simple Astrolabe Programs

You can write a simple program to calculate the dimensions of the astrolabe components. Following is a little QBASIC program that calculates the centers and radii of the plate almucantars and azimuth arcs. The program asks for the radius of the Tropic of Capricorn and the latitude for the plate in decimal degrees and displays the results on the screen.

```
' Calculate Astrolabe Plate

DIM CM(18), RM(18)                        'Altitude Center Distances and Radii
DIM NP(18), RA(18)                        'Azimuth Center Distances and Radii
PI = 3.141593: PI2 = PI / 2               'Pi and Pi/2
RAD = PI / 180                            'Degree to Radian Conversion
OBR = 23.4397222# * RAD                   'Obliquity of Ecliptic in Radians

CLS : INPUT "Radius of Tropic of Capricorn"; RCAP   'Get RCAP

' Main Loop

ASTROLABE:
INPUT "Latitude"; LAT: IF LAT = 0 THEN END 'Get plate latitude
PHI = LAT * RAD                           'Convert Latitude to Radians
FAC = TAN((PI2 - OBR) / 2)                'Common Factor for Tropics
REQ = RCAP * FAC                          'REQ  =  Radius of Equator
RCAN = REQ * FAC                          'RCAN  =  Radius of Tropic of Cancer
YZ = REQ * TAN((PI2 - PHI) / 2)           'YZ  =  Zenith Distance
YN = -REQ * TAN((PI2 + PHI) / 2)          'YN  =  Nadir Distance

' Calculate Altitude Circle Center and Radius

FOR I = 0 TO 18                           'Calculate for Each 5 Degrees
    ALT = I * 5 * RAD                     'Altitude in Radians
    YU = REQ / TAN((PHI + ALT) / 2)       'Upper Edge of Almucantar Circle
    YL = -REQ * TAN((PHI - ALT) / 2)      'Lower Edge of Almucantar Circle
    CM(I) = (YU + YL) / 2                 'Center is Halfway Between Edges
    RM(I) = (YU - YL) / 2                 'Radius is Half of Difference
NEXT

' Calculate Azimuth Circle Center and Radius

N = ABS((YZ + YN) / 2)                    'Line of Azimuth Centers
Y = YZ + ABS(N)                           'Distance from Zenith to Line of Centers
FOR I = 0 TO 17                           'Each 5 Degrees From 0 to 85 Degrees
    FAC = 5 * I * RAD                     'Common Factor
    NP(I) = Y * TAN(FAC)                  'Center Distance from Meridian
    RA(I) = Y / COS(FAC)                  'Radius = Center to Zenith
NEXT

' Print Calculated Values

CLS : KEY OFF
PRINT SPC(20); : PRINT USING "Astrolabe for Latitude  =  ##.## Deg.";  LAT; :
PRINT
PRINT USING "Capricorn  =  ###.###"; RCAP;
PRINT USING "  Equator    =  ###.###"; REQ;
PRINT USING "  Cancer     =  ###.###"; RCAN

PRINT SPC(15); "ALTITUDES"; SPC(21); "AZIMUTHS"
PRINT  SPC(10);  "Deg";  SPC(4);  "Radius";  SPC(8);  "Center";  SPC(9);  "Radius";
SPC(7); "Center"
PRINT SPC(11); "0"; SPC(4);
```

```
PRINT USING "###.###"; RM(0);
PRINT USING "      ###.###"; CM(0)
FOR I = 1 TO 18
    PRINT SPC(10); : PRINT USING "##"; 5 * I;
    PRINT SPC(4); : PRINT USING "###.###"; RM(I);
    PRINT SPC(6); : PRINT USING "###.###"; CM(I);
    PRINT SPC(8); : PRINT USING "####.###"; RA(18 - I);
    PRINT SPC(4); : PRINT USING "####.###"; NP(18 - I)
NEXT

' Wait for key to be pressed

SPIN: A$ = INKEY$: IF A$ = "" THEN GOTO SPIN ELSE CLS : GOTO ASTROLABE
```

This program executes once and waits for input of another latitude. It ends when a latitude of 0 is entered.

The program can be extended and modified in a number of ways. For example, the calculation of the almucantar radius and center could be replaced to use the more compact calculations:

```
FOR I = 0 TO 18                         'Calculate for Each 5 Degrees
    ALT = I * 5 * RAD                   'Altitude in Radians
    FAC = SIN(PHI) + SIN(ALT)
    CM(I) = REQ * COS(PHI) / FAC        'Almucantar center
    RM(I) = REQ * COS(ALT) / FAC        'Almucantar radius
NEXT
```

You can also print directly to a printer to get a hard copy, or calculate the almucantars at higher resolution, such as every two degrees instead of five degrees, or you could prompt for the equator radius and calculate the tropics from this value. It would also be more elegant to enclose the calculation and printing code in a WHILE loop. One of the nice features of BASIC is how easy it is to experiment.

A similar program in C++ follows. The only C++ feature used in the program is output streams to simplify the printing. Other than this, it is a C program. This program is just a bit more sophisticated in that it calculates the obliquity of the ecliptic for the year of the instrument and takes the latitude in degrees and minutes instead of decimal degrees.

```
//==============================================================================
//    Print Astrolabe Plate Calculated Values.
//
//==============================================================================

#include   <iostream.h>
#include   <iomanip.h>
#include   <fstream.h>
#include   <stdio.h>
#include   <float.h>
#include   <math.h>
#include   <conio.h>

#define PI 3.14159265359
#define PI_2 (PI/2.0)

// Function profiles for called routines
double Julian(int mo, int day, int year, double ut);  // Julian day for date
double obl(double T);                       // Obliquity of ecliptic
double radian (double x);                   // convert degrees to radians
double ipart(double x);                     // return the integer part

//===== Main Program ===========================================================

void main(void)
{
    int i ;                                 //Loop counter
    int latd, latm ;                        //Latitude degrees and minutes
```

```
int MO = 6, DAY = 15, YEAR ;

double obr, jday, T ;               //Obliquity in radians
double req, rcan, rcap ;            //Tropics
double angr, fact;                  //Working variables
double lat, latr ;                  //Latitude - degrees and radians
double zenith, nadir, cline, yaz;   //Azimuth values
double center[19], radius[19] ;     //Almucantar center and radius
double np[19], rm[19] ;             //Azimuth center and radius

//-------- Code -------------------------------------------------------------

// Instrument year
cout << "Year for astrolabe: " ;
cin >> YEAR ;

jday = Julian(MO,DAY,YEAR,0.0) ;     //Julian day for date
T = (jday - 2451545.0) / 36525.0 ;   //T for obliquity calc
obr = radian ( obl (T) ) ;           //Obliquity for year

// Get latitude

cout << "Latitude (DD MM): " ;
cin >> latd >> latm ;

// Convert latitude to floating point

lat = abs(latd) + fabs(latm/60.0);   //Convert to dec. deg.
latr = radian (lat);                 //Latitude in radians

// Get Capricorn radius

cout << "Enter Capricorn radius: " ;
cin >> rcap ;

// Calculate tropics

req = rcap * tan ((PI_2 - obr) / 2.0) ;//Equator radius
rcan = req * tan ((PI_2 - obr) / 2.0) ;//Tropic of Cancer

//  Almucantars

for ( i = 0 ; i <= 18 ; i++ ) {
     angr = radian(i * 5) ;
     fact = sin (latr) + sin (angr);
     center[i] = req * cos (latr) / fact ;
     radius[i] = req * cos (angr) / fact;
     }

//   Azimuths

zenith = req * tan (( PI_2 - latr ) / 2.0 ) ;    //Center to zenith
nadir = -req * tan ((PI_2 + latr ) / 2.0) ;      //Center to nadir
cline   = fabs ( zenith + nadir ) / 2.0 ;        //Line of centers
yaz = zenith + cline ;                           //Center line to zenith

for ( i = 1; i <= 17 ; i++ )  {
    fact = radian(90 - i * 5) ;
    np[i] = yaz * tan (fact) ;
    rm[i] = yaz / cos (fact) ;
    }

np[18] = np[0] = rm[0] = 0 ;                 //90 deg azimuth center
rm[18] = yaz ;                               //   and radius

//Output header and labels
printf ("      Astrolabe Plate for %d%c %d' for %d\n\n",latd, 0xF8, latm, YEAR) ;
printf ("Capricorn = %6.3f  Equator = %6.3f  Cancer = %6.3f\n", rcap, req, rcan) ;
printf ("\t\tAlmucantars\t\t\tAzimuths\n") ;
printf("\tRadius\t\tCenter\t\tRadius\t\tCenter\n") ;
```

```
    for ( i = 0 ; i <= 18 ; i++ )
        printf("   %d    \t   %6.3f    \t   %6.3f    \t   %6.3f    \t   %6.3f\n",(i*5),
                        radius[i],center[i], rm[i], np[i]) ;

    getch() ;

    return ;
}

//===== Subroutines ==========================================================

// Julian day for date

double Julian (int mo, int day, int year, double ut)
{
    int jm, jy ;
    double jday, wv, dayj ;

    if (mo > 2)
        {jm = mo + 1 ; jy = year ;}
    else
        {jm = mo + 13 ; jy = year - 1 ;}

    dayj = day + ut/24.0 ;
    jday = ipart(365.25 * jy) + ipart(30.6001 * jm) + dayj + 1720994.5 ;

    //Return Julian calendar jday if before 10/4/1582
    //if (jday <= 2299160.0)
    //Modification to return jday if before 9/2/1752 English Gregorian calendar
    if (jday <= 2361220.0)
     return (jday) ;
    else
     wv = ipart(0.01*year) ; return (jday + 2 - wv + ipart(wv*0.25)) ;
}

 // Obliquity of ecliptic (degrees)

double obl(double T)
{
  return (((5.036111e-07 * T - 1.638889e-07) * T - .013004167) * T + 23.4392911) ;
}

// degrees to radians

double radian (double x)
{
    return (x * PI/180.0) ;
}

// return the integer part

double ipart(double x)
 {
    return ((x >= 0) ? floor(x) : ceil(x)) ;
 }
```

Some useful routines

There are several special routines that are essential if you want to draw or print a complete astrolabe. The following discussion is intended for experienced programmers, but might be interesting to a novice who wants to know how certain things might be done. The subroutines are in the C programming language, but can easily be converted to BASIC or some other language. The coding style in these routines is the author's and includes lots of comments, so the logic can be reconstructed, and white space to make the code more readable.

Drawing a circle from three points

Some of the astrolabe elements require you to draw a circle based on knowing three points, notably the unequal hour curves and the Houses of Heaven. This is a fairly easy, but time consuming procedure to do by hand using simple geometry; you simply draw a line between the points and erect a perpendicular in the center of the line. The point where the perpendiculars meet is the center and the distance from any of the points is the radius.

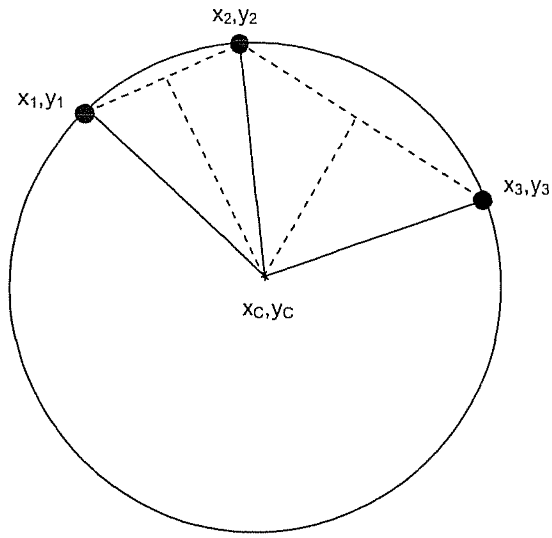


Figure 26-4. Circle generated by three points

This is not convenient to do in a program, so another method is required. The following procedure looks a lot more complicated than it is, and is the sort of problem one might have assigned in the first week of a course in computer graphics.

A circle can be constructed to pass through any three non-colinear points. Our objective is to find the center of a circle, x_c,y_c, and its radius, r, given three points, x_1,y_1, x_2,y_2, x_3,y_3.

Recall the equation of a circle is: $x^2 + y^2 = r^2$.

The resulting circle will pass through each of the three points and have the equation:

$$(x_c - x)^2 + (y_c - y)^2 = r^2$$

Therefore, $(x_c - x_1)^2 + (y_c - y_1)^2 = (x_c - x_2)^2 + (y_c - y_2)^2 = (x_c - x_3)^2 + (y_c - y_3)^2$

Taking any pair of points, we can expand this relationship and collect terms to get:

$$x_c (x_2 - x_1) + y_c (y_2 - y_1) = [(x_2^2 - x_1^2) + (y_2^2 - y_1^2)] / 2$$

Since x_1,y_1 and x_2,y_2 are known, we can solve the equation and write it in the form:

$$a\, x_c + b\, y_c = d$$

Where $a = x_2 - x_1$, $b = y_2 - y_1$, and $c = [(x_2^2 - x_1^2) + (y_2^2 - y_1^2)] / 2$.

We do this for two sets of points, which gives us two equations:

$$a\,x_c + b\,y_c = c$$
$$d\,x_c + e\,y_c = f$$

with d, e and f defined as a, b and c but using another pair of points.

We solve these two simultaneous equations using Gaussian elimination to get:

$$x_c = (c / b - f / e) / (a / b - d/e)$$

$$y_c = (c - a \times x_c) / b$$

$$r = \sqrt{[(x_c - x_1)^2 + (y_c - y_1)^2]}$$

The following C routine provides this function. The routine `circle3` is passed the coordinates of the three points (`xa, ya, xb, yb, xc, yc`) and stores the radius and center coordinates in `rc, xctr and yctr`.

```
//==========================================================================
//  Calculate center (XCTR,YCTR) and radius (rc) of circle fitting three points

void circle3 (double xa, double ya, double xb, double yb, double xc, double yc,
               double *rc, double *xctr, double *yctr)
{

     double a, b, c, d, e, f;

     //   1. Find point equidistant from XA,YA  XB,YB and XC,YC

     a = xb - xa ;
     b = yb - ya ;
     c = (xb * xb - xa * xa + yb * yb - ya * ya) / 2.0 ;
     d = xc - xa ;
     e = yc - ya ;
     f = (xc * xc - xa * xa + yc * yc - ya * ya) / 2.0 ;

     // 2. Solve two simultaneous equations for X using Gaussian elimination

     *xctr = ((c / b) - (f / e)) / ((a / b) - (d / e));

     //    Substitute to solve for Y. R = SQR of sum of square

     *yctr = (c - a * *xctr) / b;

     *rc = sqrt(((*xctr - xa) * (*xctr - xa)) + ((*yctr - ya) * (*yctr - ya)));
}
```

This routine is called as:

```
     double x1, y1, x2, y2, x3, y3, rc, xc, yc ;

     circle3(x1, y1, x2, y2, x3, y3, &rc, &xc, &yc ) ;
```

Note `circle3` does not return a value, but sets the values of rc, xc and yc directly. The & in front of rc, xc and yc means "address of" so the subroutine will know where to store the results. The reference to these variables in the subroutine is preceded by * which means, "value of" in C speak.

Clipping Circles

Clipping is the computer graphics term for drawing only that part of something within some defined area. An obvious application of clipping is the altitude circles on the astrolabe plate; only the part of the circle within the Tropic of Capricorn is drawn. Both the Tropic of Capricorn and the horizon clip the azimuth arcs. You must clip these circles, and others, if you intend to draw your astrolabe on the computer screen or print it. It is even useful to clip these circles if you are using PostScript, because it speeds the printing significantly. It is also very useful for determining the location of various labels on the plate such as the degree of an almucantar. This is, perhaps, the most useful specialized subroutine for drawing an astrolabe.

This description does not exactly match the computer science definition of clipping, which concentrates on deciding whether a given point should be drawn or not based on whether it lies within the clipping area. Instead, we will "preclip" our circle by drawing only the visible part.

The basic problem is to find what part of the circle to draw. To do this, we find the intersection points of the circle we want to draw and the circle defining the clipping area.

In Figure 26-5, the base circle is the circle to be drawn. We want to draw only the part of the circle within the intersection circle. This figure is typical for drawing an azimuth arc. Needed are the coordinates of the intersection points and the angles of intersection relative to the base circle. The angles are needed for drawing a circle and the coordinates of the intersections may be useful for labeling.

The center of the base circle is at x,y and it has radius r_1. The intersection circle has its center at x_0,y_0 and radius r_2. The objective is to find the coordinates of the points x_1,y_1 and x_2,y_2 where the circles intersect and the angles a_1 and a_2 of the intersection points. In the figure, a_1 is the angle from r_1 to the radius to x_1,y_1 and a_2 is the angle from r_1 to the radius to x_2,y_2.

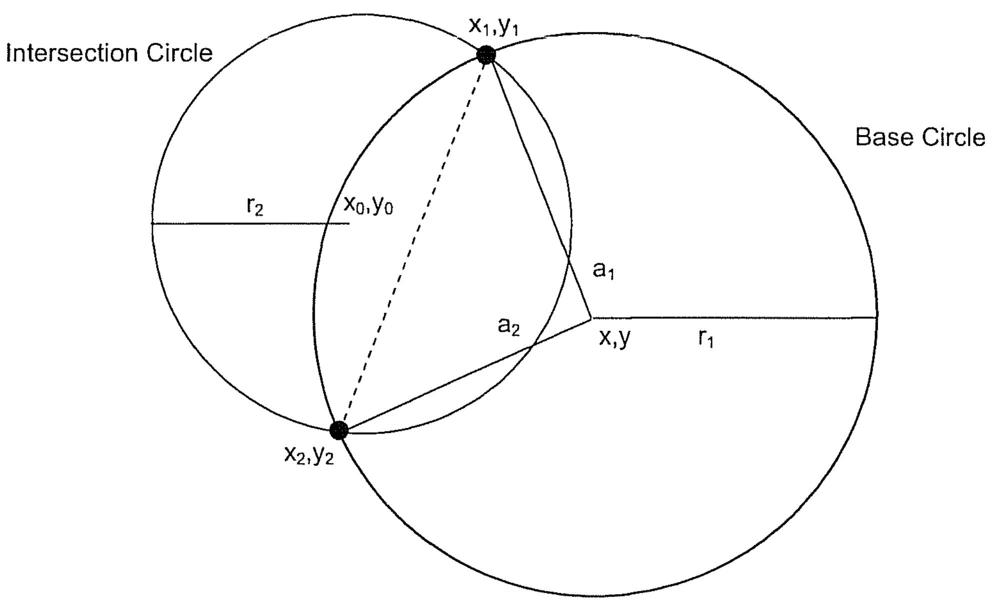


Figure 26-5. Clipping circles

The equation for the base circle is $x^2 + y^2 = r_1^2$ and the equation of the intersection circle is $(x-x_0)^2 + (y-y_0)^2 = r_2^2$. Note $x^2 = r_1^2 - y^2$. We expand the equation of the intersection circle, substitute and collect terms. The solution is in the form of a straight line: $y = mx + b$, where m is the slope of the line and b is the y-intersect. The intersection points are on a straight line with:

$$m = - x_0 / y_0$$
$$b = - 1/2y_0 (r_2^2 - r_1^2 - y_0^2 - x_0^2)$$

$$x = [-2mb \pm \sqrt{(4 m^2 b^2 - 4(m^2 + 1) (b^2 - r_1^2)}] / 2 (m^2 + 1)$$

Note the solution gives two values for x.

Once x is found, $y = mx + b$.

The intersection angles are calculated from $\tan a = y / x$

Some logic is required to select the roots in the correct order.

