Chapter 8

CELESTIAL CONCEPTS

Section 8A—Introduction to Celestial Concepts

8.1. Basics. Celestial navigation is a universal aid to dead reckoning (DR). Because it is available worldwide and is independent of electronic equipment, it is a very reliable method of fixing the position of the aircraft. It cannot be jammed and emanates no signals. Each celestial observation yields one line of position (LOP). In the daytime, when the sun may be the only visible celestial body, a single LOP may be all you can get. At night, when numerous bodies are available, LOPs obtained observing two or more bodies may be crossed to determine a fix.

8.1.1. It is impossible to predict, in so many miles, the accuracy of a celestial fix. Celestial accuracy depends on the navigator's skill, the type and condition of the equipment, and the weather. With the increase in aircraft speed and range, celestial navigation is very demanding. Fixes must be plotted and used as quickly as possible.

8.1.2. You don't have to be an astronomer or mathematician to establish a celestial LOP. Your ability to use a sextant is a matter of practice, and specially designed celestial tables have reduced the computations to simple arithmetic.

8.1.3. Although you don't need to understand astronomy in detail to establish an accurate celestial position, celestial work and celestial LOPs mean more if you understand the basics of celestial astronomy. Celestial astronomy includes the navigational bodies in the universe and their relative motions. Although there are an infinite number of heavenly bodies, celestial navigation utilizes only 63 of them: 57 stars, the moon, the sun, Venus, Jupiter, Mars, and Saturn.

8.2. Assumptions. We make certain assumptions to simplify celestial navigation. These assumptions help you obtain accurate LOPs without a detailed knowledge of celestial astronomy. However, celestial positioning is more than extracting numbers from various books. A working knowledge of celestial concepts will help you crosscheck your computations.

8.2.1. First, assume the earth is a perfect sphere. That puts every point on the earth's surface equidistant from the center, forming the terrestrial sphere. Next, assume the terrestrial sphere is the center of an infinite universe. Finally, assume all other bodies, except the moon, are an infinite distance from the terrestrial sphere. Imagine them on the inside surface of an enormous concentric sphere, the celestial sphere. If the stars, planets, and sun are infinitely distant from the earth's center, then the earth's surface (or aircraft's altitude) is approximately the center of the universe.

8.2.2. Ptolemy proposed the celestial concept of the universe in AD 127. He said the earth is the center of the universe, and all bodies rotate about the earth from east to west. In the relatively short periods of time involved with celestial positioning, you can assume that all bodies on the celestial sphere rotate at the same rate. In actuality, over months or years, the planets move among the stars at varying rates.

8.2.3. Establishing an artificial celestial sphere with an infinite radius simplifies computations for three reasons. First, since the terrestrial and celestial spheres are geometrically similar, every point on the

celestial sphere has a corresponding point on the terrestrial sphere and; conversely, every point on the terrestrial sphere has a corresponding point on the celestial sphere.

8.2.4. Second, the celestial sphere's infinite radius dwarfs variations in the observer's location. An infinite radius means all light rays from the celestial body arrive parallel, so the angle is the same whether viewed at the earth's center, on the surface, or at the aircraft's altitude.

8.2.5. Third, the relationships are valid for all bodies on the celestial sphere. Because the moon is relatively close to the earth, it must be treated differently. With certain corrections, the moon still provides an accurate LOP. This will be addressed in Chapter 10.

8.2.6. Because the celestial sphere and terrestrial sphere are concentric, each sphere contains an equator, two poles, meridians, and parallels of latitude or declination. The observer on earth has a corresponding point directly overhead on the celestial sphere called the zenith. A celestial body has a corresponding point on the terrestrial sphere directly below it called the subpoint or geographic position. At the subpoint, the light rays from the body are perpendicular to the earth's surface. See Figures 8.1 and 8.2.

8.2.7. Consistent with the celestial assumptions, the earth and the celestial meridians don't rotate. All bodies on the celestial sphere rotate 15° per hour past the celestial meridians. The moon moves at approximately 14.5^o per hour.

Section 8B—Motion of Celestial Bodies

8.3. Basics. All the celestial bodies have two types of motion, absolute and apparent. Apparent motion is important to navigators. Apparent motion is the motion of one celestial body as perceived by an observer on another moving celestial body. Since apparent motion is relative, it is essential to establish the reference point for that motion. For example, the apparent motion of Venus would be different if observed from the earth or the sun.

8.4. Apparent Motion. The earth's rotation and revolution causes the apparent motion of the celestial bodies. Rotation causes celestial bodies to appear to rise in the east, climb to a maximum height, then set in the west. All bodies appear to move along a diurnal circle, approximately parallel to the plane of the equator.

8.4.1. The apparent effect of rotation varies with the observer's latitude. At the equator, the bodies appear to rise and set perpendicular to the horizon. Each body is above the horizon for approximately 12 hours each day. At the North and South Poles, a different phenomenon occurs. The same group of stars is continually above the horizon; they neither rise nor set, but move on a plane parallel to the equator. This characteristic explains the periods of extended daylight, twilight, and darkness at higher latitudes. The remainder of the earth is a combination of these two extremes; that is, some bodies will rise and set, while others will continually remain above the horizon.

8.4.2. The greater the northerly declination of a body, the higher it appears in the sky to an observer at the North Pole. Polaris, with a declination of almost 90°, appears overhead. Bodies with southern declination are not visible from the North Pole.

Figure 8.1. Celestial Points and Subpoints on Earth Have the Same Relationship.
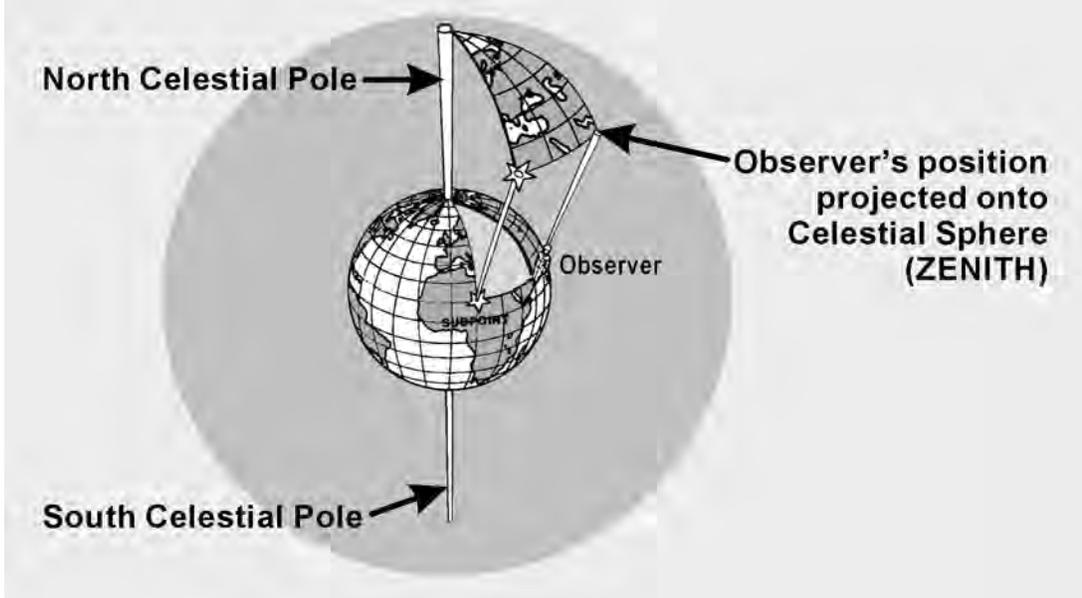
Figure 8.2. Elements of the Celestial Sphere.
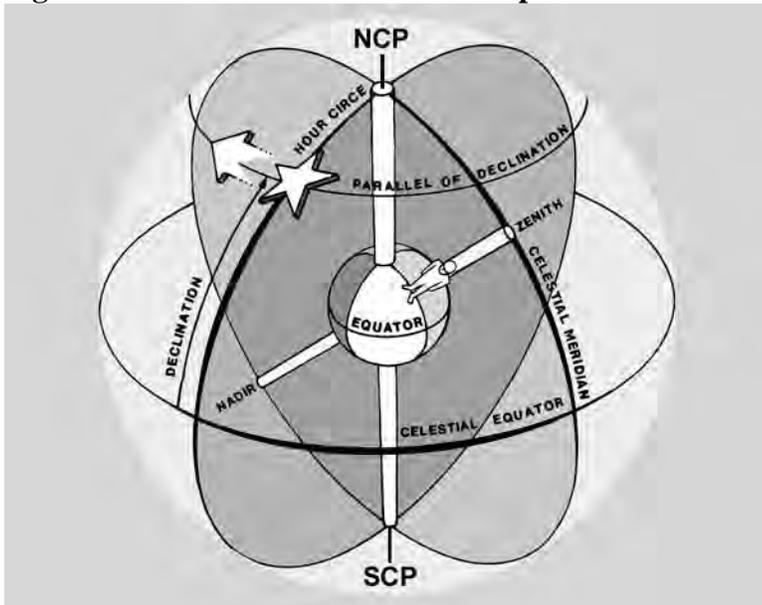
8.4.3. A circumpolar body appears to revolve about the pole and never set. If the angular distance of the body from the elevated pole is less than the observer's latitude, the body is circumpolar. For example, the declination of Dubhe is 62° N. Therefore, it is located at an angle of $90^o - 62^o$ from the North Pole, or 28°. So, an observer located above 28° N will view Dubhe as circumpolar. Although Figure 8.3 uses the North Pole, the same characteristics can be observed from the South Pole.

Figure 8.3. Some Bodies Are Circumpolar.
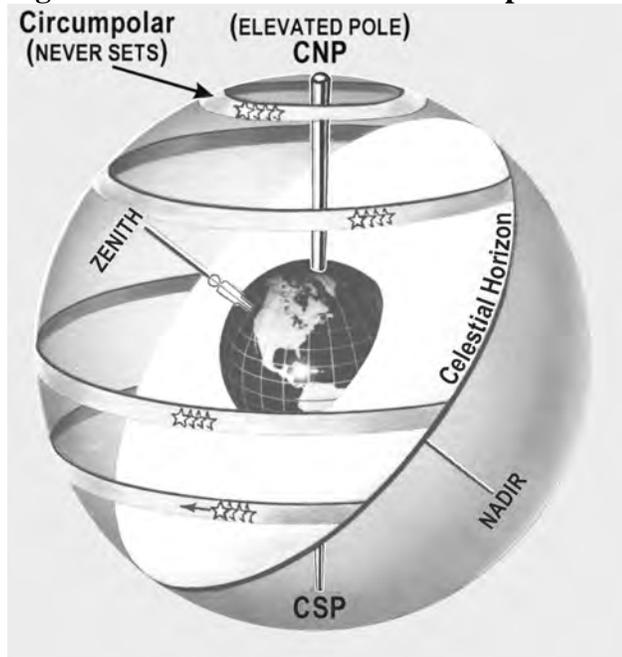
8.4.4. If the earth stopped rotating, the effect of the earth's revolution on the apparent motion of celestial bodies would be obvious. The sun would appear to circle around the earth once each year. It would cover 360° in 365 days, or move eastward at slightly less than 1 degree per day. The stars would move at the same rate. (That's why different constellations are visible at different times of the year. Every evening the same star appears to rise 4 minutes earlier.)

8.4.5. After half a year, when the earth reached the opposite extreme of its orbit, its dark side would be turned in the opposite direction in space, facing a new field of stars. Hence, an observer at the equator would see an entirely different sky at midnight in June than the one that appeared at midnight in December. In fact, the stars seen at midnight in June are those that were above the horizon at midday in December.

8.5. Seasons. The annual variation of the sun's declination and the consequent change of the seasons are caused by the revolution of the earth (Figure 8.4). If the celestial equator coincided with the ecliptic, the sun would always be overhead at the equator, and its declination would always be zero. However, the earth's axis is inclined about 66 $1/2^o$ to the plane of the earth's orbit, and the plane of the equator is inclined about 23 $1/2^o$. Throughout the year, the axis points in the same direction. That is, the axis of the earth in one part of the orbit is parallel to the axis of the earth in any other part of the orbit (Figure 8.5).

8.5.1. In June, the North Pole is inclined toward the sun so that the sun is at a maximum distance from the plane of the equator. About June 22, at the solstice, the sun has its greatest northern declination.

8.5.2. The solstice brings the long days of summer, while in the Southern Hemisphere, the days are shortest. This is the beginning of summer for the Northern Hemisphere and of winter for the Southern Hemisphere. Six months later, the axis is still pointing in the same direction; but, since the earth is at the opposite side of its orbit and the sun, the North Pole is inclined away from the Sun. At the winter

solstice, about December 21, the sun has its greatest southern declination. Days are shortest in the Northern Hemisphere, and winter is beginning.

8.5.3. Halfway between the two solstices, the axis of the earth is inclined neither toward nor away from the sun, and the sun is on the plane of the equator. These positions correspond to the beginning spring and fall.

Figure 8.4. Seasonal Changes of Earth's Position.
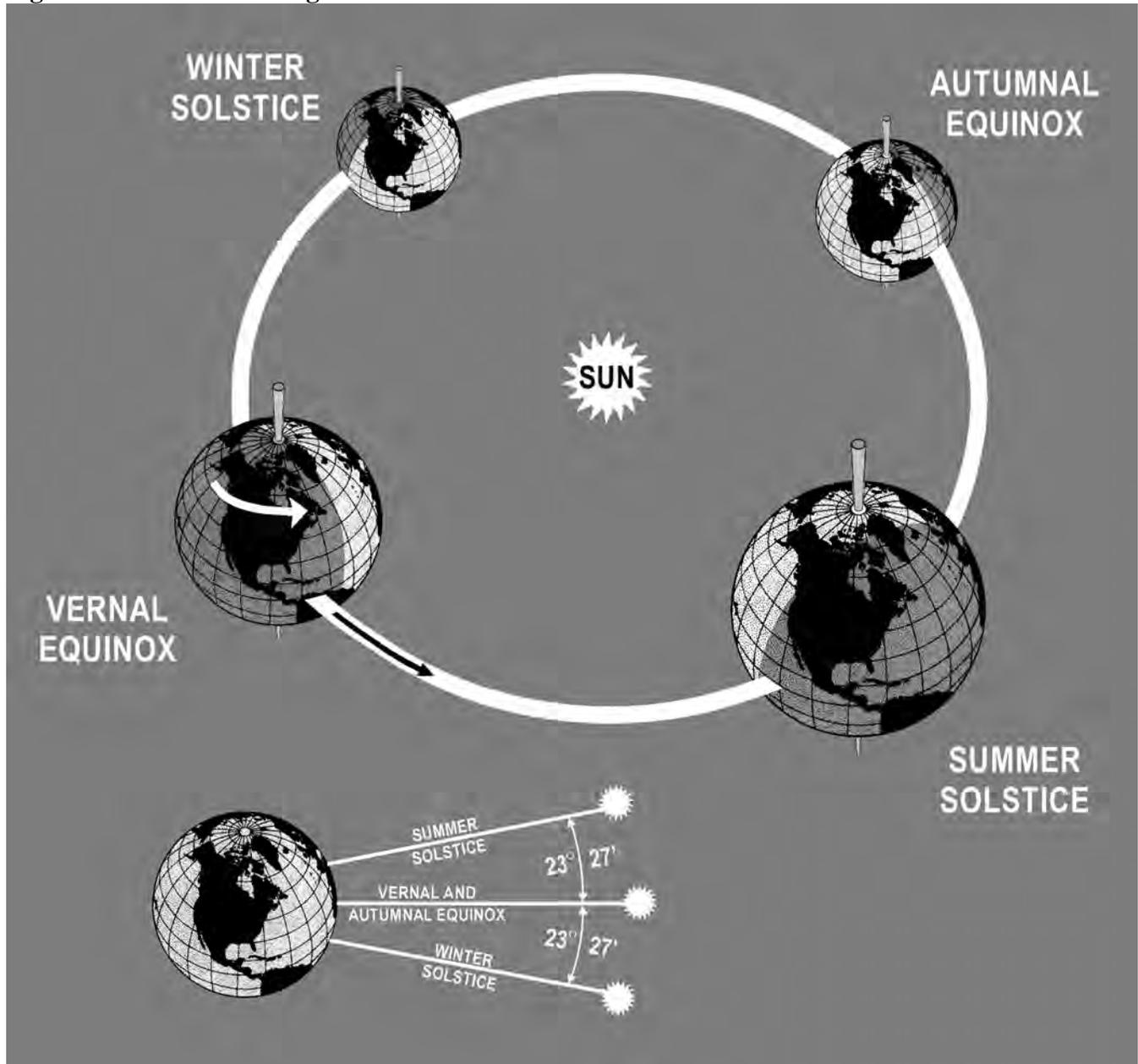
Figure 8.5. Ecliptic With Solstices and Equinoxes.
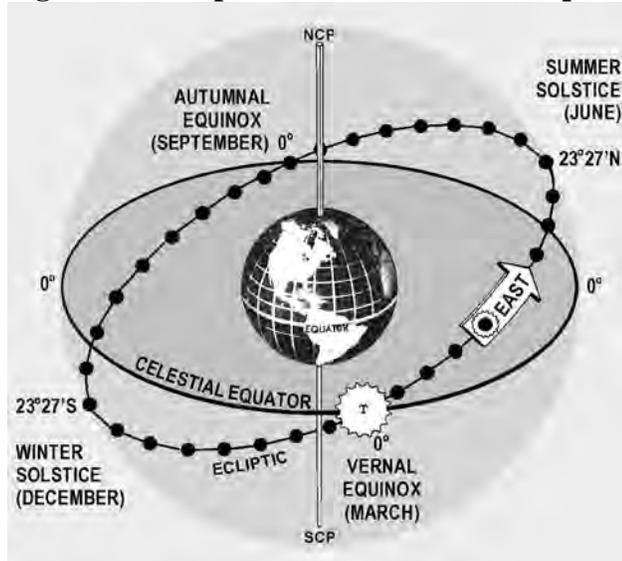
Section 8C—Celestial Coordinates

8.6. Basics. Celestial bodies and the observer's zenith may be positioned on the celestial sphere, using a coordinate system similar to that of the earth. Terrestrial lines of latitude correspond to celestial parallels of declination. Lines of longitude establish the celestial meridians.

8.6.1. The observer's celestial meridian is a great circle containing the zenith, the nadir, and the celestial poles (Figure 8.2). A line extended from the observer's zenith, through the center of the earth, intersects the celestial sphere at the observer's nadir, the point on the celestial sphere directly beneath the observer's position. The poles divide the celestial meridians into upper and lower branches. The upper branch contains the observer's zenith. The lower branch contains the nadir.

8.6.2. A second great circle on the celestial sphere is the hour circle. The hour circle contains the celestial body and the celestial poles. Unlike celestial meridians, which remain stationary, hour circles rotate 15° per hour. Hour circles also contain upper and lower branches. The upper branch contains the body. Again, the moon's hour circle moves at a different rate. The subpoint is the point on the earth's surface directly beneath the celestial body.

8.6.3. You can locate any body on the celestial sphere relative to the celestial equator and the Greenwich meridian using declination and Greenwich hour angle.

8.7. Declination (Dec). Declination is the angular distance a celestial body is north or south of the celestial equator measured along the hour circle. It ranges from 0° to 90° and corresponds to latitude.

8.8. Greenwich Hour Angle (GHA). GHA is the angular distance measured westward from the Greenwich celestial meridian to the upper branch of the hour circle. It has a range of 0° to 360°. The *Air Almanac* lists the GHA and the Dec of the sun, moon, four planets, and Aries. The subpoint's latitude matches its Dec, and its longitude correlates to its GHA, but not exactly. GHA is always measured

westward from the Greenwich celestial meridian, and longitude is measured in the shortest direction from the Greenwich meridian to the observer's meridian.

8.8.1. The following are examples of converting a body's celestial coordinates to its subpoint's terrestrial coordinates. If the GHA is less than 180°, then the subpoint is in the Western Hemisphere and GHA equals longitude. When the GHA is greater than 180°, the subpoint is in the Eastern Hemisphere and longitude equals 360° – GHA. Again, Dec and latitude are equal (Figure 8.6).

Dec S13^o-15'—Lat 13°-15' S
GHA 135°-00'—Longitude 135°-00' W
Dec N11^o-32'—Lat 11°-32' N
GHA 290°-00'—Longitude 070°-00' E

Figure 8.6. Declination of a Body Corresponds to a Parallel of Latitude.
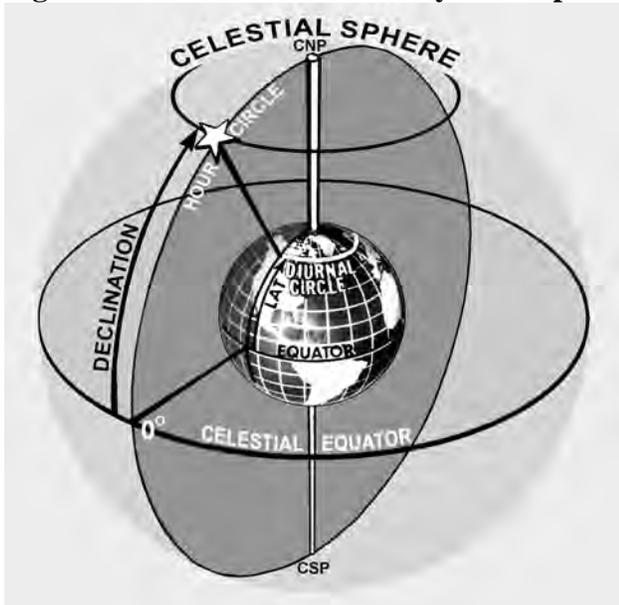
8.8.2. You will use two other hour angles in celestial navigation in addition to GHA (Figure 8.7), local hour angle (LHA), and sidereal hour angle (SHA). LHA is the angular distance from the observer's celestial meridian clockwise to the hour circle. LHA is computed by applying the local longitude to the GHA of the body. In the Western Hemisphere, LHA equals GHA - W Long, and in the Eastern Hemisphere, LHA equals GHA + E Long (Figure 8.8). When the LHA is 0, the body's hour circle and the upper branch of the observer's celestial meridian are collocated, and the body is in transit. If the LHA is 180, the hour circle is coincident with the lower branch of the observer's celestial meridian. SHA is used with the first point of Aries. The first point of Aries is the point where the sun appears to cross the celestial equator from south to north on the vernal equinox or first day of spring. Though not absolutely stationary relative to the stars, Aries moves so slowly that we consider it fixed on the celestial equator for as long as a year. The SHA is the angular measurement from the hour circle of Aries to the star's hour circle (Figure 8.9). Aries and the stars move together so the SHA remains constant for a year.

Figure 8.7. Greenwich Hour Angle.
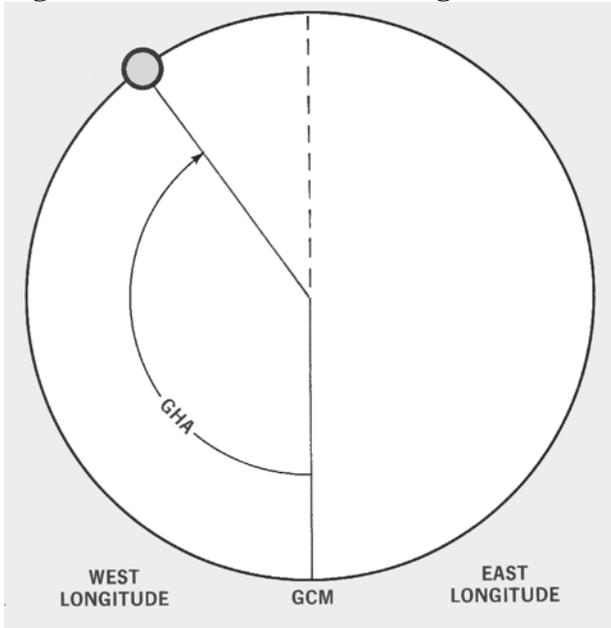
Figure 8.8. Local Hour Angle.
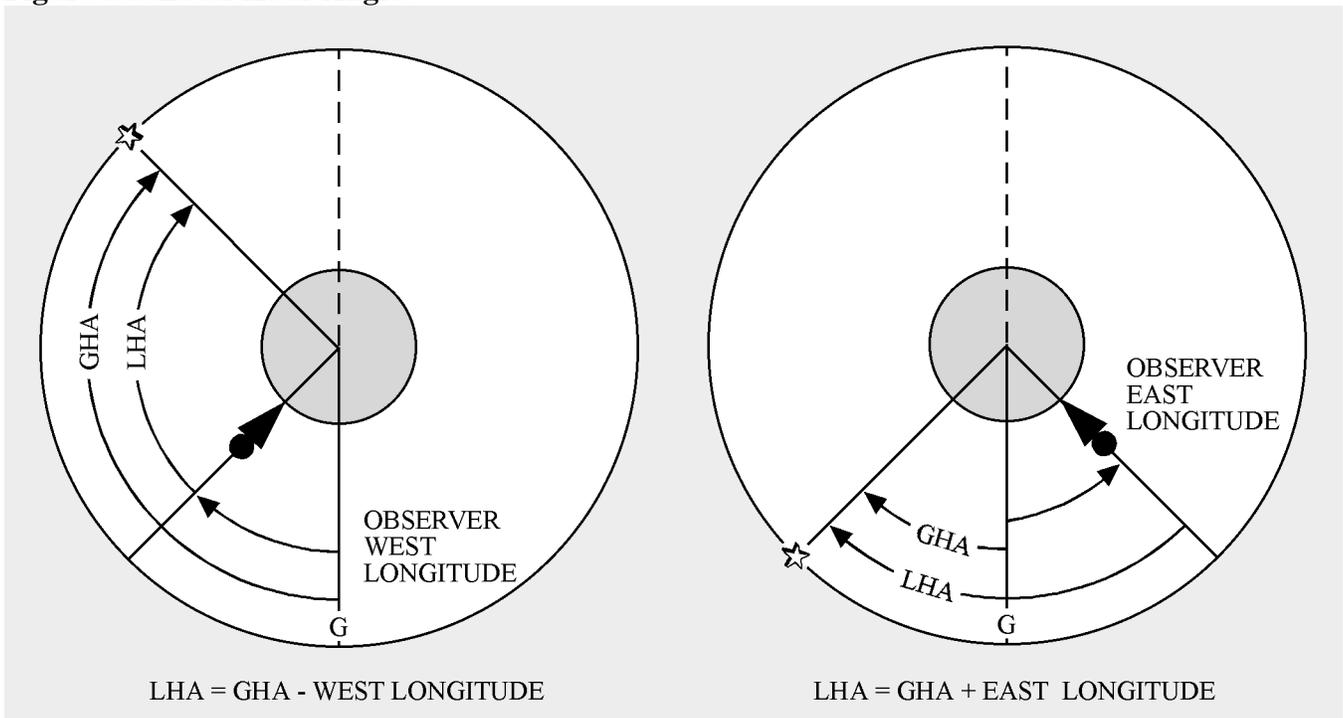
Figure 8.9. Sidereal Hour Angle.
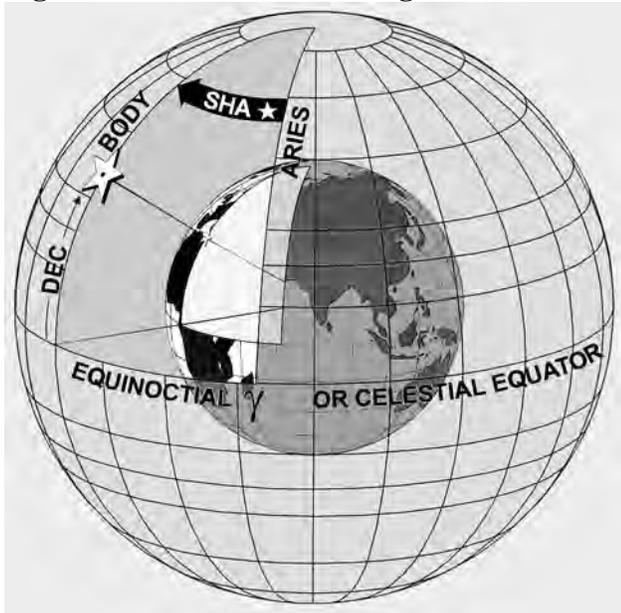
Section 8D—Use of the Air Almanac

8.9. Basics. Although the *Air Almanac* contains astronomical amounts of data, most of it is devoted to tabulating the GHA of Aries and the GHA and Declination (Dec) of the sun, moon, and the three navigational planets most favorably located for observation. Enter the daily pages with Greenwich date and GMT to extract the GHA and Dec of a celestial body.

8.10. Finding GHA and Dec. The GHA is listed for 10-minute intervals on each daily sheet. If the observation time is listed, read the GHA and Dec directly under the proper column opposite the time.

8.10.1. For example, find the sun's GHA and Dec at GMT 0540 on 11 August 1995 (Figure 8.10). The GHA is 263°-41' and Dec is N 15°-24'. (Extractions of GHA and Dec are to the nearest whole minute.) To convert these values to the subpoint's geographical coordinates, latitude is North 15°-24'. When GHA is greater than 180°, subtract it from 360° to get east longitude. The subpoint's longitude in this example is (360°-00' minus 263°-41') East 96°-19'.

8.10.2. When you don't observe at a 10-minute interval, use the time immediately before the observation time. Then use the Interpolation of GHA table on the inside front cover of the *Air Almanac* or the back of the star chart and add the increment to the GHA (Figure 8.11).

8.10.3. For example, on 11 August 1995, you observe the sun at 1012 GMT. Enter Figure 8.10 to find the GHA listed for 1010 (331°-11'). Since the observation was 2 minutes after the listed time, enter the Interpolation of GHA table (Figure 8.11) and find the correction listed for 2 minutes of time (30'). Add this correction to the listed GHA to determine the sun's exact GHA at 1012 (331°-41'). The Dec for the same time is N 15°-21'. Thus, at the time of the observation, the subpoint of the sun is at latitude 15°-21' N, longitude (360°-00' minus 331°-41') 028°-19' E.

Figure 8.10. Daily Page From *Air Almanac*—11 August 1995.

(DAY 223) GREENWICH A. M. 1995 AUGUST 11 (FRIDAY)　　　445

UT (GMT)	☉ SUN GHA	Dec.	ARIES GHA ♈	MARS 1.4 GHA	Dec.	JUPITER-2.3 GHA	Dec.	SATURN 0.9 GHA	Dec.	☾ MOON GHA	Dec.
h m	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
00 00	178 40.2	N15 28.3	318 59.5	127 27	S 4 49	75 09	S20 40	323 50	S 4 35	356 59	S10 01
10	181 10.3	28.2	321 29.9	129 57		77 40		326 21		359 24	9 59
20	183 40.3	28.1	324 00.3	132 27		80 10		328 51		1 48	57
30	186 10.3 ·	27.9	326 30.8	134 57 ·	·	82 40 ·	·	331 22 ·	·	4 13 ·	56
40	188 40.3	27.8	329 01.2	137 28		85 11		333 52		6 37	54
50	191 10.3	27.7	331 31.6	139 58		87 41		336 22		9 02	52
01 00	193 40.3	N15 27.6	334 02.0	142 28	S 4 50	90 12	S20 40	338 53	S 4 35	11 26	S 9 50
10	196 10.3	27.4	336 32.4	144 58		92 42		341 23		13 51	49
20	198 40.4	27.3	339 02.8	147 28		95 12		343 54		16 15	47
30	201 10.4 ·	27.2	341 33.2	149 59 ·	·	97 43 ·	·	346 24 ·	·	18 40 ·	45
40	203 40.4	27.1	344 03.6	152 29		100 13		348 55		21 04	43
50	206 10.4	27.0	346 34.0	154 59		102 44		351 25		23 29	42
02 00	208 40.4	N15 26.8	349 04.5	157 29	S 4 50	105 14	S20 40	353 55	S 4 35	25 54	S 9 40
10	211 10.4	26.7	351 34.9	159 59		107 44		356 26		28 18	38
20	213 40.5	26.6	354 05.3	162 29		110 15		358 56		30 43	37
30	216 10.5 ·	26.5	356 35.7	165 00 ·	·	112 45 ·	·	1 27 ·	·	33 07 ·	35
40	218 40.5	26.3	359 06.1	167 30		115 16		3 57		35 32	33
50	221 10.5	26.2	1 36.5	170 00		117 46		6 28		37 56	31
03 00	223 40.5	N15 26.1	4 06.9	172 30	S 4 51	120 16	S20 40	8 58	S 4 35	40 21	S 9 30
10	226 10.5	26.0	6 37.3	175 00		122 47		11 29		42 46	28
20	228 40.6	25.9	9 07.7	177 30		125 17		13 59		45 10	26
30	231 10.6 ·	25.7	11 38.1	180 01 ·	·	127 48 ·	·	16 29 ·	·	47 35 ·	24
40	233 40.6	25.6	14 08.6	182 31		130 18		19 00		49 59	23
50	236 10.6	25.5	16 39.0	185 01		132 48		21 30		52 24	21
04 00	238 40.6	N15 25.4	19 09.4	187 31	S 4 51	135 19	S20 40	24 01	S 4 35	54 48	S 9 19
10	241 10.6	25.2	21 39.8	190 01		137 49		26 31		57 13	17
20	243 40.7	25.1	24 10.2	192 32		140 20		29 02		59 38	16
30	246 10.7 ·	25.0	26 40.6	195 02 ·	·	142 50 ·	·	31 32 ·	·	62 02 ·	14
40	248 40.7	24.9	29 11.0	197 32		145 20		34 02		64 27	12
50	251 10.7	24.8	31 41.4	200 02		147 51		36 33		66 51	10
05 00	253 40.7	N15 24.6	34 11.8	202 32	S 4 52	150 21	S20 40	39 03	S 4 35	69 16	S 9 09
10	256 10.7	24.5	36 42.3	205 02		152 51		41 34		71 41	07
20	258 40.8	24.4	39 12.7	207 33		155 22		44 04		74 05	05
30	261 10.8 ·	24.3	41 43.1	210 03 ·	·	157 52 ·	·	46 34 ·	·	76 30 ·	03
40	263 40.8	24.1	44 13.5	212 33		160 23		49 05		78 54	02
50	266 10.8	24.0	46 43.9	215 03		162 53		51 35		81 19	9 00
06 00	268 40.8	N15 23.9	49 14.3	217 33	S 4 53	165 23	S20 40	54 06	S 4 35	83 44	S 8 58
10	271 10.8	23.8	51 44.7	220 03		167 54		56 36		86 08	56
20	273 40.9	23.6	54 15.1	222 34		170 24		59 07		88 33	55
30	276 10.9 ·	23.5	56 45.5	225 04 ·	·	172 55 ·	··	61 37 ·	·	90 57 ·	53
40	278 40.9	23.4	59 16.0	227 34		175 25		64 07		93 22	51
50	281 10.9	23.3	61 46.4	230 04		177 55		66 38		95 47	49
07 00	283 40.9	N15 23.2	64 16.8	232 34	S 4 53	180 26	S20 40	69 08	S 4 35	98 11	S 8 48
10	286 10.9	23.0	66 47.2	235 04		182 56		71 39		100 36	46
20	288 40.9	22.9	69 17.6	237 35		185 27		74 09		103 00	44
30	291 11.0 ·	22.8	71 48.0	240 05 ·	·	187 57 ·	·	76 40 ·	·	105 25 ·	42
40	293 41.0	22.7	74 18.4	242 35		190 27		79 10		107 50	41
50	296 11.0	22.5	76 48.8	245 05		192 58		81 41		110 14	39
08 00	298 41.0	N15 22.4	79 19.2	247 35	S 4 54	195 28	S20 40	84 11	S 4 35	112 39	S 8 37
10	301 11.0	22.3	81 49.6	250 05		197 59		86 41		115 04	35
20	303 41.0	22.2	84 20.1	252 36		200 29		89 12		117 28	33
30	306 11.1 ·	22.1	86 50.5	255 06 ·	·	202 59 ·	·	91 42 ·	·	119 53 ·	32
40	308 41.1	21.9	89 20.9	257 36		205 30		94 13		122 17	30
50	311 11.1	21.8	91 51.3	260 06		208 00		96 43		124 42	28
09 00	313 41.1	N15 21.7	94 21.7	262 36	S 4 55	210 31	S20 40	99 14	S 4 35	127 07	S 8 26
10	316 11.1	21.6	96 52.1	265 07		213 01		101 44		129 31	25
20	318 41.1	21.4	99 22.5	267 37		215 31		104 14		131 56	23
30	321 11.2 ·	21.3	101 52.9	270 07 ·	·	218 02 ·	·	106 45 ·	·	134 21 ·	21
40	323 41.2	21.2	104 23.3	272 37		220 32		109 15		136 45	19
50	326 11.2	21.1	106 53.8	275 07		223 03		111 46		139 10	17
10 00	328 41.2	N15 20.9	109 24.2	277 37	S 4 55	225 33	S20 41	114 16	S 4 35	141 35	S 8 16
10	331 11.2	20.8	111 54.6	280 08		228 03		116 47		143 59	14
20	333 41.2	20.7	114 25.0	282 38		230 34		119 17		146 24	12
30	336 11.3 ·	20.6	116 55.4	285 08 ·	·	233 04 ·	·	121 47 ·	·	148 49 ·	10
40	338 41.3	20.5	119 25.8	287 38		235 35		124 18		151 13	09
50	341 11.3	20.3	121 56.2	290 08		238 05		126 48		153 38	07
11 00	343 41.3	N15 20.2	124 26.6	292 38	S 4 56	240 35	S20 40	129 19	S 4 35	156 03	S 8 05
10	346 11.3	20.1	126 57.0	295 09		243 06		131 49		158 27	03
20	348 41.3	20.0	129 27.4	297 39		245 36		134 20		160 52	01
30	351 11.4 ·	19.8	131 57.9	300 09 ·	·	248 07 ·	·	136 50 ·	·	163 16	8 00
40	353 41.4	19.7	134 28.3	302 39		250 37		139 20		165 41	7 58
50	356 11.4	19.6	136 58.7	305 09		253 07		141 51		168 06	56
Rate	15 00.1	S0 00.7	15 01.0		S0 00.6	15 02.4	0 00.0	15 02.6	S0 00.1	14 27.6	N0 10.5

Moonrise

Lat. N	Moon-rise h m	Diff. m
72	20 18	−04
70	20 09	−01
68	20 02	+02
66	19 55	04
64	19 50	06
62	19 45	08
60	19 41	10
58	19 37	11
56	19 34	12
54	19 31	13
52	19 29	14
50	19 26	15
45	19 21	17
40	19 16	18
35	19 13	20
30	19 09	21
20	19 03	23
10	18 58	24
0	18 53	26
10	18 48	28
20	18 43	30
30	18 37	32
35	18 34	33
40	18 30	34
45	18 25	36
50	18 20	38
52	18 17	39
54	18 15	40
56	18 11	41
58	18 08	42
60	18 04	+44
S		

Moon's P. in A.

