Conditions for Positive Altitudes in Bygrave/MHR1 Calculations

In general for an observer at latitude, L, the altitude, h, of a celestial body with declination, δ, and local hour angle, LHA, is given by

$$\sin h = \sin\delta \sin L + \cos\delta \cos L \cos \mathrm{LHA}$$

From this it follows that the object is on the horizon ($h = 0$) when

$$\cot\left(90^\circ - L\right) = -\cot\delta \cos \mathrm{LHA}$$

But ignoring the minus sign this is just the calculation used in the procedure for finding y in the Bygrave instructions (x in the MHR1). So y represents the limiting colatitude at which a celestial body of given declination and altitude is just on the horizon.

By considering the various possible combinations of parameters it is found that the altitude obtained from using a Bygrave or MHR1 is positive (i.e. above the horizon or zenithal distance ZD < 90°) if and only if any one of three conditions is satisfied

Bygrave Notation

L and d	H	c and y
same name	less than 90°	No restriction
	greater than 90°	Y = c + y less than 180°
contrary name	less than 90°	c greater than y

MHR1 Notation

φ and δ	t	b and x
same name	less than 90°	No restriction
	greater than 90°	y = b + x less than 180°
contrary name	less than 90°	b greater than x

φ und δ	t	b und x
gleichnamig	kleiner 90°	keine Einschränkung
	größer 90°	y = b + x kleiner 180°
ungleichnamig	kleiner 90°	b größer x

The conditions look asymmetric due to the practice of not retaining signs in the Bygrave/MHR1 calculations. Subtractions always produce positive values. It also uses the convention that the hour angle falls in the range 0° to 180° and is qualified as being E or W rather than a value in the range 0° to 360° as is common practice today. In more modern notation and assuming the sign in the Bygrave variable Y (MHR1 variable y) was kept these conditions can be written

L and d	cos(H)	Y
same sign	+	No restriction
	-	Y < 180°
opposite sign	+	Y > 0°