The following C subroutine performs these calculations:

```
//=============================================================================
// Subroutine to Calculate Points of Intersection of Two Circles

// int xsect (double xa, double ya, double r2, double xb, double yb, double r1,
//            double *x1, double *y1, double *x2, double * y2,
//            double *a1, double *a2)


// The most common use of xsect is to find the intersection points between
// a circle to be drawn and another circle so the drawn circle can be
// clipped. It is also used to find points of intersection between two
// circle for inserting labels.

// Calling parameters:

// The first circle parameters are for the circle to be drawn.
// Base circle: center at xa, ya. Radius = r2
// Intersecting circle: center at xb, yb. Radius = r1

// Returns coordinates of intersection points, (XI1,YI1) and (XI2,YI2) and
// angles of intersectionsm a1 and a2 (radians).

// Given base circle to draw at origin: x^2 + y^2 = r1^2

// Intersection Circle at (x0,y0): (x-x0)^2 + (y-y0)^2 = r2^2

// Intersection points between the circles lie on a line:

//       y = mx + b

// Where:  m = -x0/y0  (perpendicular to line through centers)

//       b = - 1/2y0 (r2^2 - r1^2 - y0^2 - x0^2)

//       x = (-mb +/- SQR(m^2 b^2 - (m^2+1)(b^2-r1^2)) / (m^2 + 1)

// If the roots of x are imaginary (i.e. the radical is negative) then
// there is no intersection and A1 and A2 are set to 0 and 2Pi.

// Returned intersection angles are in radians.

// N.B. Returned x,y are relative to center of base circle, not in
//      screen coordinates.

// Returns: 1 = Interssection values stored.
//          0 = No intersection: x1 = x2 = y1 = y2 = 0, a1 = 0, a2 = 2Pi

int xsect (double xa, double ya, double r2, double xb, double yb, double r1,
             double *x1, double *y1, double *x2, double * y2,
             double *a1, double *a2)
{
   int dy0 = FALSE ;

   double X1, Y1, X2, Y2, A1, A2 ;
   double x0, y0, m, b, ac, b2, f1, f2, f3 ;
```

```
// Initialize return values
*x1 = *x2 = *y1 = *y2 = *a1 = 0.0 ; *a2 = 2 * PI ;

//Check for vertical intersection line - special case where ya = yb must be
//treated separately to avoid divide by 0

if (ya == yb) {
    dy0 = TRUE ;
    ya = yb + (xb - xa) ;
    xa = xb ;                              //Rotate base circle vertically
    }

//Intersection line not vertical = normal case

//Set origin to base circle. x0,y0 is center of xsect circle rel. to base
x0 = xa - xb ;
y0 = ya - yb ;
m = -x0 / y0 ;                             //Slope of intersection line

//Y-Intercept of Intersection Line

b = -((r2*r2) - (r1*r1) - (y0*y0) - (x0*x0)) / (2.0 * y0) ;

ac = 4.0 * (1.0 + m*m) * (b*b - r1*r1) ;  //4AC Part of Discriminanat

b2 = 4.0 * m*m * b*b ;                     //B^2 Part of Discriminant

if ( ac > b2 ) return (0) ;                //No roots = no intersect

if ( m == 0.0 ) {                          //Symmetrical roots

  X2 = sqrt(r1*r1 - b*b) ;
       X1 = -X2 ;
       }

  else {
       f1 = -2.0 * m * b ;          //Parts of quadratic solution
       f2 = sqrt(b2 - ac) ;
       f3 = 2.0 * (1.0 + m*m) ;
       X1 = (f1 + f2) / f3 ;        //Roots are intersection coordinates
       X2 = (f1 - f2) / f3 ;
       }

  Y1 = m * X1 + b ;                 //Y coordinates where int. line meets x
  Y2 = m * X2 + b ;

  //Adjust intersection to base circle origin
  X1 = X1 - x0 ;
  X2 = X2 - x0 ;
  Y1 = Y1 - y0 ;
  Y2 = Y2 - y0 ;

  // Calculate intersection angles
  if ( X1 != 0.0 ) {                       //Calculate first angle
       A1 = polar( Y1 , X1) ;
       }
  else {
       if ( Y1 > 0.0 ) A1 = PI_2 ;         //x1 = 0 => a = 90 or 270

       else  A1 = 3.0 * PI / 2.0 ;
       }

  if ( X2 != 0.0)                          //Calculate second angle
       A2 = polar( Y2 , X2 ) ;

  else {
       if (Y2 > 0.0) A2 = PI / 2.0 ;

       else A2 = 3.0 * PI / 2.0 ;
       }
```

```
    //Vertical intersection case: rotate angles and calculate x,y for base
    if (dy0) {
         swap(&A1, &A2) ;
         A1 = A1 + PI_2 ;
         if (A1>2*PI) A1-=2*PI ;              //Normalize intersection angle
         while (A1<0) A1+=2*PI ;

         A2 = A2 + PI_2 ; if (A2>(2*PI)) A2-=2*PI ; while (A2<0) A1+=2*PI ;

         //Intersection coordinates relative to center of base circle
         X1 = r2 * cos(A1) ; X2 = X1 ;
         Y1 = r2 * sin(A1) ; Y2 = -Y1 ;
         }

    // Store returned values
    *a1 = A1; *x1 = X1; *y1 = Y1 ;
    *a2 = A2; *x2 = X2; *y2 = Y2 ;

    // Sort returned values
    if ( *a1 > *a2) {
         swap (a1,a2) ;
         swap (x1,x2) ;
         swap (y1,y2) ;
         }

    return(1);  //Return shows there is an intersection
    }
```

In the routine above, the following preprocesor directives, which are not the same for all C compilers, are assumed:

```
#include  <math.h>
#define PI M_PI
#define PI_2 M_PI_2
#define TRUE -1
#define FALSE 0
```

The following routine is called:

```
// Swap two values

 void swap (double *a, double *b)
 {
     double hold ;

     hold = *a ;
     *a = *b ;
     *b = hold ;
 }
```

Note the use of `polar(y,x)` to return a value for the angle between 0 and 2π.

```
//   Polar arctangent

double polar ( double num, double den )   // Polar Arctangent
{
     double ang;

     ang = atan2( num , den );
     if (ang < 0.0) ang = ang + 2.0 * PI;
     return ang;
 }
```

An example of the use of xsect is drawing the azimuth arcs on an astrolabe plate. The intersection angle of of the azimuth arc and the horizon is needed. In the following example, `center` is the x-coordinate of the center of the azimuth circle, `cline` is the y-coordinate and the radius is `radius`. The horizon is centered at (`0,hcenter`) with radius `hradius`.

```
xsect(center, -cline, radius, 0.0, hcenter, hradius, &x1, &y1, &x2, &y2, &a1, &a2) ;
```

The limiting angle chosen to draw the azimuth depends on the direction of the arc. For azimuths with the center to the left of the plate center, the arc will start at a_1. For arcs with the center to the right of the plate center, the arc will end at a_2.

To draw the azimuth arc, call xsect again to get the intersection with the Tropic of Capricorn.

xsect is a very useful routine and has many applications for drawing the various plate arcs. It is also very useful for finding the locations of labels. As useful as it is, the use of a routine such as xsect to find the angles for drawing circles is not without complications. The orientation of the intersection points may require additional logic to handle cases where the relative positions of the circle centers change for different circles of the same type.

The calculations are quite a bit simpler if the intersecting circles have the same x-coordinate for their centers, such as finding the intersection points of almucantars and Capricorn. Here, the almucantar is the base circle (r_1). The coordinates of the intersection are:

$$y = -\frac{1}{2y}(r_2^2 - r_1^2 - y_2^2)$$ where $r_1 = R_{Cap}$, r_2 = almucantar radius, y_2 = almucantar center

$x = \pm\sqrt{r_1^2 - y^2}$ and the intersection angle is **arctan (y / x)**.

This case is handled by the more general xsect routine, but it can be useful for special cases.

Ecliptic Division

Most of the instruments in this book require the ecliptic to be divided by the zodiac. The geometric method of dividing the ecliptic described in an earlier chapter can be expressed arithmetically for use in a program.

There are probably several ways to do this. The following description applies to the ecliptic projection when it is oriented with Capricorn 0° on the meridian (Aries 0° is on the east side of the right horizon). There is nothing special about this ecliptic orientation. It is merely a convenience for the calculations. This method is a direct algebraic representation of the graphical method. The objective is to calculate the coordinates of each ecliptic division.

Calculate the following values once (R_{cap} = radius of Tropic of Capricorn, R_{can} = radius of Tropic of Cancer, R_{eq} = Equator radius, ε = obliquity of ecliptic) :

1. Ecliptic circle radius: $R_{ecl} = (R_{cap} + R_{can}) / 2$
2. Position of ecliptic pole: $y_{ep} = R_{eq} \tan(\varepsilon / 2)$
3. Position of unrotated ecliptic center: $y_{ec} = R_{cap} - R_{ecl}$

For each division:

4. Solve for the coordinates of a point on the ecliptic circle by finding the x,y coordinates of the point where the line from the ecliptic pole to the longitude, λ, on the equator intercepts the ecliptic using the equation of a straight line: $y = mx + b$ where:

$$m = \frac{R_{eq}\sin\lambda - y_{ep}}{R_{eq}\cos\lambda}$$

5. Solve the quadratic equation: $y = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

   where:

   $a = 1 + m^2$

   $b = -2(y_{ep} + m^2\, y_{ec})$

   $c = y_{ep}^2 + m^2\,(y_{ec}^2 - R_{ecl}^2)$

6. Solve for $x = (y - y_{ep}) / m$

7. The angle of the line from the center is found from atan(y / x).

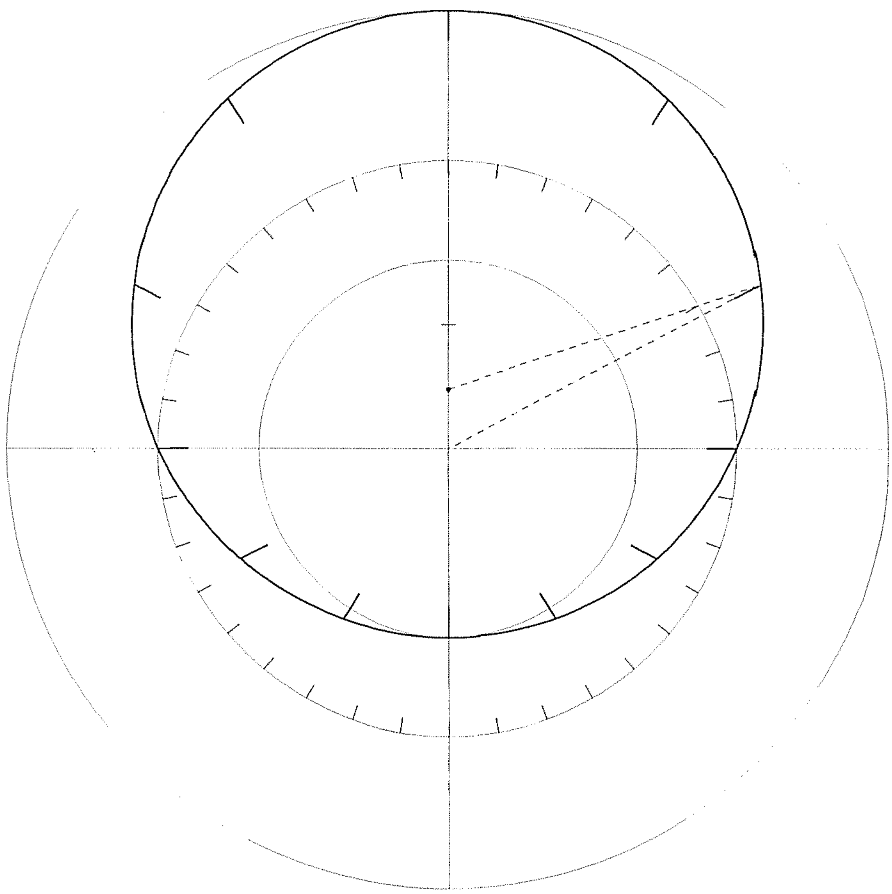


Figure 26-6. Ecliptic Division

This procedure looks more formidable than it really is and it is not difficult to program, although some logic is required to find the correct roots to use for each quadrant. Note the ecliptic divisions for half the zodiac can be determined and the other side drawn by symmetry.

To divide the ecliptic by the zodiac, calculate the ecliptic divisions for each degree from 1 to 360, divide the divisions with longer tics for 5, 10 and 30 degrees.

To divide the ecliptic by the calendar you calculate the Sun's longitude for each day of the year and use the procedure above to find the corresponding ecliptic line. Draw the tics with longer divisions for each 5 and 10 days and a long tic between months.

The following C subroutine calculates the coordinates and angle of an ecliptic division:

```
//===========================================================================
// Divide Ecliptic. Return coordinates for longitude gl.

// Uses globals rcap, rcan, req, yep (ecliptic pole) and yec (ecliptic center)

//   Description of method:

//   The graphical method of laying out the ecliptic on an astrolabe is:

//   a. Divide equator in arcs equal to the Sun's geocentric longitude.
//      For example, the zodiac signs are defined by lines 30 degrees apart.
//   b. Draw a line from the ecliptic pole, through the point on the equator
//      defined in a., to the ecliptic circle.
//   c. Draw a line segment from the point on the ecliptic circle found in
//       b toward the center of the astrolabe.

//   The method used in this module is an exact reproduction of this method.

//   The steps are:

//   a. Radius of ecliptic circle = RECL = (RCAP + RCAN) / 2
//   b. Ecliptic Pole is ECLP = REQ * tan (Obliquity/2)
//   c. Location of unrotated ecliptic center = ECLC = RCAP - RECL
//   d. To find point xa,xb on ecliptic circle for geocentric longitude, a,

//      1. Slope of line from ecliptic pole, through equator is
//             m = (REQ*SIN A - ECLP)/REQ*COS A
//      2. Solve quadratic equation for yb with:
//             a = 1 + m^2
//             b = -2(ECLP + m^2 * ECLC)
//             c = ECLP^2 + m^2 (ECLC^2 - ECLR^2)
//      3. xb = (yb-ECLP)/m
//      4. Angle of line from center = b = arctan (yb/xb)
//
//      Note: considerable logic is needed to select the correct root of the
//             quadratic solution and to choose the line end points.

//      Calling sequence: gl = geocentric longitude

//      returns: nothing. Sets values of b = angle of line from center,
//                           xb, yb = coordinates of point on ecliptic
//                           rr = distance of point from center

void ecliptic (double gl, double *b, double *xb, double *yb, double *rr )
{
     double a, m1, qa, qb, qc, ac, f1, f2, f3 ;
     double x1, x2, y1, y2, b1, b2 ;

     if ( gl%90 == 0 ) }                 //Points at quadrants
         *b = radian (gl) ;
         *xb = req * sgn ( 90 - gl ) + req * (gl == 270) ; //x = req or 0
         if ( (gl ==180) | (gl ==0) ) *yb = 0.0 ;
         else if (gl == 90) *yb = rcap ;
         else if (gl == 270) *yb = -rcan ;
         }
     else {                              //Points not at quadrants
        a = radian(gl) ;
        m1 = ( req * sin(a) - yep ) / (req * cos(a)) ; //Slope from ecl pole to equ
        qa = 1.0 + m1*m1 ;        //Quadratic Terms
        qb = -2.0 * (yep + yec * m1*m1) ;
        qc = yep*yep + m1*m1 * ( yec*yec - rec*rec ) ;
        ac = 4.0 * qa * qc ;             //4AC Part of Discriminanat
        b2 = qb*qb ;                     //B^2 Part of Discriminant
```

```
        f1 = -qb ;
        f2 = sqrt((b2 - ac)) ;
        f3 = 2.0 * qa ;                         //Parts of Quadratic Solution
        y1 = (f1 + f2) / f3 ;
        y2 = (f1 - f2) / f3 ;           //Roots are Y-Coordinates of Intersection
        x1 = (y1 - yep) / m1 ;          //X-Coordinates are Where Int. Line Meets Y
        x2 = (y2 - yep) / m1 ;
        b1 = polar(y1,x1) ;             //Angles
        b2 = polar(y2,x2) ;

        //Determine correct roots
        if ( gl <= 180.0) {
          if (b1 < b2) {
                *b = b1 ;
                *xb = x1 ;
                *yb = y1 ;
                }
          else {
                *b = b2 ;
                *xb = x2 ;
                *yb = y2 ;
                }
          }
        else {
          if (b1 > b2) {
                *b = b1 ;
                *xb = x1 ;
                *yb = y1 ;
                }
          else {
                *b = b2 ;
                *xb = x2 ;
                *yb = y2 ;
                }
          }
        if ( (gl > 90.0) & (*b < PI_2) ) {
          if (b1 > b2) {
                *b = b1 ;
                *xb = x1 ;
                *yb = y1 ;
                }
          else {
                *b = b2 ;
                *xb = x2 ;
                *yb = y2 ;
                }
        }
    }
    // Calculate radius vector

    *rr = sqrt( (*xb)*(*xb) + (*yb)*(*yb)) ;
}
```

The called routine `polar()` is an arctangent routine that returns an angle $[0,2\pi]$.

```
    // Polar arctangent

     double polar ( double num, double den)    // Polar Arctangent
     {
       double ang ;

       ang = atan2( num , den ) ;
       if (ang < 0.0) ang = ang + 2.0 * PI ;
       return ang ;
     }
```

The ecliptic routine is flexible enough to be called in many different ways. For flexibility in drawing planispheric astrolabes with the ecliptic divided by either the zodiac or the calendar, it can be called from the main line rete drawing routine through an intermediate routine that stores the locations of the tic marks

The main line routine calls the function `calendar` or `zodiac` depending on the required division and then draws the ecliptic division lines. The x,y coordinates of both ends of each tic mark are stored in the array `ecl [ ]` which must be large enough to hold the maximum number of lines and the count of the number of tics is in the variable, `lines`.

```
int lines ;
double ecl[366][4] ;          //Ecliptic division lines
double label[24][3] ;         //Ecliptic degree or day labels

int dom[12] = {31, 28, 31, 30, 31, 30, 31, 31, 30, 31, 30, 31} ;

if (decision variable for selecting ecliptic division) calendar(T) ;
else zodiac() ;
```

The ecliptic divisions are then drawn with a simple loop that is independent of the ecliptic division method:

```
for (i = 0 ; i <= lines ; i++) {

    (Draw one division from ecl[i][0], ecl[i][1] to ecl[i][2], ecl[i][3])

    }
```

The function `ZODIAC` fills in `ecl [] []` with the coordinates of each division tic mark for each degree of longitude. It makes the line at Aries 0° extend to the Tropic of Capricorn as the sidereal time pointer. It also saves the coordinates and angle of the 10° and 20° divisions for inserting labels. The variables `LONGL`, `TENL`, `MEDIUM` and `SHORT` define the lengths of the tic marks for the value of the tic mark and are defined in whatever units are being used to draw the rete.

```
//==============================================================================
// Divide Ecliptic by zodiac

// zodiac calculates the end points for the lines that represent each degree
// of solar longitude around the ecliptic and stores the coordinates in
// the array "ecl". The coordinates of the inner end of the lines at 10 and
// 20 deg are stored in the array "label" to define the position of the
// labels.

// The ecliptic divisions are the same for north and south latitudes.

void zodiac(void)
{
     double lin, gl, b, xb, yb, rr, xro, yro ;
     int i, j, tenl ;

     //    Sidereal time pointer
     ecl[0][0] = -rcap ; ecl[0][1] = 0.0 ;
     ecl[0][2] = -req + LONGL ; ecl[0][3] = 0.0 ;
     lines = 1 ; j = 0 ;                              //Count ecliptic lines

        // Calculate geocentric longitude for each degree from Aries 0

     for (i = 1; i <= 359; i++) {

           gl = (i + 180)%360 ;

           if (gl == 0) {
             ecl[lines][0] = req ;
             ecl[lines][1] = 0.0 ;
             ecl[lines][2] = req - LONGL ;
             ecl[lines][3] = 0.0 ;
```

```
            lines++ ;
            continue ;
            }

          ecliptic(gl, &b, &xb, &yb, &rr);   //Get line for longitude

          //   Calculate line ends and tic length

          tenl = FALSE ;
          if (i%30 == 0) lin = LONGL ;

          else {
            if ((i%10) == 0){lin = TENL ; tenl = TRUE ;}
            else
            if ((i%5) == 0) lin = MED ;
            else
            lin = SHORT ;
            }

          ecl[lines][0] = xb ;
          ecl[lines][1] = yb ;
          xro = (rr - lin) * cos(b) ; ecl[lines][2] = xro ;
          yro = (rr - lin) * sin(b) ; ecl[lines][3] = yro ;
          lines++ ;

          if ( tenl ) }              //Save 10 deg positions for labels
            label[j][0] = degree(b) ;
            label[j][1] = xro ;
            label[j][2] = yro ;
            j++ ;
            }
     }
     --lines ;                 //Adjust line count to final value
}
```

The routine CALENDAR calculates the ecliptic divisions for each day of the year with longer tics for the first day of each month and each 5 and 10 days. It uses the array `dom[]` to control the length of each month. T is the date parameter for calculating the Sun's geocentric longitude for each day of the year. See the appendix on astronomical calculations for and explanation.

```
int dom[12] = {31, 28, 31, 30, 31, 30, 31, 31, 30, 31, 30, 31} ;

//=========================================================================
//   Divide ecliptic by calendar

void calendar (double T)
{
     double dt, td, lc ;
     double lin, gl, b, xb, yb, rr, xro, yro ;
     int i, j, k, tenl ;

     //   Sidereal time pointer
     ecl[0][0] = -rcap; ecl[0][1] = 0.0 ;
     ecl[0][2] = -req; ecl[0][3] = 0.0 ;
     lines = 1; j = 0;                  //Count ecliptic lines

     //  Calculate geocentric longitude for each day from Jan 0

     dt = 1.0 / 36525.0 ;               //Change in T per day
     lc = lng / (360.0 * 36525.0) ;     //Longitude correction

     td = T + lc ;                      //Starting T is Jan 0

     //Calculate ecliptic for each month of year
     for (i = 0; i <= 11; i++) {

           //Calculate ecliptic tics for each day of month
           for (k = 0; k <= (dom[i] - 1); k++) {
                gl = geolong(td) ;
                gl -= 180.0 ;
```

```
            if (gl < 0.0) gl = gl + 360.0 ;}

            ecliptic(gl, &b, &xb, &yb, &rr) ;     //Get line for longitude
            td += dt ;

            tenl = FALSE;
            if (k == 0) {          //Line length
               lin = LONGL ;
               }
            else {
                if ((k%10) == 0) {lin = TENL ; tenl = TRUE ;}
                else
                   if ((k%5) == 0) lin = MED ;
                   else lin = SHORT ;
                }

         if (k == 30) {lin = SHORT ; tenl = FALSE ;}

         ecl[lines][0] = xb ;
         ecl[lines][1] = yb ;
         xro = (rr - lin) * cos(b) ; ecl[lines][2] = xro ;
         yro = (rr - lin) * sin(b) ; ecl[lines][3] = yro ;
         lines++;

         if ( tenl) {        //Save 10 deg positions for labels
            label[j][0] = degree(b) ;
            label[j][1] = xro ;
            label[j][2] = yro ;
            j++ ;
            }
         }
    }
    lines--;                  //Adjust line count to final value

}
```

PostScript

A basic introduction to PostScript®[144] follows for completeness. We can only touch the surface of PostScript programming in this section. PostScript is not the only tool available for drawing high quality graphics, but it is by far the most accessible and requires the minimum investment. Anyone wanting to draw a finished astrolabe or make masters for etching a working instruments should consider PostScript.