Alt °	Corr ′	Alt °	Corr ′
0	+	53	+
3	60	54	35
11	59	55	34
15	58	56	33
18	57	58	32
21	56	59	31
23	55	60	30
26	54	61	29
28	53	62	28
30	52	63	27
32	51	64	26
33	50	65	25
35	49	66	24
37	48	67	23
38	47	68	22
40	46	69	21
41	45	70	20
43	44	71	19
44	43	72	18
45	42	73	17
47	41	74	16
48	40	75	15
49	39	76	14
51	38	77	13
52	37	78	12
53	36	79	11
54	35	80	10

Sun SD 15′.8
Moon SD 16′
Age 15d

8.10.4. You can find the GHA and Dec of a planet in almost the same way as the sun. Because the planet's Dec change slowly, they are recorded only at hourly intervals. Use the Dec listed for the entire hour. For example, to find the GHA and Dec of Jupiter at 1109 GMT, 11 August 1995, enter the correct daily page (Figure 8.10) for the time of 1100 GMT. The GHA is 240°-35' and the Dec is S20^o-40'. Enter the Interpolation of GHA table under sun, etc., (Figure 8.11) to get the adjustment for 9 minutes of time, 2°-15'. Therefore, GHA is 242°-50'. Jupiter's subpoint is at latitude 20°-40' S, longitude (360°-00' minus 242°-50') 117°-10' E.

8.10.5. If you need to find an accurate GHA and Dec without the *Air Almanac*, you can find the procedures and applicable tables in Pub. No. 249, Volume 1 for Aries or Volume 2 or 3 for the sun.

8.11. Finding GHA and Dec of Moon. The moon moves across the sky at a different rate than other celestial bodies. In the Interpolation of GHA table, the intervals for the moon are listed in the right column where the values for the sun, Aries, and the planets are in the left column.

8.11.1. The interpolation of GHA table is a critical table and the increment is opposite the interval in which the difference of GMT occurs. If the difference (for example, 06'-31" for the moon) is an exact tabular value, take the upper (or right) of the two possible increments (that is, 1°-34'). The up or right rule applies to all critical tables.

8.11.2. For example, at 1136 GMT on 11 August 1995 you observe the moon. The following information is from the *Air Almanac* (Figures 8.10 and 8.11):

> GHA of moon at 1130 GMT 163° 16'
> GHA correction for 6 minutes 1° 27'
>
> GHA 164° 43'
> Dec S8^o 00'

Thus at 1136Z, the moon's subpoint is located at S 8°-00', longitude 164°-43'W.

8.12. Finding GHA and Dec of a Star. The stars and the first point of Aries remain fixed in their relative positions in space, so the gas of the stars and Aries change at the same rate. Rather than list the GHA and Dec of every star throughout the day, the *Air Almanac* lists the GHA of Aries at 10-minute intervals and gives the sidereal hour angle (SHA) of the stars. The GHA of a star for any time can be found by adding the GHA of Aries and the SHA of the star. The GHA of a star is used to precomp any star that falls within 29° (declination) of the equator using Volume 2 or Volume 3.

8.12.1. The table, STARS, is inside the front cover of the almanac and on the back of the star chart. This table lists navigational stars and the following information for each star: the number corresponding to the sky diagram in the back, the name, the magnitude or relative brightness, the SHA, the Dec, whether used in Pub. No. 249, and stars that can be used with Dec tables. *NOTE:* If you need a higher degree of accuracy, the SHA and Dec of the stars are listed to tenths of a degree in the *Air Almanac's* appendix.

8.12.2. For example, at 0124 GMT on 11 August 1995, you observe Altair. To find the GHA and Dec look at the extracts from the tables in Figures 8.11 and 8.12.

GHA at 0120 GMT:	339°-03'
GHA correction for 4 minutes:	1°-00'
GHA for 0124:	340°-03'
SHA Altair	62°-21'
GHA for Altair at 0124:	42°-24'
Dec Altair	N8°-52'

Thus the subpoint of Altair is 08°-52' N 042°-24' W.

Figure 8.11. Interpolation of Greenwich Hour Angle, *Air Almanac.*

Figure 8.12. Sidereal Hour Angle Obtained From Table.

STARS, SEPT.-DEC 1981

No.	Name	Mag.	S.H.A.	Dec.
			° ′	° ′
7*	Acamar	3·1	315 45	S.40 26
5*	Achernar	0·6	335 52	S.57 24
30*	Acrux	1·1	173 50	S.62 55
19	Adhara †	1·6	255 40	S.28 55
10*	Aldebaran †	1·1	291 30	N.16 27
32*	Alioth	1·7	166 52	N.56 08
34*	Alkaid	1·9	153 27	N.49 29
55	Al Na'ir	2·2	28 28	S.47 08
15	Alnilam †	1·8	276 22	S. 1 13
25*	Alphard †	2·2	218 31	S. 8 31
41*	Alphecca †	2·3	126 41	N.26 50
1*	Alpheratz †	2·2	358 20	N.28 55
51*	Altair †	0·9	62 43	N. 8 47
2	Ankaa	2·4	353 50	S.42 29
42*	Antares †	1·2	113 10	S.26 2?
3*	Schedar	2·5	350 21	N.56 21
45*	Shaula	1·7	97 10	S.37 05
18*	Sirius †	−1·6	259 05	S.16 40
33*	Spica †	1·2	159 09	S.10 59
23*	Suhail	2·2	223 19	S.43 1?
49*	Vega	0·1	81 03	N.38 45
39	Zuben'ubi †	2·9	137 45	S.15 54

INTERPOLATION OF G.H.A.

Increment to be added for intervals of G.M.T. to G.H.A. of: Sun, Aries (♈) and planets ; Moon

SUN, etc.		MOON	SUN, etc.		MOON	SUN, etc.		MOON
m s	° ′	m s	m s	° ′	m s	m s	° ′	m s
00 00	0 00	00 00	03 17	0 50	03 25	06 37	1 40	06 52
01	0 01	00 02	21	0 51	03 29	41	1 41	06 56
05	0 02	00 06	25	0 52	03 33	45	1 42	07 00
09	0 03	00 10	29	0 53	03 37	49	1 43	07 04
13	0 04	00 14	33	0 54	03 41	53	1 44	07 08
17	0 05	00 18	37	0 55	03 45	06 57	1 45	07 13
21	0 06	00 22	41	0 56	03 49	07 01	1 46	07 17
25	0 07	00 26	45	0 57	03 54	05	1 47	07 21
29	0 08	00 31	49	0 58	03 58	09	1 48	07 25
33	0 09	00 35	53	0 59	04 02	13	1 49	07 29
37	0 10	00 39	03 57	1 00	04 06	17	1 50	07 33
41	0 11	00 43	04 01	1 01	04 10	21	1 51	07 37
45		00 47	05	1 01	04 14	25	1 51	07 42
05	0 46	03 12	25	1 36	06 39	45		2 27
09	0 47	03 16	29	1 37	06 44	49		2 28
13	0 48	03 20	33	1 38	06 48	53		2 29
17	0 49	03 25	37	1 39	06 52	09 57		2 30
03 21	0 50	03 29	06 41	1 40	06 56	10 00		

* Stars used in H.O. 249 (A.P. 3270) Vol. 1.
† Stars that may be used with Vols. 2 and 3.

8.13. Summary. All the celestial concepts and assumptions you've learned may help you obtain a celestial LOP. A celestial LOP is simply a circle plotted with the center at the subpoint and a radius equal to the distance from the observer to the subpoint. To accurately compute this distance and the direction to the subpoint of the body, you must initially position the subpoint and then measure the angular displacement of the body above the horizon. GHA and Dec position the body, and the sextant measures the height above the horizon. A basic knowledge of celestial theory and LOPs will help you appreciate celestial navigation and detect errors. The next section explains how angular displacement is measured.

Section 8E—Celestial Horizon

8.14. Basics. You use a sextant to measure a body's angular displacement above the horizon. The celestial horizon is a plane passing through the earth's center perpendicular to the zenith-nadir axis. The visual horizon approximates this plane at the earth's surface. Figure 8.13 depicts the zenith-nadir axis and the celestial horizon. The angular displacement you see through a sextant is the height observed, or Ho. Ho is measured along the vertical circle above the horizon. The vertical circle is a great circle containing the zenith, nadir, and celestial body. The body's altitude is the same whether measured at the earth's surface from an artificial horizon or at the center of the earth from the celestial horizon because these horizons are parallel and the light rays from the body are essentially parallel. Figure 8.14 shows

that the infinite celestial sphere makes the difference in angle for light rays arriving at different points on the earth infinitesimal.

Figure 8.13. Celestial Horizon is 90° From Observer Zenith and Nadir.
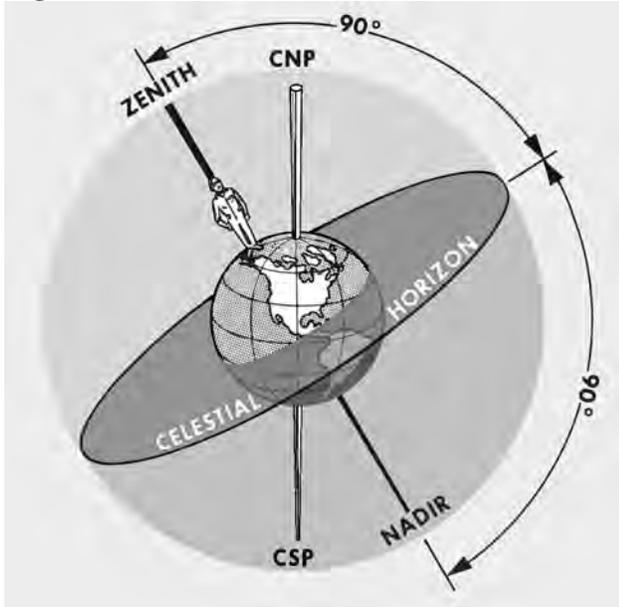
Figure 8.14. Parallel Lines Make Equal Angles With Parallel Planes.
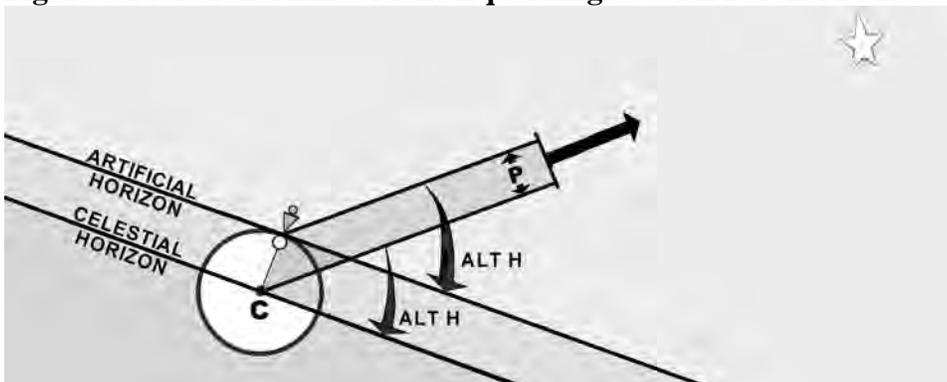
8.14.1. The angle between light rays is called parallax. In Figure 8.15, parallax is shown at its maximum; that is, when the observer and the subpoint are separated by 90°. Since the earth's radius is tiny compared to the infinite distance to the stars, the angle p is very small. For the sun, angle p is a negligible 9 seconds of arc or 0.15 NM. Observed altitudes from either the artificial or celestial horizon are practically the same.

8.14.2. The bubble in a sextant or artificial horizon is most used by navigators. As in a carpenter's level, a bubble indicates the apparent vertical and horizontal. With the bubble, the navigator can level the sextant and establish an artificial horizon parallel to the plane of the celestial horizon. Figure 8.16 shows that the plane of the artificial (bubble) horizon and the plane of the celestial horizon are parallel and

separated by the earth's radius. Compared to the vast distances of space, the radius of the earth is inconsequential. Thus, the artificial horizon and the celestial horizon are nearly identical.

Figure 8.15. Parallax.
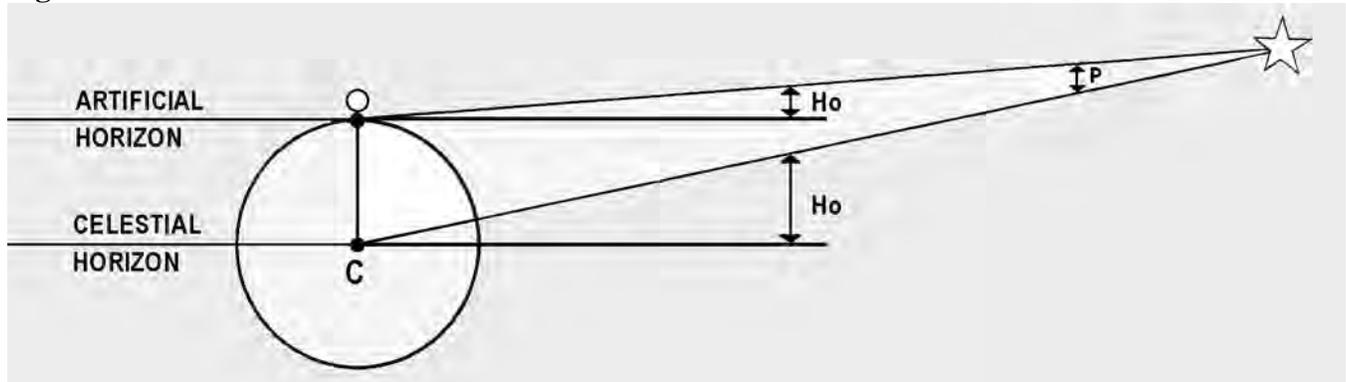
Figure 8.16. The Two Planes Are Parallel.
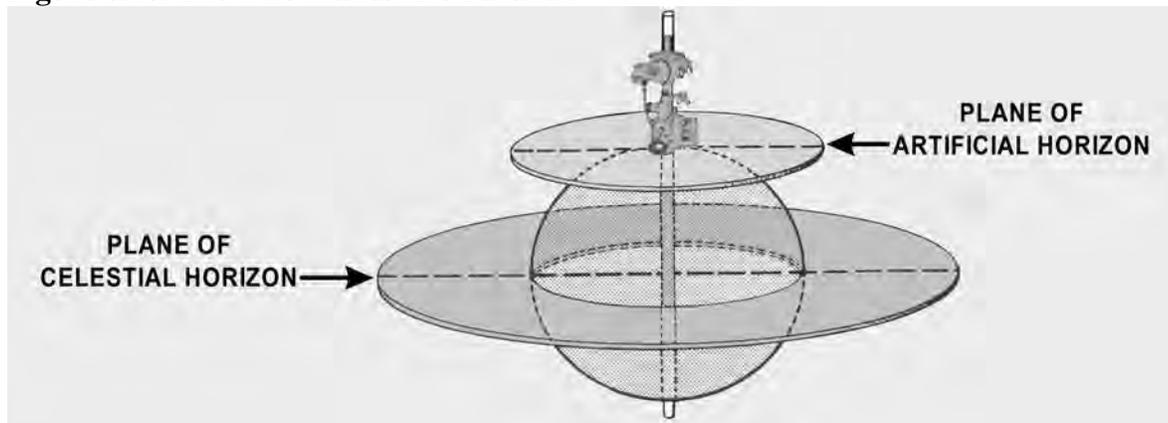
8.15. Observed Altitude. The distance of the observer from the subpoint and a body's Ho are related. (Figure 8.17). When the body is directly overhead, the Ho is 90°, and the subpoint and the observer's position are collocated. When the Ho is 0°, the body is on the horizon and the subpoint is 90° (5,400 NM) from the observer's position (See Figure 8.18, where C is the center of the earth, AB is the observer's horizon, and S is the subpoint of the body). Since the sum of the angles in a triangle equals 180°, the angle OX is equal to $180^o - (Ho + P)$. The sum of the angles on a straight line equals 180°, so angle OXC is equal to Ho + P. The horizon AB being tangent to the earth at O is perpendicular to OC, a radius of the earth. Thus, angle OCX equals 90° (Ho + P). The preceding discussion showed that angle P is negligible, so this angle becomes $90^o - Ho$. The arc on the surface subtended by the angle OCX at the center of the earth is arc OS. This arc then is equal to $90^o - Ho$.

Figure 8.17. Measure Altitude From Celestial Horizon Along Vertical Circle.

Figure 8.18. Finding Observed Altitude.

8.15.1. The distance from the subpoint to the observer is the zenith distance or co-alt and is computed using the astronomical triangle described in Chapter 9. Basically, the zenith distance equals 90° minus the Ho (Figure 8.19). The figures are then converted to NM by multiplying the number of degrees by 60 and adding in the odd minutes of arc.

Ho	= 37°26'
therefore, co-alt	= 90° – 37°26'
therefore, co-alt	= 52°34'
Zenith Distance	= (52° x 60) + 34'
Zenith Distance	= 3,154 NM

Zenith distance is the radius of the circle which becomes the celestial LOP.

8.15.2. This circle is called the circle of equal altitude (Figure 8.20), as anyone located on it will view an identical Ho. Now that you can determine the distance to the subpoint, you must next find the direction.

Figure 8.19. Co-Altitude and Zenith Distance.
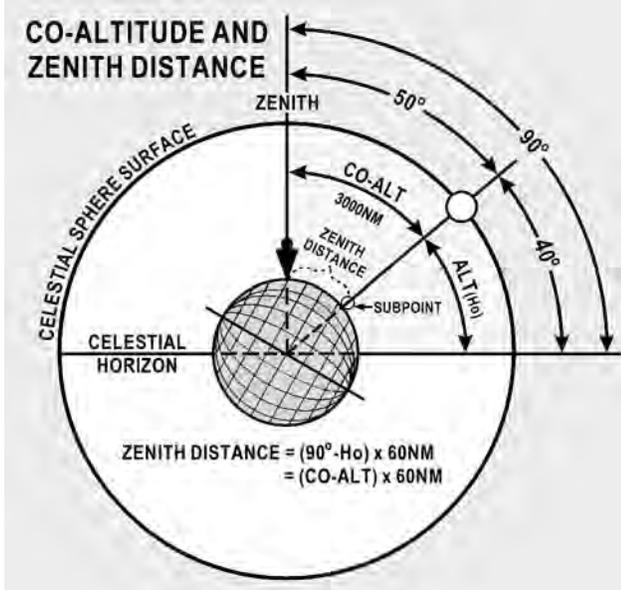
Figure 8.20. Constructing a Circle of Equal Altitude.
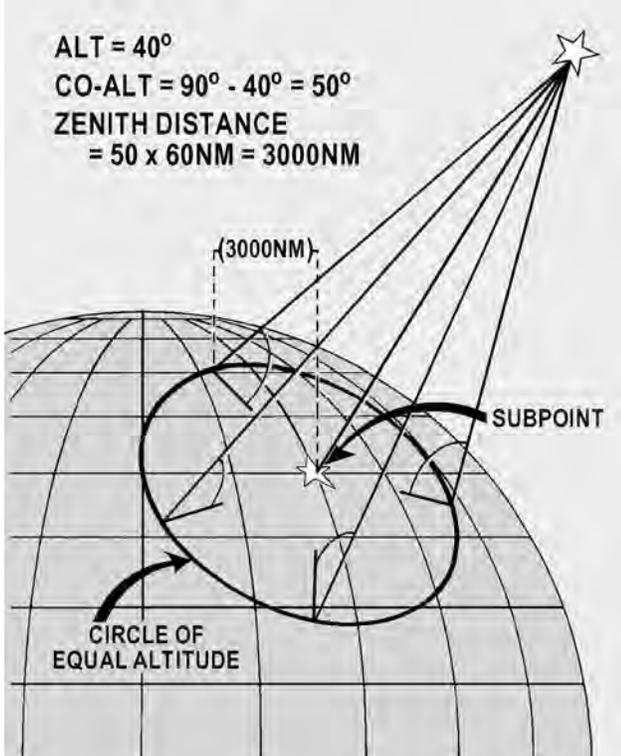
8.16. True Azimuth (Zn). The direction to a body from an observer is called Zn. A celestial body's Zn is the true bearing (TB) to its subpoint. The Zn is the angle measured at the observer's position from true north (TN) clockwise through 360° to the great circle arc joining the observer's position with subpoint, as illustrated in Figure 8.21. If you could measure the Zn when you measure its altitude, you could have a fix. Unfortunately, there is no instrument in the aircraft which will measure Zn accurately enough. Except in the case of a very high body (85-90 degrees), if you observe a body with a Ho of 40° and you mismeasure the Zn by 1 degree, the fix will be 50 NM off.

Figure 8.21. Relationship of True Azimuth to an Observer.
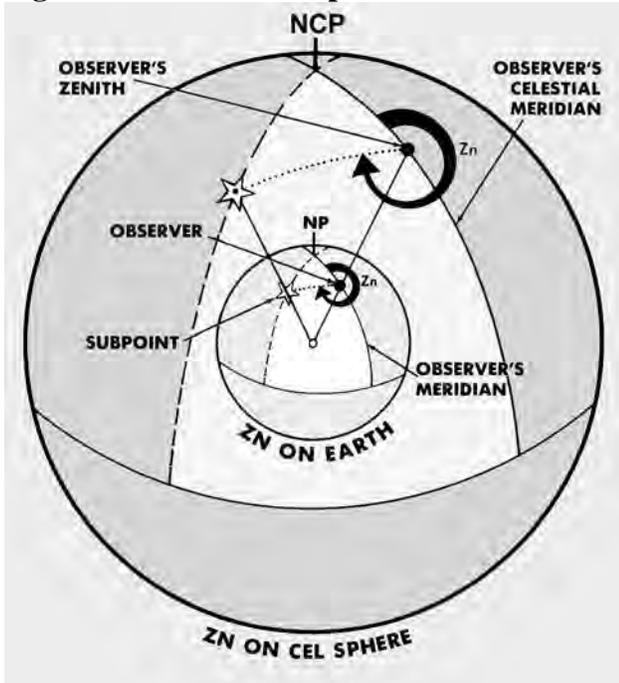
8.17. Celestial Fix. Since you cannot normally fix off a single body, you will usually need to cross two or more LOPs. The fix position is the intersection of the LOPs. A celestial LOP is a circle as shown in Figure 8.22. When two celestial LOPs are plotted, they intersect at two points, only one of which can be your position. In practice, these two intersections usually are so far apart that dead reckoning removes all doubt as to which is correct.

Figure 8.22. Celestial Fix With Two Bodies.
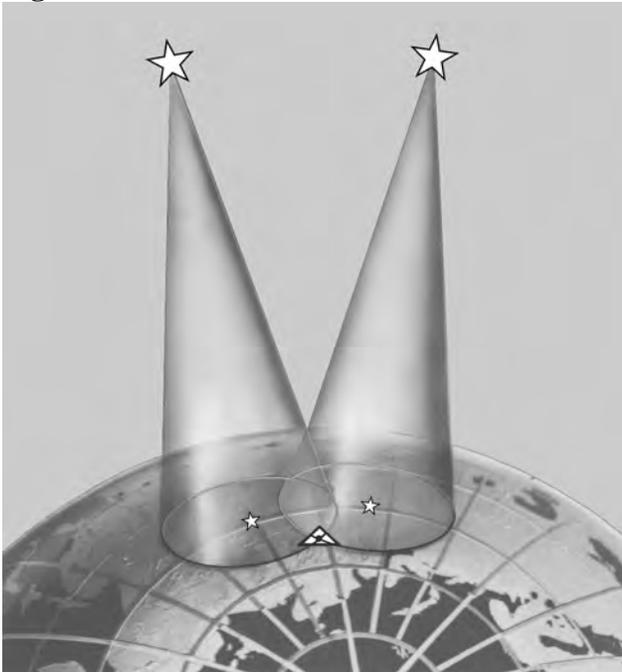
Chapter 9

COMPUTING ALTITUDE AND TRUE AZIMUTH

Section 9A—Introduction

9.1. Basics. This chapter discusses the procedures and some of the tables used to compute a celestial line of position (LOP). Some of the tables used to resolve the LOP, including the *Air Almanac*, were previously mentioned. First, we'll discuss the astronomical triangle upon which the tables are based. Then we'll cover how to determine the local hour angle (LHA) of Aries and the LHA of a star.

Section 9B—LHA and the Astronomical Triangle

9.2. Basics. The basic principle of celestial navigation is to consider yourself to be at a certain assumed position at a given time; then, by means of the sextant, determining how much your basic assumption is in error. At any given time, an observer has a certain relationship to a particular star. The observer is a certain number of nautical miles away from the subpoint, and the body is at a certain true bearing called true azimuth (Zn), measured from the observer's position (Figure 9.1).

Figure 9.1. Subpoint of a Star.
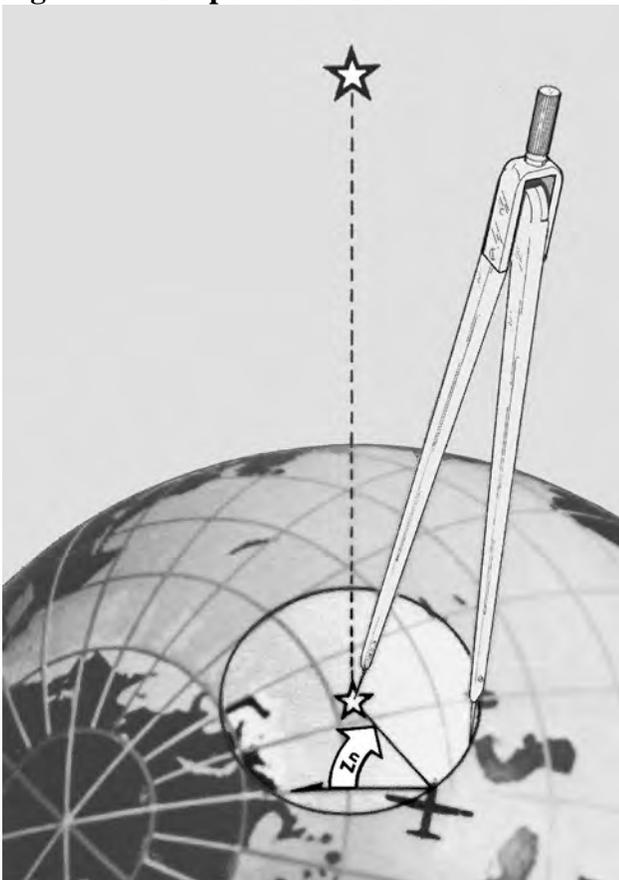
9.3. Intercept. Assume yourself to be at a given point (called the assumed position). At a given time, there exists at that instant a specific relationship between your assumed position and the subpoint. The various navigational tables provide you with this relationship by solving the astronomical triangle for you. From the navigational tables, you can determine how far away your assumed position is from the subpoint and the Zn of the subpoint from the assumed position. This means, in effect, that the tables give you a value called computed altitude (Hc) which would be the correct observed altitude (Ho) if you were anywhere on the circle of equal altitude through the assumed position. Any difference between the Hc determined for the assumed position and the Ho as determined by the sextant for the actual position is called intercept. Intercept is the number of NM between your actual circle of equal altitude and the circle of equal altitude through the assumed position. It is by means of the astronomical triangle that you can solve for Hc and Zn in the Pub. No. 249 tables.

9.4. Construction of the Astronomical Triangle. Consider the solution of a star as it appears on the celestial sphere. Start with the Greenwich meridian and the equator. Projected on the celestial sphere, these become the celestial meridian and the celestial equator (called equinoctial) as shown in Figure 9.2. Notice also in the same illustration how other known information is derived, namely the LHA of the star Aries—equal to the Greenwich hour angle (GHA) of Aries minus longitude west. You can also see that if the LHA of Aries and sidereal hour angle (SHA) of the star are known, the LHA of the star is their sum. It should also be evident that the GHA of Aries plus SHA of the star equals GHA of the star. Also, the GHA of the body minus west longitude (or plus east longitude) of the observer's zenith equals LHA of the body. These are important relationships used in the derivation of the Hc and Zn.

Figure 9.2. Astronomical Triangle.
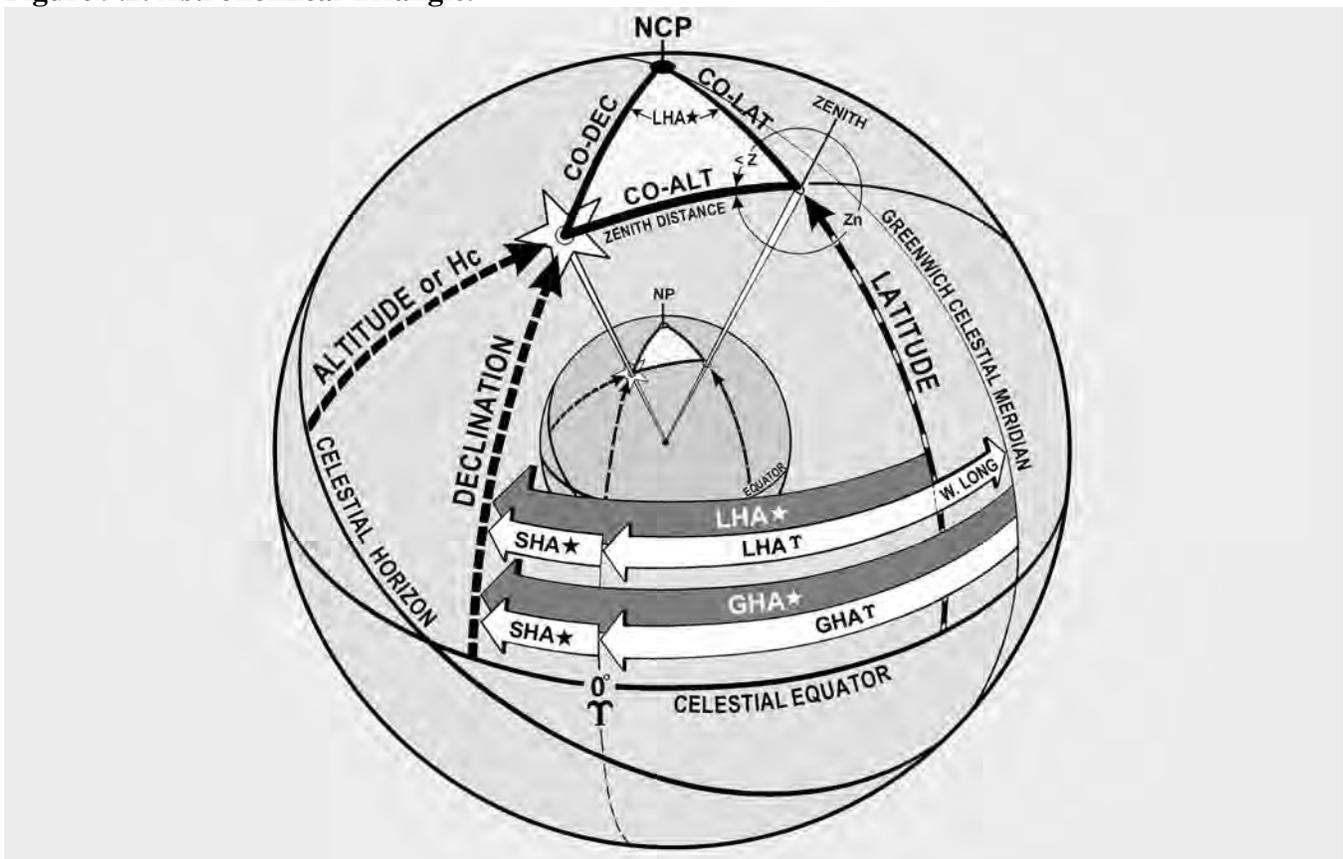
9.4.1. Figure 9.3 shows part of the celestial sphere and the astronomical triangle. Notice that the known information of the astronomical triangle is the two sides and the included angle; that is, Co-Dec, Co-Lat and LHA of the star. Co-Dec, or polar distance, is the angular distance measured along the hour circle of the body from the elevated pole to the body. The side, Co-Lat, is 90° minus the latitude of the assumed position. The included angle in this example is the LHA. With two sides and the included angle of the spherical triangle known, the third side and the interior angle at the observer are easily solved. The third side is the zenith distance, and the interior angle at the observer is the azimuth angle (Z). Instead of listing the zenith distance, the astronomical tables list the remaining portion of the 90° from the zenith, or the Hc. Hc equals 90° minus zenith distance of the assumed position, just as zenith distance of the assumed position equals 90° – Hc. Note that when measured with reference to the celestial horizon, zenith distance is synonymous with co-altitude. Figure 9.4 is a side view of this solution.

Figure 9.3. Celestial—Terrestial Relationship.
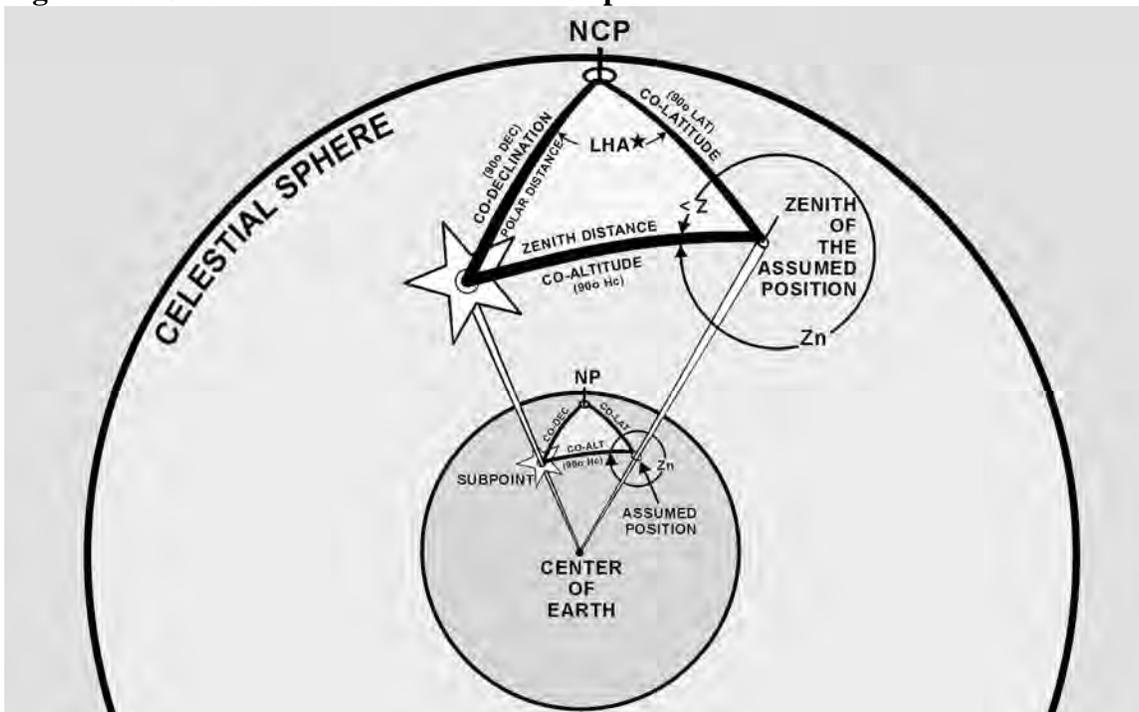
9.4.2. So far, the astronomical triangle has been defined only on the celestial sphere. Refer again to Figure 9.3 and notice the same triangle on the terrestrial sphere (earth). The same triangle with its corresponding vertices may be defined on the earth as follows: (1) celestial pole—terrestrial pole; (2) zenith of assumed position—assumed position; and (3) star—-subpoint of the star. The three interior angles of this triangle are exactly equal to the angles on the celestial sphere. The angular distance of each of the three sides is exactly equal to the corresponding side on the astronomical triangle. Celestial and terrestrial terms are used interchangeably. For example, refer to Figure 9.3 and notice that Co-Lat on the terrestrial triangle is also called Co-Lat on the celestial triangle. To be perfectly correct, the term on the celestial sphere corresponding to latitude on the earth is declination (Dec); therefore, the celestial side could well be called co-declination of the zenith of the assumed position.

