

Longitude from Lunar Altitudes Simplified

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ABSTRACT: *Described is a simple method of observing and calculating longitude from lunar altitudes that does not require a succession of approximations. The method is unique in that the observations and calculations employed are standard procedure for practising navigators. No special tables or plotting, except for normal sight reduction, are required and an illustrative example is included.*

INTRODUCTION

Historically, interest began to wane in lunar observations when the chronometer became an integral part of the ship's navigation equipment; with the death knell being the inauguration of radio time signals at the beginning of the 20th century. Interest was revived in 1966 when Francis Chichester [1] published a note in which he discussed the predicament he might be in at sea if he lost his correct time and was without a radio. He likened the situation to the pre-chronometer era when the most accurate method was to measure the angle between the Moon and the Sun or a planet, i.e., lunar distances. The method also required the altitudes of these bodies to be observed followed by some involved calculations. In all, he was horrified by the prospect of doing this on a small boat and suggested an alternative: observing altitudes and calculating using standard sight reduction techniques, thus employing processes familiar to practising navigators. His suggestion was as follows. Latitude was first determined from a meridian altitude observation of the Sun before or after a Sun/Moon fix. The value of this latitude was then advanced or retired to the time of fix using the courses steered and distances run by the boat. A graphical method was then employed to find a clock time when the fix would cut the known parallel of latitude. A small scale chart covering 15 deg of longitude was used in the latter process. Sadler [2], the former Director of Her Majesty's Nautical Almanac Office, thought that the method was original although its accuracy was inherently inferior to that of conventional lunar distances. He went on to investigate the conditions which should pertain for an optimum result. As a general guide, Moon observations should not be taken in the

vicinity of the meridian but near the prime vertical. There is no doubt that lunar altitude methods are inferior to the lunar distance method but the advantage of the former is that we can use techniques which are familiar to navigators. However, the accuracy does depend upon the reliability of altitude measurement such as a clear horizon and reliable refraction and dip corrections.

Most proposed methods require plotting or refining an initial estimate of the watch correction or longitude. Luce [3] provides a comprehensive list of such methods.

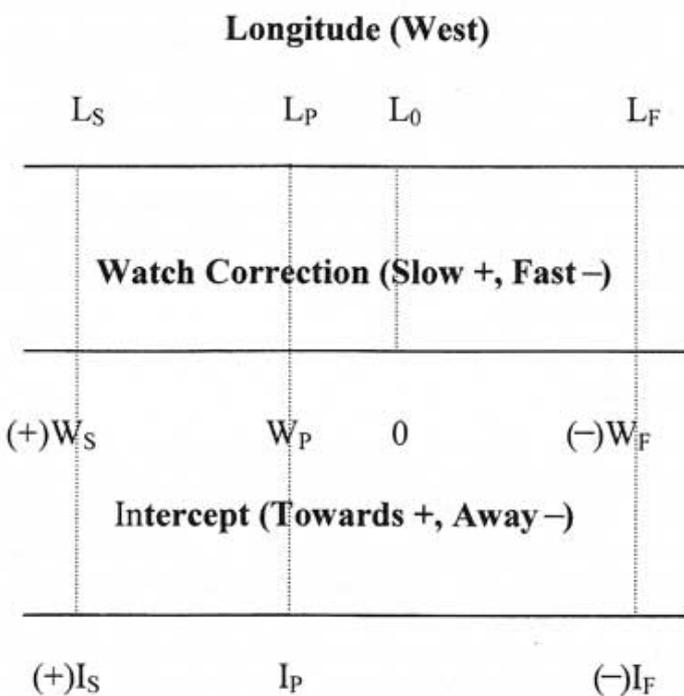
METHOD

In what follows, Moon observations are not required to be made at twilight or in conjunction with any other body. The watch should be set on the best estimate of Greenwich Mean Time (GMT) and a conventional set of timed altitude observations should be made at morning or evening twilight to determine the local position (latitude and longitude) of the observer. The latitude result is unaffected by any uncertainty in the unknown watch correction and the value of longitude so obtained assumes that the watch correction is zero. A group of three or more star observations, well-distributed in azimuth, should give a reliable result. These observations should be made as close as possible to a set of timed altitude observations on the Moon which should be made when the Moon is in the vicinity of the prime vertical. A compass bearing will indicate this direction. The latitude and longitude resulting from the star fix are advanced or retired to the epoch of the Moon observation by whatever technique is normally used.

An alternative to the previous combination of observations may be found more convenient. Instead of a star fix, a meridian altitude of the Sun or a minimum of two sun observations, with a reasonable

separation in azimuth, to effect a latitude and a local longitude fix, may be used. For example a sequence of observations such as Sun, Sun, Moon or Sun, Moon, Sun, or Moon, Sun, Sun, then advancing or retiring the sun observations to the epoch of the Moon observation, may suit the situation.