Learning PostScript to the level required to produce the type of pictures in this book is not terribly difficult. If you want to learn PostScript you will need several references. The one absolutely required reference is *PostScript Language Reference Manual*, Adobe Systems Incorporated. This book is universally called "The Red Book" by PostScript people. It contains the exact details of all of the PostScript language elements. The programming language reference manual can be downloaded as a PDF from the Adobe web site. Another very useful book is *PostScript Language Tutorial and Cookbook* ("The Blue Book"), also from Adobe Systems Incorporated. This book contains a very well written basic tutorial and several extremely useful routines. *PostScript by Example* by McGilton and Campione, Addison-Wesley (1992) is an excellent tutorial and learning aid.

There are also a lot of tutorial materials and examples on the web.

It is far beyond the scope of this book to present a complete PostScript tutorial, but a few very basic concepts will be useful to allow you to follow the content of the following examples.

[144] PostScript® is a registered trademark of Adobe Systems Incorporated.

PostScript is a programming language executed directly in a PostScript printer. The entire program is sent to the printer. There is no compiler. PostScript programs describe pages of printed output. Multiple pages can be created in a single program, but the page is the basic element. The program "describes" the contents of a page.

PostScript was originally developed to provide high quality output for desktop publishing. Therefore, some of the terminology is derived from printing technology. The printing industry measures the sizes of things on a page in terms of "points". You have probably encountered point sizes in relation to computer fonts. One PostScript point is exactly 1/72 inch[145]. All dimensions in a PostScript program are defined in points. You can use measurements in inches, cm or other convenient measure by providing the appropriate conversion factors in your program.

PostScript is a so-called "postfix" language[146]. That is, you specify the parameters for an operation before you specify the operation itself. You may have encountered this type of language if you have ever used an HP calculator. For example, to add 7 and 5 you code;

```
7 5 add.
```

PostScript allows you to draw just about anything you can visualize, if you have the required skill and experience. Drawing simple figures, such as those in this book, does not require nearly as much background as is needed to create publication-ready documentation.

For example, to draw a line you code something like:

```
10 20 moveto 50 60 lineto stroke
```

to draw a line from (10,20) in points, to (50,60) on the page. The `moveto` action locates the "'current point"' to the specified coordinates and the `lineto` action specifies there is a line from the current point to the point specified. Many line segments can be connected by coding a series of `lineto` commands. `stroke` actually draws the line and resets the current point.

Similarly, circles can be drawn with;

```
x y r a1 a2 arc
```

where `x,y` are the coordinates of the center, `r` is the radius and `a1` and `a2` are the angles of the circular arc, 0 360 for a complete circle (PostScript uses angles in degrees).

PostScript allows you to define variables to simplify repetitive actions. For example, to specify a conversion from inches to points you might code:

```
/in {72 mul} def
```

After making this definition you can use inch measurements in your code like:

```
2 in 3 in moveto 5 in 6.21 in lineto
```

PostScript operands are stored on a "stack", which can be thought of as memory that holds anything in the order it is specified. Parameters are taken off of the stack as commands are executed. For example, `10 12 moveto` stores a 10 on the stack and then a 12. When the `moveto` command is encountered it sees the top two stack values as [12, 10]. `moveto` uses these operands and discards them. There are several commands that operate only on the stack to

[145] A classic printer's point is 72.27 to the inch.
[146] Also called RPN for "Reverse Polish Notation".

```
2 pel setlinewidth
%intersection of projection line and sphere
%Draw projection line
H radius mul neg 0 moveto wid wid lineto stroke

%line down
wid wid moveto wid 0 lineto stroke
%line across
0 radius 2 div moveto wid radius 2 div lineto stroke

%Diagonal in box
0 0 moveto wid wid lineto stroke

end cleartomark
showpage
```

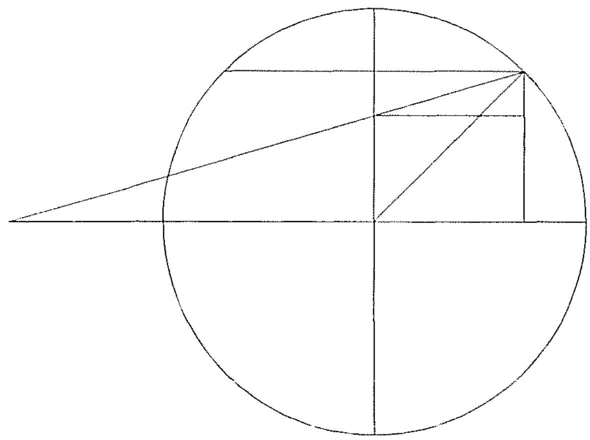


Figure 26-7. PostScript Example

This little program has several elements. It was written to be embedded in a larger document so it is formatted as an EPS file. The comments beginning with %% are required for an EPS file. Following the EPS prefix are instructions to define a local dictionary to hold variables defined in this program, and to mark the stack to identify where this routine starts. There is a `cleartomark` command at the end of the program to make sure nothing was accidentally left on the stack.

Then there are commands to center the figure on the page and define the conversion factor so the program can use measurements in inches. The variables used in the program are defined next. Each variable defined can be a value or a rather complex subprogram. These variables just calculate the constants needed for the picture. Each procedure is enclosed in braces and `def` tells PostScript you are finished. `bind` tells PostScript to save the value calculated as a number instead of having to re-execute the routine each time it is referenced. This saves some time when printing. The rest of the program just draws lines and circles in the right places. It should not be difficult to see what it does by studying the code. The end statement at the end of the program destroys the local dictionary since we no longer need it and it just takes up space.

Getting a little bit more sophisticated, the following PostScript routines draw and label the limb divisions on Gunter's quadrant for each half-degree. This set of routines shows how PostScript is a powerful drawing tool and can do a lot in very few statements. This code is a small part of a much larger program.

```
%Degree scale
%Scale boxing circle radii
/c10base 543.6 def
/c10top 536.4 def
/cmid 550.8 def
```

```
/cdeg 554.4 def
%Draw scales boxing circles
0 0 c10base 270 360 arc stroke
0 0 cmid 270 360 arc stroke
0 0 cdeg 270 360 arc stroke

%Label font
/Palatino-BoldItalic findfont 11 scalefont setfont

%Center labels over 10 degree tics
/adjust {dup stringwidth pop 2 div neg 0 rmoveto} def
/str 3 string def

%Routine to label tics at each 20 tic marks
/ticlabel { tic 20 mod 0 eq {0 c10base neg 2 add moveto tic 2 div cvi str cvs
adjust show } if } def

%Routine to draw tics of different lengths depending on angle
/ticdraw {0 size neg moveto
0 cdeg neg
tic 2 mod 0 eq {pop cmid neg} if
tic 10 mod 0 eq {pop c10base neg} if
tic 20 mod 0 eq {pop c10top neg} if
lineto stroke} def

%Draw and label degree scale by half degrees
gsave
0.5 rotate
1 1 180 {/tic exch def ticdraw ticlabel 0.5 rotate} for
grestore
```

For some tasks it is much easier to write a computer program that writes a PostScript program.

The following C++ code fragment was extracted from the program that draws the Prophatius quadrant. It is only part of a much larger program, but it illustrates why it is more convenient to write such a PostScript generator than to code the PostScript by hand. The astronomy calculations could be done in PostScript, but it would be complicated and slow the printing process dramatically. The called routines are the same ones presented earlier.

```
jday = Julian(MO,DAY,YEAR,0.0);              //Julian day for date
T = (jday - 2451545.0) / 36525.0;            //T for obliquity calc
obr = radian ( obl (T) );                    //Obliquity for year in radians

req = rcap * tan ((PI_2 - obr) / 2.0);       //Equator radius
rcan = req * tan ((PI_2 - obr) / 2.0);       //Tropic of Cancer
rec = (rcap + rcan) / 2.0;                   //Radius of Ecliptic
yep = req * tan(obr / 2.0);                  //Location of Ecliptic Pole
yec = rcap - rec;                            //Location of Ecliptic Center
resalt = 1 ;                                 //Resolution of alumcantars
 .
 .
ofstream prnt;
prnt.open("PROPHAT.EPS", ios::out);

//Draw altitude arcs
line = 0 ;
for ( i = resalt ; i <= 89 ; i += resalt ) {
     al = radian(i) ;                        //Almucantar angle
     fact = sin (latr) + sin (al) ;          //Common factor
     center = req * cos (latr) / fact ;      //Almucantar center on meridian
     radius = req * cos (al) / fact ;        //Almucantar radius
     linewas = line ;                        //Select line width
     if ( i%10 == 0 ) line = 6 ;             //Wide line for 10 deg. almucantars
     else if ( i%5 == 0 ) line = 3 ;         //Narrower line for 5 deg.
     else line = 1 ;                         //Thin line for 1 deg.
     //Print setlinewidth command only if needed
     if (line != linewas) prnt << line << " pel setlinewidth\n" ;
     prnt << "0 " << -center << " " << radius << " 270 90 arc stroke\n" ;
     }
```

This routine could have calculated the limits of each arc and drawn only that part. Instead, there is code in another part of the program to set up a "clipping path" that PostScript uses to clip the arcs. PostScript's clipping facility was used for this figure since there is no requirement for printing it many times and the processing penalty was considered to be acceptable. The programs that draw the astrolabe plates in this book all use the clipping routine discussed earlier to draw only the part of the almucantars and azimuth curves inside the Tropic of Capricorn.

PostScript has the ability of doing mathematical calculations and you can, theoretically, offload a lot of logic to the printer's PostScript interpreter. There are several considerations for using PostScript to do math. Some PostScript interpreters restrict integers to ± 32768 and floating point numbers accurate to only about 7 significant digits. This range is adequate for graphics work, but precision is lost when doing astronomical calculations. Also, PostScript is not very self-documenting, and constructing PostScript subroutines and flow logic can get quite involved. It may not be elegant PostScript, but it seems reasonable for this type of work to concentrate the program logic and math in the fast computer and let the printer print. Some PostScript gurus do not necessarily agree with this point-of-view. These are the same gurus who produce unreadable PostScript.

We conclude with a rather advanced topic that is actually beyond our scope, but will be very useful if you decide to draw an astrolabe with PostScript. Following is a PostScript routine that performs a vital function needed to draw the rete: centering the stars accurately at a specific point. The precessed x,y coordinates for the star are calculated in a C++ program. We need to draw a little star for each point with the center of the star exactly at the calculated x,y coordinates. This turns out to be harder than it sounds. PostScript has a number of operators that work with fonts, such as an operator (`stringwidth`) that provides the location of the current point after the character string is printed, from which the actual width of the string can be derived and used to center the string horizontally. However, the star glyphs in fonts such Zapfdingbats are not in the vertical center of the character box so we have to calculate the center of the star glyph using the relatively advanced `pathbbox` operator.

`pathbbox` provides the x,y locations of the corners of the smallest box that completely encloses the glyph. At first glance, it would appear all we need to center our star is to find the center of the glyph bounding box and move the glyph by that amount. But, the glyph bounding box may extend below or to the right or left of the current point specified by the x,y coordinates. Therefore, we also have to adjust the print position to account for the exact location of the glyph bounding box relative to the current point. One other consideration for this routine is that we may want to draw a line from the center of one star to the center of the next star to form constellation asterisms. We need to save the x,y current point for the line drawing.

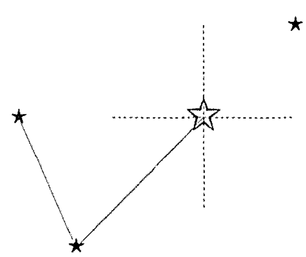
The following PostScript routine centers a star (or any other glyph) at the current point:

```
/star {gsave gsave                          %Save current point
dup false charpath flattenpath pathbbox     %stack: (top) uy ux ly lx H
%Calculate glyph offset - vertical
3 -1 roll exch sub 2 div                    %dy ux lx H
%Calculate glyph offset - horizontal
exch 3 -1 roll exch sub 2 div exch          %ux dy lx H -> dx dy H -> dy dx H
%adjust to current point
pathbbox pop pop grestore currentpoint      %cy cx ly lx dy dx
3 -1 roll sub                               %cdy cx lx dy dx
exch 3 -1 roll sub                          %cdx cdy dy dx
4 -1 roll add                               %dx cdy dy
3 1 roll add                                %dy dx
rmoveto                                     %Move glyph center to current point
show grestore } def                         %Restore current point after show
```

This routine is executed by setting the current point to the calculated x,y coordinates with `moveto` and calling the star routine for a string containing the octal index of the star character in the font. The current point for the star is still valid after placing the star glyph and can be used as the point starting a line as shown below.

```
300 400 translate

% Lines to show star is centered
gsave [1 1] 0 setdash
-.5 in 0 moveto .5 in 0 lineto
0 .5 in moveto 0 -.5 in lineto stroke
grestore

% Draw big star with hollow center

/ZapfDingbats 18 selectfont

%Draw glyph using bounding box routine

0 0 moveto                               %Set current point to origin
(\111) star                              %Big star at origin

%Smaller stars

/ZapfDingbats 8 selectfont

%line connecting star centers and another star

-50 -50 lineto stroke        %Connect line to previous star
-50 -50 moveto
(\110) star                  %Small star at (-50, -50)

-1 in 0 lineto stroke        %Line connecting previous star
-1 in 0 moveto
(\110) star                  %Small star at (-1 in, 0)

.5 in .5 in moveto           %Standalone star at (.5 in,  .5 in)
(\110) star
```

The result of the code above is:



The dotted lines through the big star just illustrate how the glyph is centered.

\110 means the glyph with the octal index of 110 (decimal 58) in the Zapfdingbats PostScript font, which is a black, five-pointed star. Finding the index of this sort of glyph can be a bit problematical. A number of utility programs that will print the contents of of PostScript fonts are available on the web or in PostScript tutorial books. The standard font encoding is included in the Red Book.

This routine can be modified easily to center entire strings horizontally and vertically. Also, don't let the apparent complexity of this routine deter you from getting started with PostScript. As with any programming language, it takes some experience before you can "think" in PostScript, but your toolkit will grow rapidly once you reach that level.

Another PostScript reference of value is *PostScript Language Program Design* from Adobe Systems, universally known as the "Green Book". The Green Book is quite technical, but it discusses the tradeoffs between which functions are best performed in a generator and which are best done in a PostScript program. Quoting the Green Book, "The PostScript programming language is an easy one to learn, and graphics programs may be written by hand to produce high quality text and images. However, the language is intended for *machine generation*. PostScript language programs are generally produced by other software rather than by programmers."[147]

There is a facility available that makes it much easier to write and test PostScript. It is called "Ghostscript", and there is a companion program called "Ghostview". Ghostscript is a fully functional PostScript interpreter that runs on a PC (there is also a Unix version). Ghostscript supplies a multitude of functions including displaying PostScript programs, creating PDF files, inserting a graphical version of the result of the program so you preview the results from other programs, etc. Ghostview is a Windows program that provides a very easy to use "front-end" to Ghostscript that provides the facility for displaying the results of a PostScript routine very easily. These programs were written under a government contract so they cannot carry a charge, although there is also a supported fee version. You can download them from the web.

There are also some rather expensive programs available that will display the results of PostScript as you write the code. Such tools are not of great value for PostScript generated by a separate program, but can be of significant value for debugging a complex PostScript program.

The weakest element of PostScript is the lack of debugging aids. All you get when a PostScript program fails is the name of the error and the contents of the stack when the failure occurred. It is often useful to include the values of calculated values in the PostScript program as comments so you can see the values in use at the time of the error. You may also have to insert an invalid command (I use "`blow`") in the code so you can see the contents of the stack. Debugging PostScript is no more difficult than debugging any other language once you are familiar with the most common errors.

The easiest way to write a PostScript program is to write some code, use Ghostview to show the result, write some more, see the result and so on, until finished. PostScript programs are not written; they evolve.

You will rapidly accumulate lots of routines that will be used over and over once you get familiar with PostScript basics. It's fun and very rewarding to be able to produce high quality graphics.

[147] *PostScript Language Program Design*, Adobe Systems Incorporated, 1988. p. 1.

Chapter 27 - Design, Layout and Fabrication

We have noted several times that the only way to completely understand an instrument is to make one. This certainly applies to the astrolabe, as the process of learning the procedures and making the necessary design decisions builds a deeper understanding of the instrument itself and contributes to appreciating the skill and dedication of the old instrument makers. However, the decision to make an astrolabe is one that both should and should not be taken lightly. On the one hand, learning the procedures and making an astrolabe for practice is well within the capabilities of anyone with even the most modest artistic skills. On the other hand, the time, effort and, possibly, expense of building a complete working instrument, particularly one of metal, is a challenge to all but the most dedicated. The result, if well done, will bring you much pride and the astrolabe may even be quite valuable.

We will discuss the design decisions and general procedure for laying out a planispheric astrolabe of the usual European style from the 15th or 16th century as an aid to making your first astrolabe and as a guide for helping you decide whether to devote the considerable effort of making a permanent instrument. It is assumed you are already familiar with general astrolabe principles, terminology and use.