Figure 9.4. Co-Altitude Equals 90 Minus Hc.
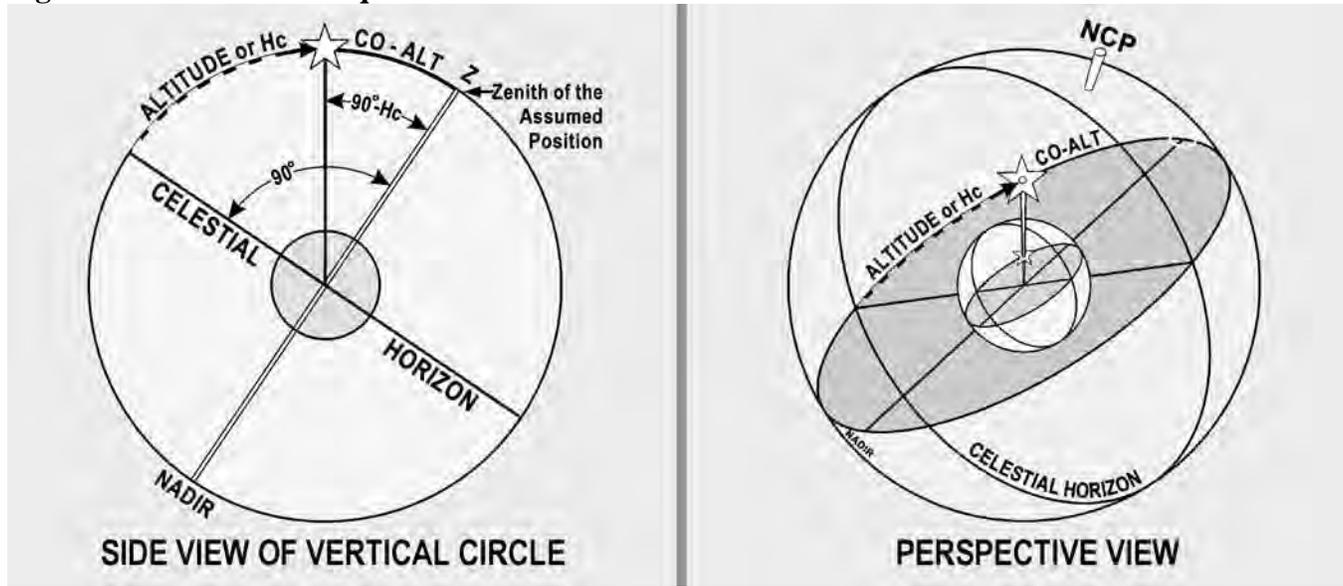
9.4.3. Rather than have this confusion, the terrestrial term Co-Lat is also used with reference to the celestial sphere, just as latitude of the subpoint is considered to be the Dec amount from the equator. Latitude is used when referring to the observer or zenith, and Dec is used when referring to the star or its subpoint. The distance between the subpoint and the assumed position is generally referred to as zenith distance (Co-Alt) rather than the segment of the vertical circle joining the subpoint and the assumed position. These angular distance terms are interchangeable on the celestial and terrestrial spheres.

9.4.4. The values of the Zn and the interior angle (Z) are listed in the Pub. No. 249 tables depending upon whether or not a Dec solution is desired. Pub. No. 249, Volume 1, lists the Zn rather than the interior angle. Pub. No. 249, Volumes 2 and 3, list the interior angle (Z). It is necessary to follow rules printed on each page to convert the interior angle (Z) to true azimuth (Zn).

9.5. Pub. No. 249, Volume 1. This volume deals solely with the solution concerning selected stars and is considered separately from Volumes 2 and 3. Volume 1 provides complete worldwide coverage from pole to pole for each degree of latitude. The LHA of Aries is listed in 1° increments from latitudes of 0° to 69° North and South inclusive. From 70° through 89° of latitude, the meridians are so close together that it is only necessary to tabulate the values of the LHA of Aries in even 2° increments. There are two pages devoted to each whole degree of latitude between latitudes 69° N and 69° S inclusive. From there to the pole, only one page is devoted to each whole degree of latitude. The three stars marked by diamonds on each page provide sets for fixing purposes, which are favorably situated in altitude and azimuth.

9.5.1. The entering arguments are the assumed latitude and the LHA of Aries (to whole degrees). At any one time, the navigator has the choice of the seven listed stars for that latitude plus Polaris. The names of the stars are in capital letters if the star is of first magnitude or brighter; the second magnitude stars are printed in small letters. The names of the stars are listed every 15° of LHA of Aries (every 30° in the polar latitudes). For the time the navigator expects to make an observation, commonly called a shot, they look up the GHA of Aries and apply the approximate longitude to get a whole degree LHA of Aries. The

navigator then enters Pub. No. 249, Volume 1, with the latitude closest to the dead reckoning (DR) latitude and the LHA of Aries to select the stars that will be shot.

9.5.2. Since single celestial observation results in only one LOP, it is necessary to shoot two or more bodies to obtain a fix. Suppose the navigator wants to shoot at approximately 0230 Greenwich Mean Time (GMT), he or she looks up the GHA of Aries (in the *Air Almanac*) and finds it to be 196°. The DR position for this time is 31°48' N, 075°26' W. A quick calculation shows the LHA of Aries is approximately 121°, and the closest latitude is 32° N. Notice in the portion of the tables reproduced in Figure 9.5 the available stars at this position are Alkaid, Regulus, Alphard, Sirius, Rigel, Aldebaran, and Capella. Using Sirius, a shot is taken at 0231 and the Ho obtained is 37°50'.

GHA Aries for 0230 GMT	196°06'
Correction for l minute	15'
GHA Aries for 0231 GMT	196°21'
Closest longitude to DR for whole LHA	W075^{o}21' (assumed longitude)
LHA Aries for 0231 GMT	121°

Figure 9.5. Enter Tables With LHA Aries and Latitude.

LAT 32°N

							Alkaid		REGULUS		Alphard		SIRIUS		RIGEL		ALDEBARAN		CAPELLA	
Zn	Hc	Zn	Hc	Zn	Hc	Zn	LHA ϒ	Hc	Zn	Hc	Zn	Hc	Zn	Hc	Zn	Hc	Zn	Hc	Zn	Hc	Zn
							120	25 51	046	54 49	117	44 36	150	38 00	203	33 21	232	41 00	264	55 21	306
							121	26 28	046	55 24	118	45 01		37 40	205	32 41	233	40 09	265	54 40	306
							122	27 05		56 19	119	45 26	152	37 18	206	32 00	234	39 19	265	53 58	306
							123	27 42	047	57 03	121	45 49	153	36 55	207	31 19	235	38 28	266	53 17	306
							124	28 19	047	57 47	122	46 12	155	36 32	208	30 37	236	37 37	267	52 35	305
							125	28 57	047	58 30	123	46 33	156	36 08	209	29 55	236	36 46	267	51 54	305
							126	29 34	047	59 12	124	46 53	157	35 43	210	29 12	237	35 56	268	51 12	305
							127	30 11	048	59 54	126	47 12	159	35 17	211	28 29	238	35 05	268	50 31	305
							128	30 49	048	60 35	127	47 30	160	34 50	212	27 46	239	34 14	269	49 49	305
							129	31 27	048	61 15	129	47 47	162	34 23	213	27 03	239	33 23	269	49 07	305
							130	32 05	048	61 54	130	48 02	163	33 55	214	26 19	240	32 32	270	48 26	305
							131	32 43	048	62 33	132	48 17	164	33 26	215	25 34	241	31 41	270	47 44	305
							132	33 21	049	63 10	133	48 30	166	32 56	216	24 50	242	30 50	271	47 02	305
							133	33 59	049	63 47	135	48 42	167	32 26	217	24 05	242	29 59	272	46 21	305
							134	34 37	049	64 22	137	48 52	169	31 55	218	23 20	243	29 09	272	45 39	305

9.5.3. The closest whole degree of latitude is 32° N; therefore, it is used as the assumed latitude. The assumed longitude is selected as the closest point, resulting in an LHA of Aries that is a whole degree (no minutes). The Hc of Sirius is listed as 37°40'. The Zn is 205°.

9.5.4. The second shot was taken at 0234 using Regulus, the Ho being 55°30'. A new DR position could be obtained for 0234 GMT, but the 0230Z DR position will suffice for this determination of Hc and Zn.

GHA Aries for 0230 GMT	196°06'	
Correction for 4 minutes	1°00'	
GHA Aries for 0234Z	197°06'	
Closest longitude for whole LHA	W075°06'	(assumed longitude)
LHA Aries for 0234Z	122°	

9.5.5. The assumed latitude is still 32° N and, in this case, 075°06' W is the assumed longitude since this is the closest longitude to the DR longitude that results in the LHA of Aries being a whole degree. The Hc of Regulus is listed as 56°19', and the Zn is 119°. The various corrections that must be applied as well as the plotting of the fix are discussed later.

9.6. Postcomputation Method. The steps in this procedure are as follows:

9.6.1. Determine the GHA of Aries for the time of observation from the *Air Almanac*.

9.6.2. Assume a position as close as possible to the DR position at the time of the shot so the latitude and LHA of Aries in whole degrees may be determined.

9.6.3. Turn to the page in Pub. No. 249 for the assumed latitude and, opposite the LHA of Aries, select the stars to be shot. In making the selection, assume the LHA of Aries will change 1° every 4 minutes of time.

9.6.4. Shoot the body and record the time, Ho, and name of the body.

9.6.5. Obtain the GHA of Aries for the time of the observation, and apply the assumed longitude to determine the LHA of Aries.

9.6.6. Turn to the pages for the assumed latitude and, opposite the LHA of Aries in the column headed by the name of the star, find and record the Hc and Zn.

9.7. Pub. No. 249, Volumes 2 and 3. Volume 1 consists of tables of Hc and Zn for selected stars. Because the Dec and SHA of each star change slowly, these tables may be used for many years with only small corrections. The Dec and SHA of a nonstellar body change rapidly, making a permanent format similar to Volume 1 impossible for the sun, moon, and planets.

9.7.1. Volumes 2 and 3 have Dec tables adequate for determining the Hc and Zn of any celestial body within the Dec range of 30° N to 30° S. They are intended primarily for use when observing nonstellar (solar system) bodies. Volume 2 provides latitudes between 39° N and 39° S, and Volume 3 provides for latitudes from 40° N or S to the poles.

9.7.2. Provision is made for observed altitudes from 90° above to 3° below the horizon (7° from latitudes 70° to the pole). In view of refraction and of possible long intercepts, the tables are actually extended 2° below these limits.

9.8. Entering Arguments. Volumes 2 and 3 are entered with the LHA of the body, in contrast to Volume 1, which is entered with the LHA of Aries. The range extends from 0° through all LHAs applicable within the altitude limits of the body. Between latitude 70° and the pole, the LHA interval is 2°; for latitudes below 70°, the interval is 1°. Arguments of LHA of the body less than 180° appear on the left margin, and arguments greater than 180° appear on the right.

9.8.1. Several pages are devoted to each degree of latitude. Each page has 15 declination columns and is labeled with its value at the top and bottom. Each page is also marked Declination Contrary Name to Latitude or Declination Same Name as Latitude.

9.8.2. The entering arguments of LHA of the body, for declination of contrary name to latitude, always increase from the bottom of the page on the left side, and decrease on the right. The opposite arrangement exists on pages where Dec and latitude has the same name. Occasionally, one page will be blank in the middle and the top half will cover Declination Same Name as Latitude, while the bottom half will be Declination Contrary Name to Latitude.

9.8.3. Azimuth angle (Z) is listed instead of true azimuth (Zn). Since Zn is used for plotting, it is necessary to convert Z to Zn. The rules for conversion are listed on the left-hand side at the top and bottom of every page. Notice that LHA and Zn will never occur on the same side of 180°.

9.8.4. In addition to Hc and Z, a value of d is also listed. This d-value is the change in altitude (Hc) with a 1° increase in Dec. If the LHA and Dec of the body and the latitude of the assumed position are each a whole number of degrees, the Hc and Z are found in the correct Dec column opposite the LHA of the body on the page marked by the proper latitude value.

9.8.5. For example, refer to the portion of the table shown in Figure 9.6. At the latitude 40° N, if the LHA of a body is 86° and its Dec is 5° N, the Hc is $06^o16'$ and the azimuth angle (Z) is 089°. The rule in the upper left-hand corner of the page applies for the conversion of Z to Zn. Zn = $(360^o – Z)$ or $(360^o – 089^o) = 271^o$. Here again the position is assumed so that latitude and LHA are whole numbers.

9.9. Interpolation for Declination (Dec). When the Dec of a body is a number of minutes in addition to a whole number of degrees, the altitude (Hc) is extracted for the whole number of degrees and corrected by interpolation for the additional minutes. There is rarely a need for interpolation of Z, which is given only to the nearest degree.

9.9.1. Interpolation for Hc should always be made in the direction of increasing Dec in accordance with the sign of the d-value. Not all of the signs are printed; the sign is given at least once in each block of five entries, and can always be found by looking either up or down the column from the value of d in question. The correction to altitude for additional minutes of Dec is proportional to d and proportional to the number of additional minutes.

9.9.2. In the previous example, the latitude was 40°, the LHA of the body was 086°, and the Dec was 5° N. Suppose the Dec had been $5^o17'$ N. The basic figures obtained would be $06^o16'$ Hc and 089° Z as before, and the true azimuth (Zn) would still be 271°. The Hc of $06^o16'$ is not correct for a Dec of $5^o17'$ N, but is correct for 5° N. The Hc change for an additional 1° of Dec (d-value) is +39 minutes of altitude. However, the correction needed in this case is for 17 minutes of Dec, not a whole degree. Consequently, the additional correction is 17/60 of 39'. To the closest whole number, this would be +11 minutes of altitude.

Figure 9.6. Enter Tables With Latitude, Declination, and LHA.

N. Lat. {LHA greater than 180°......... Zn = Z
{LHA less than 180°........... Zn = 360 − Z

DECLINATION (0°-14°) SAME NAME AS LATITUDE

LHA	0° Hc d Z	1° Hc d Z	2° Hc d Z	3° Hc d Z	4° Hc d Z	5° Hc d Z	6° Hc d Z	7° Hc d Z	8° Hc d Z	9° Hc d Z	10° Hc d Z	11° Hc d Z
70	15 11 .40 103	15 51 .40 102	16 31 .40 102	17 11 .39 101	17 50 .39 100	18 29 .39 99	19 08 .39 98	19 47 .38 98	20 25 .39 97	21 05 .38 96	21 42 .37 95	22 19 .38 94
71	14 27 39 103	15 06 40 102	15 46 39 101	16 25 40 100	17 05 39 99	17 44 39 99	18 23 38 98	19 01 39 97	19 40 38 96	20 18 38 95	20 56 38 95	21 34 37 94
72	13 42 39 102	14 21 40 101	15 01 39 100	15 40 39 100	16 19 39 99	16 58 39 98	17 37 39 97	18 16 38 96	18 54 39 96	19 32 38 95	20 10 38 94	20 38 37 93
73	12 57 39 101	13 36 39 100	14 15 40 100	14 55 39 99	15 34 39 98	16 13 38 97	16 51 39 96	17 30 38 96	18 08 38 95	18 46 38 94	19 24 38 93	20 02 37 92
74	12 11 40 100	12 51 39 100	13 30 39 99	14 09 39 98	14 48 39 97	15 27 39 97	16 06 38 96	16 44 38 95	17 22 38 94	18 00 38 93	18 28 38 93	19 16 37 92
75	11 26 .39 100	12 05 .40 99	12 45 .39 98	13 24 .39 97	14 03 .38 97	14 41 .38 96	15 20 .38 95	15 58 .38 94	16 36 .38 94	17 14 .38 93	17 52 .38 92	18 30 .37 91
76	10 41 39 99	11 20 39 98	11 59 39 98	12 38 39 97	13 17 38 96	13 56 38 95	14 34 38 94	15 12 39 94	15 51 38 93	16 29 37 92	17 06 38 91	17 44 37 90
77	09 55 40 98	10 35 39 98	11 14 38 97	11 52 39 96	12 31 38 95	13 10 38 95	13 48 38 94	14 26 39 93	15 05 38 92	15 43 37 91	16 20 38 91	16 58 37 90
78	09 10 39 98	09 49 39 97	10 28 39 96	11 07 38 96	11 45 39 95	12 24 38 94	13 02 39 93	13 41 38 92	14 19 38 92	14 57 37 91	15 34 38 90	16 12 37 89
79	08 24 39 97	09 03 39 96	09 42 38 96	10 21 38 95	10 59 39 94	11 38 38 93	12 16 39 93	12 55 38 92	13 33 38 91	14 11 37 90	14 48 38 89	15 26 38 89
80	07 39 .39 97	08 18 .38 96	08 56 .39 96	09 35 .39 94	10 14 .38 93	10 52 .38 93	11 30 .38 92	12 09 .38 91	12 47 .38 90	13 25 .39 89	14 03 .37 89	14 40 .38 88
81	06 53 39 96	07 32 39 95	08 11 38 94	08 49 39 94	09 28 38 93	10 06 39 92	10 45 38 91	11 23 38 90	12 01 38 90	12 39 38 88	13 17 37 88	13 54 38 87
82	06 07 39 95	06 46 39 94	07 25 38 94	08 03 39 93	08 42 38 92	09 20 39 91	09 59 38 91	10 37 38 90	11 15 38 89	11 53 38 88	12 31 37 87	13 08 38 87
83	05 21 39 95	06 00 39 94	06 39 38 93	07 17 39 92	07 56 38 91	08 34 39 91	09 13 38 90	09 51 38 89	10 29 38 88	11 07 38 88	11 45 37 87	12 22 38 86
84	04 36 38 94	05 14 39 93	05 53 38 92	06 31 39 92	07 10 38 91	07 48 39 90	08 27 38 89	09 05 38 89	09 43 38 88	10 21 38 87	10 59 38 86	11 37 37 85
85	03 50 .38 93	04 28 .39 92	05 07 .38 92	05 45 .39 91	06 24 .38 90	07 02 .39 89	07 41 .38 89	08 19 .38 88	08 57 .38 87	09 35 .38 86	10 13 .36 96	10 51 .38 85
86	03 04 38 93	03 42 39 92	04 21 38 91	04 59 39 90	05 38 38 90	06 16 38 89	06 55 33 88	07 33 38 87	08 11 38 86	08 49 38 86	09 27 38 85	10 05 38 84
87	02 18 38 92	02 56 39 91	03 35 39 90	04 14 38 90	04 52 38 89	05 30 39 88	06 09 38 87	06 47 38 87	07 25 38 86	08 03 38 85	08 41 38 84	09 19 38 83
88	01 32 38 91	02 10 39 91	02 49 38 90	03 28 38 89	04 06 38 88	04 44 38 87	05 23 38 87	06 01 38 86	06 39 38 85	07 18 38 84	07 56 38 84	08 34 38 82
89	00 46 39 91	01 25 38 90	02 03 39 89	02 42 38 88	03 20 39 88	03 59 38 87	04 37 38 86	06 15 39 85	05 54 38 85	06 32 38 84	07 10 38 83	07 48 38 82
90	00 00 .39 90	00 39 .38 89	01 17 .39 88	01 56 .38 88	02 34 .39 87	03 13 .38 86	03 51 .39 85	04 30 .38 85	05 08 .38 84	05 46 .39 83	06 25 .38 82	07 03 .38 82
91	-0 46 39 89	-0 07 38 89	00 31 39 88	01 10 38 87	01 48 39 86	02 27 38 86	03 05 39 85	03 44 38 84	04 22 39 83	05 01 38 82	05 39 38 82	06 17 39 81
92	-1 32 39 89	-0 53 38 88	-0 15 39 87	00 24 38 86	01 02 39 86	01 41 39 85	02 20 38 84	02 58 39 83	03 37 38 83	04 15 39 81	04 54 38 81	05 32 38 80
93	-2 18 39 88	-1 39 38 87	-1 01 39 87	-0 22 39 86	00 17 38 85	00 55 39 84	01 34 39 83	02 13 38 83	02 51 39 82	03 30 38 81	04 08 39 80	04 47 38 80

	0° Hc d Z	1° Hc d Z	2° Hc d Z	3° Hc d Z	4° Hc d Z	5° Hc d Z	6° Hc d Z	11° Hc d Z	12° Hc d Z	13° Hc d Z	14° Hc d Z	
88	00 46 39 91	00 07 38 91	-0 31 39 92	-1 10 38 93	-1 48 38 94	-2 27 38 94	-3 05 39 95	5 32 38 100	-6 10 39 100	272		
87	01 32 38 92	00 53 38 92	00 15 39 93	-0 24 38 94	-1 02 39 94	-1 41 38 95	-2 20 38 96	4 47 38 100	-5 25 39 101	-6 04 38 102	273	
86	02 18 39 92	01 39 38 93	01 01 38 93	00 22 39 94	-0 17 38 95	-0 55 39 96	-1 34 39 9	4 02 38 101	-4 40 38 102	-5 19 38 102	-5 57 38 103	274
85	03 04 39 93	02 25 39 93	01 46 38 94	01 08 39 95	00 29 38 96	-0 10 38 96	-0 48 38 9	3 16 39 102	-3 55 39 102	-4 34 38 103	-5 12 39 104	275
84	03 50 38 93	03 11 38 94	02 32 38 95	01 54 39 96	01 15 38 96	00 36 39 97	-0 03 38 9					
84	04 36 39 94	03 57 39 95	03 18 38 95	02 39 39 96	02 00 38 97	01 22 39 98	00 43 39 98	31 39 102	-3 10 39 103	-3 49 39 104	-4 28 38 105	276
83	05 21 38 95	04 43 39 95	04 04 39 96	03 25 39 97	02 46 39 98	02 07 39 98	01 28 39 99	47 39 103	-2 26 38 104	-3 04 39 104	-3 43 39 105	277
82	06 07 39 95	05 28 38 96	04 50 39 97	04 11 39 97	03 32 39 98	02 53 39 99	02 14 40 10	02 39 104	-2 20 39 105	-2 59 39 106	-3 22 39 106	278
81	06 53 39 96	06 14 39 97	05 35 39 97	04 56 39 98	04 17 39 99	03 38 39 100	02 59 39 10	17 39 104	-0 56 40 105	-1 36 39 106	-2 15 39 106	279
80	07 39 39 97	07 00 39 97	06 21 39 98	05 42 40 99	05 02 39 100	04 23 39 100	03 44 39 10	27 39 105	-0 12 40 106	0 52 39 106	-1 31 38 107	280
79	08 24 39 97	07 45 39 98	07 06 39 99	06 27 39 99	05 48 40 100	05 08 39 101	04 29 39 10	12 40 105	-0 07 40 107	-0 47 40 108	281	
78	09 10 39 98	08 31 39 99	07 52 40 99	07 12 39 100	06 33 39 101	05 54 40 102	05 14 40 10	56 40 106	01 16 40 107	00 36 39 108	-0 03 40 108	282
77	09 55 39 98	09 16 39 99	08 37 39 100	07 58 40 101	07 18 40 102	06 38 39 102	05 59 40 10	2 40 40 107	02 00 40 108	01 20 40 108	00 40 40 109	283
76	10 41 39 99	10 02 40 100	09 22 39 101	08 43 40 101	08 03 40 102	07 23 39 103	06 44 40 10	3 24 40 107	02 44 40 108	02 04 40 109	01 24 41 110	284
75	11 26 39 100	10 47 40 101	10 07 39 101	09 28 40 102	08 48 40 103	08 08 40 104	07 28 40 10	08 41 108	03 27 40 109	02 47 40 110	02 07 41 110	285
74	12 11 39 100	11 32 40 101	10 52 39 102	10 13 40 103	09 33 40 104	08 53 40 104	08 13 40 10	4 51 40 109	04 11 41 109	03 30 40 110	02 50 41 111	286
73	12 57 40 101	12 17 40 102	11 37 40 103	10 57 40 103	10 17 40 104	09 37 41 105	08 57 40 10	5 35 41 109	04 54 41 110	04 13 40 111	03 33 41 112	287
72	13 42 40 102	13 02 40 103	12 22 40 103	11 42 40 104	11 02 40 105	10 22 41 106	09 41 40 10	6 18 41 110	05 37 41 111	04 56 41 112	04 15 41 112	288
71	14 27 40 103	13 47 40 103	13 07 41 104	12 26 40 105	11 46 41 106	11 06 41 106	10 25 40 10	7 01 41 111	06 20 41 112	05 39 41 112	04 58 42 113	289
70	15 11 40 103	14 31 40 104	13 51 40 105	13 11 41 106	12 30 40 106	11 50 41 107	11 09 41 10	7 44 41 111	07 03 42 112	06 21 41 113	05 40 42 114	290

0°	1°	2°	3°	4°	5°	6°	11°	12°	13°	14°

S. Lat. {LHA greater than 180°........ Zn=180−Z
{LHA less than 180°............ Zn=180+Z

DECLINATION (0°-14°) CONTRARY NAME TO LATITUDE   LAT. 40°

LAT. 40°

9.9.3. This multiplication can be done on the slide rule face of the DR computer or by means of a table found in back of Pub. No. 249, Volumes 2 and 3. A portion of this table is shown in Figure 9.7. Notice that there are no signs listed. The proper sign for the answer from this table is the same sign as the basic d-value. A rule of thumb is the correction will be a plus (+) for declination of same name as latitude and a negative (-) for declination of contrary name as latitude. Values of d are given across the top of the table, and additional minutes of Dec are given down the side of the table. In the table, the correction 11' is found by looking across 17' for Dec and down 39' for d to their intersection at 11'. Since the sign of the d-value is plus, this correction is added to the tabulated Hc. The correct Hc value then becomes 06°16' + 11' or 06°27'.

9.9.4. Following is a sample problem illustrating the solution. Refer to the portion of the tables in Figure 9.8 for the solution. Suppose the sun is observed at 1005 GMT. The DR position is 38°12' N, 101°47' E, and the Ho of the sun is 10°52'.

Dec of the Sun for 1000Z	S7°37'	
GHA Sun for 1000Z	326°53'	
Correction to GHA for 5 minutes	+1°15	
GHA Sun for l005Z	328°08'	
Closest longitude for whole degree LHA	+E101°52'	(assumed longitude)
	430°00'	
	–360°00'	
LHA Sun for 1005Z	070°00'	

Figure 9.7. Table Performs the Multiplication.

Figure 9.8. Declination 0-14° Contrary Name to Latitude.

9.9.5. The closest whole degree of latitude is 38° N and is used as the assumed latitude. Since the assumed latitude is north and Dec is south, the navigator must use Pub. 249, Volume 2, page for 38° latitude which is headed Declination (0°–14°) Contrary Name to Latitude. Following LHA 070° across the page to 7° Dec, the navigator extracts:

Tab Hc	11°-06'
d-value	-40'
Z	108°
d-correction from Pub. No. 249, Volume 2	-25'
Corrected Hc	10°-41'
Zn using rule in the upper left-hand corner of the page	252°

9.10. Postcomp Summary. Before proceeding, review the procedures for finding the Hc and Zn of a body whose Dec lies between 30° N and 30° S, using Pub. No. 249, Volume 2 or 3.

9.10.1. Shoot the body and record the time of observation, the body's name, and the Ho.

9.10.2. From the *Air Almanac*, extract GHA and Dec of the body for the time of the observation.

9.10.3. Assume a position close to the DR position so that the latitude is a whole number of degrees and the longitude combined with the GHA of the body gives a whole number of degrees of LHA of the body. Find the LHA of the body for this position.

9.10.4. Select the correct volume (2 or 3) and page that contains the correct arguments of Dec and LHA of the body, temporarily disregarding the odd additional minutes of Dec. Thus, if the Dec were N19^{o}55', use the column for 19°. Select the table labeled Declination Same Name as Latitude, if Dec and latitude are both north or both south, or select the table labeled Declination Contrary Name to Latitude, if one is north and the other south. Opposite the LHA of the body, read the tabulated altitude, d-value, and Zn in the column headed by the whole degrees of Dec.

9.10.5. If the Dec is not a whole number of degrees, determine the correction for the additional minutes of Dec. Enter the table in the Pub. No. 249 volume with the value d and the number of additional minutes of Dec. Apply the correction to the tabulated altitude (Hc) according to the sign of d. This is the corrected Hc.

9.10.6. Convert azimuth angle (Z) to true azimuth (Zn) by means of the rule at the top or bottom of the page.

9.10.7. This completes the solution for the Dec tables. Keep in mind this solution is computed after the observation. Because of the speeds involved in air navigation, we will explain a way to compute the solution before the shot in the next chapter.

Section 9C—Precession and Nutation

9.11. Basics. The earth's axis does not maintain a fixed direction in space. Actually, the earth is like a slow running gyro that is wobbling. There are several separate patterns that the wobble makes. Some of those patterns have short cycles, while others take hundreds of years to complete. Two of the many patterns are shown in Figure 9.9. One involves small nodding motions while at the same time completing a larger circular path. You must use a correction called precession and nutation to account for these variations in the apparent position of the stars. This correction is applied only to celestial LOPs determined with Pub. No. 249, Volume 1.

Figure 9.9. Earth's Axis Wobble.
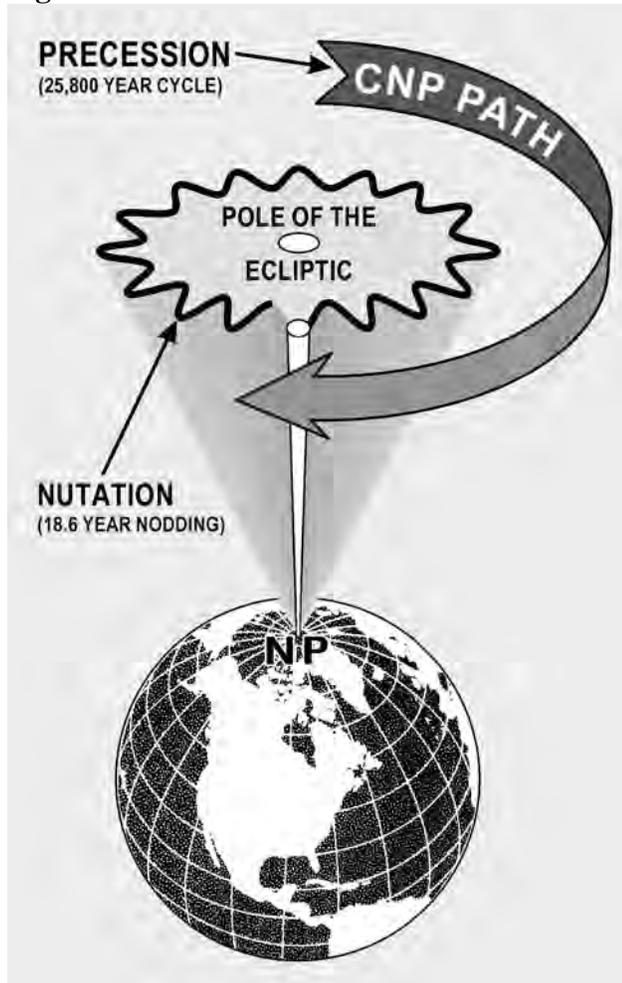
9.12. Precession. Because of the equatorial bulge, the attractive forces of other solar system bodies, principally the moon, are unbalanced about the center of the earth. The imbalance is directed toward aligning the equator with the plane of the ecliptic. However, the rotation of the earth transforms this force into an effect acting 90° away in the direction of rotation—a precessional effect. The result is that the poles travel in a conical path westward around the ecliptic poles, as shown in Figure 9.10 (the point 90° from the ecliptic). Consequently, the points of intersection of the equator with the ecliptic, or the equinoxes, travel in a westerly direction along the ecliptic. This travel is called precession of the equinoxes, and it amounts to approximately five-sixths (5/6) of a minute (50.26") annually. The equinoxes complete one revolution along the ecliptic in approximately 25,800 years. The equator is used as a reference for Dec and its movement, due to precession of the equinoxes, causes slight changes in the celestial coordinates of the stars, which otherwise appear fixed in space.

9.13. Nutation. As the relative positions and distances from the earth to the sun, moon, and planets vary so does the rate of precession. The only variation of importance in navigation is nutation. Nutation is a nodding of the poles, one oscillation occurring in about 18.6 years.

Figure 9.10. Precession of the Equinoxes.
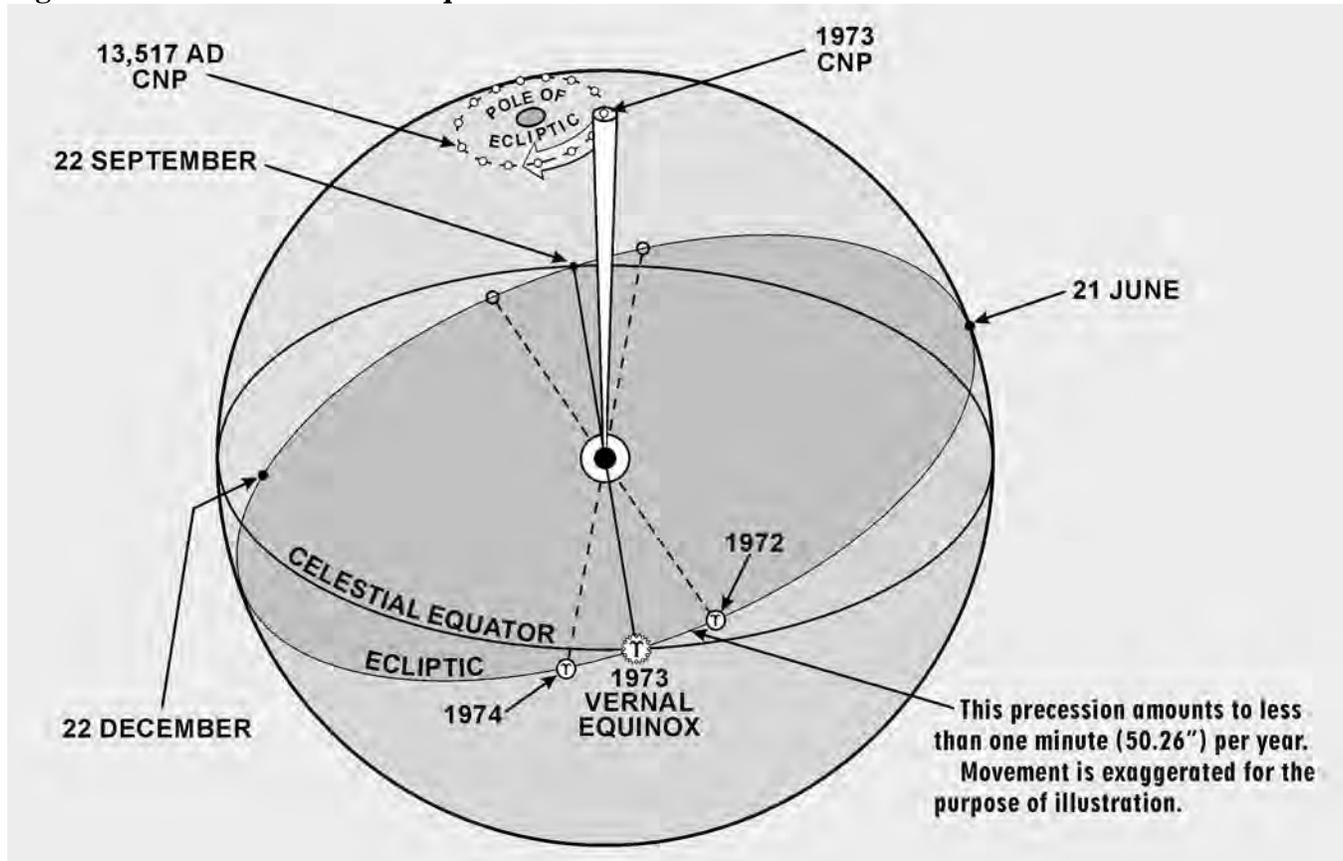
9.13.1. In Figure 9.11, you can see that if the stars remain fixed and the equinoctial moves up and down, the Dec of these bodies is changing.

