Simple Method to Calculate the GHA and Declination of the Sun for Navigation – Updated Approach.

In *The Calculator Afloat,* Shufeldt and Newcomer describe a simple way to calculate the GHA and declination of the sun for any date and time during a year using two parameters specific to that calendar year (Schfeld, pp172-179). They tabulated values for those parameters for the years 1979 through 1999. Harris, in *Astro Navigation by Pocket Computer*, updated those parameters to include the years 1988 through 2010 (Harris, pp 69). These calculations are quite practical using just an inexpensive scientific calculator. Here we provide updated parameters for the years 2025 – 2045 and make some additional adjustments to Improve the accuracy of the results.

This method is a simplification of the formulas in *Astronomical Algorithms* by Meeus for low accuracy solar coordinates (Meeus, pp163-5). Using the nomenclature of Harris, the parameter P is the Mean Anomaly of the Sun at the start of the calendar year, and the parameter Q is P minus the Mean Longitude of the Sun at the start of the year. Harris also provides an annual parameter R, which is the GHA Aries at the beginning of the year and is useful for working star sights. B represents the date and time for the required Solar GHA and declination in days and fractional days since the start of the year. There are a number of tables giving the integral day of the year (DOY) by date. Such a table is often included in the Nautical Almanac. The value of B is found by the formula B = DOY + GMT/24. In the convention used here, the value of B for January 1 at 0h GMT is 1, corresponding to day 1 of the year. The values of P and Q are not really "at the start of the year", but rather from the day before – calculated from from Meeus formulas 25.3 and 25.2 using the value of T (Julian centuries since the epoch 2000) corresponding to 12/31 0h GMT of the previous year.

Below is the formula for calculating the GHA and declination of the Sun, where M is the Mean Anomaly, L is the Apparent Longitude, E is the declination and F is the GHA of the sun:

M = .985647B + P
L = M + 1.913sinM + .02sin2M – Q
E = ArcSin(.3977sinL)
F = ArcTan(.9175tanL)
If cosL < 0. Then F = F -` 180
F = 360Frac(B) + M – F – 180 - Q

The term 1.913sinM + .02sin2M is called the Equation of Center. That term added as a correction to the mean longitude, M – Q, yields an approximation of the true longitude L. The arcTan(L) must be in the same quadrant as L. If Cos(L) is < 0 then F = F – 180. Frac(B) is the remainder of B after the integer number of days is subtracted (fraction of day). In line 4, F is the Right Ascension; in line 6, F is converted to GHA.

The inclusion of the R parameter allows the calculation of the GHA Aries by the formula:

GHA Aries = R + 360.985647B = R + 360Frac(B) + .985647B

Line 6 could also be written: F = GHA Aries – F; or F = R + 360Frac(B) + .985647B - F , which seems more intuitive to me.

Example: Calculate GHA and declination of the sun for April 12, 2030 at 22:15:15 GMT

For 2030, P = -3.7414 Q = 76.5479 R = 99.7099
4/12 is DOY 102. B = 102 + 22:15:15/24 = 102.9272569
M = .985647X102.92725 – 3.7414 = 97.7085
L = 97.7085 + 1.913sin(97.7085) + .02sin(2X97.7085) – 76.5479 = 23.0510
E = ArcSin(.3977sin23.0510) = 8.9585 = 8deg 57’ 31”
F = ArcTan(.9175tan23.0510) = 21.3266
Cos23.0510 > 0, so F is unchanged
F = 360(.9272569) + 97.7085 – 21.3266 – 180 – 76.5479 = 153.6469 = 153deg 38’ 48”
Or:
F = 99.7099 + 360(.9272569) + .985647X102.9272569 -21.3266 = 153.6457 = 153deg 38’ 45”

The Actual values from truncated VSOP87 theory are: GHA = 153deg 38’ 40” and Dec = 8deg 57’ 32”
Since the fractional part of B is multiplied by 360, one needs to include sufficient number of decimal places to maintain accuracy. Using the calculator memories to store intermediate results greatly facilitates the calculation and reduces rounding error.

Updates and Modifications to Improve Accuracy

The formula for the Mean Anomaly M is not really .985647B + P; it is actually .985600B + P, which is the factor used by Harris (.9856). However, to simplify calculation, the variable value of M already calculated is used to replace the variable value of the Mean Longitude L in line 2. The correct factor for the Mean Longitude is .985647, and since the accuracy of the Mean Longitude is much more important than the accuracy of M (which comes in only as small correction factors involving sinM and sin2M), we use .985647 everywhere.

The parameters Q listed below include a term to correct for aberration and nutation (Meeus p164). Nutation has a period of 18.6 years, so any error over the year in this correction is less than 3” arc, and is usually less than 2”.

Accuracy and Utility of This Method

The main errors of this simplified method come from lack of accounting for perturbations of the earth’s orbit by the gravitational effects of the other planets. I checked a sample of results from this simplified method for each year against results for GHA and declination of the sun calculated from a truncated VSOP87 theory utilizing 50 periodic terms (Meeus), as well as an on-line almanac. All the errors I found were no more than 0.3’ minutes of arc, and usually less than 0.2’minutes. The average absolute error in 25 calculations was 7.5” arc for GHA and 4.4” for declination. This is better than one can expect to actually measure from a small boat with a sextant, so this method is quite suitable for small boat navigation.

A Casio fx-300MS calculator with several user memories is available on Amazon for less than $10. Such a calculator is quite practical for these calculations. It also makes quick work of solving the formulas for the Navigational Triangle, thus replacing annual Almanacs and volumes of tables for much celestial work. It is easy to enter this algorithm into a programmable calculator. In 1992, I programmed a small Radio Shack calculator with limited memory with this and several other useful navigation algorithms and navigated a sailboat to Hawaii, never having had to crack a book or mark a plotting sheet. Modern calculators have a lot of memory and can hold the precise algorithms for the sun, moon, planets and stars, although entering those hundreds of periodic terms for the Solar bodies is quite tedious.

Annual Parameter Updates

Year	P	Q	R
2025	-3.4475	76.6378	99.9140
2026	-3.7034	76.6191	99.6768
2027	-3.9593	76.6006	99.4393
2028	-4.2152	76.5826	99.2015
2029	-3.4855	76.5649	99.9488
2030	-3.7414	76.5479	99.7099
2031	-3.9973	76.5314	99.4705
2032	-4.2532	76.5153	99.2306
2033	-3.5235	76.4995	99.9761
2034	-3.7794	76.4839	99.7358
2035	-4.0333	76.4683	99.4955
2036	-4.2912	76.4525	99.2554
2037	-3.5615	76.4363.	100.0013
2038	-3.8174	76.4197	99.7620
2039	-4.0733	76.4025	99.5232
2040	-4.3292	76.3848	99.2850
2041	-3.5995	76.3666.	100.0329
2042	-3.8554	76.3481	99.7955
2043	-4.1113	76.3293	99.5583
2044	-4.3672	76.3105	99.3212
2045	-3.6375	76.2918.	100.0696

References

1) Shufeldt, H and Newcomer, *The Calculator Afloat*, Naval Institute Press, 1980
2) Harris, M, *Astro Navigation by Pocket Computer*, Adlard Coles, 1989
3) Meeus, J, *Astronomical Algorithms*, Willmann-Bell, 1998