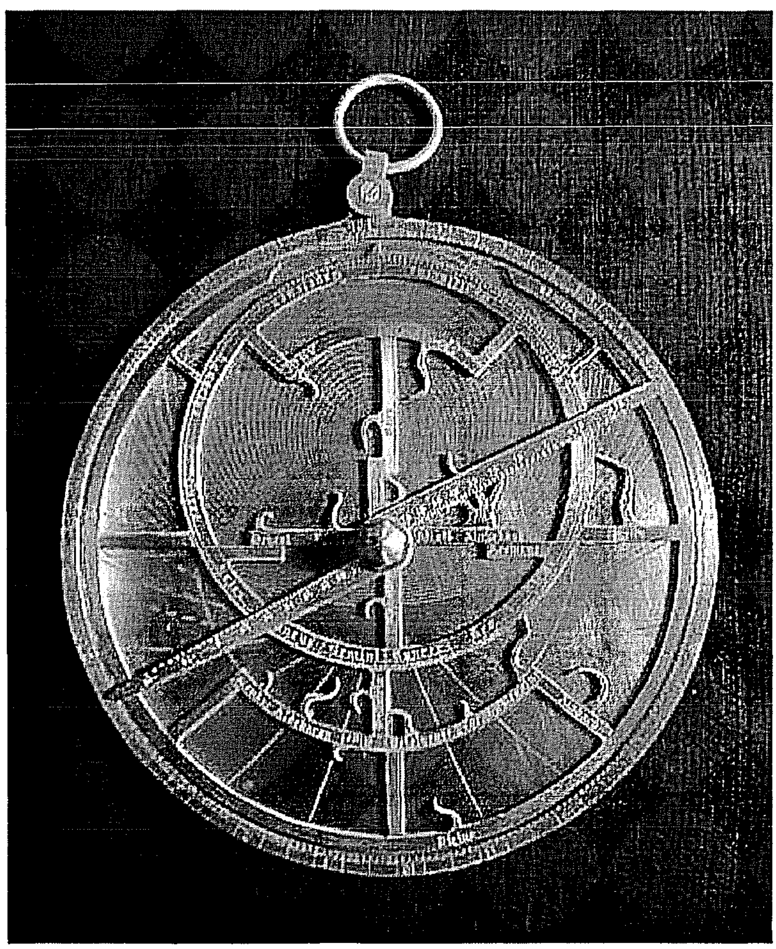


Figure 27-1. Astrolabe made by Dr. John Jarvis

The astrolabe in Figure 27-1 is an updated reproduction of a Fusoris astrolabe and was made by Dr. John Jarvis, who is an expert in chemical milling.

Design

The resulting astrolabe will be greatly influenced by the initial design decisions. Basic design points are:

Date for instrument
Size of the Tropic of Capricorn
Overall size
Limb numerals (Roman or Arabic)
Scales to include on plate
Throne and suspension style
Rule style
Rete style
Number of plates and method for securing
Other limb scales
Back calendar scale style (concentric or eccentric)
Alidade style
Other back scales
Mechanical considerations

Date for Instrument

The instrument's date influences several factors because astronomical values used to define the astrolabe components change over time. The correct astronomical values for the date of the instrument must be used in order for the result to be accurate. Specifically, the basic astronomical values used for design are the obliquity of the ecliptic for that date (which defines the size of the equator and Tropic of Cancer), the eccentricity of the Earth's orbit (used on the calendar/longitude conversion scale on the back), the longitude of perihelion and star positions. The exact year of the astrolabe defines the moment of the vernal equinox, which is the basis for all celestial longitudes. Therefore, it is generally best to select a year halfway between leap years for the date, since this gives a vernal equinox that is an "average". The time of the 2006 vernal equinox is March 20, at 1820 UTC. The time must be corrected for time zones. For example, the 2006 vernal equinox in California will be at 1820 - 8 hours = 1020 PST. The obliquity of the ecliptic in 2006 is 23° 26′ 18.4″. The eccentricity of the Earth's orbit in 2006 is 0.01671. Star positions for a 2006 instrument can use the J2000.0 coordinates, since any precession over six years would not be sensible on the finished instrument. (N.B., Precessed star positions MUST be used for any date more than 25 years from J2000.0). The 2006 longitude of perihelion is 283.049°.

Size of the Tropic of Capricorn

The Tropic of Capricorn size must be selected before the overall instrument size is chosen, since this value determines the size of the plates and all plate calculations use this value as the starting point. It is best to select a size that is an even measure (inches or cm) to make subsequent drawing easier. Any convenient size can be chosen. Typical astrolabes were rather small (about six inches overall), but a bigger one is easier to use. A good diameter for Capricorn might be 6 or 7 inches. You will find you will probably be limited to the size of paper your printer can handle.

Overall size

The overall size will be chosen based on the room needed for the limb to contain the chosen numerals and scales (if any) and the availability of raw materials. An instrument much more than eight inches in diameter may cause problems with standard paper sizes or machine tools. Be sure the overall size will fit your equipment before starting to make plates.

Limb Numerals

Old instruments always used Roman numerals on the limb, but Arabic numbers might be chosen for a modernized style. It is a matter of taste. If Roman numerals are chosen, care must be taken to select the aspect ratio of the numerals (i.e., the ratio of height and width). Roman numerals look more stylish if they are rather tall for the width. A feel for elegant numerals can be gained by looking at the dials of old clocks. Note, however, the numerals on the chapter ring of an old tall case clock were the primary focus of the entire machine and were proportionally larger than they should be for an astrolabe.

Most of the figures in this book use a character cell for limb numerals with the height 1.5 times the width. This aspect gives good readability and balance with the overall size of the instrument. A thinner numeral might also work, but a taller character would place too much emphasis on the limb. Note the character for 4 o'clock. On clock dials this numeral is usually shown as IIII instead IV to balance the VIII on the other side. It is a matter of taste and tradition.

Room must be provided below the numerals for the minute scale and any other scales desired (e.g. degrees). The minute scale can be left open or boxed as a matter of overall style. Boxed scales are elegant if the mass does not overload the outside of the instrument. The length of the minute strokes must be selected carefully for mass and readability. The type face chosen for the numerals requires some thought. Roman styles are traditional and elegant. A sans-serif face can be used to give a more modern appearance. Italics or exotic faces should be avoided except for novelty instruments.

VIII VI IV	VIII VI IV
Times-Roman	Palatino-Roman

Figure 27-2. Limb Numeral Scale

Figure 27-2 shows three numeral scales in two type faces. Each face is shown in three aspect ratios. The first is "normal". The second one has the height 1.5 times the width, and the third has an aspect ratio of 2.0. Most of the limb numerals in this book use a 1.5 aspect ratio. It might be fun to use the "Golden Ratio" [148], $\phi = 1.618034$ as the aspect ratio. The difference with an aspect of 1.5 is small, but it gives an amusing talking point. The two type faces are very similar, but Palatino-Roman is a bit more delicate with narrower strokes. The choice of the type face is purely a matter of taste. Most of the examples in this book use Times or TimesNewRoman, which are very similar. Gunter's quadrant figures use Palatino. The Sutton's quadrant figure uses Palatino for all elements except the Roman numerals, which are Times-Roman. To my eye, Palatino looks "old", which makes the instrument look more classic. I just like the way Times-Roman looks for Roman numerals.

The width of the limb is key style element. A very narrow limb gives the instrument a sense of great delicacy, and a wide limb adds robustness. A limb that is too wide looks bulky and awkward, and a too narrow limb appears fussy and hard to read.

[148] Huntley, H. E. *The Divine Proportion*, Dover, New York, 1970.

Plates

To be called an astrolabe, the plate must include the equator, Tropic of Cancer, N-S, E-W lines, horizon and some altitude arcs. Azimuth arcs are somewhat optional, although most old European instruments carried them. Many Islamic instruments did not include the azimuth arcs. The resolution of the altitude arcs (almucantars) used to be a sign of the quality of an astrolabe, with plates marked for every degree being considered the best. In fact, too many arcs can make the instrument hard to read, and it is better to have fewer arcs than to make them so close together they are hard to use. A good choice is to have arcs for each two degrees and the degree in between can easily be estimated. Old astrolabes were made with 3° altitudes, but our numbering system makes this inconvenient. A plate with arcs for each 5° cannot be considered useable. 2° is a reasonable compromise, but note carefully the slightest inconsistency will be visible with circles spaced so closely. Great care is needed.

A key design decision is selecting which plate latitudes to include. Classic astrolabes were generally custom made, and the plate latitudes were (presumably) selected by the owner. In general, a plate can be used within a few degrees of its design. Therefore, if plates were chosen for, say, 35°, 40°, and 45°, the 35° plate would be fairly accurate from 32.5° to 37.5°, the 40° plate from 37.5° to 42.5°, etc. The range of latitudes in North America run from about 25° in south Florida and Texas to 50° or so in southern Canada. This range can be done on six plates (three disks with a plate on each side). However, it is also advisable to include at least one specialty plate, such as a plate of horizons, and a plate of longitudes or a plate for 0°- 90°, The mater must be made deep enough to accommodate all the planned plates. A single plate instrument for personal use is just fine.

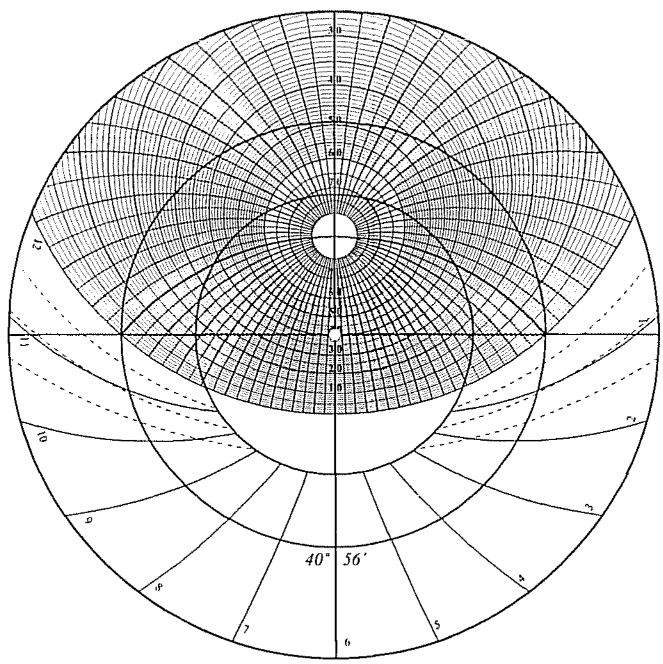


Figure 27-3. Plate for Hellespont, 1° almucantars, 5° azimuths

It is worth noting that some classicists insist the plates for the traditional "climates" be included. The "climates" (or climata) were defined by the latitudes where the length of the longest day of the year differed by one half hour. The traditional seven climates and their latitudes are: I. Meroe (16° 27′), II. Soene (23° 51′), III. Lower Egypt (30° 22′), IV. Rhodes (36° 0′), V. Hellespont (40° 46′), VI. Mid-Pontus (45° 1′) and VII. Mouth of Borysthia (48° 32′). The longest day at

Meroe is 13h, 13.5h at Soene, etc. The defined list of climates actually included 39 latitudes from the equator to the North Pole, but only the key seven in the middle were used very much. There is some logic in using the climate latitudes, but one will have to choose whether the added explanation is worth the trouble. Using the climates would add an element of authenticity if you are making a reproduction of a very old instrument.

Note, however, the climates calculated by Ptolemy used an obliquity of the ecliptic of 23° 51′ 20″ so the exact climate limits would have to be recalculated for the modern value of 23° 26′ 26″. The modern values are:

		Modern	Ptolemy	
I	Meroe	16° 46′	16° 27′	13 h
II	Soene	24° 14′	23° 51′	13 ½ h
III	Lower Egypt	30° 51′	30° 22′	14 h
IV	Rhodes	36° 34′	36° 0′	14 ½ h
V	Hellespont	41° 26′	40° 56′	15 h
VI	Mid-Pontus	45° 35′	45° 1′	15 ½ h
VII	Mouth of Borysthinia	49° 5′	48° 32′	16 h

Azimuths can be done is several ways. Many astrolabes have azimuths in 5° compass bearings. Another way is to have azimuths only for the compass "winds" (i.e., NNE, NE, etc.). Since the azimuth scale is used less often, it is fine to have coarser resolution than for altitudes. 5° or 10° is OK.

The crepuscular arc of -18° had religious meaning. A modern instrument might have crepuscular arcs for -6° and -12° to show the levels of astronomical twilight. It is a matter of choice and style, depending on how cluttered the plate looks. Note the crepuscular arcs are used only with the Sun and go only from Capricorn to Cancer, never inside Cancer.

The plate is always engraved with the latitude. The location of the engraving is optional but is traditionally just below the equator.

Traditional instruments have the unequal hour arcs below the horizon. An instrument for modern use would eliminate them, but they would be included in a reproduction. Several methods of counting equal hours have been used. For example, "Italian Hours" are equal hours starting at sunset. Some old astrolabes showed such styles of equal hours, but they are not common and tend to clutter the area below the horizon.

Figure 27-3 shows a plate for the Hellespont with a full complement of almucantars, azimuths and crepuscular arcs. It is very dense but usable.

Many, but not all, old European astrolabe plates included the "Houses of Heaven." They are totally optional depending on the intended use of the astrolabe. They look rather nice, but do add an element of complexity and, of course, are an overt acknowledgment of astrology.

It is important to think through how the plates will be secured when making an instrument with multiple plates. The instrument will be useless if the plates move around. Old instruments often used a mortise on the plates, but this must have been very hard to do accurately. A better method might be to use a peg in the mater with matching holes in the plates.

Just as an aside, the so-called "Golden Ratio" is used in art and architecture as a model for balance and proportion. The hoizon intersection that divides the meridian line by the Golden Raio is for latitude 39° 33′ 47.1″ which lies just north of Athens. Artistically, this latitude can be used for an astrolabe plate of pleasing proportions.

Throne and suspension style

The throne and suspension offer many opportunities for design expression. There is not a traditional style that must be followed. The only requirements are the throne be accurately located over the center of the instrument and is sufficiently robust to stay in place without wobble. The suspension can be as simple as a ring or as elaborate as a gimbaled shaft. Whatever works and looks balanced with rest of the instrument is fine. Persian astrolabes often had thrones (called the *kursi* on their instruments) that were so elaborate they seemed an end in themselves. By contrast, some instruments had thrones that were so small and simple they look anemic and out of place. It's all a question of style. The only critical point is the throne must attach securely to the body of the astrolabe, which is often not as simple as it sounds. The throne must be rather robust if you are making a paper astrolabe since a small, stylish throne would be bent easily. It may be necessary to sacrifice style for utility in this case.

Rule style

There are several possible rule styles. The simplest rule is like a clock hand:, its length is a radius of the instrument. This type of rule is adequate for many uses, but makes use of the unequal hour arcs awkward since it cannot show the Sun's nadir during the day. It is, however, a little easier to make. Full length rules can have either a single fiducial edge that runs across the face of the instrument or a counterchanged style. The counterchanged style is more common. The edge of the rule should be graduated by declination. The rule center hole must be exactly the same diameter as the pin holding the instrument together to avoid inaccuracies in readings.

Rete style

The rete offers many opportunities for artistic expression without affecting function. Several design factors are critical. First, the rete design must allow for sufficient piercing so the plate can be seen under it. The piercing cannot, however, be so extensive that it sacrifices structural rigidity. Second, the stars selected must be bright stars that are well distributed over the sky. Not too many, but enough so at least one star is in an observable position at all times. Some stars are in positions that create special challenges. For example, Regulus is very close to the ecliptic and Sirius is very near the edge. Creativity is needed to place such stars in a useable way. The ecliptic is always divided by the zodiac on classic instruments. The width of the ecliptic band should be about 12° of latitude to define the zone in which the planets are found, but this is not critical. A *muri* should be included at Capricorn 0°.

The rete structure provides unlimited opportunity for artistic expression. Classic instruments have had retes of many designs including the straightforward Moorish Gothic style, trefoil/quaterfoil, "Y" type and simplified Gothic retes with many counterchanges. Renaissance retes should be studied for ideas on rete design. The objective is to provide a symmetrical structure that supports the star pointers and is artistically pleasing at the same time.

Many types of star pointers have been used depending on the taste and artistic objectives of the maker. Examples are simple pointers carved from the rete plate, shaped straight and curved pointers, zoömorphic pointers made in the form of vines or animals, teeth, birds, etc. The pointers should be robust enough to prevent accidental damage, and all stars should be labeled. The rete can also be decorated with non-functional ornamentation to add to the overall sophistication of the instrument's appearance.

Many more stars can be included if the rete is made of transparent material, although too many stars can be hard to see unless they are arranged in constellation asterims. Only the bright stars need to be labeled if constellations are shown.

The rete shown in Figure 27-4 was designed by artist Laura DeAngelis and was made of ceramics for inclusion in a sculpture. This extraordinary design incorporates a fully anatomical heart with all of the chambers along with the normal ecliptic and 32 stars. This rete combines functionality with artistic expression in a unique and beautiful way.

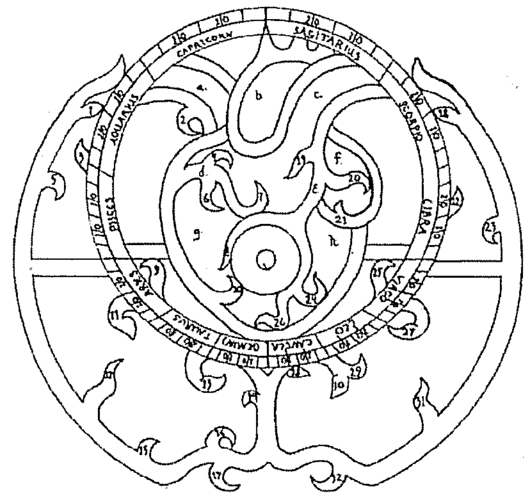


Figure 27-4. Heart Rete

The Back

The astrolabe back offers an opportunity for technical and artistic expression. The only required scales are for converting calendar date to solar longitude. There is a lot of space remaining for creativity.

The solar longitude scale is always concentric with the instrument center and shows two scales, one for longitude and one for altitudes.

The calendar scale can be either eccentric or concentric. The eccentric scale is more elegant and educational, but many old makers used a concentric calendar. The distance between days on a concentric calendar scale is not constant, and it is very hard to draw the varying scale accurately. It is important to have a strategy when drawing either kind of scale in order to detect errors before it is too late. Make the first divisions at least 30° apart. Then divide each section into progressively smaller divisions, say 10°. Then draw the 5° divisions, etc. Same for the calendar. In this way errors will be immediately obvious before they are propagated around the entire scale. It's actually easier to lay out the calendar scale on a computer.

Classic instruments almost always included a shadow scale. Different scale divisions were used depending on where and when the astrolabe was made. For modern use, dividing half the scale in feet and the other half in inches makes sense. For completeness, consider including a shadow scale extension that shows cotangents.

The content of the rest of the back is a matter of taste and intended use, but the scales should be useful and complete. Many old astrolabes were made with scales for unequal hours and equal hour conversions with no divisions on the alidade for using them. By implication, the scales may have been more decorative than useful. You would want to have alidade divisions if you have this scale.

The style of the alidade should match the style of the rule (i.e. straight or counterchanged). Don't underestimate the mechanical challenge of installing sights on the alidade. The peep holes in the alidade sights have to be perfectly in line with the center of the instrument or the readings will be in error.

Fabrication Alternatives

Making an astrolabe is not an idle undertaking. Even the simplest instrument takes a lot of work to do well. Following are some thoughts about possible approaches.

Your first try should be to make a simple, single plate instrument on paper. Just draw the components on good quality drawing paper and paste the result to a piece of stiff cardboard, masonite or formica. The cardboard used to mat pictures is of high quality and easily available at picture framing shops. Wood can be used if it well seasoned dry plywood with a solid coat of varnish (otherwise it will warp). 3M makes a terrific spray glue for mounting the paper on a board. The quality of the paper is important because it must be dense enough to keep the glue from bleeding through. Imagine the frustration of losing all that hard work because the glue caused the ink to run. It would be a good idea to test the glue on a scrap before committing. Varnish the paper with thin, clear varnish. The holes should be **drilled**, not punched. Drill a very small pilot hole for alignment but don't drill the final center hole until after final assembly unless you are sure you can get everything aligned.

You can create the components on a computer if you have the programming skills. Digital plotters do a nice job of printing the components. It is, however, not easy to write the programs. Expect to write your own code for drawing clipped circles as few compiler libraries have appropriate routines. Also, labeling is very hard and you may have to create your own font to allow rotating the characters. Some printers will accept plotter commands. You can try this, but the plotting language may not have sufficient resolution to get good clipped circles. The facilities are fine for pie charts but do not always allow the precision needed for more demanding applications.