9.13.2. Nutation, being approximately perpendicular to the ecliptic, has an appreciable influence on Dec. It is caused by complex gravitational forces among the sun, moon, and earth because the moon's orbit does not always lie in the plane of the ecliptic. The change in Dec of the celestial bodies caused by the resulting wobble of the earth's axis is called nutation.

9.14. Position Corrections. Because of precession and nutation, Hc and Zn for a star are accurate only at the instant, or epoch, at which the LHA and Dec for the computations are correct. A position obtained at any other time with that Hc and Zn requires a correction. Pub. No. 249, Volume 1, contains Hcs and Zns calculated for an epoch year (midnight, 1 January, of that year) so, if the volume is used in any other year, the resultant position must be corrected. The precession and nutation corrections are combined and given in Pub. No. 249, Volume 1, Table 5.

9.14.1. Entering arguments for the table is year, latitude, and LHA of Aries, and the correction is presented in the form of a distance and direction to move the fix. The tabulated values show the distance, parallel to the ecliptic, between the observer's position in the year of the fix and the position in the epoch year at the latitude and LHA of Aries.

Figure 9.11. Nutation Changes the Declination.
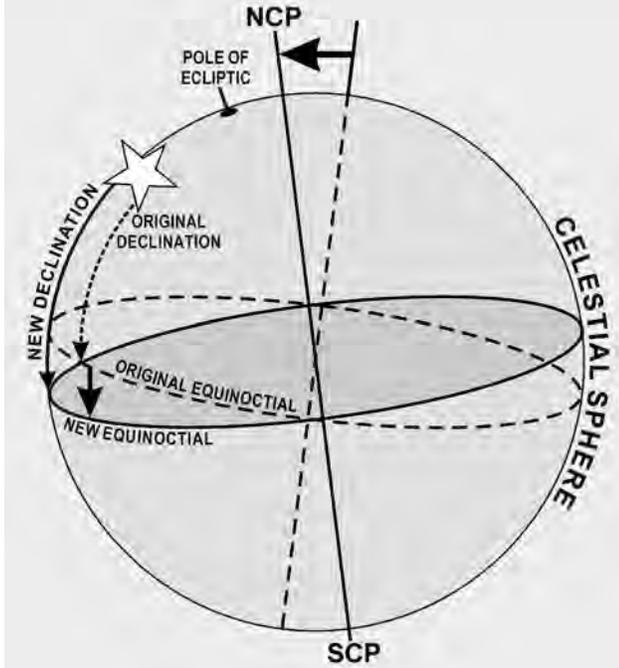
9.14.2. Directions for using Table 5 are printed in the introduction of Pub. No. 249, Volume 1. One point needs emphasis here: The table is to be used only for observations plotted with the aid of Volume 1, never in conjunction with Volumes 2 or 3.

9.15. Summary. This chapter has dealt with the astronomical triangle and how the Pub. No. 249 volumes you use in resolving the astronomical triangle. We've discussed obtaining solutions involving celestial bodies using Pub. No. 249, Volumes 1, 2, and 3. Succeeding chapters will discuss plotting of the celestial LOPs and techniques of precomputation.

Chapter 10

CELESTIAL PRECOMPUTATION

Section 10A—Introduction

10.1. Basics. Celestial precomputation is neither new nor revolutionary. The tables necessary to do precomputation have been available since 1940; however, there was no operational requirement for precomputation at that time. With present day high-speed aircraft, however, the picture has changed radically. By "postcomping," a great deal of work must be done after the last celestial observation. The fix could easily be 15 minutes old by the time it is plotted on the chart. At 450 knots groundspeed (GS), a fix that is 15 minutes old is over 100 miles behind the aircraft and is of questionable value. Another factor necessitating precomputation in high-speed aircraft lies in the method of shooting celestial. With the limited field of view of the sextant, the correct star is difficult to find unless you know where to look.

10.2. Presetting the Sextant. Precomputation greatly reduces both of the problems just mentioned. By completing most of the computations before shooting, the navigator reduces the time necessary to plot the fix after the last observation. Also, the problem of finding the star in the optics of the sextant is simplified. The procedure for finding the star is similar to the heading check performed with the periscopic sextant, using the true bearing (TB) method as explained in Chapter 12. In this case the true azimuth (Zn) is set into the sextant mount and the computed altitude (Hc), which will approximate the sextant altitude (Hs), is set into the sextant. Now, instead of sighting the body to determine the true heading (TH), set the TH under the vertical crosshair to find the selected body, hopefully very close to the crosshairs in the sextant field of view. Use the inverse relative bearing (IRB) method to avoid erroneous settings in the azimuth window and to increase speed in setting up the sextant. In this method, the azimuth window remains permanently at 000.0^o and the IRB is computed by the formula: IRB = TH – Zn. The body should be found at its computed altitude when its IRB appears under the crosshairs.

Section 10B—Precomputation Techniques

10.3. Basics. There are many acceptable methods of precomputation in general usage. However, these methods are basically either graphical, mathematical, or a combination of both methods. Selection is largely based on individual navigator preference and assigned command.

10.3.1. Celestial corrections which are used in precomputation include atmospheric refraction, parallax of the moon, instrument and acceleration errors, Coriolis and rhumb line, precession and nutation, motion of the observer, and wander. With precomputation, new corrections and terminology are introduced, which include fix time, solution time, observation time, scheduled time, and motion of the body adjustment.

10.3.2. Fix time is the time for which the LOPs are resolved and plotted on the chart. Solution time is the time for which the astronomical triangle is solved. Observation time is the midtime of the actual observation for each celestial body. Scheduled time is the time for which the astronomical triangle is solved for each LOP in the graphic method. Motion of the body correction is used to correct for the changing altitude of the selected bodies from shot to fix time and may be applied either graphically or mathematically.

10.4. Motion of the Body Correction. Motion of the body correction can be applied graphically by moving the assumed position eastward or westward for time. This is possible because the Greenwich hour angle (GHA) and the subpoint of the body move westward at the rate of 1° of longitude per 4 minutes of time. In the graphic method, a scheduled time of observation is given to each body. If shooting is off schedule, the following rules apply: For every minute of time that the shot is taken early, move the assumed position 15' of longitude to the east; for every minute of time that the shot is taken late, move the assumed position 15' of longitude to the west.

10.4.1. When the latitude of the assumed position and the Zn of the body are known, the motion of the body can be computed mathematically. For 1 minute, the formula is: 15(cos lat)/(sin Zn). This correction is shown in tabular form in Figure 10.1. In Pub. No. 249, the local hour angle (LHA) increases 1° in 4 minutes of time. Thus, the Hc for an LHA that is 1° less than the LHA used for precomputation is the Hc for 4 minutes of time earlier than the solution time. The difference between the two Hcs is the value to apply to the Hc or Hs to advance or retard the line of position (LOP) for 4 minutes of time. If the Hc decreases (Zn greater than 180°), the body is setting and the sign is minus to advance the LOP if the value is applied to the Hs. If the Hc increases (Zn less than 180°), the body is rising and the sign is plus to advance the LOP if the value is applied to the Hs.

10.4.2. In addition, motion corrections may be determined by using a modified MB-4 computer. This modification allows for greater accuracy and speed in computation of combined motions (motion of the observer and motion of the body) than the Pub. No. 249 tables. For a discussion of this modification, see Chapter 12.

10.5. Special Celestial Techniques. The main difference between the basic methods of precomputation is the manner in which the motion of the observer and the motion of the body corrections are applied. In the graphic method, both corrections are applied graphically by movement of the assumed position or the LOP. In the mathematical method, both corrections are applied mathematically to the Hc, the Hs, or the intercept after being obtained from tables, a modified MB-4 computer, or the Pub. No. 249.

10.6. Celestial Computation Sheets. The format in Figure 10.2 is a typical celestial precomputation and illustrates one acceptable method of completing a precomputation. The explanation is numbered to help locate the various blocks on the celestial sheets (Figure 10.2). *NOTE:* Not all blocks apply on every precomputation.

Figure 10.1. Correction for Motion of the Body.

True Zn	Correction for 4 Minutes of Time — Latitude																											True Zn
°	0°	8°	16°	20°	24°	28°	30°	32°	34°	36°	38°	40°	42°	44°	46°	48°	50°	52°	54°	56°	60°	64°	68°	72°	76°	80°	84°	°
090	+60	+59	+58	+56	+55	+53	+52	+51	+50	+49	+47	+46	+45	+43	+42	+40	+39	+37	+35	+34	+30	+26	+22	+19	+15	+10	+6	090
095	60	59	57	56	55	53	52	51	50	48	47	46	44	43	42	40	38	37	35	33	30	26	22	18	14	10	6	085
100	59	59	57	56	54	52	51	50	49	48	47	45	44	43	41	40	38	36	35	33	30	26	22	18	14	10	6	080
105	58	57	56	54	53	51	50	49	48	47	46	44	43	42	40	39	37	36	34	32	29	25	22	18	14	10	6	075
110	56	56	54	53	52	50	49	48	47	46	44	43	42	41	39	38	36	35	33	32	28	25	21	17	14	10	6	070
115	54	54	52	51	50	48	47	46	45	44	43	42	40	39	38	36	35	33	32	30	27	24	20	17	13	9	6	065
120	+52	+51	+50	+49	+47	+46	+45	+44	+43	+42	+41	+40	+39	+37	+36	+35	+33	+32	+31	+29	+26	+23	+19	+16	+13	+9	+5	060
125	49	49	47	46	45	43	43	42	41	40	39	38	37	35	34	33	32	30	29	27	25	22	18	15	12	9	5	055
130	46	46	44	43	42	41	40	39	38	37	36	35	34	33	32	31	30	28	27	26	23	20	17	14	11	8	5	050
135	42	42	41	40	39	37	37	36	35	34	33	33	32	31	29	28	27	26	25	24	21	19	16	13	10	7	4	045
140	39	38	37	36	35	34	33	33	32	31	30	30	29	28	27	26	25	24	23	22	19	17	14	12	9	7	4	040
145	34	34	33	32	31	30	30	29	29	28	27	26	26	25	24	23	22	21	20	19	17	15	13	11	8	6	4	035
150	+30	+30	+29	+28	+27	+26	+26	+25	+25	+24	+24	+23	+22	+22	+21	+20	+19	+18	+18	+17	+15	+13	+11	+9	+7	+5	+3	030
155	25	25	24	24	23	22	22	22	21	21	20	19	19	18	18	17	16	16	15	14	13	11	9	8	6	4	3	025
160	21	20	20	19	19	18	18	17	17	17	16	16	15	15	14	14	13	13	12	11	10	9	8	6	5	4	2	020
165	16	15	15	15	14	14	13	13	13	13	12	12	12	11	11	10	10	10	9	9	8	7	6	5	4	3	2	015
170	10	10	10	10	10	9	9	9	9	8	8	8	8	7	7	7	7	6	6	6	5	5	4	3	3	2	1	010
175	+5	+5	+5	+5	+5	+5	+5	+4	+4	+4	+4	+4	+4	+4	+4	+3	+3	+3	+3	+3	+3	+2	+2	+2	+1	+1	+1	005
180	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	000
185	−5	−5	−5	−5	−5	−5	−5	−4	−4	−4	−4	−4	−4	−4	−4	−3	−3	−3	−3	−3	−3	−2	−2	−2	−1	−1	−1	355
190	10	10	10	10	10	9	9	9	9	8	8	8	8	7	7	7	7	6	6	6	5	5	4	3	3	2	1	350
195	16	15	15	15	14	14	13	13	13	13	12	12	12	11	11	10	10	10	9	9	8	7	6	5	4	3	2	345
200	21	20	20	19	19	18	18	17	17	17	16	16	15	15	14	14	13	13	12	11	10	9	8	6	5	4	2	340
205	25	25	24	24	23	22	22	22	21	21	20	19	19	18	18	17	16	16	15	14	13	11	9	8	6	4	3	335
210	30	30	29	28	27	26	26	25	25	24	24	23	22	22	21	20	19	18	18	17	15	13	11	9	7	5	3	330
215	−34	−34	−33	−32	−31	−30	−30	−29	−29	−28	−27	−26	−26	−25	−24	−23	−22	−21	−20	−19	−17	−15	−13	−11	−8	−6	−4	325
220	39	38	37	36	35	34	33	33	32	31	30	30	29	28	27	26	25	24	23	22	19	17	14	12	9	7	4	320
225	42	42	41	40	39	37	37	36	35	34	33	33	32	31	29	28	27	26	25	24	21	19	16	13	10	7	4	315
230	46	46	44	43	42	41	40	39	38	37	36	35	34	33	32	31	30	28	27	26	23	20	17	14	11	8	5	310
235	49	49	47	46	45	43	43	42	41	40	39	38	37	35	34	33	32	30	29	27	25	22	18	15	12	9	5	305
240	52	51	50	49	47	46	45	44	43	42	41	40	39	37	36	35	33	32	31	29	26	23	19	16	13	9	5	300
245	−54	−54	−52	−51	−50	−48	−47	−46	−45	−44	−43	−42	−40	−39	−38	−36	−35	−33	−32	−30	−27	−24	−20	−17	−13	−9	−6	295
250	56	56	54	53	52	50	49	48	47	46	44	43	42	41	39	38	36	35	33	32	28	25	21	17	14	10	6	290
255	58	57	56	54	53	51	50	49	48	47	46	44	43	42	40	39	37	36	34	32	29	25	22	18	14	10	6	285
260	59	59	57	56	54	52	51	50	49	48	47	45	44	43	41	40	38	36	35	33	30	26	22	18	14	10	6	280
265	60	59	57	56	55	53	52	51	50	48	47	46	44	43	42	40	38	37	35	33	30	26	22	18	14	10	6	275
270	−60	−59	−58	−56	−55	−53	−52	−51	−50	−49	−47	−46	−45	−43	−42	−40	−39	−37	−35	−34	−30	−26	−22	−19	−15	−10	−6	270

Observations Earlier than Solution Time - SIGNS AS GIVEN

Observations Later than Solution Time - SIGNS REVERSED

Figure 10.2. Typical Celestial Precomputation Format.

CELESTIAL PRECOMPUTATION		SHEET NUMBER

PRECOMPUTATION - PERISCOPIC SEXTANT

NAVIGATOR		ALT MSL 20	DATE (Z) 1	FIX TIME 2 z

STAR SELECTION BY AZIMUTH	TRACK	18	BODY	3		Polaris		
(compass diagram 0/30/60/90/120/150/180/210/240/270/300/330)	GS	19	BASE GHA	5				
	CORIOLIS	21 R/L	CORR	6				
	PREC/NUT	22	+ 360	6				
	DR LAT	4 (N/S)	GHA	7				
	DR LONG	4 E/(W)	ASSUM LONG (W)/+E	8				

MOTION OF OBSERVER	24					LHA	9				
MOTION OF BODY	25					ASSUM LAT	10 N/S				
4 MIN ADJUST	26					DEC	11 N/S	N/S	N/S	N/S	N/S
OFF-FIX TIME	27 E/L	E/L	E/L	E/L	E/L	PLANNED MID-TIME					
TOTAL MOT. ADJUST	28					ACTUAL MID-TIME					
POLARIS Q / MOON PA SD	32 / 31		33			TAB Hc	12				
REF (-)	29					D / DEC CORR	13 / 14	15				
PERS / SEXT	30					CORR Hc	16				
TOTAL ADJ →	34					TOTAL → ADJ	34				
TH / GH						ADJ Hc	16 ± 34				
Zn / GZn (-)						OFF TIME MOTION	35				
IRB						Hc	16±34 ±35				

IRB		REFRACTION TABLE (Condensed)					Ho	36				

REFRACTION TABLE (Condensed)

Ro	ALTITUDE MSL (Thousands of feet)					
	0	20	25	30	35	40
1	63	46	41	36	31	26
2	33	19	16	14	11	9
3	21	12	10	8	7	5
4	16	8	7	6	5	3-10
	12	7	5	4	3-10	2-10

Zn / GZn (+)		INT	37 T/A	T/A	38 T/A	T/A	T/A
TII / GII		Zn	17				
TRACK T/G	18	CONV + W ANGLE - E	39				
Zn	17	GRID Zn	40				
REL Zn	23		TK	DC	TH	VAR	MH	DEV	CH

1. DATE. Place the Zulu date of the *Air Almanac* page used in this block.

2. FIX TIME. GMT (coordinated universal time) of the computation.

3. BODY. The celestial body being observed.

4. DR LAT LONG. The DR position for the time of the observation.

5. GHA. The value of GHA extracted from *Air Almanac* (10-minute intervals).

6. CORR. The GHA correction for additional minutes of time added to the GHA in block 5 and, if necessary, the 360° addition required establishing the LHA. SHA-When a star is precomped with Volume 2 or 3, SHA is placed in this block.

7. GHA. Corrected GHA (sum of blocks 5 and 6).

8. ASSUM LONG (-W/+E). The assumed longitude required to obtain a whole degree of LHA.

9. LHA. LHA of the body (or Aries).

10. ASSUME LAT. The whole degree of latitude nearest the DR position.

11. DEC. The declination of the celestial body (not used with Volume 1).

12. TAB Hc. The Hc from the appropriate page of Volume 2 or 3.

13. D. The d correction factor found with previous Hc. Include + or -, as appropriate. The value is used to interpolate between whole degrees of Dec.

14. DEC. Minutes of declination from block 11.

15. CORR. The correction from the Correction to Tabulated Altitude for Minutes of Declination table in Volume 2 or 3, using blocks 13 and 14 for entering arguments.

16. CORR Hc. This is the corrected Hc—sum of blocks 12 and 15 or extracted from Volume 1.

17. Zn. True azimuth of the celestial body from the formula in Volume 2 or 3, or directly from Volume 1.

18. TRACK. The true course (track) of the aircraft.

19. GS. The groundspeed of the aircraft.

20. ALT MSL. Aircraft altitude.

21. CORIOLIS. The Coriolis correction extracted from Pub. No. 249, the *Air Almanac*, or a Coriolis/rhumb line table.

22. PREC/NUT. Precession and nutation correction computed from the table in Volume 1.

23. REL Zn or Zn. The difference between Zn and track, used to determine motion of the observer correction.

24. MOTION OF OBSERVER (MOO). Motion of the observer correction for either 1 minute (using 1-minute motion correction table) or 4 minutes (using 4-minute correction table in Pub. No. 249) of time.

25. MOTION OF BODY (MOB). Motion of the body correction for either 1 minute (using 1-minute motion correction table) or 4 minutes (using tabulated Hc change for 1° of LHA or 4-minutes correction table in Pub. No. 249) of time.

26. 4-MINUTE ADJUST. Algebraic sum of 24 and 25; for use of 4-minute motion corrections extracted from Pub. No. 249.

27. X-Time. Time in minutes between planned shot time and fix time.

28. TOTAL MOT ADJUST/ADV/RET. Correction based on combined motion of observer and body, for the difference between the time of the shot and fix time. The sign of this correction will be the same as the sign in block 26 if the observation was taken prior to the computation time. If it was taken later, the sign will be reversed.

29. REFR. Correction for atmospheric refraction.

30. PERS/SEXT. Sextant correction or personal error.

31. SD. Semidiameter correction for Sun or Moon.

32. PA. Parallax correction for Moon observation.

33. POLARIS/Q CORR. The Q correction for the time of the Polaris observation (extracted from Pub. No. 249 or the *Air Almanac*).

34. Total ADJ. Algebraic sum of blocks 28-33 as applicable.

35. OFF-TIME MOTION. Motion adjustment for observation other than at planned time.

36. Ho. Height observed (sextant reading).

37. INT. Intercept distance (NM) is the difference between the final Hc and Ho. Apply the HOMOTO rule to determine direction (T or A) along the Zn.

38. LAT. Polaris latitude.

39. CONV ANGLE (W/-E). Convergence angle used in grid navigation.

40. GRID Zn. The sum of blocks 17 and 39.

10.7. Corrections Applied to Hc. In some methods of precomputation, corrections are applied in advance to the Hc to derive an adjusted Hc. When using corrections that are normally applied to Hs, the signs of the corrections are reversed if applied to Hc. For example:

Corrections Applied to Hs

Hs	31° 05
REFR	-01
PERS/SEXT	-05
Ho	30° 59
Hc	30° 40
INT	19T

Corrections Applied to Hc

Hc	30° 40
REFR	+01
PERS/SEXT	+05
ADJ Hc	30° 46
Hs	31° 05
INT	19T

10.7.1. This example demonstrates that corrections may be applied to either Hs or Hc. As long as they are applied with the proper sign, the intercept remains the same. The following sample precomp uses a common fix time (though computation times are different) and common observation times to facilitate comparison. *NOTE:* Atmospheric refraction correction must be extracted for the actual Hs. It may then be applied to either Hc or Hs using the proper sign. Extracting the value for Hc may cause large errors, especially when the body is near the horizon. Figure 10.3 is a sample three-star precomputation using the mathematical format. Corrections to altitude of the body are applied to the Hc and the sign of the correction has been reversed in this process, so the fix can be plotted prior to the computation time. All shots are early shots, allowing the navigator to resolve the fix and alter at fix time. However, any minor errors in interpolation for motions are multiplied for the two earliest shots and may cause inaccuracies in the fix.

10.7.2. Figure 10.4 shows a three-star precomputation using a three-LHA or graphical solution. The assumed position will then be moved for track and groundspeed to accommodate LOPs shot off time. Each observation is taken on time and then plotted out of its own plotting position. This precomp is easier and faster to accomplish with relatively few opportunities for math errors to occur. The three assumed positions required for this solution, on the other hand, often cause large intercepts and may make star identification difficult if care is not taken in choosing the precomp assumed position.

Figure 10.3. Mathematical Solution.

CELESTIAL PRECOMPUTATION		SHEET NUMBER

PRECOMPUTATION - PERISCOPIC SEXTANT

NAVIGATOR	CAPT WATTERS		ALT MSL 330	DATE (Z) 20 APR 79	FIX TIME 0704 z

STAR SELECTION BY AZIMUTH

(azimuth compass diagram: 0, 30, 60, 90, 120, 150, 180, 210, 240, 270, 300, 330)

					BODY	VEGA	SPICA	POLLUX		
TRACK	120									
GS	450				BASE GHA	312-46				
CORIOLIS	10 (R/L)				CORR	1-00				
PREC/NUT	0/000				+ 360	----				
DR LAT	38-14 (N/S)				GHA	313-46				
DR LONG	120-50 (E/W)				ASSUM LONG (W) +E	120-46				
MOTION OF OBSERVER	+15	+19	-29			LHA	193				
MOTION OF BODY	+41	+09	-45			ASSUM LAT	38 N/S				
4 MIN ADJUST	+56	+28	-1-14			DEC	N/S	N/S	N/S	N/S	N/S
OFF-FIX TIME	12 (E/L)	8 (E/L)	4 (E/L)	E/L	E/L	PLANNED MID-TIME	0652	0656	0700		
TOTAL MOT. ADJUST	+2-48	+56	-1-14			ACTUAL MID-TIME					
POLARIS Q / MOON PA SD						TAB Hc					
REF (-)	-01	0	-01			D/DEC CORR					
PERS / SEXT	0	0	0			CORR Hc	25-19	40-22	26-31		
TOTAL ADJ →	+2-47	+56	-1-15			TOTAL →ADJ	-2-47	-56	+1-15		
TH / GH						ADJ Hc	22-32	39-26	27-46		
Zn / GZn (-)						OFF TIME MOTION					
IRB						Hc	22-32	39-26	27-46		
IRB						Ho	23-04	39-20	27-24		
Zn / GZn (+)						INT	32 (T/A)	6 (T/A)	22 (T/A)	T/A	T/A
TII / GII						Zn	059	170	286		
TRACK T/G	120	170	286			CONV + W ANGLE - E					
Zn	059	120	120			GRID Zn					
REL Zn	061	050	166				TK	DC	TH	VAR	MH	DEV	CH

REFRACTION TABLE (Condensed)

Ro	ALTITUDE MSL (Thousands of feet)					
	0	20	25	30	35	40
1	63	46	41	36	31	26
2	33	19	16	14	11	9
3	21	12	10	8	7	5
	16	8	7	6	5	3-10
4	12	7	5	4	3-10	2-10

Figure 10.4. Graphical Solution.

BODY			
TIME			
LAT			
DEC			
GHA			
± 360			
GHA			
LONG $\frac{-W}{+E}$			
LHA			
HC			
CORR			
HC			
HO			
INT (T_A)			
ZN			
LONG/CONV			
GRID ZN			
BODY	VEGA	SPICA	Pollux
TIME	0652	0656	0700
LAT			39N
DEC			
GHA			312-46
± 360			—
GHA			312-46
LONG $\frac{-W}{+E}$			120-46W
LHA	190	191	192
HC	23-49	39-05	27-32
CORR	+01	0	+01
HC	23-50	39-05	27-33
HO	23-04	39-20	27-24
INT (T_A)	46 A	15 T	9 A
ZN	058	167	285
LONG/CONV			
GRID ZN			
	DRIFT DATA		
TIME(C-5)			
GROSS WT			
PAGE NO.			
FUEL REM			
O/H FUEL			
DIFF			
FUEL ETE			
ETE DEST			
EXTR TIME			
ENDUR			

10.8. Limitations. Precomputational methods lose accuracy when the assumed position and the actual position differ by large distances. Another limiting factor is the difference in time between the scheduled and actual observation time. The motion of the body correction is intended to correct for this difference. The rate of change of the correction for motion of the body changes very slowly within 40° of 090° and 270° Zn and the observation may be advanced or retarded for a limited period of time with little or no error. When the body is near the observer's meridian, however, the correction for motion of the body

changes rapidly due in part to the fast azimuth change and it's inadvisable to adjust such observations for long (over 6 minutes) periods of time. *NOTE:* Errors in altitude and azimuth creep into the solution if adjustments are made for too long an interval of time. Because of these errors, the navigator should attempt to keep observation time as close as possible to computation time.

Section 10C—Preplotting True Azimuth (Zn)

10.9. Basics. To speed up fix resolution, some navigators preplot the Zns of the bodies. This technique works best when used on a constant scale chart and using a technique of precomputation that will give one assumed position. Before making any observations, plot the assumed position, correct it for Coriolis and precession and/or nutation (if required) and draw the Zns of the bodies through this point. Label each Zn as the 1st, 2d, or 3d as shown in Figure 10.5, or use the name of the bodies. Use arrowheads to identify the direction of the body. Suppose the corrected assumed position is 30°40' N, 117°10' W and the following Zns were computed for the bodies:

    1st shot Zn 020°
    2d shot Zn 135°
    3d shot Zn 270°

The original assumed position of 31° N; 117°08' W has been corrected for precession and/or nutation and for Coriolis or rhumb line error to obtain the plotting position. When the first intercept is found to be 10A, second intercept 40A, and the third intercept 50T, the fix may be plotted quickly by constructing perpendicular lines at the correct point on the respective Zn line. This greatly reduces the time necessary to plot the fix.

10.10. Summary. Celestial precomputation methods have been brought to the forefront with the proliferation of high-speed aircraft. Aircraft speeds make it necessary to minimize the time between shooting and fixing. Since the sextant may be the only means of viewing the body, it is necessary to precompute the altitude and azimuth of a body in order to locate it. Remember corrections may be applied to the Hc, Ho, or intercept, and pay close attention to the sign of the correction. In addition to precomputation, the fix may be resolved faster by preplotting the true azimuths of the bodies.

Figure 10.5. Fix Can Be Plotted Quickly.
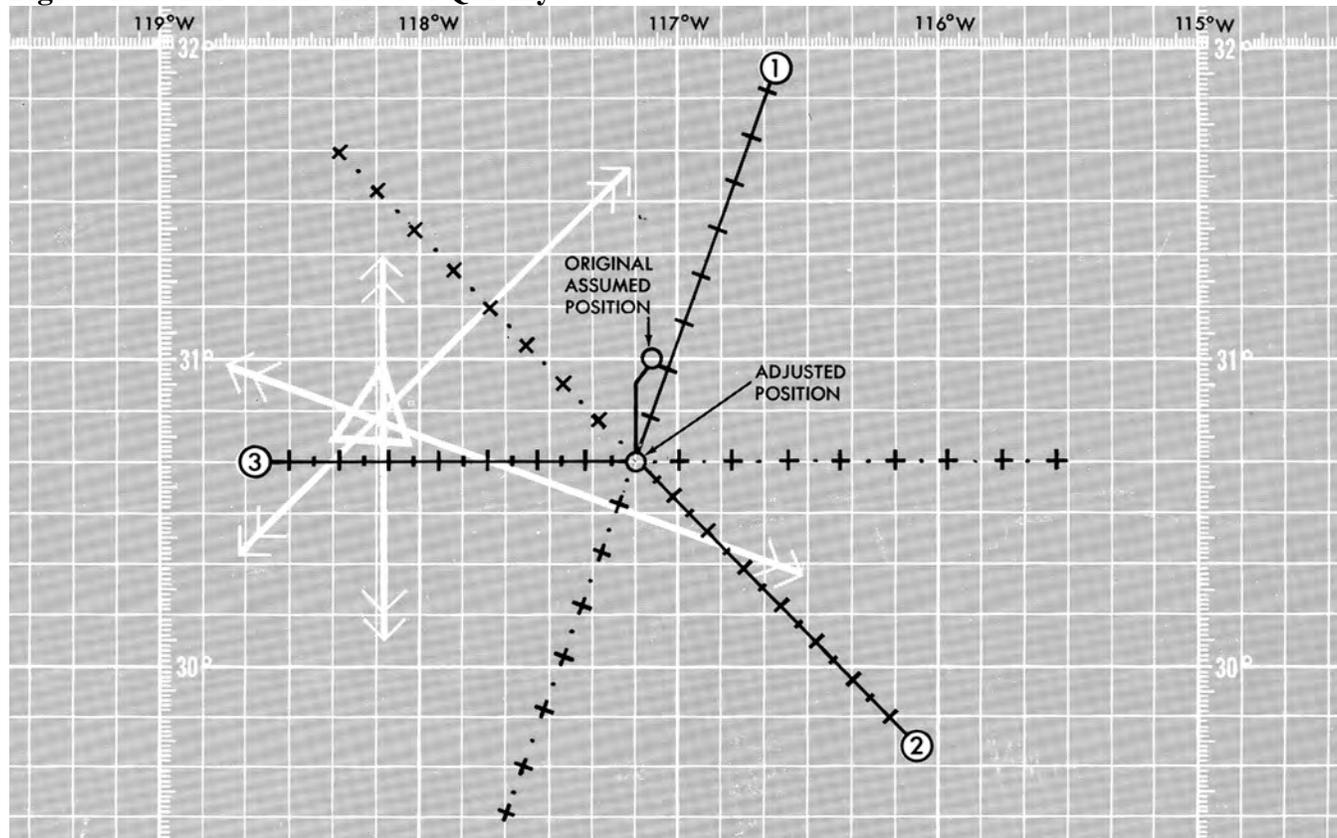
Chapter 11

PLOTTING AND INTERPRETING THE CELESTIAL LINE OF POSITION

Section 11A—Introduction

11.1. Basics. This chapter explains the methods that transform the tabulated and in-flight observation values into an aircraft position. The navigator is faced with two tasks: plotting the resultant information onto a chart and resolving this information into an aircraft position. There are two basic methods of obtaining a line of position (LOP): the subpoint method and the intercept method.

11.2. Subpoint Method (Figure 11.1). A detailed explanation of the theory concerning this method is in Chapters 9 and 10.

Figure 11.1. The Subpoint Method.
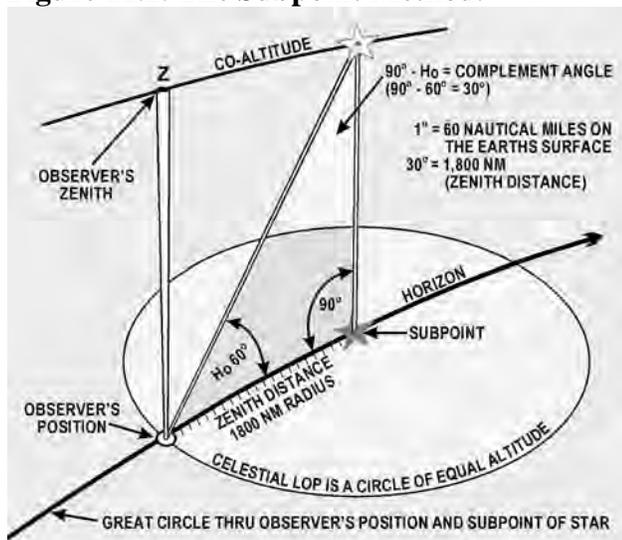
11.2.1. Here is a summary of the steps involved:

11.2.1.1. Positively identify the body and measure the altitude using a sextant.

11.2.1.2. Because no tabulated information for azimuth or elevation is required for this method, corrections for refraction, parallax, semidiameter, wander error, and sextant correction are applied directly to the Ho.

11.2.1.3. The resultant measurement is subtracted from 90° to obtain the co-altitude (co-alt). To convert to NM (1°=60 NM), multiply the number of degrees times 60. Any fractional portion of degrees is added to the previous value.

11.2.2. Example: Vega is observed at an altitude (Ho) of $88^o23'$. Sextant correction is -03'.

$88^o23' - 03' = 88^o20'$
$90^o - 88^o20' = 1^o40'$
$1^o 40' = 60' + 40' = 100$ NM

11.2.3. In this example, 100 NM represents the distance from the observer's position to the subpoint of the body. The coordinates of the body are its corresponding declination (Dec) and Greenwich hour angle (GHA). For this example, Vega's Dec is N38^{o}46'. The GHA is obtained by applying the sidereal hour angle (SHA) of Vega to the GHA of Aries.

> *EXAMPLE:*
>   SHA = 080°59'
>   GHA Aries = 039°18'
>   GHA Vega = 120°17'

11.2.4. Subpoint of Vega is located at 38°46' N l20^{o}17' W. The observer is now ready to apply the information:

11.2.4.1. Plot the subpoint on an appropriate chart.

11.2.4.2. With dividers or compass, span the co-alt distance; in this case 100 NM.

11.2.4.3. Use the body's subpoint (38°46' N l20^{o}17' W) as the center and 100 NM (co-alt) as the radius. The circle is called the circle of equal altitude and the observer is located on that portion of the circle nearest the DR position. There are definite advantages to this method. It requires no precomp values and plotting is very simple if the observer and body are reasonably close together. When the observer and body are separated by great distances, some disadvantages appear.

11.2.5. If a body is observed at 20° above the horizon, the observer is 4,200 NM from its subpoint. To swing a LOP from this subpoint, the subpoint and the arc must be plotted on the same chart. To permit plotting of any LOP, the chart must cover an area extending more than 4,000 miles in every direction from the DR position. This means that the chart must be either of such large size that it cannot be spread out on a table in the aircraft, or of such small scale that plotting on it is inaccurate. To cover an area 8,000 miles across, a chart 4 feet square must be drawn to a scale of about 1:10,000,000. Furthermore, measuring would be difficult because of distortion.

11.2.6. Since a celestial LOP cannot always be drawn by the subpoint method, the intercept method, based on the same principles, is often used.