If the observer's longitude and watch correction, both of which are unknown, were to be used in the calculation of an intercept for the Moon, the result would be zero. Exact values of these unknowns may be found by calculating intercepts for a pair of values of watch correction and longitude. These intercepts indicate the change in altitude of the Moon caused by its change in Right Ascension and Declination on the celestial sphere. The watch correction and longitude that correspond to a zero intercept are found by calculating the ratios of line segments from the following diagram with its associated sign conventions:



Where

L_0 is the longitude run up from the star fix, L_P , W_P and I_P are the longitude, watch correction and intercept respectively for the observer, W_S and W_F are chosen values of watch correction and L_S and L_F are their corresponding values of longitude, and

I_S and I_F are intercepts calculated using W_S and L_S , and W_F and L_F respectively.

Let

$$F = \frac{I_S - I_P}{I_S - I_F} = \frac{W_S - W_P}{W_S - W_F} = \frac{L_S - L_P}{L_S - L_F}$$

Therefore $W_P = W_S - F(W_S - W_F)$ and $L_P = L_S - F(L_S - L_F)$

The choice of value of the watch corrections (Slow or Fast 20min used in the following example) does not affect the accuracy of the solution, i.e., one could use 40min or 1h, it is immaterial. The calculations may be further simplified by substituting L_0 for L_S or L_F . All the calculations should be familiar to navigators with the exception of the trivial arithmetic used to find W_P and L_P .

EXAMPLE

This example is situated near where Chichester was located at DR position $N41^\circ$, $W15^\circ$ on 22 October 2005. The watch was purportedly keeping GMT, the Height of Eye 3.0 m and the sextant had an Index Correction of $+2.0'$. The vessel's speed was 6.0 knots on a course of $050^\circ(T)$.

Observations and Calculations

At morning civil twilight, three stars, Hamal, Procyon and Alioth were observed.

Body	Watch Time	Observed Altitude	Azimuth	Intercept
Hamal	6h 54min 25.3s	$17^\circ 30.6'$	286.7°	T30.2
Procyon	6 55 07.0	53 48.2	187.9	A11.6
Alioth	6 55 33.7	41 15.8	045.8	A8.7
Fix at 6h 55min		$N41^\circ 15.5'$, $W15^\circ 36.0'$		

At 9h 07min 55.0s the upper limb of the Moon was observed at a sextant altitude of $37^\circ 09.7'$.

Advance (Course 050° , Speed 6.0 knots)

Watch Time	Dist. miles	Latitude	Longitude
6h 55min	13.3	$N41^\circ 15.5'$	$W15^\circ 36.0'$
9 08		$N41^\circ 24.1'$	$W15^\circ 22.5(L_0)$

Moon Intercept Calculations at 9h 07min 55.0s, Latitude $N41^\circ 24.1'$, Sextant Altitude $37^\circ 09.7'$.

Watch Correction	Longitude	Intercept
Slow (+)20min(W_S)	$W20^\circ 22.5'(L_S)$	A10.7(I_S)
0(W_F)	$W15^\circ 22.5(L_0)$	A2.4(I_F)

Note: $20\text{min} = 5^\circ$ $I_P = 0$ $L_F = L_0$ $W_F = 0$
 $L_S = W15^\circ 22.5' + 5^\circ = W20^\circ 22.5'$

Azimuth of the Moon 277° —not required.

$$F = \frac{I_S}{I_S - I_F} = \frac{10.7}{8.3}$$

$$W_S - W_F = 20\text{min} \quad L_S - L_F = 5^\circ$$

Required Watch Correction $W_P = 20\text{min} - F \times 20\text{min} = -5\text{min } 47\text{s (Fast)}$

Required Longitude $L_P = W20^\circ 22.5' - F \times 5^\circ = W13^\circ 56'$

Optional Check

Calculate an intercept based on the new values of the Watch Correction and Longitude together with the original data. The intercept should be zero.

CONCLUSION

The method does not appear to have been used before and has the advantage of being simple to observe and calculate using standard celestial navigational procedures. The success of the technique depends on the appreciation that watch correction and longitude are not independent. For a particular pair of values of these variables there is a corresponding

value of a Moon intercept, which can only be zero for the correct values of watch correction and longitude. The example has been made up to demonstrate the principles of the method.

REFERENCES

1. Chichester, F., *Longitude Without Time*, Journal of the Institute of Navigation, Vol. 19, London, 1966, pp. 106–107.
2. Sadler, D. H., *Comments*, Journal of the Institute of Navigation, Vol. 19, London, 1966, pp. 107–109.
3. Luce, J. W., *Longitude without Time*, NAVIGATION, Journal of the Institute of Navigation, Vol. 24, No. 22, Summer 1977, pp. 112–115.