Excellent astrolabe renditions can be printed on a PostScript printer. The computer programs generate the PostScript program that the printer executes. Programs that create PostScript to draw all the astrolabe components are not simple. PostScript is an excellent choice for this application because it allows clipping, font scaling and writing text around a circle and the printed output from a high quality printer looks good. The investment in time to learn PostScript and write and the test the programs is significant.

There may be an inexpensive CAD program that will allow you to create a nice astrolabe. Professional tools, such as CATIA, are very expensive for the amateur. You would have to calculate all the values and position the various arcs and lines but the result could be very nice. Calculating the circle radii and centers is easy and can be done with a small program in, say, QBASIC, or you could do it with a spreadsheet program with a modest effort (a day or so of work).

The price of brass or aluminum astrolabe reproductions is high. The least expensive ones are about $300 and are rather plain. Expect to pay $800 to several thousand dollars for a high quality reproduction. Old astrolabes start at $40,000 and go up to over a million. The pewter reproductions of "Chaucer's Astrolabe" are just costume jewelry. I have not heard of a good affordable metal astrolabe, so if you want a brass astrolabe, you will have to make one.

I have never made a brass astrolabe, but I have made a lot of clock parts and I have a feeling for how hard it would be. I would guess it would take me three months working steadily, and I have four lathes, a milling machine, metal band saw and metal scroll saw, plus all the clockmaker's hand tools. In other words, it is a meaningful project. Each step presents its own rewards and challenges. Cutting out and turning the components just takes patience and some skill with a fret saw and file. The engraving is the hardest part, since engraving is an art that requires lots of practice and excellent artistic skills. Alternatives to engraving include acid etching, chemical milling, anodizing and machine engraving.

To do acid etching or chemical milling, you need a full size master from which you make a contact print. The part to be engraved is coated with photoresist, exposed with the negative and then etched in an acid bath. Clock dials are made this way, so it is possible with some experimentation. Once etched, the limb numerals can be filled with clock dial wax and polished on the lathe. The fumes from acid etching can be poisonous. Great care is required to be safe.

Anodizing is possible. As I understand it, you can lay out the letters and numbers on aluminum or brass and have them made permanent at a shop that does anodizing. Anodizing creates a very rugged sapphire surface. I don't know much about this technique but would like to know more.

Machine engraving is a definite possibility. A pantograph engraver can do very good work with a skilled operator. Don't plan on buying one; they cost about $3,000. You could make one that uses a Dremel tool as the engraving head as an exercise. Every mall has a store that engraves trophies and such, and there is a technology that will convert a photograph into commands for an automatic engraver. This might be a possibility, but you would, of course, have to supply the finished parts ready for engraving.

Inexpensive Reproductions

I have made a working model of every instrument described in this book[149]. My intention has been to make accurate instruments that can be made inexpensively using easily found materials. Making your own reproduction is fun, and it is rewarding to take them outside and see that they actually work.

I generate the components with computer programs that generate PostScript programs and print the components on a 1200 dpi PostScript laser printer on card stock (110 lb. Index weight). The paper is available in a many colors, but I have not found one that looks very much like brass. I use either a Buff color paper or Canary. Buff can look fairly authentic. Canary has a more modern look, and is a little easier to read due to the higher contrast. The printed output includes ¼" registration circles in identical locations on both parts for alignment. A ¼" hole can be punched with a standard hand hole punch from the office supply store. Some effort was needed to make sure the printed circles on the instruments are absolutely round.

I print the front and back of the astrolabe separately, punch out the center, align the parts and glue the components together using spray glue. The rete is printed on overhead transparency material from the office supply store. The rule and alidade are a single thickness of paper. I made a little jig for the registration holes to align the front and back. I made a 0.110" punch and die for the holes that is inserted into a drill press chuck, but they can be drilled into a piece of scrap wood just as easily.

The components are cut out after assembly, laminated with a 5 mil laminating pouch and the holes are repunched. I sometimes have to run the completed instrument body through the laminating machine twice for it to come out flat. Laminating pouches come up to 10 mil, but the 10 mil result is very hard to cut. A 3 mil pouch is too flimsy.

Mounting the front and back on something with a bit of weight makes it easier to use your astrolabe in the wind, but laminating then become difficult. I glued a lead fishing weight on the astrolabe on my boat to make it possible to use in a breeze. All astrolabes are next to impossible to use when it is windy.

I made aluminum templates as a cutting guide, but you can also cut the components out with scissors if you are careful. You need about 1/8" of lamination around the edges so the lamination won't separate and curl up.

[149] They all work.

I have never found a good way to mount sights on a laminated paper astrolabe alidade. Bent paper clips can work. In use, I have found that sights are not really needed. You can see the edge of the alidade at night if there is any ambient light. I use the shadow of the nut in the daytime with good results.

Accessories for the instruments can be problematic. I use book rings, available from any office supply store, for the astrolabe suspension. Two rings, one 1" and one ¾", can be interlocked to provide a usable and inexpensive suspension. I use a #4-40 pan head nylon screw and self-locking nut to assemble the astrolabe. A #35 drill (0.110 inch) is perfect for this size screw. The pan head screw does not catch on things. No washer is needed with the nylon hardware, but might be needed if using metal screws. The self-locking nut tension can be tightened with your fingers to get a smooth action.

My quadrants were mounted on matting cardboard using dry mount, a material that bonds papers with heat. The local art supply store does the mounting for a small fee. They are then laminated and trimmed right up to the edge. It took a lot of shopping to find materials for the quadrants. Sights are the most difficult. I ended up using 0.5 inch earring backs from the local crafts shop. You clip the earring back onto the side of the quadrant and the little holes can be used for taking readings. They work, but are not ideal. I'm still looking for something that is both readily available and inexpensive for this function. After much experimentation, too much actually, I found 1 mm leather cord from the craft shop and 10/0 beads work nicely for the quadrant string. I use lead fishing weights for the bob.

The picture below shows some of the astrolabe reproductions I have made. My total investment in materials for the laminated reproductions is less than $20. The investment in time is, shall we say, astronomical.

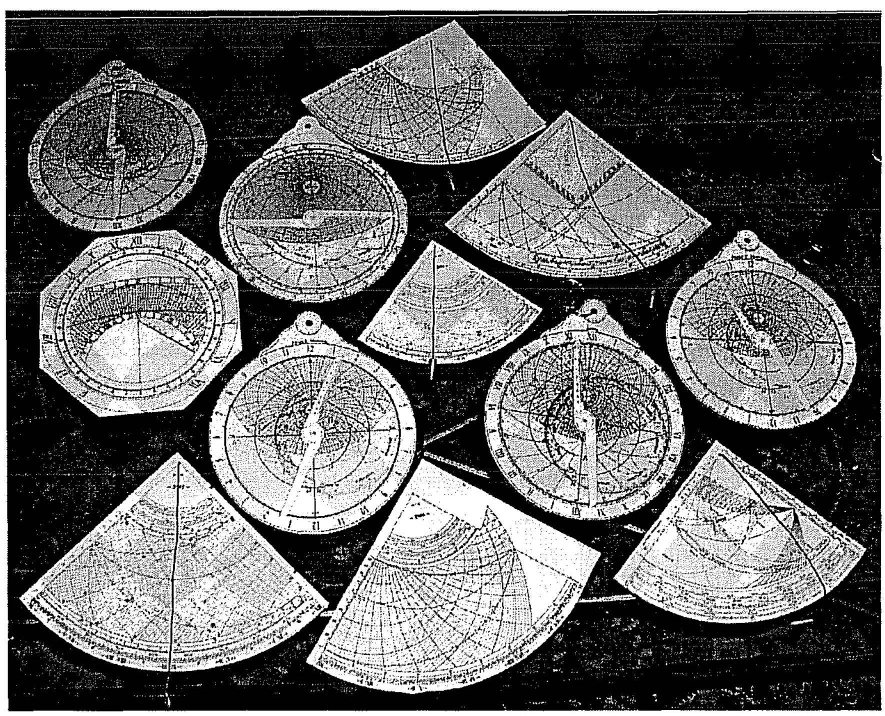


Figure 27-5. Author's Homemade Instruments

Appendix - Star Positions

Following is a list of J2000.0 star coordinates arranged by constellation. The right ascension, declination, **Bayer letter**, name, if any, and magnitude are shown for each star. These are the stars used on the rete in the examples. They were chosen for inclusion only for their role in defining the constellation asterism. Not all asterisms are complete. Bright stars are shown in **bold.** Stars commonly included on old astrolabes are also *italic.* Navigation stars are underlined.

RA h	m	s	Declination °	′	″	Bayer	Name	Magnitude
Andromeda								
0	8	23.2	29	5	26	α	*Alpheratz*	2.06
1	9	43.9	35	37	14	β	Mirach	2.06
2	3	53.9	42	19	47	γ	Almach	2.18
0	49	8.8	41	4	44	ν		4.53
1	9	30.1	47	14	31	φ		4.25
Aquarius								
22	5	46.9	-0	19	22	α		2.96
21	31	33.3	-5	34	16	β		2.91
22	21	39.3	-1	23	14	γ		3.84
22	54	38.9	-15	49	15	δ		3.27
Aquila								
19	**50**	**46.9**	**8**	**52**	**6**	α	***Altair***	**0.77**
Ara								
17	31	50.4	-49	52	34	α		2.98
17	25	17.9	-55	31	47	β		3.66
17	31	5.9	-60	41	1	δ		3.62
16	59	35.0	-53	9	38	ε		4.06
16	58	37.1	-55	59	24	ζ		3.13
16	49	47.0	-59	2	29	η		3.76
18	6	37.7	-50	5	30	θ		3.66
Aries								
2	7	10.3	23	27	45	α	Hamal	2.00
1	54	38.3	20	48	9	β		2.64
Auriga								
5	**16**	**41.3**	**45**	**59**	**53**	α	***Capella***	**0.08**
5	59	31.7	44	56	51	β	Menkalinan	1.90
5	26	17.5	28	36	27	γ	El-Nath	1.65
5	59	43.2	37	12	45	θ		2.62
4	56	59.6	33	9	58	ι	Hasseleh	2.69
Boötes								
14	**15**	**39.6**	**19**	**10**	**57**	α	***Arcturus***	**-0.04**

15	1	56.6	40	23	26	β	Nekkar	3.50
14	32	04.6	38	18	29	γ	Seginus	3.03
15	15	30.1	33	18	53	δ		3.47
14	31	49.7	30	22	17	ρ		3.58
Cancer								
8	58	29.2	11	51	28	α	Acubens	4.25
8	16	30.9	9	11	8	β	Altarf	3.52
8	44	41.0	18	9	15	δ		3.94
8	46	41.8	28	45	36	ι		4.02
Canis-Major								
6	**45**	**8.9**	**-16**	**42**	**58**	α	***Sirius***	**-1.46**
6	22	41.9	-17	57	22	β	Murzim	1.98
Canis-Minor								
7	**39**	**18.1**	**5**	**13**	**30**	α	***Procyon***	**0.38**
Capricorn								
20	17	38.8	-12	30	30	α		3.57
21	47	02.3	-16	7	38	δ	*Al-Giedi*	2.87
21	26	39.9	-22	24	41	ζ		3.74
21	5	56.7	-17	13	58	θ		4.07
20	51	49.2	-26	55	9	ω		4.11
Carina								
6	**23**	**57.2**	**-52**	**41**	**44**	α	**Canopus**	**-0.72**
9	13	12.1	-69	43	2	β	Miaplacidus	1.68
8	22	30.8	-59	30	34	ε	Avior	1.86
10	42	57.4	-64	23	40	θ		2.76
9	47	05.4	-59	16	31	ι		2.25
9	47	61.0	-65	4	18	ν		2.96
7	56	46.7	-52	58	56	χ		3.47
10	13	44.3	-70	2	16	ω		3.32
Cassiopea								
0	40	30.4	56	32	15	α	Shedir	2.23
0	9	10.6	59	8	59	β	*Caph*	2.27
0	56	42.4	60	43	0	γ		2.47
1	25	48.9	60	14	7	δ	Ruchbah	2.68
1	54	23.6	63	40	13	ε	Segin	3.38
Centaurus								
14	**39**	**36.2**	**-60**	**50**	**7**	α	**Alpha-Centaurus**	**-0.01**
14	3	49.4	-60	22	22	β		0.61
12	41	30.9	-48	57	34	γ		2.17
13	39	53.2	-53	27	59	ε		2.30
13	55	32.3	-47	17	18	ζ		2.55
14	35	30.3	-42	9	28	η		2.31
14	6	40.8	-36	22	12	θ		2.06
13	20	35.7	-36	42	44	ι		2.75

Crux								
12	26	35.9	-63	5	56	α		1.58
12	47	43.3	-59	41	19	β		1.25
12	31	9.9	-57	6	47	γ		1.63
12	15	8.6	-58	44	56	δ		2.80
Cygnus								
20	**41**	**25.8**	**45**	**16**	**49**	α	***Deneb***	**1.25**
19	30	45.2	27	57	55	β	Albireo	3.08
20	22	13.6	40	15	24	γ	Sadr	2.20
19	44	58.4	45	7	51	δ		2.87
20	46	12.6	33	58	13	ε	Gienah	2.46
Eridanus								
1	**37**	**42.9**	**-57**	**14**	**12**	α	**Achernar**	**0.46**
False-Cross								
8	44	42.2	-54	42	30	δ	Velae	2.02
8	22	30.8	-59	30	34	ε	Carinae	1.86
9	47	54.0	-59	16	31	ι	Carinae	1.25
9	22	6.8	-55	0	38	κ	Velae	2.50
Gemini								
7	**34**	**35.9**	**31**	**53**	**18**	α	**Castor**	**1.58**
7	**45**	**18.9**	**28**	**1**	**34**	β	**Pollux**	**1.14**
7	20	7.3	21	58	56	δ	Wasat	3.53
6	43	55.9	25	7	52	ε	Mebsuta	2.98
7	4	6.5	20	34	13	ζ	Mekbuda	3.79
7	44	26.8	24	23	53	κ		3.57
7	18	5.5	16	2	25	λ		3.58
7	11	8.3	30	14	43	τ		4.41
Grus								
22	8	13.9	-46	57	40	α	Al-Nair	1.74
22	42	40.0	-46	53	5	β		2.10
21	53	55.6	-37	21	54	γ		3.01
22	29	16.1	-43	29	45	δ		3.97
23	10	21.5	-45	14	48	ι		3.90
Hercules								
17	0	17.3	30	55	5	ε		3.92
16	41	17.1	31	36	10	ζ		2.81
16	42	53.7	38	55	20	η		3.46
17	15	2.7	36	48	33	π		3.13
Leo								
10	**8**	**22.2**	**11**	**58**	**2**	α	***Regulus***	**1.35**
11	49	3.5	14	34	19	β	Denebola	2.14
10	19	58.6	19	50	25	γ	Algeiba	2.28
11	14	6.4	20	31	25	δ	Zosma	2.56
9	45	51.0	23	46	27	ε		2.98
10	16	41.4	23	25	2	ζ		3.44
10	7	19.9	16	45	45	η		3.52

11	14	14.3	15	25	46	θ		3.34
9	52	45.8	26	0	25	μ		3.88
Libra								
14	50	41.1	-15	59	50	α	Zubenelgenubi	5.16
15	17	0.3	-9	22	58	β		2.61
15	35	31.5	-14	47	22	γ		3.91
15	12	13.2	-19	47	30	ι		4.54
Lyra								
18	**36**	**56.2**	**38**	**47**	**1**	α	***Vega***	**0.03**
Ophiucus								
17	34	55.9	12	33	36	α	Ras-Alhage	2.09
17	43	28.3	4	34	2	β	Cheleb	2.77
16	14	20.6	-3	41	39	δ		2.72
16	37	9.4	-10	34	2	ζ		2.57
17	10	22.6	-15	43	29	η	Sabik	2.46
16	57	40.0	9	22	30	κ		3.20
Orion								
5	**55**	**10.3**	**7**	**24**	**25**	α	**Betelgeuse**	**0.50**
5	**14**	**32.2**	**-8**	**12**	**6**	β	***Rigel***	**0.12**
5	**25**	**7.8**	**6**	**20**	**59**	γ	**Bellatrix**	**1.64**
5	32	0.4	0	17	4	δ	Mintaka	2.23
5	40	45.5	-1	56	34	ζ	Alnitak	2.05
5	47	45.3	-9	40	11	κ	Saiph	2.06
5	35	8.2	9	56	3	λ	Meissa	3.66
Pavo								
20	**25**	**38.8**	**-56**	**44**	**7**	α	**Peacock**	**1.94**
Pegasus								
0	8	23.2	29	5	26	α	Markab	2.06
23	3	46.4	28	4	58	β	*Scheat*	2.42
0	13	14.1	15	11	1	γ	Algenib	2.83
Perseus								
3	24	19.3	49	51	41	α	Mirfak/Algenib	1.79
3	8	10.1	40	57	21	β	*Algol*	2.12v
Pisces								
2	2	2.7	2	45	49	α		4.33
23	17	9.9	3	16	56	γ		3.69
1	31	28.9	15	20	45	η		3.62
23	27	58.0	6	22	44	θ		4.28
23	39	57.0	5	37	35	ι		4.13
23	26	55.9	1	15	20	κ		4.94
23	42	02.7	1	46	48	λ		4.50
Sagittarius								
18	5	48.4	-30	25	27	γ		2.99
18	20	59.6	-29	49	41	δ		2.70

18	24	10.3	-34	23	5	ε		1.85
19	2	36.6	-29	52	49	ζ		2.60
18	27	58.1	-25	25	18	λ		2.81
18	55	15.8	-26	17	48	σ		2.02
19	6	56.3	-27	40	14	τ		3.32
18	45	39.3	-26	59	27	φ		3.17

Scorpius

16	**29**	**24.4**	**-26**	**25**	**55**	α	***Antares***	**0.92**
16	5	26.1	-19	48	19	β	Graffias	2.55
16	0	19.9	-22	37	18	δ	Dschubba	2.34
16	50	9.7	-34	17	36	ε		2.28
16	54	34.9	-42	21	41	ζ		4.80
17	12	9.1	-43	4	21	η		3.33
17	37	19.0 -	42	59	52	θ	Sargas	1.87
17	47	35.0 -	40	7	37	ι		1.87
17	33	36.4	-37	6	13	λ	Shaula	1.62
15	58	51.0	-26	6	51	π		2.90

Taurus

4	**35**	**55.2**	**16**	**30**	**33**	α	***Aldebaran***	**0.85**
4	19	47.5	15	37	39	γ		3.65
4	28	36.9	19	10	49	ε		3.53

Triangulum-Australe

16	48	39.9	-69	1	40	α	Atria	1.92
15	55	8.4	-63	25	40	β		2.85
15	18	54.6	-68	40	46	γ		2.89

Ursa-Major

11	3	43.6	61	45	3	α	Dubhe	1.49
11	1	50.4	56	22	56	β	Merak	2.37
11	53	49.8	53	41	41	γ	hecda	2.44
12	15	25.5	57	1	57	δ	Megrez	3.31
12	54	1.7	55	57	35	ε	Alioth	1.77
13	23	55.5	54	55	31	ζ	Mizar/Alcor	2.27
13	47	32.3	49	18	48	η	*Alkaid*	1.86

Ursa-Minor

2	31	43.6	89	15	51	α	Polaris	2.02

Vela

8	9	31.9	-47	20	12	γ		1.78
8	44	42.2	-54	42	30	δ		2.02
9	22	6.8	-55	0 3	8	κ		2.50
9	7	59.7	-43	25	57	λ		2.21

Virgo

13	**25**	**11.5**	**-11**	**9**	**41**	α	***Spica***	**0.98**
11	50	41.6	1	45	53	β		3.61
12	41	39.5	-1	26	58	γ	Porrima	3.48
12	55	36.1	3	23	51	δ		3.38

Appendix – Solar Data

The following table contains calculated solar data for an entire year. The year 2006 was chosen as it is halfway between leap years and represents something of an average. The tabulated values are for 00:00 UT for the date specified.