11.3. Intercept Technique:

11.3.1. Intercept Method (Figure 11.2). You can eliminate the need for plotting the body's subpoint and still draw the arc representing the circle of equal altitude. By using the following formula, you can calculate the altitude and azimuth of the body for the DR position:

> Hc = SIN^{-1} [SIN (DEC') SIN (LDr)
>   +COS (DEC')COS (LDr) COS (LHA)]
> Z = COS (Z) = [SIN (DEC')
>   – SIN (LDR) SIN (HC)]/[COS (Hc COS (LDr)]
>
> Zn = Z if SIN (LHA) < 0
> Zn = 360 – Z if SIN (LHA) ≥ 0

Figure 11.2. Line of Position Computed by Intercept Method.
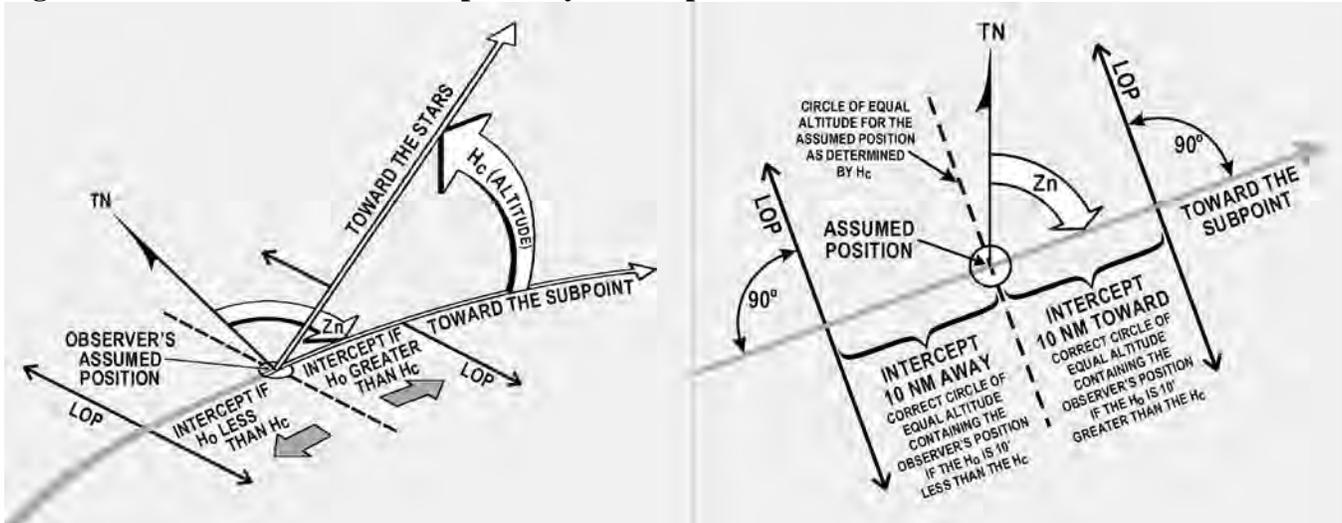
11.3.2. The calculations may be performed quickly, using a programmable calculator, or they may be extracted from the appropriate volume of Pub. No. 249. This method enables the observer to use any of the navigational bodies available at the appropriate fix time. Here is a brief review:

11.3.2.1. Compute a DR for the time of the position, using preflight or in-flight data.

11.3.2.2. Determine the necessary entering values for the Pub. 249 volume being used (Lat, LHA, Dec contrary or same) and extract all the necessary values of computed altitude (Hc), azimuth angle (Z), etc.

11.3.2.3. After making all the necessary conversions and corrections (Chapter 10), compare the Ho and corrected Hc. This difference is the intercept. If the Ho equals the corrected Hc, then the circle of equal altitude passed through the plotting position. If the Ho is greater than the Hc, the difference is plotted in the direction of the true azimuth (Zn). The Zn represents the azimuth from the observer's position to the subpoint. If the Ho is less than the Hc, plot the difference 180° from the Zn.

NOTE: If HO is MOre, plot TOward the subpoint (HO MO TO)

11.3.2.4. Example: The assumed position is 38° N, 121°30' W for a shot taken at 1015Z on Aldebaran. The Ho is 32°14'. The Hc is determined to be 32°29' and the Zn 120°. A comparison of Ho and Hc determines the intercept to be 15 NM away (15A).

11.4. Plotting LOP Using Zn Method (Figure 11.3):

11.4.1. Plot the assumed position and set the intercept distance on the dividers.

11.4.2. Draw a dashed line through the assumed position toward the subpoint.

11.4.3. Span intercept distance along dashed Zn line.

11.4.4. Place plotter perpendicular to Zn.

Figure 11.3. Celestial Line of Position Using True Azimuth Method.
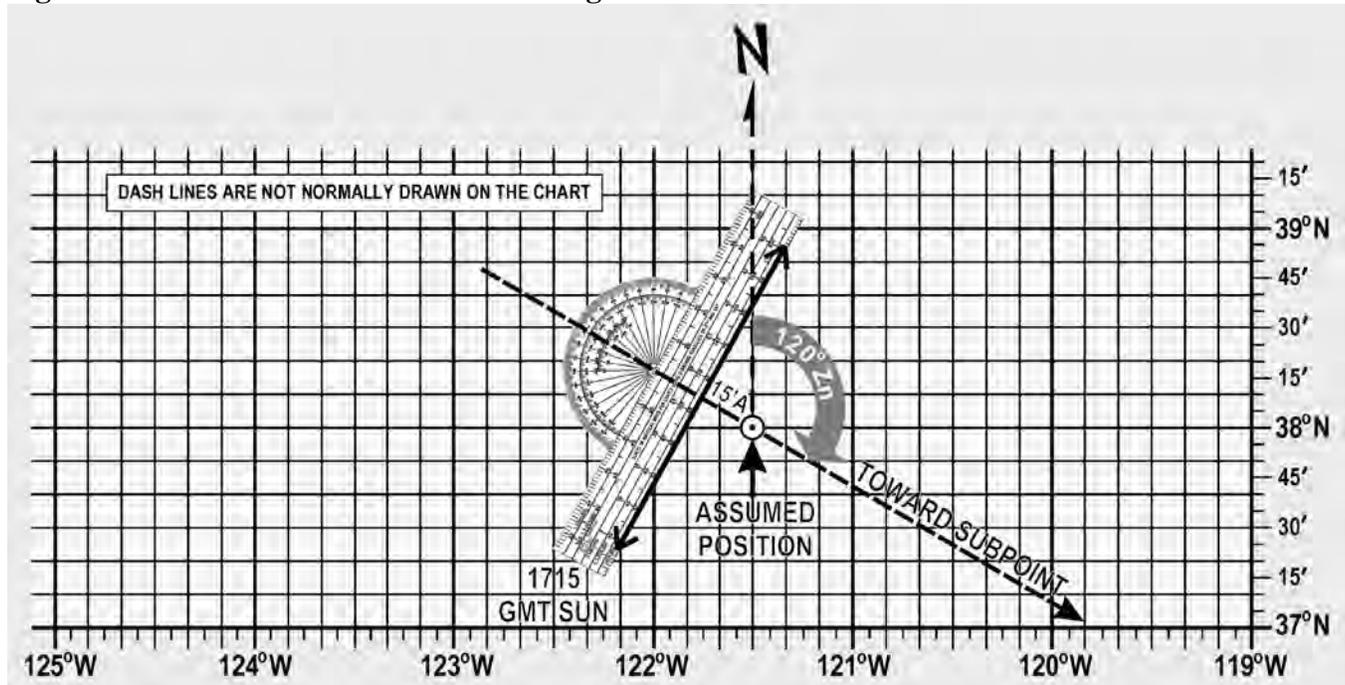
11.4.5. Draw LOP along plotter as shown.

11.5. Plotting LOP Using Flip-Flop Method (Figure 11.4):

11.5.1. Plot the assumed position and set the intercept distance on the dividers.

11.5.2. Measure 120° of the Zn with point A of the dividers on the assumed position and place point B of the dividers down, in this case, away from 120° or in the direction of 300° from the assumed position. Slide the plotter along the dividers until the center grommet and the 100/200-mile mark are lined up directly over point B of the dividers marking the intercept point.

11.5.3. Remove point A of the dividers from the assumed position, keeping point B in place. Flip point A (that was on the assumed position) across the plotter, at the same time expanding the dividers so that point A can be placed on the chart at the $90^o/270^o$ mark of the plotter.

11.5.4. Flop the plotter around and place the straight edge against the perpendicular, which is established by the dividers.

11.5.5. Draw LOP along the plotter as shown.

11.6. Summary of Intercept Method. When using the intercept method, remember:

11.6.1. For some assumed position near the DR position, find the Hc and Zn of this body for the time of the observation. This is done with the aid of celestial tables, such as Pub. No. 249, or a programmable calculator.

Figure 11.4. Plotting Celestial Line of Position Using Flip-Flop Method.
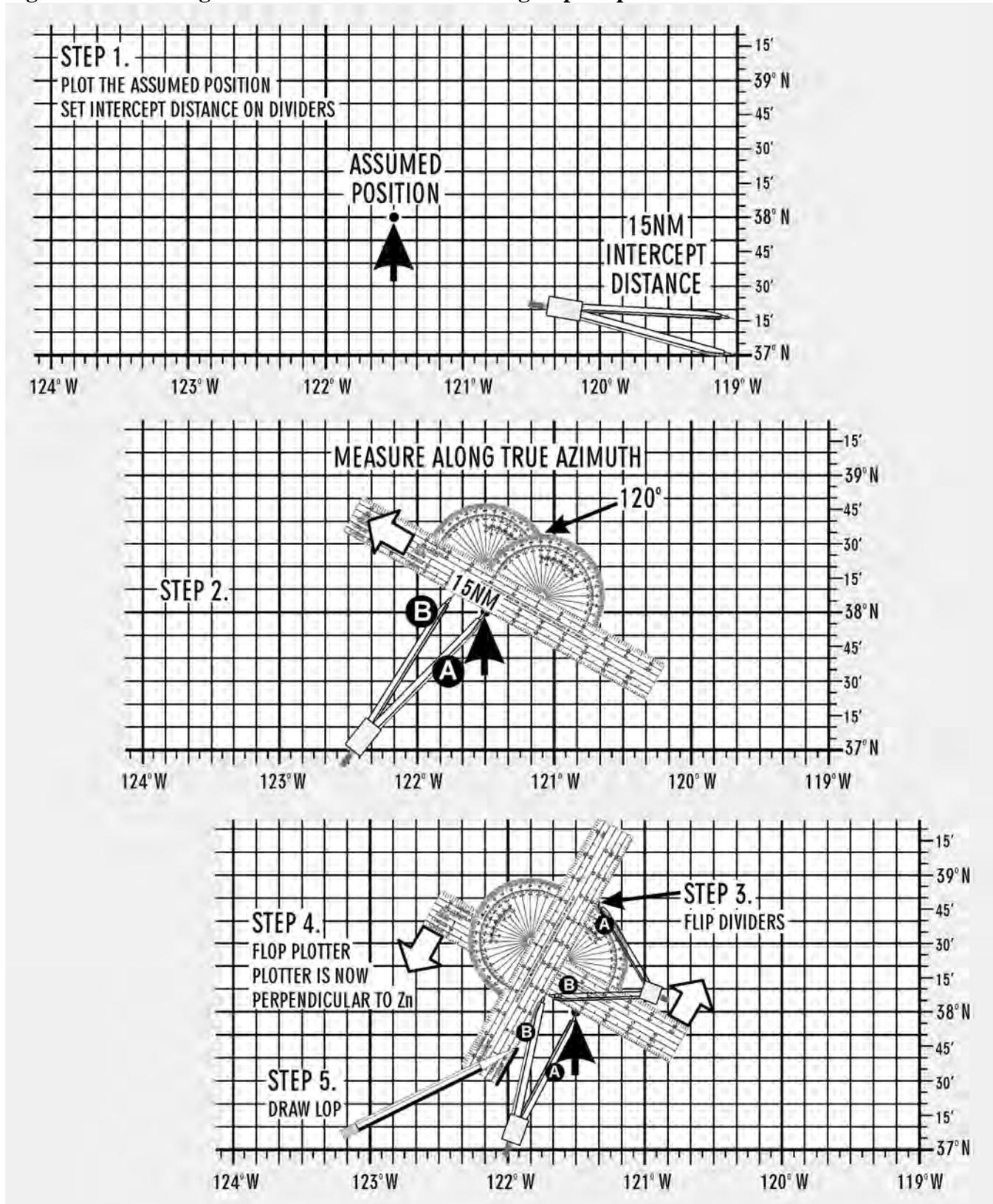
11.6.2. Obtain needed corrections, sextant correction, refraction, etc., and apply these to the Hc by reversing the sign (remember, we are striving to derive a precomputed value to ensure the correct body is shot). Measure the altitude (Ho) of the celestial body with the sextant and record the midtime of the observation.

11.6.3. Find the intercept, which is the difference between Ho and Hc. Intercept is toward the subpoint if Ho is greater than Hc and away from the subpoint if Ho is smaller than Hc.

11.6.4. From the assumed position, measure the intercept toward or away from the subpoint (in the direction of Zn or its reciprocal) and locate a point on the LOP. Through this point, draw the LOP perpendicular to the Zn.

11.7. Additional Plotting Techniques. The preceding techniques involve the basic plotting procedures used on most stars and the bodies of the solar system. However, there are certain techniques of plotting that are peculiar to their own celestial methods; for example, the plotting of LOPs obtained by using Polaris, which is discussed later. Also, certain precomputation techniques lend themselves more readily to other plotting techniques, such as preplotting the true azimuths or plotting the fix on the DR computer.

11.7.1. These last plotting techniques are discussed in Pub. No. 249 in the section on precomputation.

11.7.2. Other special techniques are discussed in the section on curves, in which the celestial observation is plotted on a graph rather than on the chart.

Section 11B—Interpretation of an LOP

11.8. Basics. Navigation has two aspects—the mechanical and the interpretive. The mechanical aspect includes operation and reading of instruments, simple arithmetical calculations, plotting, and log keeping. The interpretive aspect is the analysis of the data that have been gathered mechanically. These data are variable and subject to error. You must convert them into probabilities as to the position, track, and GS of the aircraft and the direction and speed of the wind. The more these data are subject to error, the more careful the interpretations must be and the less mechanical the work can be. LOPs and fixes especially require careful interpretation. It is convenient to think of a fix as the true position of the aircraft and of the LOP as a line passing through this position, but these definitions are optimistic. It is almost impossible to make a perfect observation and plot a perfect LOP. Therefore, an LOP passes some place near this position, but not necessarily through it and a fix determined by the intersection of LOPs is simply the best estimate of this position on the basis of one set of observations. Thus, in reality, a fix is a most probable position (MPP) and a LOP is a line of MPP.

11.8.1. The best interpretation of LOPs and fixes means they are used, to the best advantage, with DR. But good interpretation cannot compensate for poor LOPs, nor can good LOPs compensate for careless DR. To get good results, every precaution must be taken to ensure the accuracy of LOPs and exact DR calculations.

11.8.2. Intelligent interpretation requires fine judgment, which can only be acquired from experience. You can be guided, however, by certain well-established, though flexible, rules.

11.8.3. The following discussion pertains especially to celestial LOPs and fixes. It also applies to LOPs and fixes established by radio and, to some extent, to those obtained by map reading.

11.9. Single LOP. Previous discussions dealt with the basic plotting of an LOP and errors in LOPs, but they did not show the actual mechanics of the plotted corrections which must be applied. The LOP must be corrected for Coriolis or rhumb line correction and also for precession and/or nutation correction if it is based on a Volume 1 star shot. Coriolis or rhumb line correction becomes a very significant correction at higher speeds and latitudes. For example, suppose the correction determined from the Coriolis or rhumb line correction table is 9 NM right (of the track). The LOP must be moved a distance of 9 NM to the right of track. This can be done either by moving the assumed position prior to plotting, or by moving the LOP itself after it is plotted. (Remember the assumed position is not used in the plotting of the LOP obtained from a Polaris observation.) Consider Figure 11.5, which shows a track of 90°.

Figure 11.5. Two Methods of Coriolis/Rhumb Line Correction.
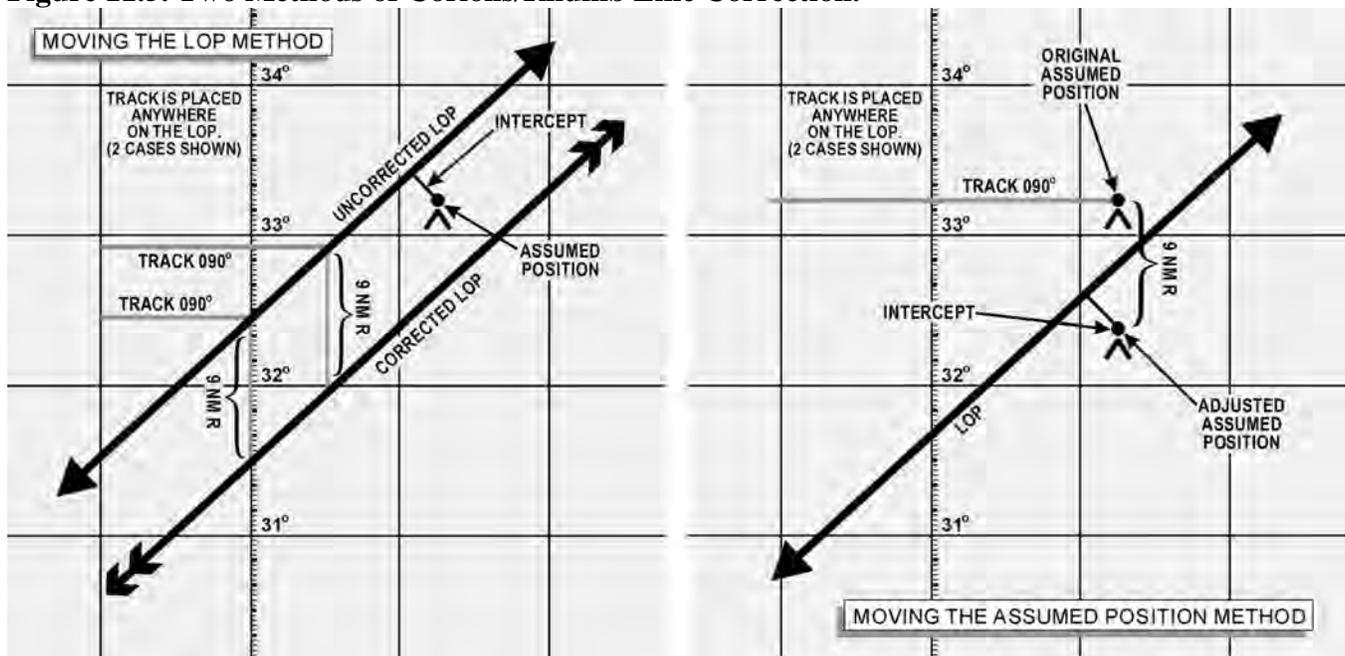
11.9.1. Notice that, in both methods, the corrected LOP is in the same place with respect to the original assumed position and that the intercept value is the same. The resultant LOP is the same regardless of the method used.

11.9.2. If, in addition to the Coriolis or rhumb line correction, a precession and/or nutation correction of 3 NM in the direction of 60° is required, it would have been further applied as shown in Figure 11.6. Again, the corrected LOP is the same, using either method, because the intercept and resultant position of the corrected LOP to the original assumed position are the same. The corrected LOP alone gives very little information; hence, a position must be arrived at only after considering the LOP and the DR position for the same time.

Figure 11.6. Two Methods of Coriolis/Rhumb Line and Precession/Nutation Correction.
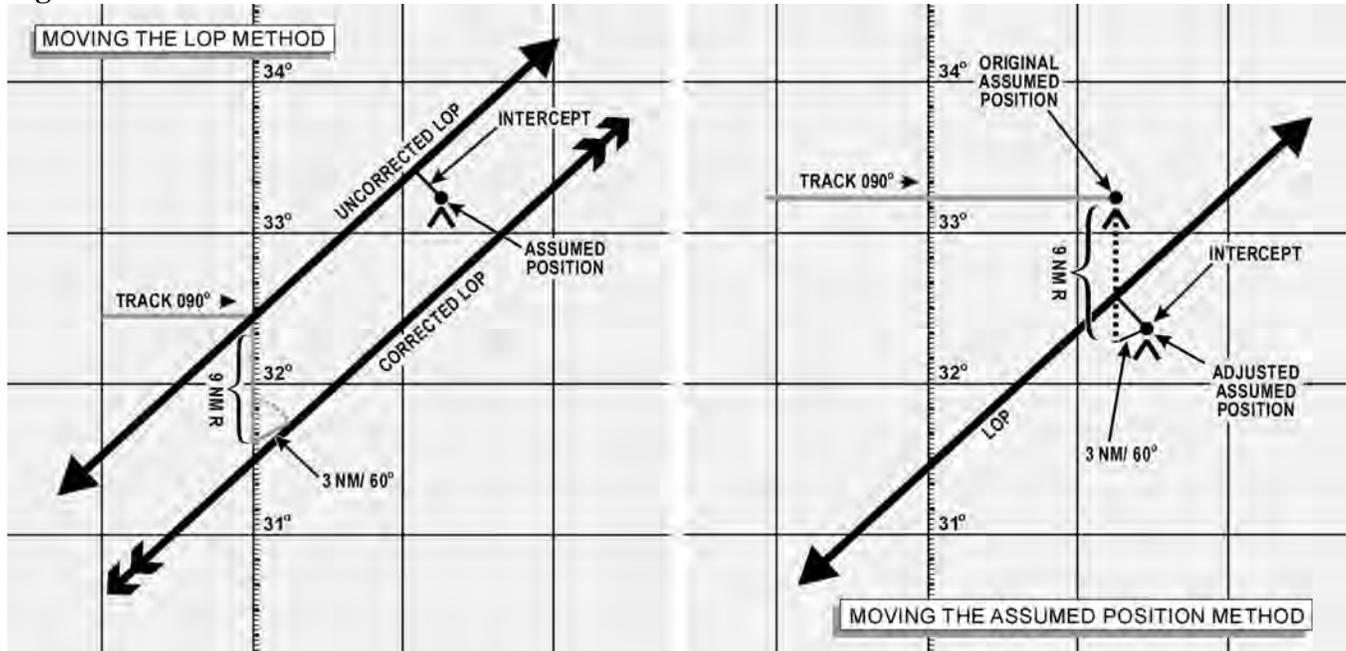
11.10. Most Probable Position (MPP) by C-Plot. The MPP is just what the name implies. It is not a fix; however, since it is the best information available, it is treated as such. Notice in A of Figure 11.7 that the DR position and celestial LOP (for the same time) do not coincide.

Figure 11.7. Most Probable Position by C-Plot.
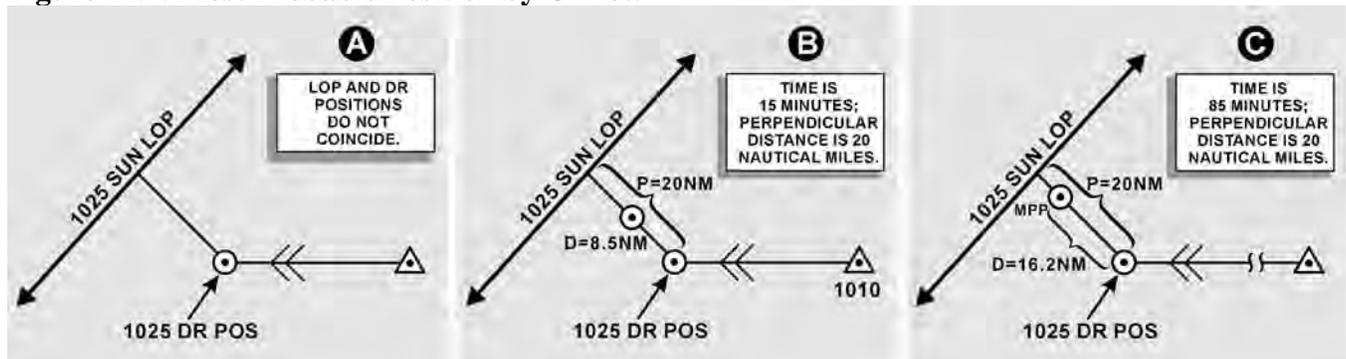
11.10.1. Obviously, the DR information or celestial information, or both is in error. Notice that the prior fix has no time on it. Suppose this prior fix had been for the time of 1010. It would then be very likely that most of the error is in the celestial information and the probable position is closer to the DR position than to the celestial LOP. On the other hand, suppose the prior fix had been for the time of 0900. Since the accuracy of the celestial information is unaffected by the time from the last fix, it would, in this case, be most likely that the actual position is closer to the LOP than to the DR position.

11.10.2. A formula has been devised to position the observer along the perpendicular to the LOP according to the time factor. The formula is:

$$\frac{d}{t} = \frac{p}{t+p}$$

where t is time in minutes, p is the perpendicular distance between the DR position and the LOP and d is the distance from the DR position for the time of the MPP measured along the perpendicular to the LOP. Look at B and C of Figure 11.7 and see how the formula works for the two problems cited above if the perpendicular is 20 NM in length. In B of Figure 11.7, t is 15 minutes and p is 20 NM, so the MPP would be located along the perpendicular about 8½ NM from the DR position.

$$\frac{d}{15} = \frac{20}{15+20} \qquad\qquad d = \frac{300}{35} = 8.57 \text{NM}$$

11.10.3. Now, consider C in Figure 11.7 where t is 1 hour 25 minutes or 85 minutes, p is 20 NM and in this case, the MPP would be over 16 NM away from the DR position along the perpendicular to the LOP.

$$\frac{d}{85} = \frac{20}{85+20} \qquad\qquad d = \frac{1700}{105} = 16.2 \text{NM}$$

11.10.4. If you prefer not to use the formula, a simple table can be easily constructed to solve for d with entering arguments of t and p as shown in Figure 11.8. The table could easily be enlarged to handle larger values of t and p. In most fixes, the DR position is so close to the LOP that the midpoint between these two can be considered the MPP. A good rule to use is to take the midpoint of the perpendicular if the total distance between the DR position and the LOP is 10 NM or less. If the value of p is greater than 10 NM, use a table or the formula to determine the MPP. Up to this point, determination of the MPP has been rather mechanical. Experienced navigators will frequently further adjust the position of the MPP for other factors not yet considered. For example, if the LOP is carefully obtained under good conditions or if it is the average of several LOPs, you may further weight the MPP in the direction of the LOP by an amount that judgment dictates. However, the reverse may be true if the LOP is obtained under adverse conditions of rough air. In the latter case, you might move the MPP closer to the DR position by some amount determined by sound judgment.

Figure 11.8. To Solve for Distance.

P	"t"—TIME IN MINUTES			
▼	10 m	15 m	30 m	60 m
12	5	7	9	10
14	6	7	10	11
16	6	8	10	13
18	6	8	11	14
20	7	9	12	15
25	7	9	14	18
30	8	10	15	20
35	8	10	16	22
40	8	11	17	24
45	8	11	18	26
50	8	12	19	27
55	9	12	19	29
60	9	12	20	30

To the closest NM values of "d" in the formula.

11.10.5. Further, consider the validity of the DR position in relation to factors other than time. A DR position at the end of 40 minutes would be more reliable with Doppler drift and GS versus one based on metro information. These factors may also adjust the original MPP closer to or farther away from the DR position, along the perpendicular. However, these last mentioned factors are judgment values that come only with experience. In fact, with experience you may mentally calculate all the factors involved and arrive at the final position of the MPP without recourse to a formula or table.

11.11. Finding a Celestial Fix Point. Up to this point, only the single celestial LOP and what to do with it have been considered. Now, the celestial fix should be considered. To establish a fix, two or more LOPs must be obtained. Since, in most cases, two or more LOPs cannot be obtained simultaneously, they must be converted to a common time. For example, a LOP obtained at 1010 must be converted to the LOP obtained at the fix time of 1014. There are several methods for making this conversion, which are discussed in this chapter. Consideration is also given to the planning of the fix and the final interpretation of the fix itself.

Section 11C—Conversion of LOPs To A Common Time

11.12. Moving the LOP. One method of converting LOPs to a common time is to move the LOP along the best-known track for the number of minutes of GS necessary for the time conversions. This method is similar to that used for correcting for Coriolis or rhumb line and precession or nutation. For example, suppose the track is 110° and the GS 300 knots. LOPs are for 1500, 1504, and 1508 and a fix is desired at 1508. This means the 1500 LOP must be moved to the time of the fix, using the track and 8 minutes of the best known GS. The 1504 LOP must be moved to the time of the fix, using the track and 4 minutes of GS. The 1508 LOP is already at the fix time, so it requires no movement. Figure 11.9 shows the method of conversion as it is completed on the chart.

Figure 11.9. Conversion of Lines of Position to a Common Time.
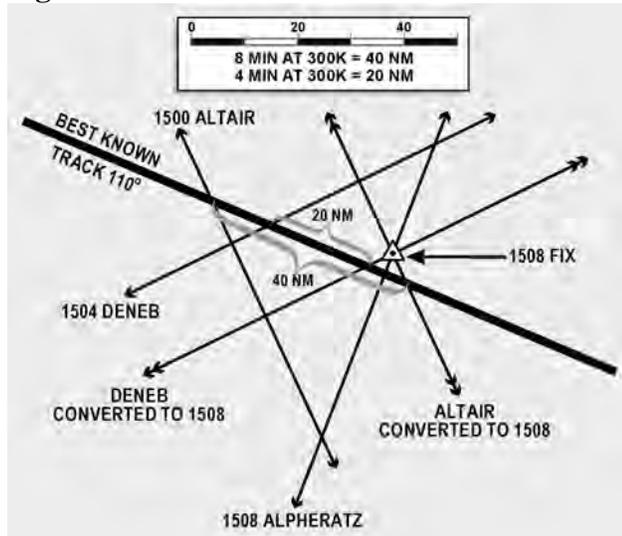
11.12.1. If, at any time, the LOP has to be retarded (moved back) to the time of the fix, use the following procedures. Using the reciprocal track and GS, obtain the correction in the regular manner for the number of minutes of difference. For example, suppose the fix is at 1800 and the last shot is at 1802.

11.12.2. Retarding the LOP 2 minutes of GS on a track of 70° would be the same as advancing it 2 minutes of GS on a track of 250°.

11.13. Motion of Observer Tables. A second method of conversion of LOPs to a common time is with a Motion of the Observer table such as the one in Pub. No. 249. This table gives a correction to be applied to the Ho or Hc so that the LOP plots in its converted position. The correction obtained from Table 1 in all volumes of Pub. No. 249 is for 4 minutes of time. An additional table allows you to get the correction for the number of minutes needed.

11.13.1. For example, suppose the LOP needs to be advanced for 11 minutes and the Ho of the body is 33° 29' and Zn is 080°. The track of the aircraft is 020° and the GS is 240 knots. In Table 1, Correction for Motion of the Observer for 4 minutes of Time (Figure 11.10), the entering arguments is Rel Zn and GS. Rel Zn is azimuth relative to course (Zn minus track or track minus Zn). Subtract the smaller angle from the larger and enter the table with the answer. In this case, Zn – track = 080° – 020° = 060° (Rel Zn) and GS is 240 knots. Entering this table with these arguments, the correction listed is +08' for 4 minutes of time.

11.13.2. Use the whiz wheel to calculate the total motion for 11 minutes. In this case, the 11-minute correction totals 22'. By applying any other correction (refraction, sextant correction), a total adjustment is derived. By changing the sign, this total may be applied to the Hc. To apply the correction to the Ho, the sign of the adjustment would remain the same. Apply the adjustment to the intercept as the rules state in Table 1. In each case, the resultant intercept would be the same.

11.13.3. Suppose the Hc was 33°57'. Applying the correction -22 yields 33°35'. Comparing this with our Ho 33°29' results in an intercept of 6 NM away. If you decide to apply the correction to the Ho, 33°29' + 22' yields 33°5l'. Comparing this to the Hc 33°57' yields the same result, 6 NM away. When using the Motion of the Observer table and when the fix time is earlier than the observation (LOP to be retarded), the rule for the sign of the correction is also printed below Table 1.

11.14. Moving the Assumed Position. Another method of converting LOPs to a common time is to move the assumed position. This method is recommended for shots 4 minutes apart computed to give all three bodies a single assumed position. However, it is not limited to that type of computation. The assumed position is moved along the best-known track at the best-known GS. For example, again suppose the track is 330° and the GS 300 knots. LOPs are for 1500, 1504, and 1508 and a fix is desired at 1508 (Figure 11.11). Since the first LOP would have to be advanced 40 NM (8 min at 300 knots), the same result is realized by advancing the assumed position 40 NM parallel to the best-known track. The 1504 LOP must be advanced 20 NM; therefore, the assumed position is advanced 20 NM miles parallel to the best-known track. The third shot requires no movement and it is plotted from the original assumed position. It should be noted that the first shot is always plotted from the assumed position, which is closest to destination. In this method, if observations are precomputed and the assumed position is moved prior to shooting, the following procedure is used when shooting is off schedule. For every minute of time that the shot is taken early, move the assumed position 15 minutes of longitude to the east. For every minute of time that the shot is taken late, move the assumed position 15 minutes of longitude to the west. In addition, the affected LOP must be moved along the best-known track for the number of minutes of GS the observation was early or late. If the shot was early, advance the LOP; if the shot was late, retard the LOP.

Figure 11.10. Entering Arguments Are Relative True Azimuth and Groundspeed.