Solar Values for 2006

Obliquity of ecliptic = 23.438446 (23d 26m 18.4s)
Eccentricity = 0.016706
Longitude of Perihelion = 103.049
Vernal Equinox = 2453815.26409 3/20/2006 18:20 UT

Date	Julian Day	Decl	RA	Longitude	Mean Anomaly	EqT
01/01	2453736.5	-23 02	18:45	280.426	357.472	-3 19
01/02	2453737.5	-22 57	18:50	281.446	358.458	-3 47
01/03	2453738.5	-22 51	18:54	282.465	359.443	-4 15
01/04	2453739.5	-22 45	18:59	283.484	0.429	-4 42
01/05	2453740.5	-22 39	19:03	284.503	1.415	-5 09
01/06	2453741.5	-22 32	19:07	285.523	2.400	-5 36
01/07	2453742.5	-22 25	19:12	286.542	3.386	-6 02
01/08	2453743.5	-22 17	19:16	287.561	4.371	-6 28
01/09	2453744.5	-22 09	19:20	288.580	5.357	-6 53
01/10	2453745.5	-22 00	19:25	289.599	6.343	-7 18
01/11	2453746.5	-21 51	19:29	290.618	7.328	-7 42
01/12	2453747.5	-21 42	19:34	291.637	8.314	-8 06
01/13	2453748.5	-21 32	19:38	292.656	9.299	-8 29
01/14	2453749.5	-21 22	19:42	293.675	10.285	-8 52
01/15	2453750.5	-21 11	19:46	294.694	11.271	-9 13
01/16	2453751.5	-21 00	19:51	295.712	12.256	-9 35
01/17	2453752.5	-20 49	19:55	296.731	13.242	-9 55
01/18	2453753.5	-20 37	19:59	297.749	14.227	-10 15
01/19	2453754.5	-20 24	20:04	298.767	15.213	-10 34
01/20	2453755.5	-20 12	20:08	299.785	16.199	-10 52
01/21	2453756.5	-19 59	20:12	300.803	17.184	-11 10
01/22	2453757.5	-19 45	20:16	301.820	18.170	-11 27
01/23	2453758.5	-19 31	20:20	302.838	19.155	-11 43
01/24	2453759.5	-19 17	20:25	303.855	20.141	-11 58
01/25	2453760.5	-19 03	20:29	304.872	21.127	-12 13
01/26	2453761.5	-18 48	20:33	305.889	22.112	-12 27
01/27	2453762.5	-18 33	20:37	306.905	23.098	-12 39
01/28	2453763.5	-18 17	20:41	307.921	24.083	-12 52
01/29	2453764.5	-18 01	20:45	308.937	25.069	-13 03
01/30	2453765.5	-17 45	20:50	309.953	26.055	-13 13
01/31	2453766.5	-17 29	20:54	310.969	27.040	-13 23

Date	Julian Day	Decl	RA	Longitude	Mean Anomaly	EqT
02/01	2453767.5	-17 12	20:58	311.984	28.026	-13 32
02/02	2453768.5	-16 55	21:02	312.999	29.011	-13 40
02/03	2453769.5	-16 37	21:06	314.014	29.997	-13 47
02/04	2453770.5	-16 20	21:10	315.028	30.983	-13 54
02/05	2453771.5	-16 02	21:14	316.042	31.968	-13 59
02/06	2453772.5	-15 43	21:18	317.056	32.954	-14 04
02/07	2453773.5	-15 25	21:22	318.069	33.939	-14 08
02/08	2453774.5	-15 06	21:26	319.082	34.925	-14 11
02/09	2453775.5	-14 47	21:30	320.095	35.911	-14 13
02/10	2453776.5	-14 28	21:34	321.107	36.896	-14 15
02/11	2453777.5	-14 08	21:38	322.119	37.882	-14 16
02/12	2453778.5	-13 48	21:42	323.131	38.867	-14 16
02/13	2453779.5	-13 28	21:46	324.142	39.853	-14 15
02/14	2453780.5	-13 08	21:50	325.153	40.839	-14 13
02/15	2453781.5	-12 48	21:54	326.163	41.824	-14 11
02/16	2453782.5	-12 27	21:58	327.173	42.810	-14 08
02/17	2453783.5	-12 06	22:01	328.183	43.795	-14 04

02/18	2453784.5	-11 45	22:05	329.192	44.781	-12 00
02/19	2453785.5	-11 24	22:09	330.201	45.767	-13 55
02/20	2453786.5	-11 03	22:13	331.210	46.752	-13 49
02/21	2453787.5	-10 41	22:17	332.217	47.738	-13 42
02/22	2453788.5	-10 19	22:21	333.225	48.723	-13 35
02/23	2453789.5	-9 57	22:24	334.232	49.709	-13 27
02/24	2453790.5	-9 35	22:28	335.239	50.695	-13 19
02/25	2453791.5	-9 13	22:32	336.245	51.680	-13 10
02/26	2453792.5	-8 51	22:36	337.250	52.666	-13 00
02/27	2453793.5	-8 28	22:40	338.256	53.651	-12 50
02/28	2453794.5	-8 06	22:43	339.260	54.637	-12 39

Date	Julian Day	Decl	RA	Longitude	Mean Anomaly	EqT
03/01	2453795.5	-7 43	22:47	340.264	55.623	-12 28
03/02	2453796.5	-7 20	22:51	341.268	56.608	-12 16
03/03	2453797.5	-6 57	22:55	342.271	57.594	-12 04
03/04	2453798.5	-6 34	22:58	343.274	58.579	-11 51
03/05	2453799.5	-6 11	23:02	344.276	59.565	-11 38
03/06	2453800.5	-5 48	23:06	345.278	60.551	-11 24
03/07	2453801.5	-5 25	23:09	346.279	61.536	-11 10
03/08	2453802.5	-5 01	23:13	347.280	62.522	-10 56
03/09	2453803.5	-4 38	23:17	348.280	63.507	-10 41
03/10	2453804.5	-4 15	23:21	349.280	64.493	-10 26
03/11	2453805.5	-3 51	23:24	350.279	65.479	-10 10
03/12	2453806.5	-3 27	23:28	351.278	66.464	-9 54
03/13	2453807.5	-3 04	23:32	352.276	67.450	-9 38
03/14	2453808.5	-2 40	23:35	353.273	68.435	-9 22
03/15	2453809.5	-2 17	23:39	354.270	69.421	-9 05
03/16	2453810.5	-1 53	23:43	355.266	70.407	-8 48
03/17	2453811.5	-1 29	23:46	356.262	71.392	-8 31
03/18	2453812.5	-1 05	23:50	357.258	72.378	-8 14
03/19	2453813.5	0 -42	23:54	358.252	73.363	-7 56
03/20	2453814.5	0 -18	23:57	359.247	74.349	-7 38
03/21	2453815.5	0 06	0:01	0.240	75.335	-7 21
03/22	2453816.5	0 29	0:05	1.233	76.320	-7 03
03/23	2453817.5	0 53	0:08	2.226	77.306	-6 45
03/24	2453818.5	1 17	0:12	3.218	78.291	-6 27
03/25	2453819.5	1 40	0:15	4.209	79.277	-6 09
03/26	2453820.5	2 04	0:19	5.200	80.263	-5 50
03/27	2453821.5	2 27	0:23	6.190	81.248	-5 32
03/28	2453822.5	2 51	0:26	7.180	82.234	-5 14
03/29	2453823.5	3 14	0:30	8.169	83.219	-4 56
03/30	2453824.5	3 38	0:34	9.158	84.205	-4 38
03/31	2453825.5	4 01	0:37	10.146	85.191	-4 20

Date	Julian Day	Decl	RA	Longitude	Mean Anomaly	EqT
04/01	2453826.5	4 24	0:41	11.133	86.176	-4 02
04/02	2453827.5	4 47	0:45	12.120	87.162	-3 44
04/03	2453828.5	5 10	0:48	13.106	88.147	-3 27
04/04	2453829.5	5 33	0:52	14.092	89.133	-3 09
04/05	2453830.5	5 56	0:56	15.077	90.119	-2 52
04/06	2453831.5	6 19	0:59	16.062	91.104	-2 34
04/07	2453832.5	6 42	1:03	17.046	92.090	-2 17
04/08	2453833.5	7 04	1:07	18.030	93.075	-2 01
04/09	2453834.5	7 27	1:10	19.012	94.061	-1 44
04/10	2453835.5	7 49	1:14	19.995	95.047	-1 28
04/11	2453836.5	8 11	1:18	20.977	96.032	-1 12
04/12	2453837.5	8 33	1:21	21.958	97.018	00 -56
04/13	2453838.5	8 55	1:25	22.939	98.003	00 -41
04/14	2453839.5	9 17	1:29	23.919	98.989	00 -25
04/15	2453840.5	9 38	1:32	24.898	99.975	00 -11
04/16	2453841.5	10 00	1:36	25.877	100.960	00 04
04/17	2453842.5	10 21	1:40	26.856	101.946	00 18
04/18	2453843.5	10 42	1:43	27.834	102.931	00 32
04/19	2453844.5	11 03	1:47	28.811	103.917	00 45
04/20	2453845.5	11 24	1:51	29.788	104.903	00 58
04/21	2453846.5	11 44	1:55	30.764	105.888	01 10
04/22	2453847.5	12 05	1:58	31.740	106.874	01 23
04/23	2453848.5	12 25	2:02	32.715	107.859	01 34

04/24	2453849.5	12 45	2:06	33.690	108.845	01 45
04/25	2453850.5	13 05	2:10	34.664	109.831	01 56
04/26	2453851.5	13 24	2:13	35.638	110.816	02 06
04/27	2453852.5	13 43	2:17	36.611	111.802	02 16
04/28	2453853.5	14 02	2:21	37.584	112.787	02 25
04/29	2453854.5	14 21	2:25	38.556	113.773	02 34
04/30	2453855.5	14 40	2:29	39.528	114.759	02 42
Date	Julian Day	Decl	RA	Longitude	Mean Anomaly	EqT
05/01	2453856.5	14 58	2:32	40.499	115.744	02 50
05/02	2453857.5	15 16	2:36	41.470	116.730	02 57
05/03	2453858.5	15 34	2:40	42.440	117.715	03 04
05/04	2453859.5	15 52	2:44	43.410	118.701	03 10
05/05	2453860.5	16 09	2:48	44.379	119.687	03 16
05/06	2453861.5	16 26	2:52	45.348	120.672	03 21
05/07	2453862.5	16 43	2:55	46.316	121.658	03 25
05/08	2453863.5	17 00	2:59	47.284	122.643	03 29
05/09	2453864.5	17 16	3:03	48.251	123.629	03 32
05/10	2453865.5	17 32	3:07	49.218	124.615	03 35
05/11	2453866.5	17 47	3:11	50.184	125.600	03 37
05/12	2453867.5	18 03	3:15	51.151	126.586	03 39
05/13	2453868.5	18 18	3:19	52.116	127.571	03 40
05/14	2453869.5	18 33	3:23	53.081	128.557	03 40
05/15	2453870.5	18 47	3:27	54.046	129.543	03 40
05/16	2453871.5	19 01	3:31	55.010	130.528	03 39
05/17	2453872.5	19 15	3:35	55.974	131.514	03 38
05/18	2453873.5	19 28	3:39	56.938	132.499	03 36
05/19	2453874.5	19 42	3:43	57.901	133.485	03 34
05/20	2453875.5	19 54	3:47	58.864	134.471	03 31
05/21	2453876.5	20 07	3:51	59.826	135.456	03 27
05/22	2453877.5	20 19	3:55	60.788	136.442	03 23
05/23	2453878.5	20 31	3:59	61.750	137.427	03 19
05/24	2453879.5	20 42	4:03	62.711	138.413	03 14
05/25	2453880.5	20 53	4:07	63.672	139.399	03 08
05/26	2453881.5	21 04	4:11	64.633	140.384	03 02
05/27	2453882.5	21 14	4:15	65.593	141.370	02 56
05/28	2453883.5	21 24	4:19	66.553	142.355	02 49
05/29	2453884.5	21 34	4:23	67.513	143.341	02 41
05/30	2453885.5	21 43	4:27	68.472	144.327	02 33
05/31	2453886.5	21 52	4:31	69.431	145.312	02 25
Date	Julian Day	Decl	RA	Longitude	Mean Anomaly	EqT
06/01	2453887.5	22 00	4:35	70.389	146.298	02 16
06/02	2453888.5	22 08	4:39	71.348	147.283	02 07
06/03	2453889.5	22 16	4:43	72.306	148.269	01 58
06/04	2453890.5	22 23	4:47	73.264	149.255	01 48
06/05	2453891.5	22 30	4:52	74.221	150.240	01 37
06/06	2453892.5	22 37	4:56	75.179	151.226	01 27
06/07	2453893.5	22 43	5:00	76.136	152.211	01 16
06/08	2453894.5	22 49	5:04	77.092	153.197	01 05
06/09	2453895.5	22 54	5:08	78.049	154.183	00 53
06/10	2453896.5	22 59	5:12	79.005	155.168	00 41
06/11	2453897.5	23 04	5:16	79.961	156.154	00 29
06/12	2453898.5	23 08	5:20	80.917	157.139	00 17
06/13	2453899.5	23 11	5:25	81.873	158.125	00 05
06/14	2453900.5	23 15	5:29	82.828	159.111	00 -8
06/15	2453901.5	23 18	5:33	83.784	160.096	00 -20
06/16	2453902.5	23 20	5:37	84.739	161.082	00 -33
06/17	2453903.5	23 22	5:41	85.694	162.067	00 -46
06/18	2453904.5	23 24	5:45	86.648	163.053	00 -59
06/19	2453905.5	23 25	5:50	87.603	164.039	-1 12
06/20	2453906.5	23 26	5:54	88.558	165.024	-1 25
06/21	2453907.5	23 26	5:58	89.512	166.010	-1 38
06/22	2453908.5	23 26	6:02	90.466	166.995	-1 51
06/23	2453909.5	23 26	6:06	91.420	167.981	-2 04
06/24	2453910.5	23 25	6:10	92.374	168.967	-2 17
06/25	2453911.5	23 24	6:15	93.328	169.952	-2 30
06/26	2453912.5	23 22	6:19	94.282	170.938	-2 43
06/27	2453913.5	23 20	6:23	95.236	171.923	-2 55

06/28	2453914.5	23 18	6:27	96.189	172.909	-3 08
06/29	2453915.5	23 15	6:31	97.143	173.895	-3 20
06/30	2453916.5	23 11	6:35	98.097	174.880	-3 32

Date	Julian Day	Decl	RA	Longitude	Mean Anomaly	EqT
07/01	2453917.5	23 08	6:39	99.050	175.866	-3 44
07/02	2453918.5	23 04	6:44	100.003	176.851	-3 55
07/03	2453919.5	22 59	6:48	100.957	177.837	-4 07
07/04	2453920.5	22 54	6:52	101.910	178.823	-4 18
07/05	2453921.5	22 49	6:56	102.864	179.808	-4 28
07/06	2453922.5	22 43	7:00	103.817	180.794	-4 39
07/07	2453923.5	22 37	7:04	104.770	181.779	-4 49
07/08	2453924.5	22 31	7:08	105.724	182.765	-4 58
07/09	2453925.5	22 24	7:12	106.677	183.751	-5 08
07/10	2453926.5	22 17	7:16	107.631	184.736	-5 16
07/11	2453927.5	22 09	7:21	108.584	185.722	-5 25
07/12	2453928.5	22 01	7:25	109.538	186.707	-5 33
07/13	2453929.5	21 53	7:29	110.492	187.693	-5 40
07/14	2453930.5	21 44	7:33	111.445	188.679	-5 47
07/15	2453931.5	21 35	7:37	112.399	189.664	-5 54
07/16	2453932.5	21 25	7:41	113.353	190.650	-6 00
07/17	2453933.5	21 15	7:45	114.307	191.635	-6 06
07/18	2453934.5	21 05	7:49	115.261	192.621	-6 11
07/19	2453935.5	20 54	7:53	116.215	193.607	-6 15
07/20	2453936.5	20 43	7:57	117.169	194.592	-6 19
07/21	2453937.5	20 32	8:01	118.124	195.578	-6 23
07/22	2453938.5	20 21	8:05	119.078	196.563	-6 25
07/23	2453939.5	20 09	8:09	120.033	197.549	-6 28
07/24	2453940.5	19 56	8:13	120.988	198.535	-6 29
07/25	2453941.5	19 44	8:17	121.943	199.520	-6 31
07/26	2453942.5	19 31	8:21	122.898	200.506	-6 31
07/27	2453943.5	19 17	8:25	123.854	201.491	-6 31
07/28	2453944.5	19 04	8:29	124.809	202.477	-6 30
07/29	2453945.5	18 50	8:33	125.765	203.463	-6 29
07/30	2453946.5	18 36	8:36	126.721	204.448	-6 27
07/31	2453947.5	18 21	8:40	127.677	205.434	-6 25

Date	Julian Day	Decl	RA	Longitude	Mean Anomaly	EqT
08/01	2453948.5	18 06	8:44	128.634	206.419	-6 22
08/02	2453949.5	17 51	8:48	129.590	207.405	-6 18
08/03	2453950.5	17 36	8:52	130.547	208.391	-6 14
08/04	2453951.5	17 20	8:56	131.505	209.376	-6 09
08/05	2453952.5	17 04	9:00	132.462	210.362	-6 03
08/06	2453953.5	16 48	9:04	133.420	211.347	-5 57
08/07	2453954.5	16 31	9:07	134.378	212.333	-5 50
08/08	2453955.5	16 14	9:11	135.336	213.319	-5 43
08/09	2453956.5	15 57	9:15	136.295	214.304	-5 35
08/10	2453957.5	15 40	9:19	137.253	215.290	-5 27
08/11	2453958.5	15 22	9:23	138.213	216.275	-5 17
08/12	2453959.5	15 04	9:26	139.172	217.261	-5 08
08/13	2453960.5	14 46	9:30	140.132	218.247	-4 58
08/14	2453961.5	14 28	9:34	141.092	219.232	-4 47
08/15	2453962.5	14 09	9:38	142.052	220.218	-4 36
08/16	2453963.5	13 51	9:41	143.013	221.203	-4 24
08/17	2453964.5	13 32	9:45	143.974	222.189	-4 11
08/18	2453965.5	13 13	9:49	144.936	223.175	-3 59
08/19	2453966.5	12 53	9:53	145.898	224.160	-3 45
08/20	2453967.5	12 34	9:56	146.860	225.146	-3 31
08/21	2453968.5	12 14	10:00	147.823	226.131	-3 17
08/22	2453969.5	11 54	10:04	148.786	227.117	-3 02
08/23	2453970.5	11 34	10:07	149.749	228.103	-2 47
08/24	2453971.5	11 13	10:11	150.713	229.088	-2 31
08/25	2453972.5	10 53	10:15	151.677	230.074	-2 15
08/26	2453973.5	10 32	10:18	152.642	231.059	-1 59
08/27	2453974.5	10 11	10:22	153.607	232.045	-1 42
08/28	2453975.5	9 50	10:26	154.572	233.031	-1 25
08/29	2453976.5	9 29	10:29	155.538	234.016	-1 07
08/30	2453977.5	9 07	10:33	156.504	235.002	00 -49

08/31	2453978.5	8 46	10:37	157.471	235.987	00 -31

Date	Julian Day	Decl	RA	Longitude	Mean Anomaly	EqT
09/01	2453979.5	8 24	10:40	158.438	236.973	00 -12
09/02	2453980.5	8 03	10:44	159.406	237.959	00 07
09/03	2453981.5	7 41	10:48	160.374	238.944	00 26
09/04	2453982.5	7 19	10:51	161.343	239.930	00 46
09/05	2453983.5	6 56	10:55	162.312	240.915	01 05
09/06	2453984.5	6 34	10:58	163.281	241.901	01 25
09/07	2453985.5	6 12	11:02	164.251	242.887	01 46
09/08	2453986.5	5 49	11:06	165.222	243.872	02 06
09/09	2453987.5	5 27	11:09	166.193	244.858	02 27
09/10	2453988.5	5 04	11:13	167.164	245.843	02 47
09/11	2453989.5	4 41	11:16	168.136	246.829	03 08
09/12	2453990.5	4 19	11:20	169.109	247.815	03 29
09/13	2453991.5	3 56	11:24	170.082	248.800	03 51
09/14	2453992.5	3 33	11:27	171.055	249.786	04 12
09/15	2453993.5	3 10	11:31	172.029	250.771	04 33
09/16	2453994.5	2 47	11:34	173.004	251.757	04 54
09/17	2453995.5	2 23	11:38	173.979	252.743	05 16
09/18	2453996.5	2 00	11:41	174.954	253.728	05 37
09/19	2453997.5	1 37	11:45	175.930	254.714	05 59
09/20	2453998.5	1 14	11:49	176.907	255.699	06 20
09/21	2453999.5	0 50	11:52	177.884	256.685	06 41
09/22	2454000.5	0 27	11:56	178.862	257.671	07 03
09/23	2454001.5	0 04	11:59	179.840	258.656	07 24
09/24	2454002.5	0 -20	12:03	180.819	259.642	07 45
09/25	2454003.5	0 -43	12:07	181.798	260.627	08 06
09/26	2454004.5	-1 06	12:10	182.778	261.613	08 26
09/27	2454005.5	-1 30	12:14	183.759	262.599	08 47
09/28	2454006.5	-1 53	12:17	184.740	263.584	09 07
09/29	2454007.5	-2 16	12:21	185.721	264.570	09 28
09/30	2454008.5	-2 40	12:25	186.704	265.555	09 48