Correction for 4 Minutes of Time — Ground Speed in Knots

Rel. Zn	90	120	150	180	210	240	270	300	330	360	390	420	450	480	510	540	570	600	630	660	690	720	750	780	810	840	870	900	Rel. Zn
000	+6	+8	+10	+12	+14	+16	+18	+20	+22	+24	+26	+28	+30	+32	+34	+36	+38	+40	+42	+44	+46	+48	+50	+52	+54	+56	+58	+60	000
005	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50	52	54	56	58	60	355
010	6	8	10	12	14	16	18	20	22	24	26	28	30	32	33	35	37	39	41	43	45	47	49	51	53	55	57	59	350
015	6	8	10	12	14	15	17	19	21	23	25	27	29	31	33	35	37	39	41	43	44	46	48	50	52	54	56	58	345
020	6	8	9	11	13	15	17	19	21	23	24	26	28	30	32	34	36	38	39	41	43	45	47	49	51	53	55	56	340
025	5	7	9	11	13	15	16	18	20	22	24	25	27	29	31	33	34	36	38	40	42	44	45	47	49	51	53	54	335
030	+5	+7	+9	+10	+12	+14	+16	+17	+19	+21	+23	+24	+26	+28	+29	+31	+33	+35	+36	+38	+40	+42	+43	+45	+47	+48	+50	+52	330
035	5	7	8	10	11	13	15	16	18	20	21	23	25	26	28	29	31	33	34	36	38	39	41	43	44	46	48	49	325
040	5	6	8	9	11	12	14	15	17	18	20	21	23	25	26	28	29	31	32	34	35	37	38	40	41	43	44	46	320
045	4	6	7	8	10	11	13	14	16	17	18	20	21	23	24	25	27	28	30	31	33	34	35	37	38	40	41	42	315
050	4	5	6	8	9	10	12	13	14	15	17	18	19	21	22	23	24	26	27	28	30	31	32	33	35	36	37	39	310
055	3	5	6	7	8	9	10	11	13	14	15	16	17	18	20	21	22	23	24	25	26	28	29	30	31	32	33	34	305
060	+3	+4	+5	+6	+6	+8	+9	+10	+11	+12	+13	+14	+15	+16	+17	+18	+19	+20	+21	+22	+23	+24	+25	+26	+27	+28	+29	+30	300
065	3	3	4	5	6	7	8	8	9	10	11	12	13	14	14	15	16	17	18	19	19	20	21	22	23	24	25	25	295
070	2	3	3	4	5	5	6	7	8	8	9	10	10	11	12	12	13	14	14	15	16	16	17	18	18	19	20	21	290
075	2	2	3	3	4	4	5	5	6	6	7	7	8	8	9	9	10	10	11	11	12	12	13	13	14	14	15	16	285
080	1	1	2	2	2	3	3	3	4	4	5	5	5	6	6	6	7	7	7	8	8	8	9	9	9	10	10	10	280
085	+1	+1	+1	+1	+1	+1	+2	+2	+2	+2	+2	+2	+3	+3	+3	+3	+3	+3	+4	+4	+4	+4	+4	+5	+5	+5	+5	+5	275
090	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	270
095	-1	-1	-1	-1	-1	-1	-2	-2	-2	-2	-2	-2	-3	-3	-3	-3	-3	-3	-4	-4	-4	-4	-4	-5	-5	-5	-5	-5	265
100	1	1	2	2	2	3	3	3	4	4	5	5	5	6	6	6	7	7	7	8	8	8	9	9	9	10	10	10	260
105	2	2	3	3	4	4	5	5	6	6	7	7	8	8	9	9	10	10	11	11	12	12	13	13	14	14	15	16	255
110	2	3	3	4	5	5	6	7	8	8	9	10	10	11	12	12	13	14	14	15	16	16	17	18	18	19	20	21	250
115	3	3	4	5	6	7	8	8	9	10	11	12	13	14	14	15	16	17	18	19	19	20	21	22	23	24	25	25	245
120	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	240
125	-3	-5	-6	-7	-8	-9	-10	-11	-13	-14	-15	-16	-17	-18	-20	-21	-22	-23	-24	-25	-26	-28	-29	-30	-31	-32	-33	-34	235
130	4	5	6	8	9	10	12	13	14	15	17	18	19	21	22	23	24	26	27	28	30	31	32	33	35	36	37	39	230
135	4	6	7	8	10	11	13	14	16	17	18	20	21	23	24	25	27	28	30	31	33	34	35	37	38	40	41	42	225
140	5	6	8	9	11	12	14	15	17	18	20	21	23	25	26	28	29	31	32	34	35	37	38	40	41	43	44	46	220
145	5	7	8	10	11	13	15	16	18	20	21	23	25	26	28	29	31	33	34	36	38	39	41	43	44	46	48	49	215
150	5	7	9	10	12	14	16	17	19	21	23	24	26	28	29	31	33	35	36	38	40	42	43	45	47	48	50	52	210
155	-5	-7	-9	-11	-13	-15	-16	-18	-20	-22	-24	-25	-27	-29	-31	-33	-34	-36	-38	-40	-42	-44	-45	-47	-49	-51	-53	-54	205
160	6	8	9	11	13	15	17	19	21	23	24	26	28	30	32	34	36	38	39	41	43	45	47	49	51	53	55	56	200
165	6	8	10	12	14	15	17	19	21	23	25	27	29	31	33	35	37	39	41	43	44	46	48	50	52	54	56	58	195
170	6	8	10	12	14	16	18	20	22	24	26	28	30	32	33	35	37	39	41	43	45	47	49	51	53	55	57	59	190
175	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50	52	54	56	58	60	185
180	-6	-8	-10	-12	-14	-16	-18	-20	-22	-24	-26	-28	-30	-32	-34	-36	-38	-40	-42	-44	-46	-48	-50	-52	-54	-56	-58	-60	180

Correction for Less Than 4 Minutes of Time
Value from 4-minute Motion Tables (For values greater than 60' see opposite page)

Interval of Time (m s)	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50	52	54	56	58	60	Interval of Time (m s)
0 10	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	2	2	0 10
20	0	0	0	0	1	1	1	1	2	2	2	2	2	2	2	3	3	3	3	3	4	4	4	4	4	4	4	5	5	5	20
30	0	0	1	1	1	2	2	2	2	2	3	3	3	4	4	4	4	4	5	5	5	6	6	6	6	6	7	7	7	8	30
40	0	1	1	1	2	2	2	3	3	3	4	4	4	5	5	5	6	6	6	7	7	7	8	8	8	9	9	9	10	10	40
50	0	1	1	2	2	2	3	3	4	4	5	5	5	6	6	7	7	8	8	8	9	9	10	10	10	11	11	12	12	12	50
1 00	0	1	2	2	2	3	4	4	4	5	6	6	6	7	8	8	8	9	10	10	10	11	12	12	12	13	14	14	14	15	1 00
10	1	1	2	2	3	4	4	5	5	6	6	7	8	8	9	9	10	10	11	12	12	13	13	14	15	15	16	16	17	18	10
20	1	1	2	3	3	4	5	5	6	7	7	8	9	9	10	11	11	12	13	13	14	15	15	16	17	17	18	19	19	20	20
30	1	2	2	4	4	4	5	6	7	8	8	9	10	10	11	12	13	14	14	15	16	16	17	18	19	20	20	21	22	22	30
40	1	2	2	4	5	5	6	7	8	8	9	10	11	12	12	13	14	15	16	17	18	18	19	20	21	22	22	23	24	25	40
50	1	2	3	5	5	6	6	7	8	9	10	11	12	13	14	15	16	16	17	18	19	20	21	22	23	24	25	26	27	28	50
2 00	1	2	3	5	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	2 00
10	1	2	3	5	5	6	8	9	10	11	12	13	14	15	16	17	18	20	21	22	23	24	25	26	27	28	29	30	31	32	10
20	1	2	4	6	6	7	8	9	10	12	13	14	15	16	18	19	20	21	22	23	24	26	27	28	29	30	32	33	34	35	20
30	1	2	4	6	6	8	9	10	11	12	13	14	15	16	18	20	21	22	24	25	26	28	29	30	31	32	34	35	36	38	30
40	1	3	4	7	7	8	9	11	12	13	15	16	17	19	20	21	23	24	25	27	28	29	31	32	33	35	36	37	39	40	40
50	1	3	4	7	7	9	10	11	13	14	16	17	18	20	21	23	24	26	27	28	30	31	33	34	35	37	38	40	41	42	50
3 00	2	3	6	8	8	9	10	12	14	15	16	18	20	21	22	24	26	27	28	30	32	33	34	36	38	39	40	42	44	45	3 00
10	2	3	5	6	8	10	11	13	14	16	17	19	21	22	24	25	27	28	30	32	33	35	36	38	40	41	43	44	46	48	10
20	2	3	5	7	8	10	12	13	15	17	18	20	22	23	25	27	28	30	32	33	35	37	38	40	42	43	45	47	48	50	20
30	2	4	5	7	9	10	12	14	16	18	19	21	23	24	26	28	30	32	33	35	37	38	40	42	44	46	47	49	51	52	30
40	2	4	6	7	9	11	13	15	16	18	20	22	24	26	28	29	31	33	35	37	38	40	42	44	46	48	50	51	53	55	40
50	2	4	6	8	10	12	13	15	17	19	21	23	25	27	29	31	33	34	36	38	40	42	44	46	48	50	52	54	56	58	50
4 00	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50	52	54	56	58	60	4 00

Time of fix	Sign from 4-min. Table	To observed altitude	To tabulated altitude	To intercept
Later than observation	+	Add / Subtract	Subtract / Add	Toward / Away
Earlier than observation	−	Subtract / Add	Add / Subtract	Away / Toward

Figure 11.11. Moving Assumed Positions.

11.15. Planning the Fix. In selecting bodies for observation, one should generally consider azimuth primarily and such factors as brightness, altitude, etc., secondarily. If all observations were precisely correct in every detail, the resulting LOPs would meet at a point. However, this is rarely the case. Three observations generally result in LOPs forming a triangle. If this triangle is not more than 2 or 3 miles on a side under good conditions and 5 to 10 miles under unfavorable conditions, there is normally no reason to suppose that a mistake has been made. Even a point fix, however, is not necessarily accurate. An uncorrected error in time, for instance, would require the entire fix to be moved eastward if observations were early and westward if observations were late, at the rate of 1 minute of longitude for each 4 seconds of time.

11.15.1. In a two-LOP fix, the ideal cut of the LOPs is 90°. In Figure 11.12, a 90° cut with a 5 NM error in one LOP will cause a 5 NM error in the fix. If the acute angle between the LOPs is 30°, a 5 NM error in one LOP will cause a 10 NM error in the fix. Thus, with a two-LOP fix, an error in one LOP will cause at least an equal error in the fix; the smaller the acute angle between the LOPs, the greater the fix error caused by a given error in one LOP. Of course, if both LOPs are in error, the fix may be thrown off even more. In a three-LOP fix, the ideal cut of the LOPs is 60° (star azimuths 120° apart). With this cut, a 3 NM error in any one LOP will cause a 2 NM error in the fix. With any other cut, a 3 NM error in any one LOP will cause more than a 2 NM error in the fix. In a three-star fix, the cut will be 60° if the azimuths of the stars differ by 60° or if they differ by 120°. If there is any unknown constant error in the observations, all the Hos will be either too great or too small. Notice in Figure 11.13 that, if stars are selected whose azimuths differ by 120°, this constant error of the Hos will cause a displacement of the three LOPs, either all toward the center or all away from the center of the triangle. In either case, the position of the center of the triangle will not be affected. If you use any three stars with azimuths outside a 180° range, any constant error in observations will tend to cancel out.

Figure 11.12. Effect of Cut on Accuracy of a Fix.
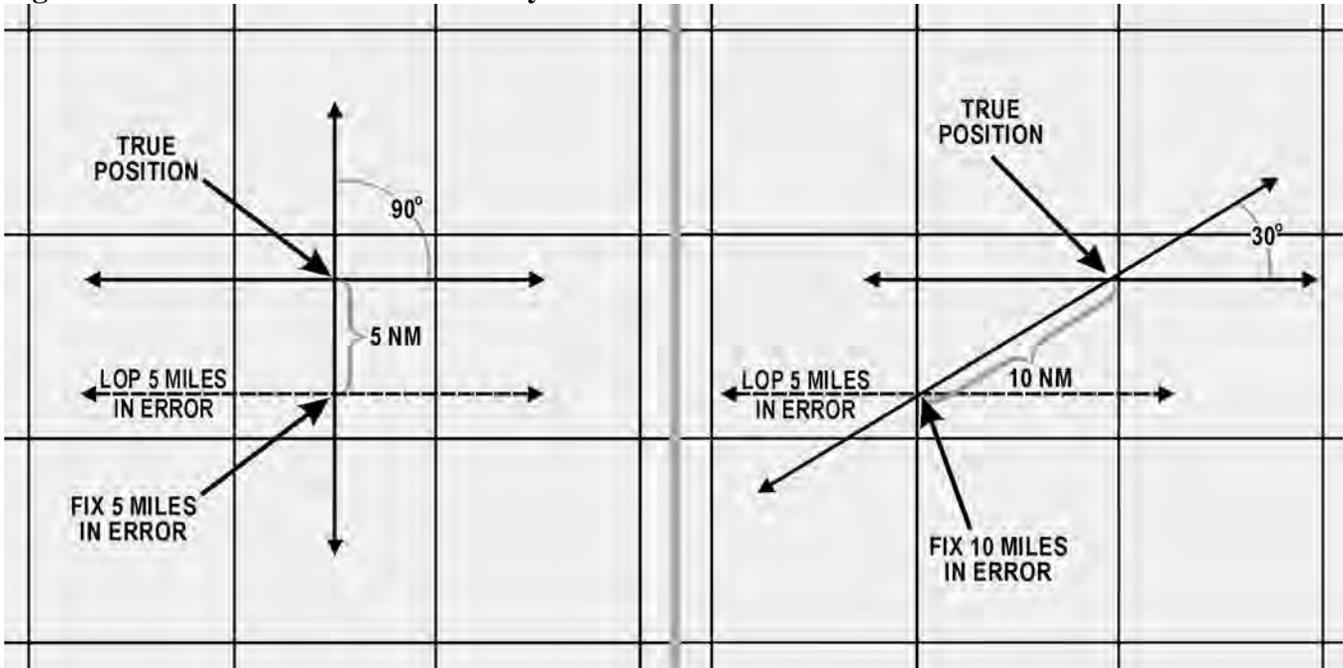
Figure 11.13. Effect of Azimuth on Accuracy of Fix.
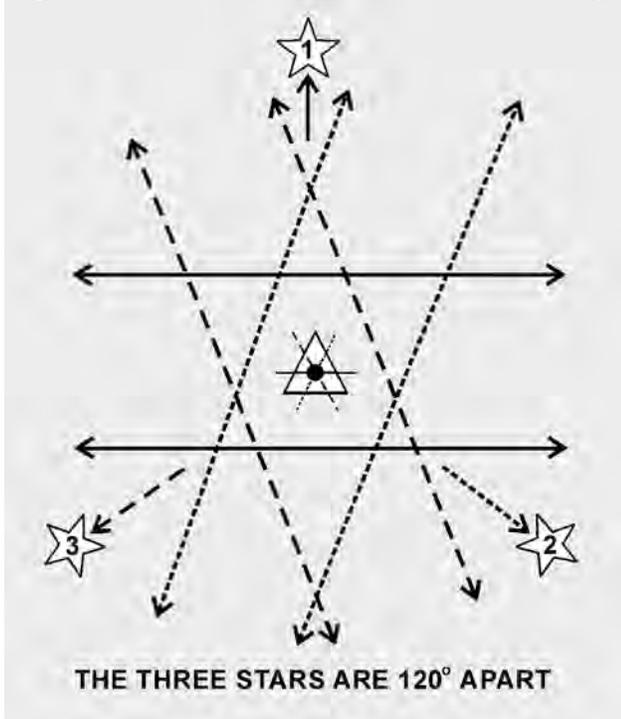
11.15.2. The three-star fix has two distinct advantages over the two-star fix. First, it is the average of three observations. Second, selecting the stars carefully can counteract the effect of constant errors of observation. There is also a third advantage. Each pair of two LOPs furnishes a rough check on the third. In resolving an observation into a LOP, you might possibly make a gross error; for example, obtaining

an LHA which is in error by a whole degree. Such an error might not be immediately apparent. Neither would such a discrepancy come to immediate attention in a two-LOP fix. However, this third advantage does not apply when a single LHA is used in solving all LOPs, such as is done when precomputing and using motion corrections to resolve all LOPs to a common time. Because of these three advantages, it is evident that a three-star fix should be used, rather than a two-star fix, when possible.

11.15.3. Whatever the number of observations, common practice, backed by logic, is to take the center of the figure formed unless there is reason for deviating from this procedure. Center is meant as the point representing the least total error of all lines considered reliable. With three LOPs, the center is considered that point, within the triangle equidistant from the three sides. It may be found by bisecting the angles, but is usually located by eye.

11.16. Summary. Because of all the factors involved, a certain amount of judgment is necessary, along with the proper use of the mechanics comprising celestial navigation. When using a single LOP or a fix, you have to take into consideration the existing conditions and weigh the DR information against the information obtained from the LOP. An accurate DR position should always be computed.

11.16.1. The C-Plot formula helps place the MPP with a single LOP, but you might want to make further adjustments to the final position. The formula is:

$$\frac{d}{t} = \frac{p}{t+p}$$

11.16.2. Remember, d is the distance measured along a perpendicular from the DR position to the LOP. In the case of the two- or three-star fix, planning plays a very important part. Selecting stars whose azimuths differ by 120° for a three-star fix will minimize errors in the fix position. In two-star fixes, the ideal azimuth separation is 90°. Also, when dealing with more than one LOP, it is necessary to resolve the LOPs to a common time. This adjustment can be accomplished by moving the assumed position, by moving the LOPs, or by applying a correction factor to the Hc or Ho.

Chapter 12

SPECIAL CELESTIAL TECHNIQUES

Section 12A—Introduction

12.1. Basics. This chapter has some techniques which may not be used every day and under all circumstances but are valuable alternatives from normal precomping procedures. Most of these techniques save time by eliminating either some extractions or computations. Some navigational techniques and planning procedures are also discussed.

12.2. Determining Availability of Celestial Bodies. By doing a quick comparison of GHA to the observer's position, it is easy to determine the availability of celestial bodies. For example, the observer anticipates being at 18° N 135° W at 0015Z on 28 September 1995. There are several bodies listed in the *Air Almanac*, but not all of them are available for observation. To determine availability, take the observer's longitude and look 80° either side of it. Within this range, compare the GHA of a body. Looking at Figure 12.1 we see that the sun, moon, Venus, and Jupiter are within the 80° range and are therefore usable. Saturn is outside of the 80° range so it is not usable. The declination of a body is not normally a factor; however, at high latitudes a body may not be available when its subpoint is near the pole opposite the observer.

Figure 12.1. A Quick Check of Body Availability.

(DAY 271) GREENWICH A. M. 1995 SEPTEMBER 28 (THURSDAY)   541

UT (GMT)	☉ SUN GHA	☉ SUN Dec.	ARIES GHA ♈	VENUS-3.9 GHA	VENUS-3.9 Dec.	JUPITER-2.0 GHA	JUPITER-2.0 Dec.	SATURN 0.7 GHA	SATURN 0.7 Dec.	☾ MOON GHA	☾ MOON Dec.	Lat.	Moon-rise	Diff.
h m	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	N	h m	m
00 00	182 15.7	S 1 45.0	6 18.1	172 23	S 4 48	117 52	S21 31	14 16	S 6 00	141 29	S15 06	°		
10	184 45.8	45.1	8 48.6	174 53		120 22		16 46		143 53	08	72	13 34	*
20	187 15.8	45.3	11 19.0	177 23		122 53		19 17		146 18	09	70	12 43	+54
30	189 45.8	· 45.5	13 49.4	179 53	· ·	125 23	· ·	21 47	· ·	148 42	· 10	68	12 11	48
40	192 15.9	45.6	16 19.8	182 23		127 53		24 18		151 06	11	66	11 48	45
50	194 45.9	45.8	18 50.2	184 53		130 24		26 48		153 31	12	64	11 29	42
01 00	197 15.9	S 1 46.0	21 20.6	187 23	S 4 49	132 54	S21 31	29 18	S 6 00	155 55	S15 13	62	11 14	40
10	199 46.0	46.1	23 51.0	189 53		135 25		31 49		158 20	15	60	11 02	39
20	202 16.0	46.3	26 21.4	192 23		137 55		34 19		160 44	16	58	10 51	38
30	204 46.0	· 46.4	28 51.8	194 52	· ·	140 25	· ·	36 50	· ·	163 09	· 17	56	10 42	37
40	207 16.1	46.6	31 22.3	197 22		142 55		39 20		165 33	18	54	10 33	36
50	209 46.1	46.8	33 52.7	199 52		145 26		41 51		167 57	19	52	10 26	36
02 00	212 16.1	S 1 46.9	36 23.1	202 22	S 4 50	147 56	S21 32	44 21	S 6 00	170 22	S15 20	50	10 19	35
10	214 46.2	47.1	38 53.5	204 52		150 27		46 52		172 46	21	45	10 05	34
20	217 16.2	47.2	41 23.9	207 22		152 57		49 22		175 11	23	40	09 53	33
30	219 46.3	· 47.4	43 54.3	209 52	· ·	155 27	· ·	51 52	· ·	177 35	· 24	35	09 43	32
40	222 16.3	47.6	46 24.7	212 22		157 58		54 23		179 59	25	30	09 35	31
50	224 46.3	47.7	48 55.1	214 52		160 28		56 53		182 24	26			

12.3. Latitude by Polaris. Polaris is the polestar, or North Star. Because Polaris is approximately 1° from the North Pole, it makes a small diurnal circle and seemingly stays in about the same place all night. This fact makes Polaris very useful in navigation. With certain corrections, it serves as a reference point for direction and for latitude in the Northern Hemisphere. Latitude by Polaris is a quick method of obtaining a latitude line of position (LOP); only the tables given in the *Air Almanac* are needed.

12.3.1. Obtaining Latitude by Polaris. A latitude by Polaris LOP is obtained by applying the "Q" correction (Figure 12.2) to the corrected observed altitude. This adjusts the altitude of the pole, which is equal to the navigator's latitude. The Q correction table is in the back of the *Air Almanac*. The entering argument for the table is exact local hour angle (LHA) of Aries. The effect of refraction is not included in Q correction, so the observed altitude must be fully corrected. When refraction is used for a latitude by Polaris LOP, it is applied to the observed altitude and the sign of the correction is negative. A Polaris LOP can also be plotted using the intercept method. In this case, the Hc is computed by reversing the sign of the Q correction and applying it to the assumed latitude (rounded off to the nearest degree). Refraction is positive when applied to get an Hc for the intercept method.

12.3.2. Obtaining Azimuth of Polaris. For either method, the azimuth of Polaris is obtained from the Azimuth of Polaris table found in the *Air Almanac* or in the Pub. No. 249 (Figure 12.2). Whether plotted as an intercept or a latitude, the assumed position should be corrected for Coriolis or rhumb line and precession or nutation. The resulting LOPs should fall in the same place for either method. To plot the LOP using the latitude method, choose the longitude line closest to the DR and plot perpendicular to the longitude line. For the intercept method, use the assumed latitude and plot the intercept normally using the azimuth of Polaris.

Figure 12.2. Polaris Q Correction and Azimuth Tables From the *Air Almanac*.

POLARIS (POLE STAR) TABLE, 1995   A167

FOR DETERMINING THE LATITUDE FROM A SEXTANT ALTITUDE

LHA Aries	Q	LHA Aries	Q	LHA Aries	Q	LHA Aries	Q	LHA Aries	Q	LHA Aries	Q	LHA Aries	Q	LHA Aries	Q
° ′	′	° ′	′	° ′	′	° ′	′	° ′	′	° ′	′	° ′	′	° ′	′
358 46		86 00		120 50		153 30		227 46		281 46		314 17		349 32	
0 50	−36	87 38	−29	122 07	− 4	154 56	+21	233 14	+44	283 10	+19	315 33	− 6	351 16	−31
3 01	−37	89 14	−28	123 23	− 3	156 23	+22	237 18	+43	284 32	+18	316 50	− 7	353 02	−32
5 20	−38	90 49	−27	124 39	− 2	157 51	+23	240 42	+42	285 55	+17	318 07	− 8	354 53	−33
7 48	−39	92 21	−26	125 54	− 1	159 21	+24	243 42	+41	287 16	+16	319 24	− 9	356 47	−34
10 29	−40	93 52	−25	127 11	0	160 53	+25	246 25	+40	288 36	+15	320 42	−10	358 46	−35
13 27	−41	95 21	−24	128 27	+ 1	162 26	+26	248 55	+39	289 56	+14	322 00	−11	0 50	−36
16 49	−42	96 49	−23	129 43	+ 2	164 01	+27	251 15	+38	291 16	+13	323 19	−12	3 01	−37
20 50	−43	98 15	−22	130 59	+ 3	165 37	+28	253 28	+37	292 35	+12	324 38	−13	5 20	−38
26 13	−44	99 40	−21	132 16	+ 4	167 17	+29	255 33	+36	293 53	+11	325 57	−14	7 48	−39
47 38	−45	101 05	−20	133 32	+ 5	168 58	+30	257 34	+35	295 11	+10	327 18	−15	10 29	−40
53 01	−44	102 28	−19	134 49	+ 6	170 43	+31	259 29	+34	296 28	+ 9	328 39	−16	13 27	−41
57 02	−43	103 51	−18	136 05	+ 7	172 31	+32	261 20	+33	297 46	+ 8	330 00	−17	16 49	−42
60 24	−42	105 12	−17	137 23	+ 8	174 22	+33	263 08	+32	299 02	+ 7	331 23	−18	20 50	−43
63 22	−41	106 33	−16	138 40	+ 9	176 17	+34	264 53	+31	300 19	+ 6	332 46	−19	26 13	−44
66 03	−40	107 54	−15	139 58	+10	178 18	+35	266 34	+30	301 35	+ 5	334 11	−20	47 38	−45
68 31	−39	109 13	−14	141 16	+11	180 23	+36	268 14	+29	302 52	+ 4	335 36	−21	53 01	−44
70 50	−38	110 32	−13	142 35	+12	182 36	+37	269 50	+28	304 08	+ 3	337 02	−22	57 02	−43
73 01	−37	111 51	−12	143 55	+13	184 56	+38	271 25	+27	305 24	+ 2	338 30	−23	60 24	−42
75 05	−36	113 09	−11	145 15	+14	187 26	+39	272 58	+26	306 40	+ 1	339 59	−24	63 22	−41
77 04	−35	114 27	−10	146 35	+15	190 09	+40	274 30	+25	307 57	0	341 30	−25	66 03	−40
78 58	−34	115 44	− 9	147 56	+16	193 09	+41	276 00	+24	309 12	− 1	343 02	−26	68 31	−39
80 49	−33	117 01	− 8	149 19	+17	196 33	+42	277 28	+23	310 28	− 2	344 37	−27	70 50	−38
82 35	−32	118 18	− 7	150 41	+18	200 37	+43	278 55	+22	311 44	− 3	346 13	−28	73 01	−37
84 19	−31	119 34	− 6	152 05	+19	206 05	+44	280 21	+21	313 01	− 4	347 51	−29	75 05	−36
86 00	−30	120 50	− 5	153 30	+20	227 46	+45	281 46	+20	314 17	− 5	349 32	−30	77 04	−35

In critical cases, ascend

Q, which does not include refraction, is to be applied to the corrected sextant altitude of *Polaris*.
Polaris: Mag. 2.1, SHA 323° 04′, Dec N 89° 14′7

AZIMUTH OF *POLARIS*, 1995

LHA Aries	Latitude 0°	30°	50°	55°	60°	65°	70°	LHA Aries	Latitude 0°	30°	50°	55°	60°	65°	70°
°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
0	0·5	0·5	0·7	0·8	0·9	1·1	1·4	180	359·5	359·5	359·3	359·2	359·1	359·0	358·7
10	0·3	0·4	0·5	0·6	0·7	0·8	1·0	190	359·7	359·6	359·5	359·4	359·3	359·2	359·0
20	0·2	0·3	0·3	0·4	0·4	0·5	0·7	200	359·8	359·7	359·7	359·6	359·6	359·5	359·4
30	0·1	0·1	0·1	0·2	0·2	0·2	0·3	210	359·9	359·9	359·9	359·8	359·8	359·5	359·7
40	0·0	0·0	359·9	359·9	359·9	359·9	359·9	220	0·0	0·0	0·1	0·1	0·1	0·1	0·1
50	359·8	359·8	359·7	359·7	359·7	359·6	359·5	230	0·2	0·2	0·3	0·3	0·3	0·4	0·5
60	359·7	359·7	359·5	359·5	359·4	359·3	359·1	240	0·3	0·3	0·5	0·5	0·6	0·7	0·8
70	359·6	359·5	359·4	359·3	359·2	359·0	358·8	250	0·4	0·5	0·6	0·7	0·8	1·0	1·2
80	359·5	359·4	359·2	359·1	359·0	358·8	358·5	260	0·5	0·6	0·8	0·9	1·0	1·2	1·5
90	359·4	359·3	359·1	358·9	358·8	358·5	358·2	270	0·6	0·7	0·9	1·0	1·2	1·4	1·7
100	359·3	359·2	358·9	358·8	358·6	358·4	358·0	280	0·7	0·8	1·0	1·2	1·3	1·6	1·9
110	359·3	359·2	358·9	358·7	358·5	358·3	357·9	290	0·7	0·8	1·1	1·3	1·4	1·7	2·1
120	359·3	359·1	358·8	358·7	358·5	358·2	357·8	300	0·7	0·9	1·2	1·3	1·5	1·8	2·2
130	359·2	359·1	358·8	358·7	358·5	358·2	357·8	310	0·8	0·9	1·2	1·3	1·5	1·8	2·2
140	359·3	359·2	358·9	358·7	358·5	358·3	357·9	320	0·7	0·9	1·1	1·3	1·5	1·8	2·2
150	359·3	359·2	358·9	358·8	358·6	358·4	358·0	330	0·7	0·8	1·1	1·2	1·4	1·7	2·1
160	359·4	359·3	359·0	358·9	358·8	358·5	358·2	340	0·6	0·7	1·0	1·1	1·3	1·5	1·9
170	359·4	359·4	359·2	359·1	358·9	358·7	358·4	350	0·6	0·6	0·9	1·0	1·1	1·3	1·7
180	359·5	359·5	359·3	359·2	359·1	359·0	358·7	360	0·5	0·5	0·7	0·8	0·9	1·1	1·4

When Cassiopeia is left (right), *Polaris* is west (east).

Figure 12.3. Plotting the Polaris LOP.
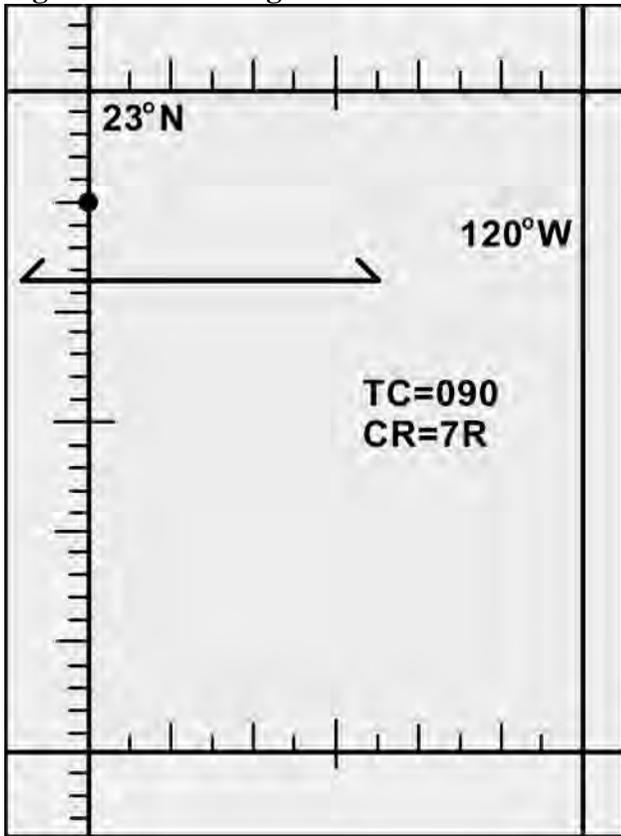
12.3.3. Latitude by Polaris Example. On 18 April 1995 for Greenwich mean time (GMT) 1600 at 23°-10' N 120° W, with an observed altitude 23°-06' at 31,000'. When doing a latitude by Polaris you must use the exact latitude and longitude. See Figure 12.3 for plotting.

GHA	086°-18'
Longitude (West)	-120°-00'
LHA	326°-18'
True Course (TC) = 090°	
Groundspeed (GS) = 400 knots	
Coriolis/rhumb line = 7R	
Corrected Observed Altitude	23°-06'
Q (based on LHA 072-44)	-15'
Refraction	-01'
Latitude	22°-50'
Azimuth (LHA 326°-18', Latitude 23°N) = 000.8°	

NOTE: If the Q correction table in Volume 1 is used, precession and nutation (P/N) and Coriolis or rhumb line must be used in plotting the LOP. This is because the Pub. No. 249 covers a 5-year period and the further the years get from the Epoch year, the greater the error is when using the Polaris table. P/N compensates for this error.

12.3.4. Intercept Method Example. Referring to the previous problem (see Figure 12.3 for plotting). *NOTE:* Applying 10A to assumed latitude gives 22°-50' N, which is the same the answer in the latitude by Polaris example.

Azimuth of Polaris = 359.5	
Coriolis/rhumb line = 7R	
Assumed Lat (rounded off)	23°-00'N
Q (Reversed Sign)	+15'
Refraction	+01'
Hc Polaris	23°-16'
Ho Polaris	23°-06'
Intercept	10A

NOTE: In these examples all information was taken from the *Air Almanac*. No P/N is required.

Section 12B—LHA Method of Fixing

12.4. LHA Method of Obtaining Three-Star Fix. The LHA technique allows you to solve the motion problem for a three-star fix by applying a correction to the assumed position rather than computing a numerical solution on the precomp. This eliminates mathematical motion calculations, therefore reducing the chance of math errors on the precomp. To accomplish a three-LHA fix, you must plan 4 minutes between the midtime of each shot (Figures 12.4 and 12.5). Because LHA changes 1 degree for every 4 minutes, the precomp will have three successive LHAs, 1 degree apart. To correct for off-time motion, adjust the assumed position based on true course and groundspeed. If a shot is planned earlier than fix time, the assumed position is advanced (down-track). For shots planned later than fix time, the assumed position is retarded (up-track).

12.4.1. The example in Figures 12.4 and 12.5 shows the LHA method for a 12-8-4 early shooting schedule. This shooting schedule allows the fix and/or MPP to be resolved before the fix time. To adjust the assumed positions, plot the fix time assumed position and then advance it for 4 minutes of track and groundspeed for each body. This will satisfy motion of the observer. When shooting the selected bodies, take care to shoot them exactly on the prescribed times. This will eliminate motion of the body.

Figure 12.4. Typical Example of Three LHA Method.

CELESTIAL PRECOMPUTATION	SHEET NUMBER *14*

PRECOMPUTATION - PERISCOPIC SEXTANT

NAVIGATOR *Major Watters*	ALT MSL *FL 200*	DATE (Z) *1 Jan 1993*	FIX TIME *0702* (Z)

STAR SELECTION BY AZIMUTH								
TRACK	*090*		BODY	*Dubhe*	*Sirius*	*Capella*		
GS	*400*		BASE GHA			*205-57*		
CORIOLIS	*7*	(R) L	CORR			*+30*		
PREC/NUT	*1 / 270*		(+ 360)					
DR LAT	*33-02*	(N) S	GHA			*206-27*		
DR LONG	*100-30*	E (W)	ASSUM LONG (W) +E			*100-27*		

MOTION OF OBSERVER						LHA	*103*	*104*	*105* / *106*		
MOTION OF BODY						ASSUM LAT	*33* (N) S				
4 MIN ADJUST						DEC	N S	N S	N S	N S	N S
OFF-FIX TIME	*12* (E) L	*8* (E) L	*4* (E) L	E L	E L	PLANNED MID-TIME	*0650*	*0654*	*0658*		
TOTAL MOT. ADJUST						ACTUAL MID-TIME					
POLARIS Q / MOON PA SD						TAB Hc					
REF (-)	*-01*	*-01*				D/DEC CORR					
PERS / SEXT						CORR Hc	*41-22*	*40-13*	*66-19*		
TOTAL ADJ →	*-01*	*-01*				TOTAL → ADJ	*+01*	*+01*			
TH / GH						ADJ Hc	*41-23*	*40-14*	*66-19*		
Zn / GZn (-)						OFF TIME MOTION					
IRB						Hc	*41-23*	*40-14*	*66-19*		
IRB						Ho	*41-15*	*39-54*	*65-59*		
Zn / GZn (+)						INT	*8* T (A)	*20* T (A)	*20* T (A)	T A	T A
TH / GH						Zn	*034*	*183*	*311*		
TRACK T/G						CONV + W ANGLE - E					
Zn						GRID Zn					

REFRACTION TABLE (Condensed)
Ro	ALTITUDE MSL (Thousands of feet)					
	0	20	25	30	35	40
1	63	46	41	36	31	26
2	33	19	16	14	11	9
3	21	12	10	8	7	5
4	16	8	7	6	5	3-10
	12	7	5	4	3-10	2-10

REL Zn						TK	DC	TH	VAR	MH	DEV	CH

Figure 12.5. Plotting Three LHA.
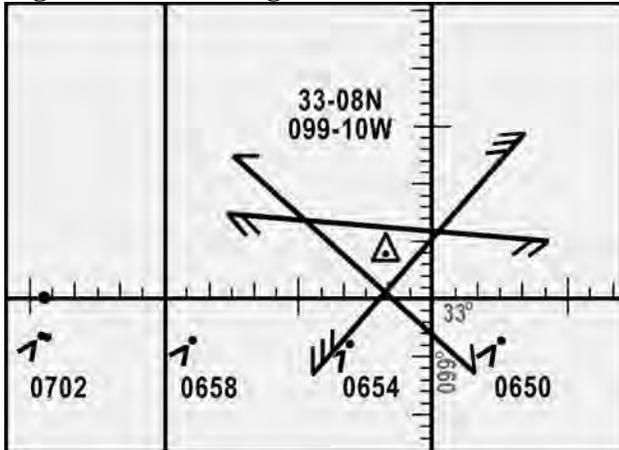
12.4.2. A variation of advancing the assumed position is to use half motions. This will enable you to plot all three LOPs from one assumed position. Table 1 from Pub. No. 249 lists corrections to position of the observer. Each correction is for 4 minutes of time. To use it, enter with your relative Zn (Zn-Track) and groundspeed. Now look at the bottom of the table and note you can apply this correction to your tabulated altitude or observed altitude. It doesn't matter which you choose but note that the sign will change dependent on where you apply it. Now take the number and multiply it by the 4-minute increment of the shot. For example, Figure 12.6 shows the precomp for a 0300 fix using 3 LHAs and half motions. The 0248 shot, Alpheratz, relative Zn, and groundspeed were used to extract a +20 correction from Table 1. Because this shot is 12 minutes early we need to multiply +20 by three before we apply it to the shot. Note the +60 correction was applied to the observed altitude and therefore kept its positive sign. The benefit of doing this is a reduction in plotting. See Figure 12.7 for the plotted LOPs. This technique can be applied to day celestial as well.

Figure 12.6. Half Motions Three LHA Format.