Date	Julian Day	Decl	RA	Longitude	Mean Anomaly	EqT
10/01	2454009.5	-3 03	12:28	187.686	266.541	10 07
10/02	2454010.5	-3 26	12:32	188.670	267.527	10 27
10/03	2454011.5	-3 49	12:35	189.653	268.512	10 46
10/04	2454012.5	-4 13	12:39	190.638	269.498	11 05
10/05	2454013.5	-4 36	12:43	191.623	270.483	11 23
10/06	2454014.5	-4 59	12:46	192.608	271.469	11 41
10/07	2454015.5	-5 22	12:50	193.594	272.455	11 59
10/08	2454016.5	-5 45	12:54	194.581	273.440	12 17
10/09	2454017.5	-6 08	12:57	195.568	274.426	12 34
10/10	2454018.5	-6 30	13:01	196.556	275.411	12 50
10/11	2454019.5	-6 53	13:05	197.544	276.397	13 06
10/12	2454020.5	-7 16	13:08	198.533	277.383	13 22
10/13	2454021.5	-7 38	13:12	199.523	278.368	13 37
10/14	2454022.5	-8 01	13:16	200.513	279.354	13 52
10/15	2454023.5	-8 23	13:19	201.503	280.339	14 06
10/16	2454024.5	-8 45	13:23	202.495	281.325	14 19
10/17	2454025.5	-9 07	13:27	203.486	282.311	14 32
10/18	2454026.5	-9 29	13:31	204.479	283.296	14 44
10/19	2454027.5	-9 51	13:34	205.472	284.282	14 56
10/20	2454028.5	-10 13	13:38	206.465	285.267	15 07
10/21	2454029.5	-10 34	13:42	207.459	286.253	15 18
10/22	2454030.5	-10 55	13:46	208.454	287.239	15 28
10/23	2454031.5	-11 17	13:50	209.449	288.224	15 37
10/24	2454032.5	-11 38	13:53	210.444	289.210	15 45
10/25	2454033.5	-11 58	13:57	211.441	290.195	15 53
10/26	2454034.5	-12 19	14:01	212.437	291.181	16 00
10/27	2454035.5	-12 40	14:05	213.435	292.167	16 06
10/28	2454036.5	-11 00	14:09	214.432	293.152	16 12
10/29	2454037.5	-13 20	14:13	215.431	294.138	16 16
10/30	2454038.5	-13 40	14:16	216.430	295.123	16 20
10/31	2454039.5	-13 59	14:20	217.429	296.109	16 24

Date	Julian Day	Decl	RA	Longitude	Mean Anomaly	EqT
11/01	2454040.5	-14 19	14:24	218.429	297.095	16 26
11/02	2454041.5	-14 38	14:28	219.430	298.080	16 28
11/03	2454042.5	-14 57	14:32	220.431	299.066	16 28
11/04	2454043.5	-15 16	14:36	221.432	300.051	16 28
11/05	2454044.5	-15 34	14:40	222.434	301.037	16 27
11/06	2454045.5	-15 52	14:44	223.437	302.023	16 26
11/07	2454046.5	-16 10	14:48	224.440	303.008	16 23
11/08	2454047.5	-16 28	14:52	225.443	303.994	16 20
11/09	2454048.5	-16 45	14:56	226.447	304.979	16 15
11/10	2454049.5	-17 02	15:00	227.452	305.965	16 10
11/11	2454050.5	-17 19	15:04	228.457	306.951	16 04
11/12	2454051.5	-17 36	15:08	229.462	307.936	15 57
11/13	2454052.5	-17 52	15:12	230.468	308.922	15 50
11/14	2454053.5	-18 08	15:16	231.475	309.907	15 41
11/15	2454054.5	-18 23	15:20	232.482	310.893	15 32
11/16	2454055.5	-18 39	15:24	233.489	311.879	15 21
11/17	2454056.5	-18 54	15:29	234.497	312.864	15 10
11/18	2454057.5	-19 08	15:33	235.505	313.850	14 58
11/19	2454058.5	-19 22	15:37	236.513	314.835	14 46
11/20	2454059.5	-19 36	15:41	237.522	315.821	14 32
11/21	2454060.5	-19 50	15:45	238.532	316.807	14 18
11/22	2454061.5	-20 03	15:49	239.542	317.792	14 02
11/23	2454062.5	-20 16	15:54	240.552	318.778	13 46
11/24	2454063.5	-20 28	15:58	241.563	319.763	13 29
11/25	2454064.5	-20 40	16:02	242.574	320.749	13 12
11/26	2454065.5	-20 52	16:06	243.585	321.735	12 53
11/27	2454066.5	-21 03	16:11	244.597	322.720	12 34
11/28	2454067.5	-21 14	16:15	245.609	323.706	12 14
11/29	2454068.5	-21 25	16:19	246.622	324.691	11 54
11/30	2454069.5	-21 35	16:23	247.635	325.677	11 33

Date	Julian Day	Decl	RA	Longitude	Mean Anomaly	EqT
12/01	2454070.5	-21 45	16:28	248.648	326.663	11 11
12/02	2454071.5	-21 54	16:32	249.662	327.648	10 48
12/03	2454072.5	-22 03	16:36	250.676	328.634	10 25
12/04	2454073.5	-22 11	16:41	251.690	329.619	10 01
12/05	2454074.5	-22 19	16:45	252.704	330.605	09 37
12/06	2454075.5	-22 27	16:49	253.719	331.591	09 12
12/07	2454076.5	-22 34	16:54	254.734	332.576	08 47
12/08	2454077.5	-22 41	16:58	255.750	333.562	08 21
12/09	2454078.5	-22 47	17:02	256.766	334.547	07 54
12/10	2454079.5	-22 53	17:07	257.782	335.533	07 28
12/11	2454080.5	-22 58	17:11	258.798	336.519	07 00
12/12	2454081.5	-23 03	17:16	259.814	337.504	06 33
12/13	2454082.5	-23 07	17:20	260.831	338.490	06 05
12/14	2454083.5	-23 11	17:25	261.848	339.475	05 36
12/15	2454084.5	-23 15	17:29	262.865	340.461	05 08
12/16	2454085.5	-23 18	17:33	263.882	341.447	04 39
12/17	2454086.5	-23 20	17:38	264.900	342.432	04 10
12/18	2454087.5	-23 23	17:42	265.917	343.418	03 41
12/19	2454088.5	-23 24	17:47	266.935	344.403	03 11
12/20	2454089.5	-23 25	17:51	267.953	345.389	02 42
12/21	2454090.5	-23 26	17:56	268.972	346.375	02 12
12/22	2454091.5	-23 26	18:00	269.990	347.360	01 42
12/23	2454092.5	-23 26	18:04	271.008	348.346	01 12
12/24	2454093.5	-23 25	18:09	272.027	349.331	00 43
12/25	2454094.5	-23 24	18:13	273.046	350.317	00 13
12/26	2454095.5	-23 23	18:18	274.064	351.303	00 -17
12/27	2454096.5	-23 20	18:22	275.083	352.288	00 -46
12/28	2454097.5	-23 18	18:27	276.102	353.274	-1 16
12/29	2454098.5	-23 15	18:31	277.121	354.259	-1 45
12/30	2454099.5	-23 11	18:35	278.141	355.245	-2 14
12/31	2454100.5	-23 07	18:40	279.160	356.231	-2 43

Glossary

This glossary contains brief definitions and/or explanations of astronomical terms that are frequently encountered in astrolabe studies. Consult an astronomical reference book for more detail.

ALTITUDE
The angle of an object in the sky above the **horizon**. For example, an object straight overhead has an altitude of 90°, and an object halfway up in the sky has an altitude of 45°.

ANNUAL EQUATION
A variation in the moon's orbit due to the varying distance of the Earth-moon system from the Sun.

ANOMALISTIC YEAR (MONTH)
The time between successive passages of an orbiting body through **periapsis**. For the Earth, one anomalistic year is 365.2596 days. An anomalistic month is 27.5546 days.

ANOMALY
Measure of position of orbiting bodies. Anomalies are measured as the angle of the body from the point where is nearest the body it is orbiting (perihelion for a planet, perigee for the Moon). Longitudes are measured from the First point of Aries. Longitude = **true anomaly** + **perihelion distance** + **longitude** of **ascending node**.

AMPLITUDE
The angle on the horizon of the rising or setting of a celestial object measured from east or west. The ortive amplitude is the angle of rising and the occiduous (or occasive) amplitude is the angle of setting. The amplitude is used in navigation to determine compass error and find true north.

APHELION
The point in a planet's orbit that is farthest from the Sun.

APOGEE
The point in the Moon's orbit that is farthest from the Earth.

ASCENDING NODE
The point on an orbit where the object (Moon, planet, comet etc.) crosses the **ecliptic** in the direction such that the **ecliptic latitude** changes from south to north. The orbital **inclination** is measured at the ascending node and the longitude of the ascending node is one of the basic orbital determination parameters. Note also that lunar eclipses can only occur near one of the nodes of the Moon's orbit.

APPARENT SOLAR TIME
Apparent solar time is time based on the Sun's local hour angle at 15° per hour. The time indicated by a sundial.

ARCS OF THE SIGNS
A scale common on Islamic astrolabes that relates the Sun's declination with the signs of the zodiac. The arcs of the signs scale is always in the upper right quadrant of the astrolabe back..

ASTERISM
An imaginary figure defined for a constellation to make it easier to visualize in the sky. An example of an asterism is the familiar "Big Dipper". Many asterisms are very old and originate

in mythology. Others, particularly in the southern sky, were invented in fairly recent times as a way to describe constellations that were not visible from northern latitudes. The history of star names and constellations is an interesting element of astronomy.

ASTROLABE
An astronomical instrument consisting of a stereographic projection of the local coordinate system on the plane of the equator with a rotating stereographic projection of the fixed stars and ecliptic. In order to be properly called an astrolabe, the instrument must also have an alidade and scale to measure the altitude of the Sun and stars and a suspension.

ASTROLABE QUADRANT
A planispheric astrolabe reduced to a quadrant. Several forms of astrolabe quadrant have been developed.

ASTRONOMICAL UNIT (AU)
The average distance of the Earth from the Sun. Used to measure large distances in the solar system such as the distance of a planet from the Sun. The length of the AU determined by radar ranging is 149,597,870 km (92,900,277 miles) or about eight light-minutes.

AZIMUTH
The compass bearing of a celestial object. Azimuth is defined as the angle from north, measured on the local horizon, of the great circle passing through the zenith and the object. In North America, due north is 0° azimuth, due east is 90° azimuth and so on. There are several methods of measuring azimuth. In Europe, azimuth is commonly measured as 0° to 360° moving west from south. Other schemes use + or - 180° from south with west taken as the positive direction.

BAYER LETTER
The Bayer letter of a star is a lower case Greek letter indicating the relative brightness of stars in a constellation. α is the brightest, star β the second brightest, etc.The Bayer designation of a star is the Bayer letter followed by the genitive of the name of the constellation. α Leonis is Regulus, the brightest star in the constellation Leo. Introduced by Johann Bayer in his star atlas in 1603.

BRACHIOLUS
A articulated indicator of two or three segments used to point to positions on universal astrolabes. Latin for "little arm".

CALLIPIC CYCLE
A period of 76 mean **tropical years**, or 27,759 days, approximately equal to 940 mean **synodic months**. Discovered by Callipus in the fourth century B.C..

CELESTIAL EQUATOR
The **great circle** on the celestial sphere perpendicular to the Earth's axis of rotation. The exact definition of the celestial equator gets rather complex. Refer to a good astronomy text for the precise definition. For practical purposes it is acceptable to consider the celestial equator to be the projection of the Earth's equator on the celestial sphere.

CELESTIAL SPHERE
The imaginary sphere with the Earth as its center used to defined the positions of celestial objects.

CHRISTIAN ERA (aka COMMON ERA)
The calendar convention adopted by most of the world begins on 1 Jan 1, Julian day 1721424. Dates BCE (Before Christian Era) are counted backward from year 1 BC. Note that the Electric Astrolabe uses the astronomical convention which includes a year 0. A historical date of 35 BC is shown as year -34. Sources must be checked carefully to ensure the correct year BCE is used. The Christian Era was introduced by Dionysius Exiguus (Denis the Little), a Roman abbot, in a table

of Easter dates in about 525 in "Liber de paschate" which contained calculated dates of Easter up to 626. The calendar was intended to start at the birth of Jesus in 1 BC but an error in counting the years of the reign of the emperor Augustus caused the result to be off by about three years (Augustus used the name Octavian in his first four years of rule). Modern research places the probable birth date of Jesus in the late summer of 3 BC. See also **Easter (Date of)**.

CIVIL TIME
The time told by an accurate clock according to established international agreement. The official civil time is called Universal Time (UT) and is defined by a mathematical formula based on the sidereal time at Greenwich. Universal time conforms very closely to the average motion of the Sun over a year. In practice, the time broadcast by national time services, such as NIST over radio station WWV, is Coordinated Universal Time (UTC) which is derived from very accurate atomic clocks. Since UT is based on the rotation of the Earth which has random irregularities, UTC and UT are kept within one second of each other by the occasional insertion of a "leap second" to UT. Time zone boundaries are also established by international agreement and do not necessarily conform to geographic longitude in order to accommodate population distributions. For example, Madrid is west of Greenwich but is in the Central European time zone so people in Madrid will be in the same time zone as the rest of Europe. The **longitude correction** for Madrid is nearly an hour and 15 minutes.

COLURE
A **great circle** on the celestial sphere passing through the poles. The most common references are to the equinoctial colure that also passes through the equinoxes and the solstitial colure which passes through the solstices.

CONJUNCTION
Phenomenon when two celestial bodies have the same **longitude** when viewed from a third body. Conjunctions normally refer to the condition when a planet has the same longitude as the Sun. Venus and Mercury are said to have inferior conjunction when they are between the Earth and the sun. Superior conjunction is when the Sun is between the planet and the Earth. Only the inner planets can have inferior conjunction.

CULMINATION
The instant that a celestial body reaches its maximum altitude. Identical to **meridian passage**. Circumpolar stars have an upper culmination or transit is when the object is closest to the observer's zenith. Lower culmination (also called "culmination below the pole") is when the object passes the meridian farther from the zenith

CURSOR
As used on a universal astrolabe, a secondary rule oriented at a right angle to the primary rule and free to slide laterally and locked in position. The cursor is divided by a scale appropriate for the instrument.

DYNAMICAL TIME
The current time measurement standard used for astronomical measurements and calculations. Adopted in 1984, dynamical time is based on time intervals from atomic clocks. There are actually two forms of dynamical time; terrestrial dynamical time (TDT) and barycentric dynamical time (TDB). They two forms differ by a very small amount due to relativistic effects due to the Earth's movement in its orbit that can be ignored for all but the most critical applications. Dynamical time is related to **universal time** by the relation, ΔT = Dynamical Time - Universal Time. ΔT depends on the instantaneous speed of rotation of the Earth and is both variable and unpredictable. The current value is about 64 seconds.

DECLINATION
The angle of a celestial object from the celestial equator. North is taken as the positive direction.

Declination is measured in degrees. *Declinatio septentrionalis* is northern declination and *declination meriodinalis* is southern declination.

DOMINICAL LETTER
The index of the first Sunday of the year from the sequence A, B, C, ..., G. If the first Sunday is, e.g. January 3, then the dominical letter for the year is C, the third letter in the sequence. All Sundays in the year have the same letter except in leap years, February 28 and 29 have the same letter and the Sunday letter changes after leap day. Thus, leap years have two dominical letters, e.g. C/D. The dominical letter was used in early methods of determining the date of Easter.

DRACONITIC PERIOD
The time between passages of an orbiting body through the ascending orbital node. Primarily used for the moon as an indication of when eclipses can happen. Also called the nodical month (27.2122 days).

EASTER (DATE OF)
The first Sunday after the first full moon after the **vernal equinox**. Note that the Gregorian calendar reform included modified rules for determining the date of Easter. These reforms use a hypothetical mean **lunation** and are used to determine the date of a hypothetical mean full moon which is in excellent agreement with the mean moon but ignore the physical moon resulting in potential differences of up to a day. If the time of full moon is very near the vernal equinox the ecclesiastical fixing of the date of Easter may vary widely from the astronomical view. While this sounds unrealistic at first glance there is justification. Full moon occurs at an instant which will be on different dates in different parts of the world. The ecclesiastical rules fix Easter for the entire world in a rational way. The current rule fixes the date of Easter as the Sunday following the 14^{th} day of the first lunation following the vernal equinox, fixed at March 21. There are other parts of the rule fixing the meridian for timekeeping, the time of the lunation and special cases, such as when the lunation falls on Sunday.

The date of Easter has an interesting history in itself. The New Testament fixes the date of Jesus' crucifixion at the beginning of Passover. Passover begins on the Jewish date of Nisan 14, which is defined as the date of the first full moon of Nisan, the spring month in the Jewish calendar. The first Council of Nicaea (325) decided that Easter should fall on Sunday, the Christian day of worship, thus divorcing Easter from the Jewish calendar. Somewhat later, the so called, "Alexandrine Rule", credited to Anatolius of Laodicea (d. ca. 282), of the 19 year lunar cycle (see **Metonic Cycle**) to calculate future Easter dates was adopted. The rules were changed to use the **Epact** and stricly alternating lunar months of 29 and 30 days when the **Gregorian Calendar** reform was implemented in 1582. The Orthodox Church did not adopt the reforms and Orthodox Easter can vary from the Latin Easter by up to five weeks. The determination of astronomically determined moveable Christian religious celebrations is called "Computus" and was required learning for every educated person in the Middle Ages and, until recently, was taught to all priests.

EPACT
The age of the moon on January 1. Used to define the ecclesiastical calendar.

EVECTION
A perturbation in the moon's orbit due to changes in the orbital **eccentricity** caused by the gravitational effect of the Earth and Sun. The moon's eccentricity can vary from .066 when a perigee passage occurs at a new or full moon to .044 when it occurs at the first or third quarter.

ECCENTRIC ANOMALY
A mathematical construction used in the solution of orbital problems. Eccentric Anomaly (E) is found from Kepler's Equation: M = E - e sin E, where M is the **mean anomaly** and e is the orbital **eccentricity**. Note that this is a transcendental equation that cannot be solved directly but

can be solved using iterative techniques. The eccentric anomaly is used to calculate the **true anomaly**.

ECCENTRICITY
The "shape" of a planetary orbit, ranging from 0 to 1. 0 means the orbit is a perfect circle with the value increasing as the orbit becomes more elliptical. The Earth's orbit is nearly circular with an eccentricity of about .0167. The orbit of comet Halley is highly elliptical with an eccentricity of about .967. The eccentricity of an orbit is defined in exactly the same way as the eccentricity of any ellipse and is defined as the ratio of the distance from the center to the focus to the length of the semi-major axis. An orbit with an eccentricity of 1 is parabolic and, thus, not periodic.

ECLIPTIC
The plane described by the orbit of the Earth around the Sun.

ECLIPTIC LATITUDE
The angle of a celestial object above the ecliptic as seen from the Sun.

ECLIPTIC LONGITUDE
The angle of a celestial object from the **vernal equinox** measured along the **ecliptic**.

ELONGATION
The angle from the Sun to a planet as seen from the Earth.

EPHEMERIS
The old name of the Astronomical Almanac, which is published jointly by the United States Naval Observatory and Royal Greenwich Observatory. Also any listing of planetary positions.

EPACT
The age of the calendar moon in days on the first day of the year. Defined as the number of days since the last astronomical full moon less 1 day. Thus the epact is the number of days since the start of the last lunar month of the previous year. Used for finding the date of the ecclesiastical vernal full moon and thus, fixing the date of Easter and other moveable Christian religious celebrations.

EQUATION OF THE CENTER
The difference in the angle of the mean Sun and the true Sun measured along the **ecliptic** = (Sun's **mean anomaly**) - (Sun's true anomaly). It is the difference in the actual angular position of a planet and the position the planet would have if it had a uniform orbital speed. Determination of the equation of the center was a critical element of Ptolemaic astronomy. It is currently used as part of the equation of time.

EQUATION OF TIME
The difference in the time of the fictitious mean Sun, upon which civil time is based, and the time of the true, physical Sun. The precise definition is the difference in right ascension of the true Sun and the fictitious mean Sun. The fictitious mean Sun is the basis for civil time and is defined as a fictitious Sun that travels at a fixed rate on the equator. The equation of time = (hour angle of the Sun) - (hour angle of mean Sun) = (right ascension mean Sun) - (right ascension of the Sun) = (Sun's mean longitude) - (Sun's right ascension) = (Sun's true longitude) - (Sun's right ascension) - (equation of the center). The term (Sun's true longitude) - (Sun's right ascension) is called the "reduction to the equator" and accounts for the time difference resulting from the fictitious mean Sun traveling on the equator instead of the ecliptic. The equation of time is used to correct the time shown by a sundial to zone time and is used in celestial navigation.

EQUATOR
The **great circle** on the surface of a body that is perpendicular to the axis of rotation.

EQUINOX
The point where the celestial equator meets the **ecliptic**. When the Earth passes through one of its equinoxes the length of the day and night are equal. The Vernal Equinox is about March 21 and the Autumnal equinox is about September 23.

EQUINOCTIAL
Latin for "equator". Also used as an adjective as related to the equinoxes or in the vicinity of the equator. Sometimes spelled "aequinocital".

EQUINOCTIAL COLURE
Great circle of the celestial sphere passing through the poles and the equinoxes.

FIRST POINT OF ARIES (♈)
The point on the celestial sphere where the equator crosses the ecliptic and the declination of the Sun changes from negative to positive. Also known as the **vernal equinox**. Note that the position of the First Point of Aries changes due to the precession of the equinoxes. Orbital positions are always measured from a specified equinox which may be fixed at some specific time (epoch) such as J2000.0 or referred to the actual equinox at a given time (the mean equinox of date). The First Point of Aries is no longer related to the constellation Aries.

FULL MOON
Condition when the Moon is at **opposition**, i.e. the longitude of the Moon differs from the longitude of the Sun by 180°.

GEOCENTRIC
Centered at the center of the Earth. Some tabulated geocentric values are actually centered at the center of mass of the Earth-Moon system. Right ascension and declination are geocentric.

GEOCENTRIC LATITUDE
The angle of a celestial object above the **ecliptic** as seen from the Earth.

GEOCENTRIC LONGITUDE
The angle of a celestial object from the First Point of Aries measured along the **ecliptic** as seen from the Earth. That is, the Earth is considered to be at the center of the ecliptic. The signs of the zodiac are divided into segments of 30° in geocentric longitude.

GOLDEN NUMBER
The year in the 19 year lunar cycle (see **Metonic Cycle**). Calculated as the remainder of [(year +1)/19] with 0 taken as 19 and expressed in Roman numerals. The Golden number for 1996 is II. Used in the calculation of the date of Easter.

GREAT CIRCLE
The plane defined by a circle on sphere that passes through the center of the sphere. Examples of great circles are the **equator** and **meridian**. A small circle does not pass through the center of the sphere. The circles defined by latitudes are small circles except for the equator.

GREGORIAN CALENDAR
The calendar proposed by Aloysisus Lilius, a physician from Naples, and adopted by Pope Gregory XIII in accordance with instructions from the Council of Trent (1545-1563) to correct for errors in the older Julian Calendar. In the Gregorian Calendar, leap years are every four years, as in the Julian Calendar, except that century years not divisible by 400 are not leap years. 2000 is a leap year. Thus, there are 97 leap years every 400 years making the average length of the **tropical year** of 365.2425 days compared to the true value of about 365.2422 days or an error of only about 26 sec. in 400 years. Presumably, the Gregorian Calendar will be updated at the appropriate time to eliminate years evenly divisible by 4000 as leap years. This modest correction will reduce the 4000 year error in the mean tropical year to 4 sec. The Gregorian

calendar was introduced in Italy, Spain, Portugal, France and Poland on October 4, 1582, a date that was followed by October 15, 1582. Other Catholic countries followed in 1583 (Holland, Flanders and the German Catholic States). Protestant countries delayed introduction with the German and Dutch Protestant states and Denmark in 1700, Britain and British dominions waiting until September 2, 1752 (followed by September 14), Sweden in 1753, Japan in 1873, China and Albania in 1912, Bulgaria in 1916, Soviet Russia in 1918, Rumania and Greece in 1924 and Turkey in 1927. Adoption of the Gregorian calendar was often accompanied by other calendar changes. For example, the British Calendar New Style Act of 1750 also included changing New Year's Day from March 25 to January 1, a custom that had been adopted in Scotland in 1600. Calendar dates must be examined carefully to determine the calendar in use at the time and even then can be very confusing. See also **Easter (Date of)**.

HELIOCENTRIC
Centered at the Sun. Ecliptic longitudes and latitudes and planetary orbital parameters are heliocentric.

HELIACAL RISING
The condition when a star rises with the Sun. Heliacal rising was the basis for many ancient calendars, the most famous of which was in Egypt where the heliacal rising of Sirius signaled the beginning of the agricultural year. Other stars have been used by other cultures. For example, there is evidence that North American plains Indians used the heliacal rising of Aldebaran to signal mid-summer.

HORIZON
The plane perpendicular to the line from the observer to the zenith. The astronomical horizon is a **great circle** and the local horizon is tangent to the surface of the Earth. They are indistinguishable for most astronomical applications.

HOUR ANGLE
The angular distance measured westward along the celestial equator from the **meridian** to the hour circle passing through the celestial object. The hour circle is a **great circle** passing through the object.

INFERIOR CONJUNCTION
The condition where Mercury or Venus is directly between the Sun and the Earth.

J2000.0
The Julian day that represents the exact beginning of the year 2000, dynamical time. The Julian ephemeris day at J2000.0 is 2451545.0 and is the basis for calculating celestial positions in the current epoch. That is, the star positions given in catalogs are for J2000.0 from which positions at other dates can be calculated.

JULIAN CALENDAR
Calendar introduced by Julius Caesar in the "Year of Confusion", 46 BC, to correct for the non-integral number of days in a year through the introduction of Leap Years. At its introduction, 80 days were added to bring the vernal equinox to the desired date and to make the new year start on January 1. In the Julian Calendar there is a leap year every fourth year in which February has 29 instead of 28 days. The "Leap Day" (probably from Old Norse *hlaupa,* "to leap") was originally inserted after February 23, which was six days before the Kalendae or beginning of March and was called *sexto-kalendae*. The leap day, when added, was the day after; *bis-sexto-kalendae*. Thus, leap years are called bissextile. The number of days in the months were also changed when the Julian calendar was introduced and ammended by Augustus in 8BC to the values in use today. Also, due to misunderstanding by the Pontifices, the leap days were inserted every three years instead of every four years for 36 years. Augustus corrected the error by omitting bissextile years from 8 BC to AD 8. Therefore, corrections must be applied to dates in

the Julian calendar from the period 46 BC to AD 8. Later superseded by the **Gregorian Calendar**. In the Julian calendar, the length of the year is taken as 365¼ days, exactly.

JULIAN DATE
A convention for timekeeping that is independent of specific calendars and is thus ideally suited for astronomical purposes since the number of days between observations is immediately known from the differences in Julian dates. Julian dates are counted in sequence from noon UT on January 1, 4713 BC. Note that Julian dates begin at noon UT. Julian dates are shown as an integer day number with a fraction of a 24 hour day. Thus, Julian date 2436116.31 corresponds to October 4, 1957 at 19:26 (the launch time of Sputnik I). The Julian Period is formed by the product of the 19 year cycle of the Moon used to determine the "golden number" of a year for use in determining the date of Easter, the solar cycle of 28 years which is the shortest period in which the same date occurs on the same day of the week in the Julian calendar and the 15 year cycle of indiction which was introduced by Constantine in 313 for setting the period of property taxation (i.e. property was reassessed each 15 years). Its length is 19 x 28 x 15 = 7980 years. In this period no two dates can have the same Julian date in all three cycles. All of these cycles began in January 1, 4713 BC so the Julian Period covers all of recorded history. Will there be another Julian date 0.0 in 3267? Many people make an intuitive association between Julian dates and the Julian calendar introduced by Julius Caesar. There is a vague relationship. The Julian Period was introduced by Joseph Justis Scaliger (1540-1609) of Leyden in *De emendatione temporum* (Paris, 1583).

LIMB
The edge of a planet or the Moon or any celestial object with a disk. On an astrolabe, the limb is the ring of the instrument outside of the Tropic of Capricorn which usually contains the hour numbers and a degree scale.

LINE OF APSIDES
The line connecting the ascending and descending **nodes** of an orbit.

LONGITUDE CORRECTION
The difference in time between a specific longitude and the center of the civil time zone. Since civil time zones are centered at 15° intervals, the difference in the sidereal time at the center of the time zone and another location in the same time zone can be significant. Madrid, for example, is over an hour from the center of the time zone used in Spain.

LUNATION
The time between new moons (29.5306 days). See also **Synodic Period.**

MEAN ANOMALY
The position of an orbiting body if it is assumed to have a circular orbit with a constant orbiting rate. Used to calculate **eccentric anomaly**.

MEAN LONGITUDE
The angle of an orbiting body from the **First Point of Aries** measured in the direction of rotation assuming the orbit is circular with a constant rotation rate.

MERIDIAN
The **great circle** passing through north, south and the **zenith** for a specific location. The meridian defines the directions of north and south and is the north-south (vertical) line on the astrolabe plate.

MERIDIAN PASSAGE
The instant that a celestial body crosses the **meridian** from east to west. Identical to **culmination** or transit.

METONIC CYCLE
The 19 year cycle of nearly 235 lunations over which the moon's phase repeats. That is, if a full moon occurs on a certain date it will occur on the same date 19 years later. Attributed to Meton of Athens ca. 432 BC but probably known earlier. The number of the year in a Metonic cycle was called "The Golden Number" and was used to fix the ecclesiastical calendar before the Gregorian calendar reform in 1582. Also called the *enneakaidekaëteris* or the *decemnovennalis cyclus*.

MURAL QUADRANT
A quadrant that is permanently mounted on a noth-south wall and equipped with an index showing angles on the quadrant degree scale. Mural quadrants were typically used to measure the zenith distance of celestial objects as they passed the local meridian which allowed the declination to be calculated.

NAVIGATION STARS
57 relatively bright stars well distributed in the sky that have been selected by celestial navigators as good targets for taking sights.

NEW MOON
Condition when the Moon is at conjunction with the Sun, i.e. the longitude of the Moon is the same as the longitude of the Sun.

NODE
The point of an orbit where the orbit crosses the **ecliptic** plane. (see also, Ascending Node).

NODIAL SCALE
Scale division method used on accurate quadrants to allow estimating values between divisions. A nodial scale usually used a diagonal line connecting the base and top of adjacent divisions cut by fine circles. Replaced by the vernier in the 18th century.

OBLIQUE HORIZON
The theoretical horizon for a location that is not on the equator. The term originates from the fact that the local horizon makes an oblique angle with the equator when the celestial sphere is viewed from the side.

OBLIQUITY OF THE ECLIPTIC
The angle the Earth's **equator** makes with the **ecliptic**.

OPPOSITION
The condition where the Earth is directly between a planet and the Sun, i.e. the longitude of the planet is 180^o from the Sun.

ORBITAL PARAMETERS
Any orbit is completely described by the following parameters: period (i.e. how long it takes to complete one orbit), eccentricity, longitude (or right ascension) of ascending node, inclination, semi-major axis and perifocus argument (perihelion argument for planets). The position of any orbiting body can be calculated for any time using these six parameters and a known position (such as perifocus passage) at some specific time.

PERIAPSIS
The general term for the point on an orbit that is closest to the body being orbited. See also perigee and perihelion. Similar words are used for orbits around Jupiter, Saturn, etc.

PERIGEE
The point in the moon's orbit that is closest to the Earth.

PERIHELION
The point in a planet's orbit that is closest to the Sun.

PERIHELION ARGUMENT (PERIHELION DISTANCE)
The angle on an orbit from the **ascending node** to **perihelion** measured in the plane of the orbit.

PHASE ANGLE
The angular distance from the Sun to the Earth as seen from the Moon or a planet.

PLATE
The part of an astrolabe display consisting of the horizon and twilight lines, altitude circles, and azimuth curves. Also known as the "tympan".

POSIGRADE MOTION
When a planet's longitude increases. That is, it moves from west to east relative to the stars.

POSITION OF BRIGHT LIMB
A measure of the orientation of the phase of the Moon or a planet. The position of the bright limb is measured as the angle from north of a line connecting the cusps of the shadow.

PRECESSION OF THE EQUINOXES
The gradual movement of the point of the equinoxes (but usually described by the movement of the vernal equinox only) in the celestial sphere. Precession is caused by the slightly irregular shape of the Earth and the fact that the moon's orbit is not exactly on the ecliptic. Precession is approximately 50″ in longitude per year.

PRIME VERTICAL
The arc in sky that starts at the east point on the local horizon, goes through the zenith and ends at the west point on the horizon.

RADIUS VECTOR
Distance of a planet from the Sun in AU's.

REGULA
Latin for "rule". The name applied to various astrolabe components used to measure distances.

RETE
The part of an astrolabe which contains pointers to the stars, or in the case of the Electric Astrolabe, the constellations and Messier objects and the ecliptic. The rete rotates once in a sidereal day. The word is pronounced "reet" by most people although some scholars prefer "reetee" as it would be pronounced in Latin. Rete is the Latin word for "net". It is also called the "spider" in various languages.

RETROGRADE MOTION
When a planet's longitude decreases. That is, it moves from east to west relative to the stars over a period of a few days. Copernicus used the retrograde motion of Mars to illustrate the validity of his heliocentric solar system theory.

RIGHT ASCENSION
The east-west position of a celestial object on the equator of the celestial sphere. Right Ascension is measured in hours and minutes beginning at the First Point of Aries, from 0:00 to 24:00, increasing to the east. The name "right ascension" derives from the rising times of the ecliptic when viewed from the equator (*sphaera recta*). At this location, the rising time for a section of the ecliptic is found directly. The "rising time at sphaera recta" is "ascensio recta" or right ascension.

RIGHT HORIZON
The horizon at the equator. The right horizon projects as a straight line going from east to west on the astrolabe because it is the same as the astrolabe's stereographic projection plane. The right horizon is also interpreted as a line because the Sun rises and sets at the same time every day at the equator (if the equation of time is ignored).

RULE
The "clock hand" on an astrolabe pointing to the current clock time. The point where the rule crosses the ecliptic circle is the position of the mean Sun in the ecliptic. Note also that the point where the rule crosses the ecliptic shows the Sun's **geocentric longitude**. The rule is sometimes graduated in declination.

RHUMB LINE
A fixed compass diretion on the Earth. A rhumb line crosses all meridians at the same angle. Also called loxodrome.

SAROS
Time period of about 18 years, 11 months after which eclipses occur with the same circumstances. The Saros is a combination of 223 synodic months of 29.5306 days (6,585.32 days), 19 Draconitic years of 346.6201 days (6,585.78 days) and 239 anomalistic months of 27.55455 days (6,585.54 days). When all three cycles coincide the Moon/Sun relationship is repeated. If the cycle is started at a solar eclipse, similar eclipses will occur 6,585 days in the future or occurred the same number of days in the past. The Saros does not exactly restore all elements and the character of consecutive eclipses gradually change finally resulting in no eclipse at all. Therefore longer cycles must be constructed. The current Saros series (numbered 136) began in 1306 and will continue until 2622 and contains 71 eclipses. Note that successive eclipses occur about 120° west of the previous one. After three Saros an eclipse will occur at the same longitude but shifted north or south. The word *Saros* derives from a Babylonian word meaning "universe". The use of this term was popularized by Halley in the late 16th century

SIDEREAL TIME
The **right ascension** of celestial bodies that are on the **meridian** for a specific place at a given instant. Mean sidereal time is referred to the mean equinox of date (i.e. corrected for precession). Apparent sidereal time is corrected for nutation (the so called equation of the equinoxes) and is referenced to the true equinox of date.

SIDEREAL DAY
The time of one rotation of the Earth; 23 hours 56 minutes 4.09054 seconds.

SIDEREAL HOUR ANGLE
A measurement used in celestial navigation that is similar to right ascension. The sidereal hour angle of a celestial body is the angle of the body, measured on the equator and expressed in degrees, increasing to the west (the opposite of right ascension). $SHA = 15° \times (24h - RA)$.

SIDEREAL YEAR (MONTH)
The time for the Earth's successive returns to the same position relative to the stars as seen from the Sun (365.2564 days). A similar definition applies to the moon. A sidereal month is 27.3217 days.

SOLSTICE
Instant when the Sun reaches its maximum northern or southern declination.

SOLSTICIAL COLURE
Great circle on the celestial sphere passing through the poles and the solstices.

SUPERIOR CONJUNCTION
The condition where the Sun is directly between a planet and the Earth.

SYNODIC PERIOD
The average period of time between successive **conjunctions** of a pair of planets or of a planet and moon. A synodic month is, therefore, the time between new moons (29.5306 days).

SYZYGY
The points on the orbit of a planet (or the moon) when it is in opposition or conjunction. Also a terrific Scrabble or Hangman word.

TROPICAL YEAR (MONTH)
The time from one **vernal equinox** to the next. Also the basis for the calendar year. Equal to 365.242191 days. For the Moon, a tropical month is the time between new moons and is equal to 29.5306 days.

TRUE ANOMALY
The angular distance on an orbit from **perihelion** to the current position measured from the focus of the orbit.

TRUE LONGITUDE
The angular distance on an orbit from the **vernal equinox** to the current position. Calculated as the **true anomaly** plus the longitude of perihelion.

UNIVERSAL TIME
Universal time is the basis for civil timekeeping. Universal time is defined as 12h plus the hour angle of the fictitious mean Sun that is assumed to orbit the Earth in a circular orbit on the equator. Time services (such as WWV) broadcast Coordinated Universal Time (UTC) which is kept in step with atomic clock times by the occasional insertion of one second steps called "leap seconds". Universal Time has become extremely complex. Interested readers are referred to *The Astronomical Almanac* for precise definitions. A good overview is in Howse, Derek, "Greenwich time and the discovery of the longitude", Oxford University Press, 1980, which also has an interesting history of timekeeping in general.

VARIATION
An inequality in the moon's orbit due to differences in the Sun's attraction during a **synodic month**.

VERSINE
versine x = 1 – cos x. That is, the "versed" (or reversed) sine; the sine turned 90°. versine x represents the residual of a radius of a circle defined by a right triangle with base angle x and hypoteneuse equal to the radius of the circle, R. If r = R cos x, versine a = R – r. The versine is almost never used today, but was very popular up to the 16th century.

ZENITH
The point in the sky directly above a location.

ZENITH DISTANCE
The angle from the zenith to a celestial object.

ZODIAC
The division of the ecliptic into the twelve astrological signs: Aries, Taurus, Gemini, Cancer, Leo, Virgo, Libra, Scorpio, Sagittarius, Capricorn, Aquarius, and Pisces. The zodiac begins at the vernal equinox (the "First Point of Aries") and continues in 30 degree sections through the twelve signs. Note that, due to the precession of the equinoxes, the constellations that give the sections of the zodiac their names no longer fall in the section with that name. Thus, the zodiac

is an arbitrary division of the ecliptic into 30 degree sections of geocentric longitude and is a convenient way to describe planetary positions.

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