<div>28SEPT95</div>

Section	Row	Col1	Col2	Col3	Col4	Col5
D R	TIME			0300		
	LAT			N27-18		
	LONG			W121-04		
P R E C O M P	BODY	ALPHERATZ	ANTARES	ALKAID		
	GHA			51-25		
	CORR			—		
	SHA			—		
	GHA			51-25		
	+360			411-25		
	ASSUM LONG -W +E			W121-25		
	LHA	287	288	289 / 290		
	ASSUM LAT			N 27-		
	DEC			—		
	TAB HC	24-56	23-45	25-03		
	d	⌐ -	⌐ -	⌐ -	⌐	⌐
	DEC	⌐	⌐	⌐	⌐	⌐
	CORR HC	24-56	23-45	25-03		
C & R O R M R O S T S	HS TIME	0248	0252	0256		
	HS	24-19	23-55	25-16		
	SEXT	O	O	O		
	REFR	-1	-1	-1		
	PA / SD / Q	—	—	—		
	MOT OBS	+60	-34	-8		
	MOT BODY	—	—	—		
	TOTAL	+59	-35	-9		
	HO	25-18	23-20	25-07		
	INTCPT	22T	25A	4T		
	ZN	069	220	315		
G R I H D D G S	CA / IFB					
	GZN / TH					
	MAG VAR					
	MAG HDG					
	DEV CORR					
	CH					
M I S C	TRACK	068	068	068		
	ZN-TR	001	152	247		
	GS	310				
	ALT	220				
	CORIOLIS	4R				
	P-N	N/A				

Figure 12.7. Plotting a Half Motions Observation.
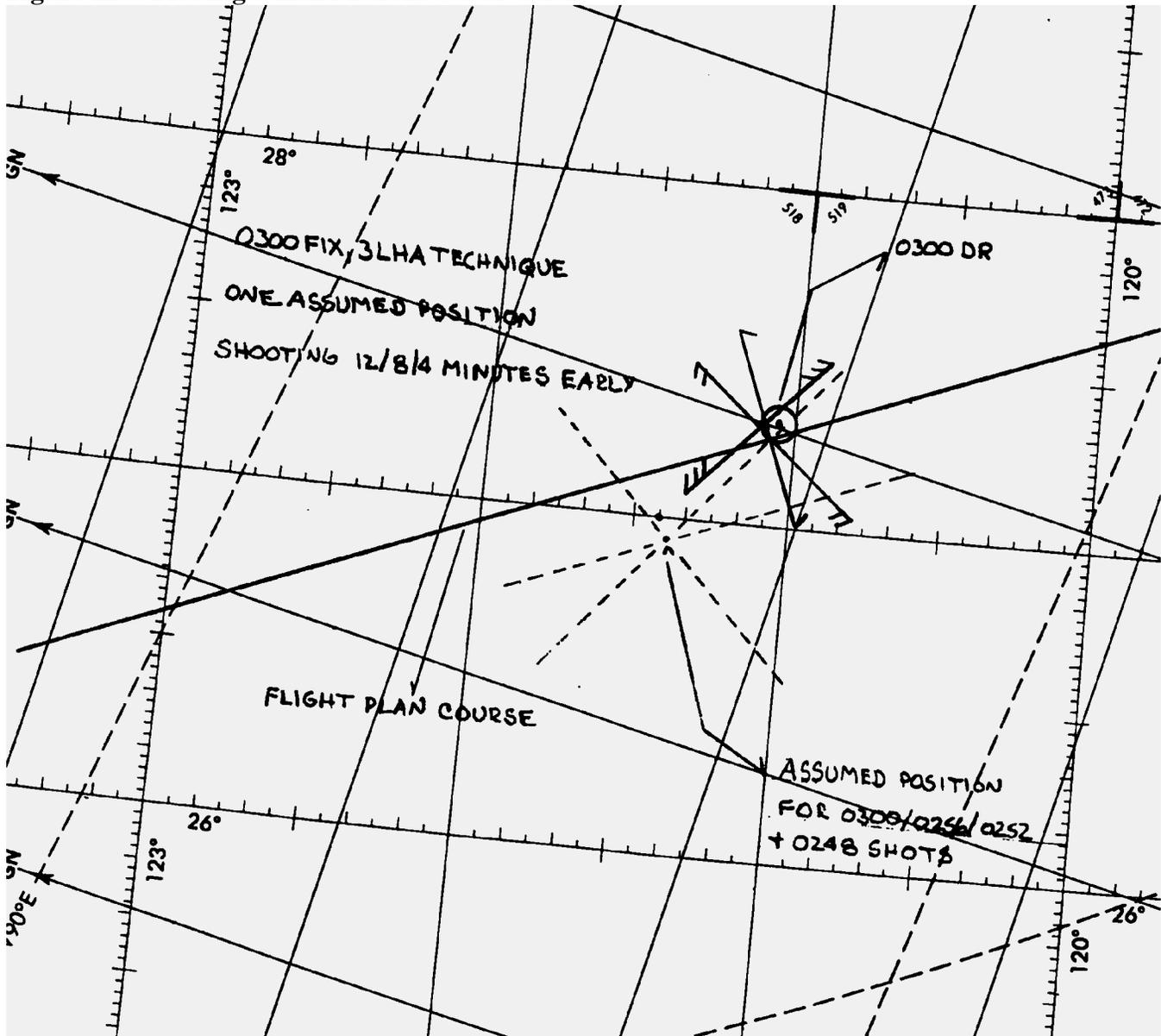
Section 12C—Daytime Celestial Techniques

12.5. Basics. Daytime fixing, using celestial techniques, is rather limited because often only one body, the sun, is visible. Ordinarily, three LOPs cannot be obtained for a fix from one body because the LOPs plot nearly parallel to each other.

12.6. The Sun Heading Shot at High Noon. The azimuth of the sun changes very rapidly when the subpoint of the sun is directly over the longitude of the observer, i.e., called the time of transit. The LHA at transit time is 360°. This phenomenon is more pronounced at lower latitudes as the subpoint of the sun passes closer to the observer. This makes it extremely difficult to get an accurate celestial heading shot at the transit time. Therefore, if you need a heading shot near the time of transit, you must take extra precaution to get the heading observation exactly at the precomputed fix time. If the moon or Venus is available, consider using these bodies for an accurate celestial heading. If using the sun, you should

weigh the increased possibility of an inaccurate heading shot. If the accuracy is questionable, get another heading shot as the sun's rate of azimuth change slows enough to allow a more accurate shot.

12.7. Intercept Method. The intercept method is normally used in obtaining a noonday fix. If the sun passes close to the observer's position, within about 4°, the subpoint method of plotting the fix may be used. This method differs from normal procedures in that three different precomps for three different times are computed. Because of the rapid change of the sun's azimuth at or near transit, this variation is necessary. The procedure is:

12.7.1. Determine the time of transit.

12.7.2. Select the LHA before and after transit for which the change in azimuth is 30° or more. Since 1° of LHA is equal to 4 minutes of time, the difference in transit LHA and the new LHA can be converted to time in minutes. Thus, the time preceding and following transit can be determined.

12.7.3. Plot the DR positions for times determined in 12.7.2. Select the appropriate assumed positions necessary for the computation and plotting of the LOPs. The assumed position for time of transit is also plotted.

12.7.4. Determine the intercepts and azimuth for each LOP. Plot these data from the respective assumed positions.

12.7.5. Resolve the LOPs to a common time, preferably that of the transit LOP.

NOTE: At 30° N latitude, the linear speed of the sun is approximately 780 knots. Thus, on westerly headings in high-speed aircraft, the DR distance involved before encountering a 30° change in azimuth will be considerable.

12.8. Subpoint Method. When the observer is within approximately 4° of the subpoint of the body, the subpoint method of solution is normally used. This is because the radius of the circle of equal altitude is so small that a straight line does not approximate the arc and a straight line will not give an accurate LOP. The procedure is:

12.8.1. Plot the subpoints of the body for the time of the observations (using GHA and/or Dec).

12.8.2. Find the co-altitude of the shots and convert it to NM ($[90^o – Alt]$ x 60 NM).

12.8.3. Advance the first subpoint and retard the third along the DR track, using best-known track and GS.

12.8.4. Set the distance found from the co-altitude and strike it off from the resolved subpoints (with a compass or pair of dividers). Do this for each observation.

NOTE: The resulting intersection or triangle will give an on-time fix. If the LOPs form a triangle, the aircraft position is probably within the triangle.

12.8.5. The subpoint method is convenient because Pub. No. 249 isn't used—only the *Air Almanac*. This method can also be used with a star near your assumed position and may be necessary if, for some reason, your Volume 1 is unavailable. The stars Dec and GHA are needed to determine if the observer is within 4° of the subpoint. The *Air Almanac* may be used to find the Dec and sidereal hour angle (SHA) of the star. The SHA of the star is added to the GHA of Aries to find the GHA of the star.

12.9. Eliminating Motions with the Bracket Technique. For sun observations, you can eliminate motion calculations by using a shooting schedule of 3 minutes early, on fix time and 3-minutes late. With this schedule, the 3-minute early and 3-minute late shots have the same magnitude of motion, but an opposite sign. Therefore, these motions cancel each other out and do not need to be computed. The on-time shot has no motions. Therefore, the three intercepts can be averaged for a single LOP. At night, shooting the same star 4 minutes early and late, with a different star shot on time can employ a similar method. In this case, the intercepts for the same star's 4-minute early or late shots can be averaged. This reduces workload, but only two LOPs are obtained.

12.10. DR Computer Modification. Rather than eliminating motions, your DR computer can be modified so both observer and body motions can be computed at one time, without entry into the Pub. No. 249. Make a GS and latitude scale as shown in Figure 12.8. After constructing these, the DR computer can be modified for quick and accurate computations of 1-minute motion adjustments.

12.10.1. Tape the GS scale (0 through 900) along the centerline of the grid scale. Match zero to zero, 300 to 50 and 600 to 100 as shown in Figure 12.8. Then, tape the latitude scale along the zero grid line so that 90° falls on the centerline and the scale extends to the left as shown. Check the accuracy of your placement: 30° latitude should fall 13 divisions left of centerline. Juggle the scale as necessary to provide the greatest accuracy between 30° and 45°.

Figure 12.8. MB-4 Motions Modification.
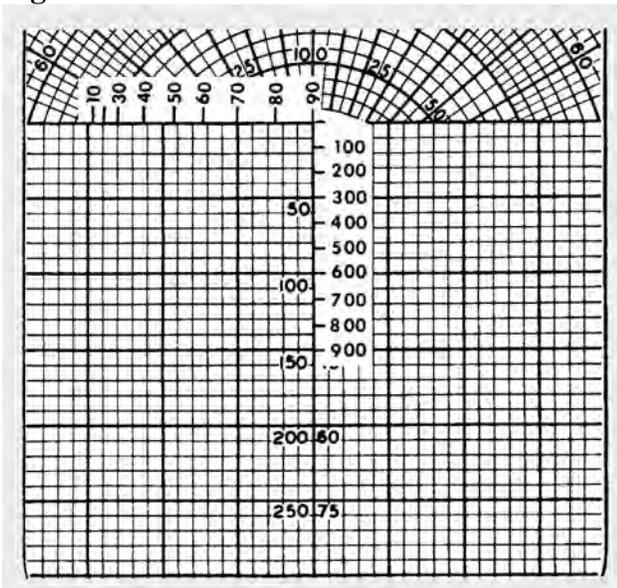
12.10.2. To use the modified MB-4 computer for motion adjustments:

12.10.2.1. Set true north under the index. If computing for grid, set polar angle (PA) under the index. In the NW and SE hemisphere quadrants PA equals convergence angle (CA). In the NE and SW quadrants

PA=360-CA. Next place the grommet over the zero grid line. Mark a cross (+) at the assumed latitude (Figure 12.9).

Figure 12.9. Celestial Motions – Step One.
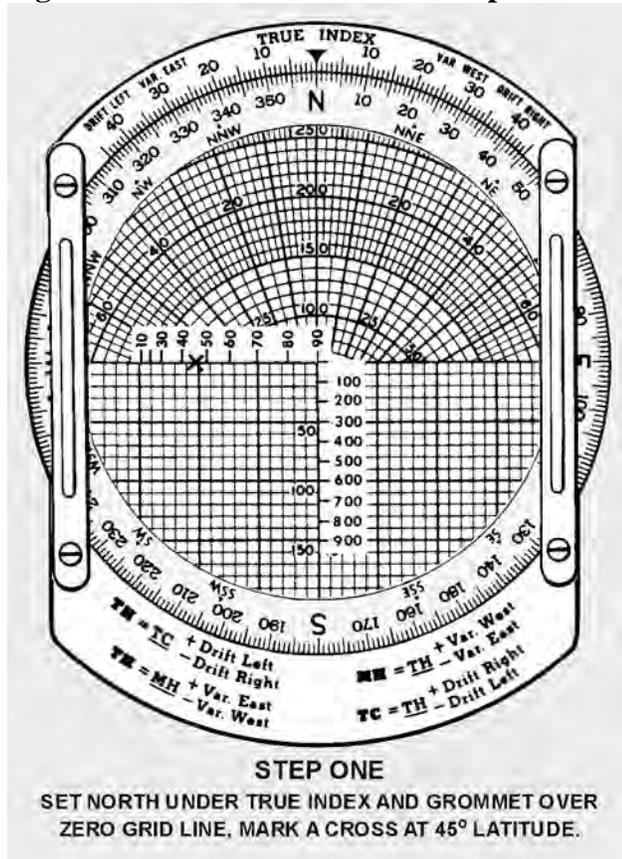
12.10.2.2. Set track (or grid track) under the index and position the slide so the GS is under the grommet. Place a dot on the zero point of the grid scale (Figure 12.10).

12.10.2.3. Place the Zn (or grid Zn) of the body under the index. Position the slide so the cross or the dot, whichever is uppermost, is on the zero line of the grid (Figure 12.11).

Figure 12.10. Celestial Motions – Step Two.

STEP TWO

SET TRACK 270 UNDER TRUE INDEX, MOVE SLIDE SO THAT GS
240 Kts IS UNDER GROMMET, MARK DOT ⊙ AT ZERO GRID LINE.
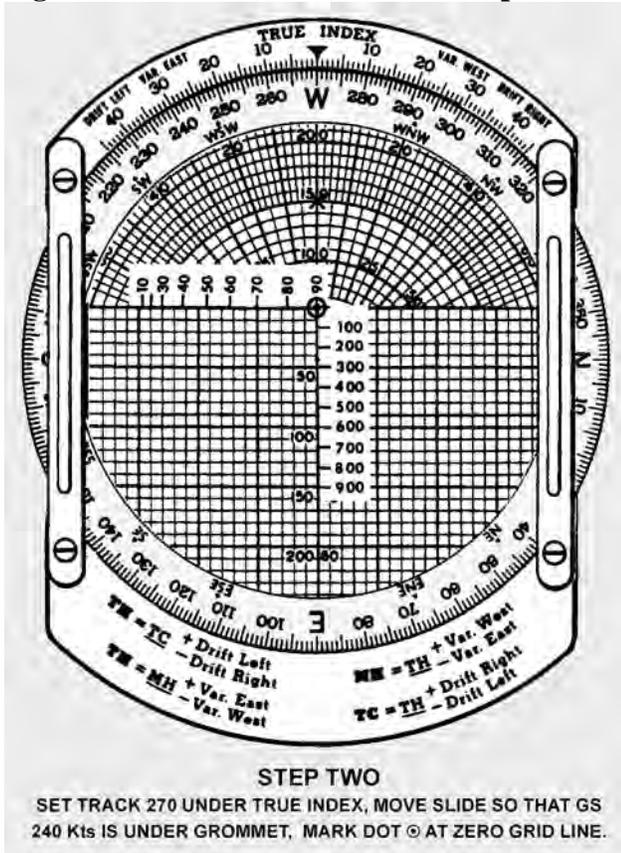
Figure 12.11. Celestial Motions – Step Three.

STEP THREE
SET Zn 171 UNDER TRUE INDEX. MOVE SLIDE
SO THAT ⊙ DOT IS ON THE ZERO GRID LINE.
READ COMBINED MOTION ADJUST OF MINUS 1.1.
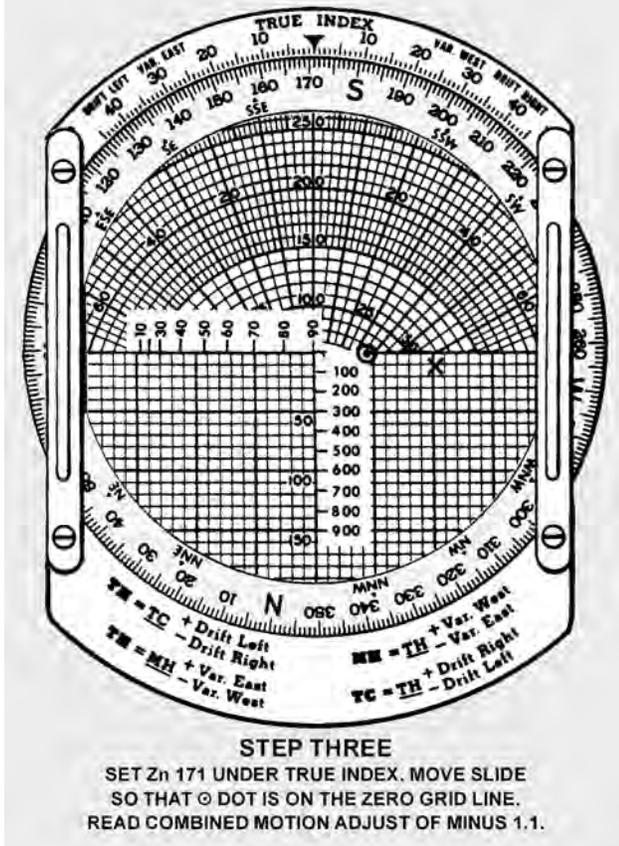
NOTE: The vertical distance between the zero line and the low mark is the combined 1-minute motion. Each line of the grid equals 1 minute of arc (1 mile). If the cross is on the zero line, the motion is positive. If the dot is on the zero line, the motion is negative. When solving for motions using grid, all directions must be grid directions!

EXAMPLE: Given the following information, find the combined 1-minute motion adjustment.

Assumed Latitude	45° 10' N
True Track	270°
GS	240 knots
True Zn	171°
Answer	+1'

12.11. Combinations of Sun, Moon, and Venus. The moon or Venus are often visible during daylight hours and can be used to obtain an LOP. Always consider fixing using these bodies during daylight celestial flights. When planning the flight, use the sky diagrams in the *Air Almanac* to determine the availability of the moon and Venus. If the bodies are available, they can be readily found by accurately precomputing their altitudes and azimuths.

12.11.1. When looking for Venus, take all the filters out of the sextant and point it at the precise location of the planet. A bright, small pinpoint of light will be visible but hard to detect, unless sky conditions

and separation from the sun are ideal. With practice, acquisition should become easier and you will be familiar with those conditions conducive to successfully making a Venus shot.

12.11.2. During the day when the sun is high, the moon or Venus, if they are available, can be used to obtain compass deviation checks. In polar regions during periods of continuous twilight, the moon and Venus will be available if their declination (Dec) is the same name as the latitude.

12.12. Duration of Light. Sunrise and sunset at sea level and at altitude, moonrise and moonset and semiduration graphs will not be discussed in detail in this chapter. It is imperative; however, to preplan for any mission where twilight occurs during the course of the flight, especially at the higher latitudes where twilight extends over longer periods of time. An excellent discussion, with appropriate examples, is provided in the *Air Almanac* and should be sufficient for those missions requiring detailed planning.

Section 12D—True Heading Celestial Observation

12.13. Basics. The periscopic sextant, in addition to measuring celestial altitudes, can be used to determine true headings (TH) and true bearings (TB). Any celestial body, whose azimuth can be computed, can be used to obtain a TH. Except for Polaris, the appropriate volume of Pub. No. 249 is entered to obtain Zn (true bearing). In the case of Polaris, the *Air Almanac* has an azimuth of Polaris table. It does not require information from the Pub. No. 249 tables. The two methods used to obtain THs with the periscopic sextant. The TB method requires precomputation of Zn. Postcomputation of Zn is possible with the inverse relative bearing (IRB) method. The procedures are as follows:

12.14. True Bearing (TB) Method:

12.14.1. Determine GMT and body to be observed.

12.14.2. Extract GHA from the *Air Almanac*.

12.14.3. Apply exact longitude, at the time of the shot, to GHA to obtain exact LHA.

12.14.4. Enter appropriate Pub. No. 249. table with exact LHA, latitude, and Dec. Interpolate if necessary and extract Zn and Hc (Figure 12.12). If Polaris is used, obtain the azimuth from the Azimuth of Polaris table in the *Air Almanac* and use your latitude instead of Hc (Figure 12.13).

12.14.5. Set Zn in the azimuth counter window with the azimuth crank and set Hc in the altitude counter window with the altitude control knob.

12.14.6. Collimate the body at the precomputed time and read the TH of the aircraft under the vertical crosshair in the field of vision. If you are using precomputation techniques, a TH is available every time an altitude observation is made.

NOTE: Shot must be taken at precomp time.

Figure 12.12. True Bearing Method (Except Polaris).

1. Precompute the Zn of the body.

2. Using the azimuth crank, set the Zn of the body in the azimuth counter window.

3. Using the altitude control knob, set Hc in the altitude counter window.

4. Locate the body by turning the sextant until the approximate TH of the aircraft falls under the vertical crosshair. Body should be in the field of vision. Bring body into collimation.

5. Read exact TH under the vertical crosshair. (060°)
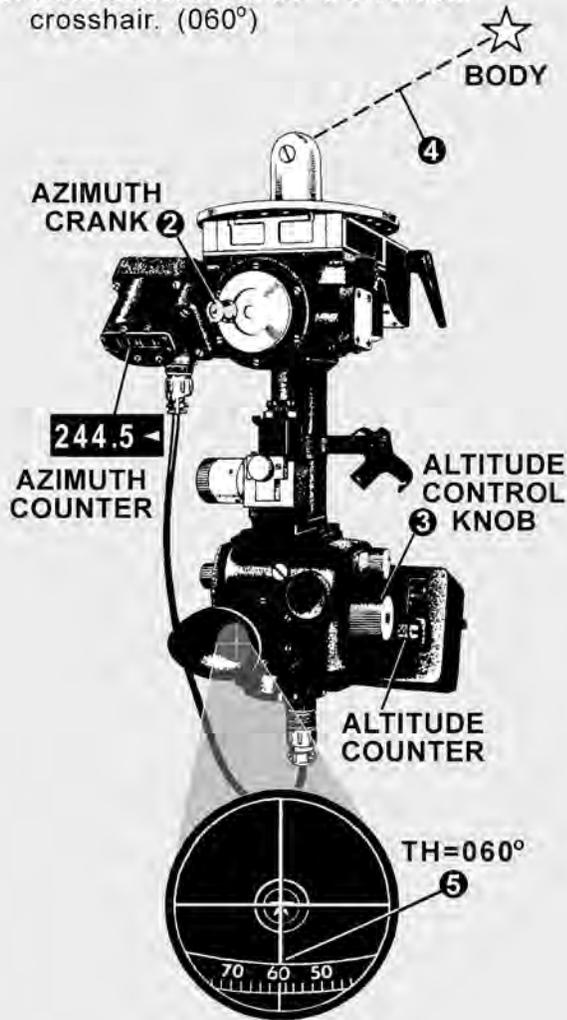
Figure 12.13. True Bearing Method (Including Polaris).

1. Precompute the Zn of the body.

2. Using the azimuth crank, set the Zn of Polaris into the azimuth counter window.

3. Using the altitude control knob, set your Latitude into the altitude counter window.

4. Locate Polaris by turning the sextant until the approximate TH of the aircraft falls under the vertical crosshair. Polaris should be in the field of vision. Bring Polaris into collimation.

5. Read the exact TH under the vertical crosshair. (050°)
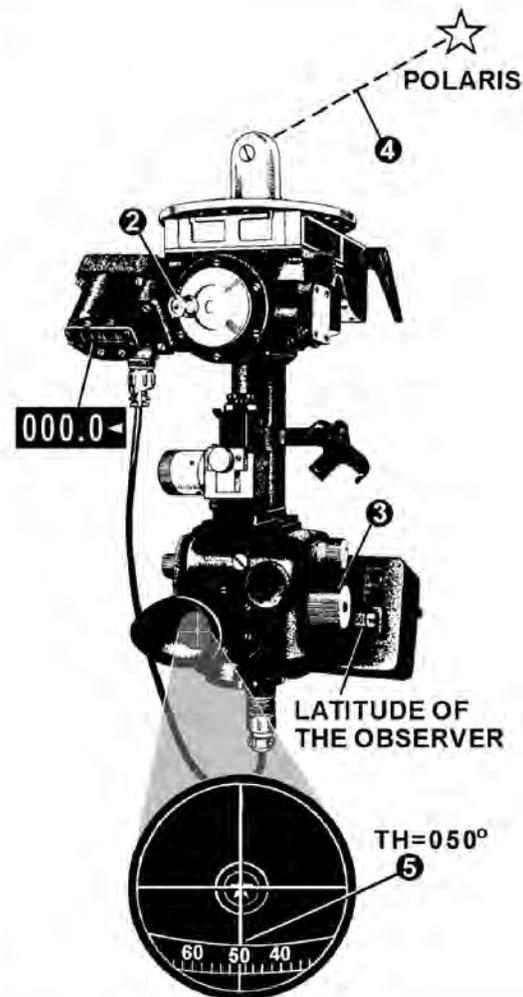
12.15. Inverse Relative Bearing (IRB) Method:

12.15.1. Set 000° in the azimuth counter window with the azimuth crank (Figure 12.14).

Figure 12.14. Inverse Relative Bearing Method.
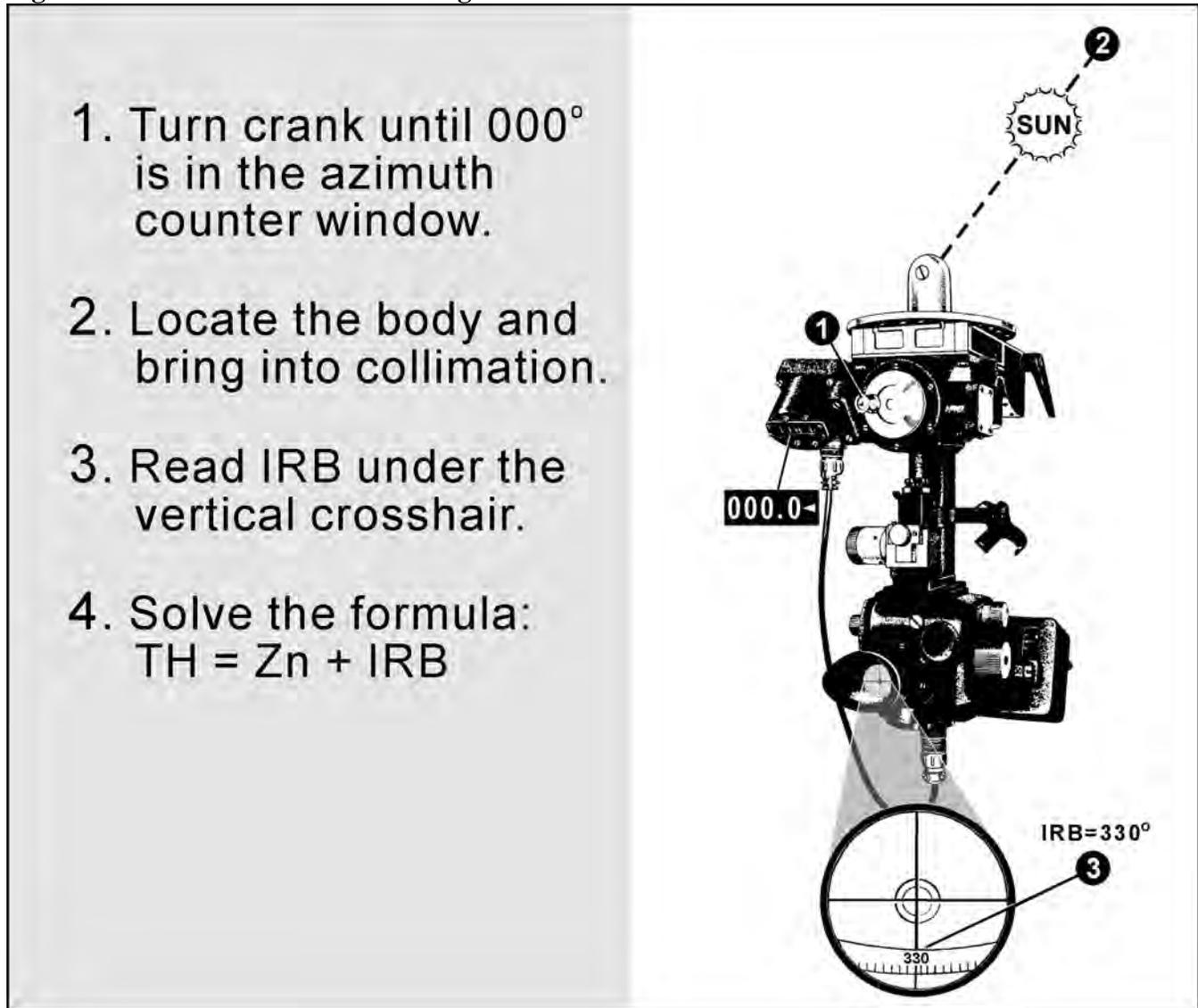
1. Turn crank until 000°
   is in the azimuth
   counter window.

2. Locate the body and
   bring into collimation.

3. Read IRB under the
   vertical crosshair.

4. Solve the formula:
   TH = Zn + IRB

12.15.2. Collimate the body. At the desired time, read the IRB under the vertical crosshair in the field of vision.

12.15.3. Compute Zn of the celestial body and use the formula:

$$TH = Zn + IRB$$

Section 12E—Celestial Navigation in High Latitudes

12.16. Basics. Celestial navigation in polar regions is of primary importance because (1) it constitutes a primary method of determining position other than by DR and (2) it provides a reliable means of establishing direction over much of the polar regions. The magnetic compass and directional gyro (DG) are useful in polar regions, but they require an independent check, which can be provided by a celestial body or other automatic systems such as INS or GPS.

12.16.1. At high latitudes, the sun's daily motion is nearly parallel to the horizon. The motion of the aircraft in these regions can easily have greater effect upon altitude and Zn of the sun than the motion of the sun itself.

12.16.2. At latitude 64°, an aircraft flying west at 400k keeps pace with the sun, which appears to remain stationary in the sky. At higher latitudes, the altitude of a celestial body might be increasing at any time of day, if the aircraft is flying toward it and a body might rise or set, at any azimuth, depending upon the direction of motion of the aircraft relative to the body.

12.17. Bodies Available for Observation. During the continuous daylight of the polar summer, only the sun is regularly available for observation. The moon is above the horizon about half the time, but generally it is both visible and at a favorable position with respect to the sun for only a few days each month.

12.17.1. During the long polar twilight, no celestial bodies may be available for observation. As in lower latitudes, the first celestial bodies to appear after sunset and the last to remain visible before sunrise are those brighter planets, which are above the horizon.

12.17.2. The sun, moon, and planets are never high in polar skies, thus making low altitude observations routine. Particularly with the sun, observations are made when any part of the celestial body is visible. If it is partly below the horizon, the upper limb is observed and a correction of -16' for semidiameter (SD) is used in the SD block of the precomputation form.

12.17.3. During the polar night, stars are available. Polaris is not generally used because it is too near the zenith in the arctic and not visible in the Antarctic. A number of good stars are in favorable positions for observation. Because of large refractions near the horizon avoid low altitudes (below about 20°) when higher bodies are visible.

12.18. Sight Reduction. Sight reduction in polar regions presents some slightly different problems from those at lower latitudes. Remember, for latitudes greater than 69° N or 69° S, Pub. No. 249 tables have tabulated Hcs and azimuths for only even degrees of LHA. This concerns you in two ways. First, it will be necessary to adjust assumed longitude to achieve a whole, even LHA for extractions. This will preclude interpolating. Second, the difference between successive, tabulated Hcs is for 2° of LHA, or 8 minutes of time, so this difference must be divided in half when computing motion of the body for 4 minutes of time.

12.18.1. For ease of plotting, all azimuths can be converted to grid. To convert, use the longitude of the assumed position to determine convergence because the Zn is for the assumed position, not the DR position. On polar charts, convergence is equal to longitude.

12.18.2. In computing motion of the observer, it is imperative that you use the difference between grid azimuth and grid track, or Zn and true track, since this computation is based on relative bearing (RB). Zn minus grid course does not give relative bearing.

12.18.3. Since low altitudes and low temperatures are normal in polar regions, refer to the refraction correction table and use the temperature correction factor for all observations.

12.18.4. In polar regions, Coriolis corrections reach maximum values and should be carefully computed.

12.19. Poles as Assumed Positions. Within approximately 2° of the pole, it is possible to use the pole as the assumed position. With this method, no tabulated celestial computation is necessary and the position may be determined by use of the *Air Almanac* alone.

12.19.1. At either of the poles of the earth, the zenith and the elevated poles are coincident or the plane of the horizon is coincident with the plane of the equator. Vertical circles coincide with the meridians and parallels of latitude coincide with Dec circles. Therefore, the altitude of the body is equal to its Dec and the azimuth is equal to its hour angle.

12.19.2. To plot any LOP, an intercept and the azimuth of the body are needed. In this solution, the elevated pole is the assumed position. The azimuth is plotted as the GHA of the body, or the longitude of the subpoint. The intercept is found by comparing the Dec of the body, as taken from the *Air Almanac*, with the observed altitude of the body. To summarize, the pole is the assumed position, the Dec is the Hc, and the GHA equals the azimuth.

12.19.3. For ease of plotting, convert the GHA of the body to grid azimuth by adding or subtracting 180° when using the North Pole as the assumed position. When at the South Pole, 360° – GHA of the body equals grid azimuth. The result will allow the use of the grid lines for plotting the LOPs. When using grid azimuth for plotting, apply Coriolis to the assumed position (in this case, the pole). Precession or nutation corrections are not necessary since current SHA and Dec are used. Motion of the observer tables may also be used in precomputation, since grid azimuth relative to grid course may be determined. Motion of the body is zero at the poles.

12.19.4. Note the exact GMT of the celestial observation. From the *Air Almanac*, extract the proper Dec and GHA. Plot the azimuth. Compare Ho and Hc to obtain the intercept. When the observed altitude (Ho) is greater than the Dec (Hc), it is necessary to go from the pole toward the celestial body along the azimuth. If the observed altitude is less than the Dec, as is the case with the sun in Figure 12.15, it is necessary to go from the pole away from the body along the azimuth. Draw the LOPs perpendicular to the azimuth line in the usual manner. Don't be concerned about large intercepts; they have no bearing on the accuracy of this type of fix. Observations on well-separated bearings give a fix that is as good close to the pole as it is anywhere else.

Figure 12.15. Using Pole as Assumed Position.


```
SUN                              MOON
Hc (Dec) 23°-12N                 Hc (Dec) 10°-57N
Ho        22°-04'                Ho        09°-56'
Intercept    68 NM away          Intercept    61 NM away
GHA          168                 GHA          236
            +180                             +180
Grid ZN      348                 Grid ZN      056
```
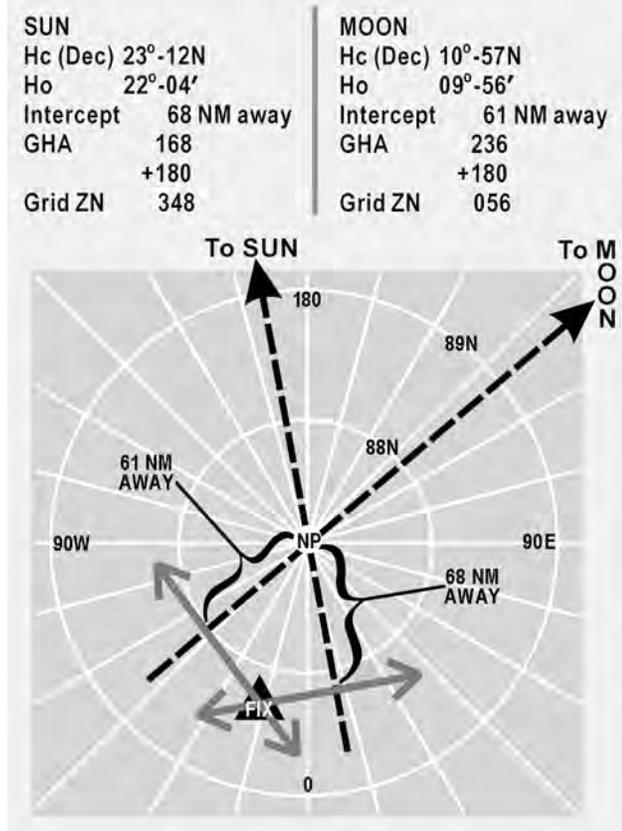
Section 12F—Adjusting Assumed Position

12.20. Adjusting Assumed Position for Off-Time Shot. There will be times when the observer does not start the shot at the prescribed time for various reasons. For example, the observer may struggle to find the body due to cloud cover. If a shot is taken off time, you can use the FEAST (Fast EAST) rule: a shot taken too fast or too early has the assumed position moved 15' of longitude east for each minute early to compensate for body motion (see example in Figure 12.16). Apply the reverse of the FEAST rule for late shots (move the assumed position west). This adjusted position is then advanced or retarded for track and GS to account for motion of the observer, applying the same concept used in the three-LHA method (Figure 12-5). This technique for solving motions is also discussed in Chapter 10.

EXAMPLE: Original assumed position: 23°-50' N 120°-00' W
             Move 15' of longitude west for 1 minute late
             Retard 6NM from track of 360° TH
             New assumed position   23°-44' N 120°-15' W

Figure 12.16. Corrections for Off-Time Shooting.
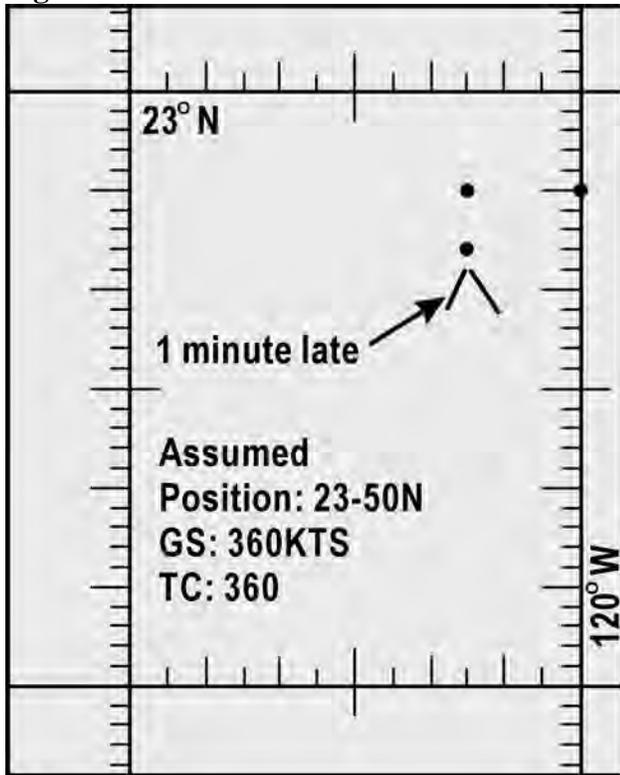
12.21. Longitude Adjustment Principle. You will occasionally make errors in your precomputations. Possibly the most common would be an extraction error of the GHA or math error while computing the LHA. If one of these numbers is incorrect, then all the extractions from the Pub. No. 249 would be based on erroneous information and the result would be an LOP error. Fortunately there is a way of compensating for this type of error without having to reenter the table and retrieving the correct data. This method is called the Longitude Adjustment Principle (LAP). You need only adjust the assumed longitude (up to 2 1/2°) to correct for a GHA extraction error, or a math error. Moving the assumed position beyond the 2 1/2° induces some error in the plotting LOP. Suppose you wanted the GHA for 1410Z (Figure 12.17), you extracted the value for 1400Z and applied it to the longitude. The resultant LHA was used and the precomp completed before you realized your error. To do the LAP first, extract the correct GHA (031-20), keep the old LHA, and adjust the longitude so that the math is correct (Figure 12.18). A math error can occur in solving for the LHA (Figure 12.19). Once you have corrected the precomp, use the adjusted longitude for your assumed longitude to plot the LOP.

Figure 12.17. LAP Using Incorrect GHA.

BODY	SUN
BASE GHA	028-50
CORR	
(+ 360)	360
GHA	388-50
ASSUM (W) LONG +E	130-50
LHA	258
ASSUM LAT	42 (N) S

Incorrect value for the GHA.

1. *Is this error within 2 1/2°?*

   031-20
   028-50
   ‾‾‾‾‾
   2-30

2. *2°30' of longitude is less than 150 NM.*

Figure 12.18. LAP Using Correct GHA.

BODY	SUN	
BASE GHA	028-50	031-20
CORR		
(+ 360)	360	360
GHA	388-50	391-20
ASSUM (W) LONG +E	130-50	133-20
LHA	258	258
ASSUM LAT	42 (N) S	

1410 GHA

Longitude is adjusted

LHA remains the same

Figure 12.19. LAP Correcting a Math Error.

BODY	SUN	
BASE GHA	320-30	
CORR		
(+ 360)		
GHA	320-30	320-30
ASSUM (W) LONG +E	121-30	122-30
LHA	198	198
ASSUM LAT	40 (N) S	

LHA should be 199.

Adjust longitude

LHA remains the same.

12.22. Summary. Any of the techniques discussed here, if used on a regular basis, can be just as accurate as normal precomping procedures and save some time as well. These techniques are not all inclusive. There are many commercial publications available as a source for celestial navigators: for example, *American Practical Navigator* by Bowditch (available through the NIMA) and the *Journal of the Institute of Navigation* (available through the Institute of Navigation).

Chapter 13

SEXTANTS AND ERRORS OF OBSERVATION

Section 13A—Sextants

13.1. Introduction. For hundreds of years, mariners have navigated the seas keeping track of their positions by use of the sextant. This instrument measured the altitude of celestial bodies (angular distance above the horizon) and the information derived from this measurement was used to determine the position of the vessel. All celestial navigation follows this rule. Today's navigator measures the altitude of the celestial bodies in much the same manner as Magellan or Columbus.

13.1.1. However, there is a difference between air and marine celestial navigation. Because marine navigators are on the surface of the ocean, they can establish their horizon by referring to the natural horizon. In an aircraft, this is impossible because altitude and aircraft attitude induce error. In the sextant designed for air navigation, a bubble, like the one in a carpenter's level, determines an artificial horizon, which is parallel to the celestial horizon. The bubble chamber is placed in the sextant so the bubble is superimposed upon the field of view. Both the celestial body and the bubble are viewed simultaneously, making it possible to keep the sextant level while sighting the body.

13.1.2. Sextants are subject to certain errors that must be compensated for when determining an LOP. Some of these errors are instrument errors while others are induced by the various in-flight conditions. The first half of this chapter discusses the sextant and the second half explains sextant errors.

13.2. The Bubble Sextant. The aircraft bubble sextant measures altitude above a horizontal plane established by a bubble. Aviators use several types of bubble sextants, all of which are indirect sighting. This means the navigator does not look directly toward the celestial body, but always looks in a horizontal direction as shown in Figure 13.1. The image of the body is reflected into the field of view when the field prism is set at the correct angle. In the bubble sextant, the bubble and body are visible in the same field of view. The sextant system consists of four parts: the mount, the sextant, the electrical cables and the carrying case.

13.3. The Mount. The mount, as shown in Figure 13.2, is fastened permanently to the top of the fuselage of the aircraft. A shutter door is built into the mount to close the opening for the tube of the periscopic sextant. This shutter door is controlled by the sextant port lever (1) on the mount. The mount has a gimbal mechanism, which allows the sextant to be tilted from the vertical in any direction. This permits a celestial body to be observed throughout the normal oscillations of an aircraft. A drain plug (2) is provided at the low point in the shutter well for draining out water, which may have collected in the mount.

NOTE: The numbers in parentheses in 13.3, 13.3.1, and 13.3.2 refer to the parts indicated in Figure 13.2.

Figure 13.1. Body Is Not Sighted Directly.
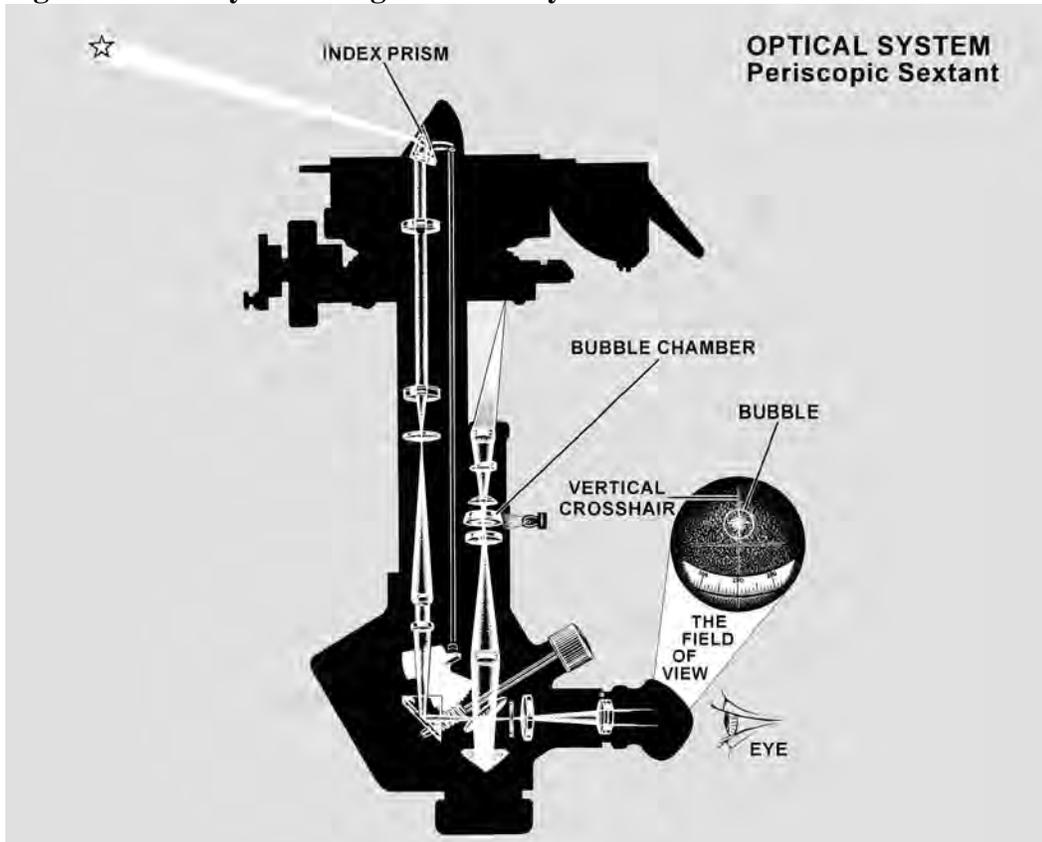
13.3.1. The sextant is held in the mount by two locking pins (4), located in a movable collar on the bottom of the mount. One pin locks the sextant into the mount and holds it in the retracted position; the other pin locks the sextant in the extended position. These pins are spring-loaded and must be pulled out to release the sextant. Located next to these locking pins is a friction clamping lever (3), which provides the observer with the option of locking the sextant at a fixed azimuth or, when the tension is released, the sextant may be rotated through 360° of azimuth. The azimuth scale (10) and azimuth counter (6) will move when the azimuth crank (5) is rotated. The azimuth scale can be read against a lubber line or index (11). The azimuth scale read against the lubber line and the azimuth counter reading should be the same.

13.3.2. Power is supplied from the aircraft through a cable connection (9) on the side of the mount. A switch (8) on the side of the mount controls power to both the mount and the sextant. The mount has one lamp that illuminates the azimuth counter window. Another cable (7) is connected to the socket on the underside of the mount and supplies power to the sextant itself.

13.4. The Periscopic Sextant. The periscopic sextant is an optical instrument which enables the navigator to determine true azimuth (Zn), relative bearing (RB), and altitude angle of a celestial body and aircraft true heading (TH). The sextant provides an angle of observation from below the horizon to directly overhead, as compared to an artificial horizon.

Figure 13.2. Periscopic Sextant Mount.

1. SEXTANT PORT LEVER
2. DRAIN PLUG
3. FRICTION CLAMPING LEVER
4. LOCKING PINS
5. AZIMUTH CRANK
6. AZIMUTH COUNTER
7. POWER TO SEXTANT
8. POWER SWITCH
9. POWER FROM AIRCRAFT
10. AZIMUTH SCALE
11. LUBBER LINE
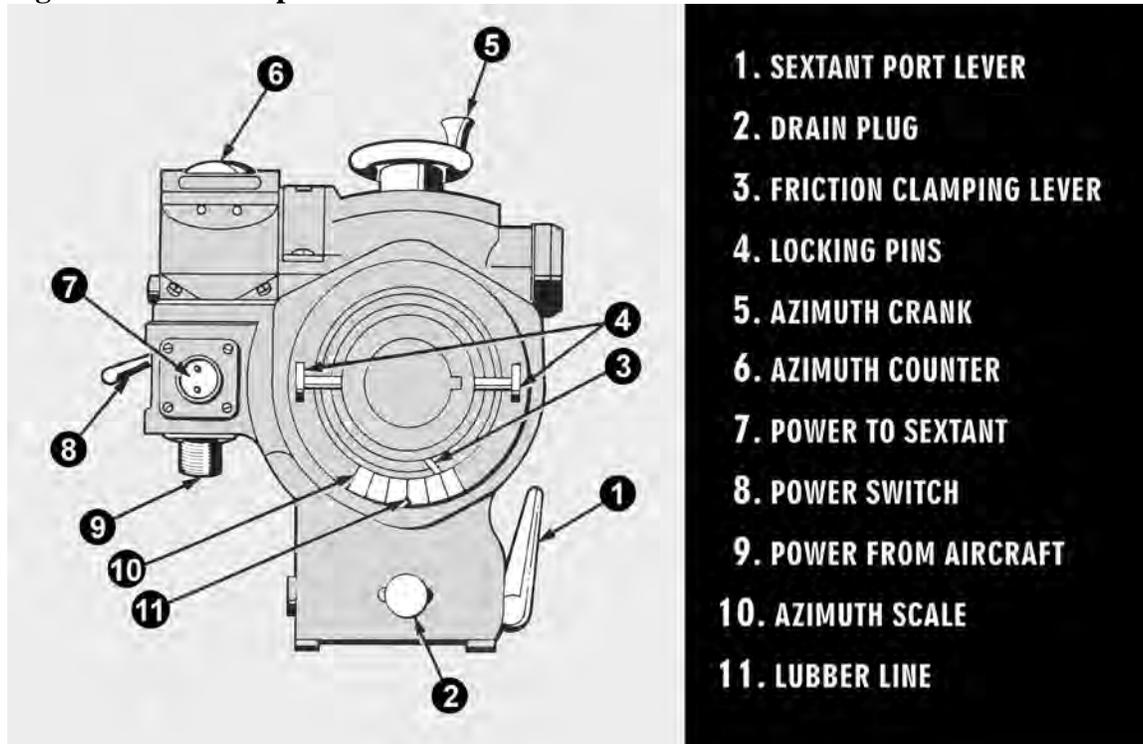
13.4.1. Proper collimation techniques and the correct size bubble are essential ingredients of accurate celestial observations. Collimation is effected when the body is placed in the center of the bubble. For greatest accuracy, the bubble should be in the center of the field, with the body in the center of the bubble. The error will be small if the bubble is anywhere on the vertical line of the field, as long as it does not touch the top or bottom of the bubble chamber. Figure 13.3 shows examples from better to worse collimation.

Figure 13.3. Correct and Incorrect Collimation.

BETTER ⟶ WORSE
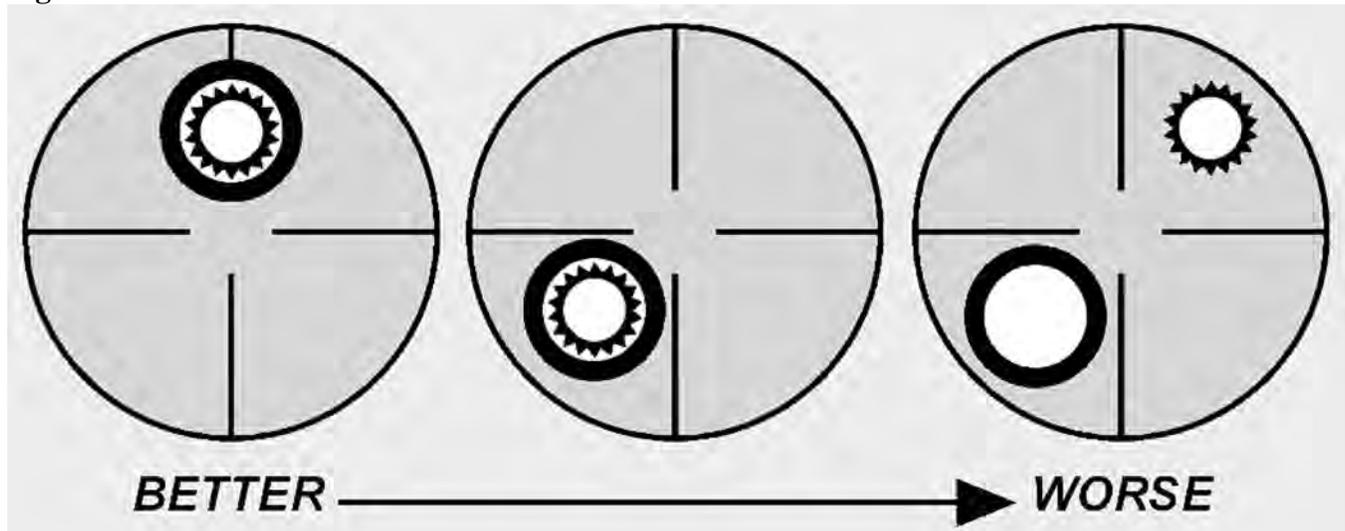
13.4.2. Bubble size affects the accuracy of a sextant observation. The ideal situation for collimation is to have a small bubble for ease in determining the center. A bubble that is too small will stick to the lens, decreasing accuracy. A bubble that is too large will move like a creature from a science fiction movie, making it difficult to find the center. Experience shows that best results are obtained with a bubble approximately one and a half times the apparent diameter of the sun or moon, or about the size of a Cheerio. The field prism is geared to an altitude scale so that when the body is collimated the altitude can be read from the scale.

13.4.3. An averaging mechanism is also incorporated which allows the navigator to take an observation over a period of time. The continuous motion of the aircraft affects the bubble and resultant artificial horizon. This movement resolves itself into a cycle in which the aircraft rolls, yaws, and pitches. To obtain an accurate reading, it is necessary to sight the body for a period of time during this cyclic movement and to average the results of a series of sightings. An averaging device has been incorporated in the sextant so an average reading can be obtained.

13.4.4. The sextant (Figure 13.4) is actually a low-power periscope with a 15° field of view. All lens surfaces in the sextant are coated to minimize light loss. To prevent condensation when the tip of the sextant is extended into cold air, the tube is filled with a dry gas and sealed. A desiccant (composed of silica gel) is used to remove moisture and check on the dryness of the gas inside the tube, and is visible in the periscopic end of the sextant, or in some models, on the sextant body. When the silica gel is pink, there is moisture in the tube and the sextant should be replaced before flight.

NOTE: The numbers in parentheses in 13.4.5 through 13.4.11 refer to the parts indicated in Figure 13.4.

Figure 13.4. Periscopic Sextant.

1. EYEPIECE
2. FILTER KNOB
3. DESICCANT
4. START SWITCH
4a. AVERAGER OPERATING LEVER
5. RESET SWITCH
5a. AVERAGER REWIND LEVER
6. BUBBLE CONTROL KNOB
7. ILLUMINATION CONTROL
8. ALTITUDE CONTROL
9. ALTITUDE COUNTER
10. HALF TIME DIAL
11. HEADING SCALE SHUTTER CONTROL
12. OBJECTIVE LENS
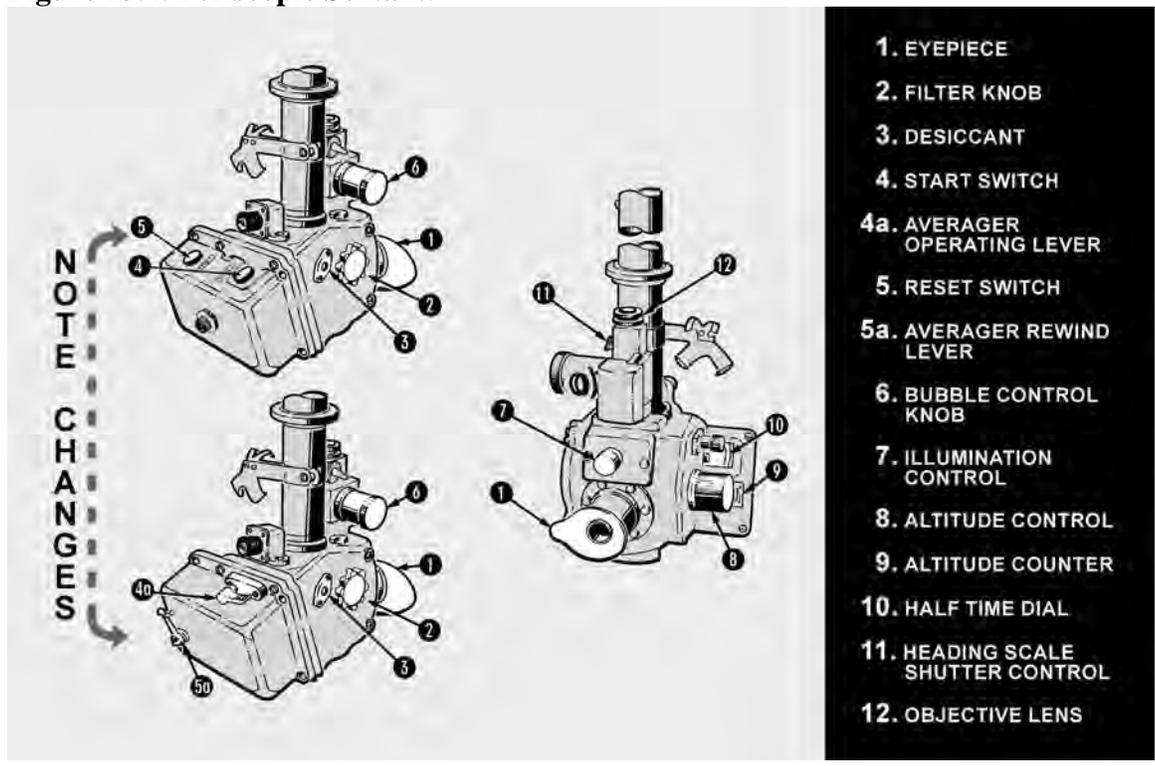
13.4.5. An eyepiece (1) rotates to correct the eyesight of the individual observer. Filters (2) are provided for selective use in the optical system so that the intensity of the sun's light might be adequately reduced. The filter control (2) is located on the left of the sextant.

13.4.6. Most sextants currently in use have been modified with an electronic device for accomplishing all the functions of the averaging mechanism. General differences in these and the unmodified sextants are addressed in this discussion.

13.4.7. A start switch (4) (a start and stop or averager operating lever [4A] on unmodified sextants) starts and stops the operation of the sextant. Adjacent to this switch is the reset switch (5) (the averager rewind lever, if unmodified [5A] located below the averager operating lever). The reset switch or averager rewind lever has four functions. When depressed and released, it does the following: (1) removes the shutter from the field of vision, (2) zeroes and resets (rewinds if unmodified) the timer, (3) zeroes the averager and places initial values in registers and data memory (realigns indices on unmodified sextants), and (4) disconnects the altitude control knob from the averager.

13.4.8. The bubble control knob (6) should be left in the maximum increase position after adjustments have been made. With the control in the maximum increase position, an aneroid is locked to the bubble chamber to compensate for changes in ambient pressure and temperature.

13.4.9. On the front of the sextant, there is a rheostat control (7) which varies the intensity of the light in the bubble chamber. The altitude knob (8) is located on the right side of the sextant. It keeps the observed body in vertical collimation during the period of the observation. At the end of the scheduled observation, it adjusts the altitude counter until the exact average indication appears, or to align the indices on unmodified sextants. The body's altitude is read in the altitude counter (9). Directly behind the altitude knob is the averager display (10) (half-time dial and indices if unmodified). The averager display or half-time dial is graduated from 0-60 and indicates the half time of the observation. The indices, when aligned, permit the direct reading of the observed altitude on the altitude dial.

13.4.10. In the periscope sextant, the averaging is accomplished by microprocessor (Deimel-Black ball integrator if unmodified), which effects a continuous moving averager over any observation period up to 2 minutes. This system has many advantages over other known averaging devices: it is very simple to operate. A single switch (or lever) sets or winds the mechanism and no other presetting of the sextant, timing mechanism, or averaging is necessary. It is continuously integrating altitude against elapsed time. After at least 30 seconds, it may be stopped at any time up to 2 minutes. The average altitude is read directly from the counter. A half-time clock will indicate the half time of the observation. The time indication may be added directly to the time of starting the observation to compute the mean time of the observation. At the end of the observation, the averager energizes a solenoid (actuates a lever if unmodified) which drops a shutter across the field of view, indicating the end of the observation. Although it is possible to utilize an instantaneous shot, the normal timed observation lasts for 2 minutes. It is impossible to time any observation for less than 30 seconds using the sextant timer.

13.4.11. A heading scale shutter (diffuser lever) control (11) provides a convenient means of blocking out the bright illumination on the azimuth scale for night celestial observations. The objective lens (12) is located just above the heading scale shutter control. The lens aligns the azimuth scale of the sextant with the longitudinal axis of the aircraft. The lens can be rotated with the fingers in order to calibrate the azimuth scale on a known bearing while looking through the eyepiece. The objectives lens can remove up to 2° azimuth error in the azimuth ring. A locking ring beneath the lens prevents accidental

movement. A dial lamp located on the right side of the sextant provides three beams of light to illuminate the averager indicators, the altitude counter, and the watch clip. The watch clip is made to hold an old-fashioned pocket watch.

13.5. Electrical Cables. Cables provide power for sextant operation and illumination. One "Y" cable provides power from the mount to the sextant for illumination and averager operation.

13.6. Sextant Case. The case provides shock-absorbent storage for the sextant when it is not in use. The sextant fits into formfitting foam blocks and is secured by straps. The case also contains spare bulbs for sextant illumination and provides storage for the electrical cable.

Section 13B—Errors of Sextant Observation

13.7. Basics. If collimation of the body with the bubble and reading the sextant were all that had to be done, celestial navigation would be simple. This would mean LOPs that are accurate to within 1 or 2 miles could be obtained without any further effort. Unfortunately, considerable errors are encountered in every sextant observation made from an aircraft. A thorough understanding of the cause and magnitude of these errors, as well as the proper application of corrections to either Hc or Hs, will help minimize their effects. Remember that any correction applied to the Hs may be applied to the Hc with a reverse sign. Accuracy of celestial navigation depends upon thorough application of these corrections together with proper shooting techniques. The errors of sextant observation may be classified into four groups: (1) parallax, (2) refraction, (3) acceleration, and (4) instrument.

13.8. Parallax Error. Parallax in altitude is the difference between the altitude of a body above a bubble horizon at the surface of the earth and its calculated altitude above the celestial horizon at the center of the earth. All Hcs are given for the center of the earth. If the light rays reaching the earth from a celestial body are parallel, the body has the same altitude at both the center and the surface of the earth. For most celestial bodies, parallax is negligible for purposes of navigation.

13.8.1. Parallax Correction for the Moon. The moon is so close to the earth that its light rays are not parallel. The parallax of the moon may be as great as 1°; thus, when observing the moon, a parallax correction must be applied to the Hs. This correction is always positive (+) and varies with the altitude and with the distance of the moon from the earth. The correction varies from day to day because the distance of the moon from the earth varies. Corrections for the moon's parallax in altitude are given on the daily pages of the *Air Almanac* and are always added, algebraically, to sextant altitudes. The values of parallax for negative altitudes are obtained from the *Air Almanac* for the equivalent positive altitudes.

13.8.2. Semidiameter Correction. This correction is found on the daily pages of the *Air Almanac*. Apply it when shooting the upper or lower limb of the moon or the sun.

13.8.2.1. It is more likely to occur on observations of the moon because, when the moon is not full (completely round), the center is difficult to estimate. Shoot either the upper or lower limb and apply the semidiameter correction listed on the *Air Almanac* page for the time and date of the observation. Subtract the correction from the Hs when shooting the upper limb; add the correction to the Hs when shooting the lower limb. Reverse the sign if applying the correction to the Hc.

13.8.2.2. Listed on the same page is the semidiameter correction for the sun, which is applied the same way as for the moon.

EXAMPLE: Using Figure 13.5 extract the corrections for the upper limb of the moon as observed on 11 August 1995 at 1100Z is 33°41'. Apply these corrections as:

Hs	33°41'
Parallax	+49'
Semidiameter	-16'
Ho	34°14'

13.9. Atmospheric Refraction Error. Still another factor to be taken into consideration is atmospheric refraction. If a fishing pole is partly submerged under water, it appears to bend at the surface. The bending of light rays as they pass from the water into the air causes this appearance. This bending of the light rays, as they pass from one medium into another, is called refraction. The refraction of light from a celestial body as it passes through the atmosphere causes an error in sextant observation.

13.9.1. As the light of a celestial body passes from the almost perfect vacuum of outer space into the atmosphere, it is refracted as shown in Figure 13.6 so that the body appears a little higher above the horizon than it really is. Therefore, the correction to the Hs for refraction is always negative. The higher the body above the horizon, the smaller the amount of refraction and, consequently, the smaller the refraction correction. Moreover, the greater the altitude of the aircraft, the less dense the layer of atmosphere between the body and the observer; hence, the less the refraction.

13.9.2. The appropriate correction table for atmospheric refraction is listed inside the back cover of all four books used for celestial computations; namely, the *Air Almanac* and each of the three volumes of Pub. No. 249. This table, shown in Figure 13.7, lists the refraction for different observed altitudes of the body and for different heights of the observer above sea level. The values shown are subtracted from Hs or added to Hc.

13.10. Acceleration Error. Presently, the only practical and continuously available reference datum for the definition of the true vertical is the direction of the gravitational field of the earth. Definition of this vertical establishes the artificial horizon. It is also fundamental that the forces caused by gravity cannot be separated by those caused by accelerations within the sextant. A level or centered bubble in the sextant indicates the true vertical only when the instrument is at rest or moving at a constant velocity in a straight line. Any outside force (changes in GS or changes in track) will affect the liquid in the bubble chamber and, consequently, displace the bubble.

13.10.1. When the sextant is moved in a curved path (Coriolis, changes in heading, rhumb line) or with varying speed, the zenith indicated by the bubble is displaced from the true vertical. This presents a false artificial horizon above which the altitude of the celestial body is measured. Since the horizon used is false, the altitude measured from it is erroneous. Therefore, the accuracy of celestial observations is directly related to changes in track and speed of the aircraft. Acceleration errors have two principal causes: changes in GS and curvature of the aircraft's path in space.

Figure 13.5. Correction for Moon's Parallax.

(DAY 223) GREENWICH A. M. 1995 AUGUST 11 (FRIDAY) — 445

UT (GMT)	SUN GHA	SUN Dec	ARIES GHA ♈	MARS 1.4 GHA	MARS Dec	JUPITER-2.3 GHA	JUP Dec	SATURN 0.9 GHA	SAT Dec	MOON GHA	MOON Dec
00 00	178 40.2	N15 28.3	318 59.5	127 27	S 4 49	75 09	S20 40	323 50	S 4 35	356 59	S10 01
10	181 10.3	28.2	321 29.9	129 57		77 40		326 21		359 24	9 59
20	183 40.3	28.1	324 00.3	132 27		80 10		328 51		1 48	57
30	186 10.3	· 27.9	326 30.8	134 57	· ·	82 40	· :	331 22	· ·	4 13	56
40	188 40.3	27.8	329 01.2	137 28		85 11		333 52		6 37	54
50	191 10.3	27.7	331 31.6	139 58		87 41		336 22		9 02	52
01 00	193 40.3	N15 27.6	334 02.0	142 28	S 4 50	90 12	S20 40	338 53	S 4 35	11 26	S 9 50
10	196 10.3	27.4	336 32.4	144 58		92 42		341 23		13 51	49
20	198 40.4	27.3	339 02.8	147 28		95 12		343 54		16 15	47
30	201 10.4	· 27.2	341 33.2	149 59	· ·	97 43	· ·	346 24	· ·	18 40	· 45
40	203 40.4	27.1	344 03.6	152 29		100 13		348 55		21 04	43
50	206 10.4	27.0	346 34.0	154 59		102 44		351 25		23 29	42
02 00	208 40.4	N15 26.8	349 04.5	157 29	S 4 50	105 14	S20 40	353 55	S 4 35	25 54	S 9 40
10	211 10.4	26.7	351 34.9	159 59		107 44		356 26		28 18	38
20	213 40.5	26.6	354 05.3	162 29		110 15		358 56		30 43	37
30	216 10.5	· 26.5	356 35.7	165 00	· ·	112 45	· ·	1 27	· ·	33 07	· 35
40	218 40.5	26.3	359 06.1	167 30		115 16		3 57		35 32	33
50	221 10.5	26.2	1 36.5	170 00		117 46		6 28		37 56	31
03 00	223 40.5	N15 26.1	4 06.9	172 30	S 4 51	120 16	S20 40	8 58	S 4 35	40 21	S 9 30
10	226 10.5	26.0	6 37.3	175 00		122 47		11 29		42 46	28
20	228 40.6	25.9	9 07.7	177 30		125 17		13 59		45 10	26
30	231 10.6	- 25.7	11 38.1	180 01	· ·	127 48	· ·	16 29	· ·	47 35	· 24
40	233 40.6	25.6	14 08.6	182 31		130 18		19 00		49 59	23
50	236 10.6	25.5	16 39.0	185 01		132 48		21 30		52 24	21
04 00	238 40.6	N15 25.4	19 09.4	187 31	S 4 51	135 19	S20 40	24 01	S 4 35	54 48	S 9 19
10	241 10.6	25.2	21 39.8	190 01		137 49		26 31		57 13	17
20	243 40.7	25.1	24 10.2	192 32		140 20		29 02		59 38	16
30	246 10.7	· 25.0	26 40.6	195 02	· ·	142 50	· ·	31 32	· ·	62 02	· 14
40	248 40.7	24.9	29 11.0	197 32		145 20		34 02		64 27	12
50	251 10.7	24.8	31 41.4	200 02		147 51		36 33		66 51	10
05 00	253 40.7	N15 24.6	34 11.8	202 32	S 4 52	150 21	S20 40	39 03	S 4 35	69 16	S 9 09
10	256 10.7	24.5	36 42.3	205 02		152 51		41 34		71 41	07
20	258 40.8	24.4	39 12.7	207 33		155 22		44 04		74 05	05
30	261 10.8	· 24.3	41 43.1	210 03	· ·	157 52	· ·	46 34	· ·	76 30	· 03
40	263 40.8	24.1	44 13.5	212 33		160 23		49 05		78 54	02
50	266 10.8	24.0	46 43.9	215 03		162 53		51 35		81 19	9 00
06 00	268 40.8	N15 23.9	49 14.3	217 33	S 4 53	165 23	S20 40	54 06	S 4 35	83 44	S 8 58
10	271 10.8	23.8	51 44.7	220 03		167 54		56 36		86 08	56
20	273 40.9	23.6	54 15.1	222 34		170 24		59 07		88 33	55
30	276 10.9	· 23.5	56 45.5	225 04	· ·	172 55	· ·	61 37	· ·	90 57	- 53
40	278 40.9	23.4	59 16.0	227 34		175 25		64 07		93 22	51
50	281 10.9	23.3	61 46.4	230 04		177 55		66 38		95 47	49
07 00	283 40.9	N15 23.2	64 16.8	232 34	S 4 53	180 26	S20 40	69 08	S 4 35	98 11	S 8 48
10	286 10.9	23.0	66 47.2	235 04		182 56		71 39		100 36	46
20	288 40.9	22.9	69 17.6	237 35		185 27		74 09		103 00	44
30	291 11.0	· 22.8	71 48.0	240 05	· ·	187 57	· ·	76 40	· ·	105 25	- 42
40	293 41.0	22.7	74 18.4	242 35		190 27		79 10		107 50	41
50	296 11.0	22.5	76 48.8	245 05		192 58		81 41		110 14	39
08 00	298 41.0	N15 22.4	79 19.2	247 35	S 4 54	195 28	S20 40	84 11	S 4 35	112 39	S 8 37
10	301 11.0	22.3	81 49.6	250 05		197 59		86 41		115 04	35
20	303 41.0	22.2	84 20.1	252 36		200 29		89 12		117 28	33
30	306 11.1	· 22.1	86 50.5	255 06	· ·	202 59	· ·	91 42	· ·	119 53	· 32
40	308 41.1	21.9	89 20.9	257 36		205 30		94 13		122 17	30
50	311 11.1	21.8	91 51.3	260 06		208 00		96 43		124 42	28
09 00	313 41.1	N15 21.7	94 21.7	262 36	S 4 55	210 31	S20 40	99 14	S 4 35	127 07	S 8 26
10	316 11.1	21.6	96 52.1	265 07		213 01		101 44		129 31	25
20	318 41.1	21.4	99 22.5	267 37		215 31		104 14		131 56	23
30	321 11.2	· 21.3	101 52.9	270 07	· ·	218 02	· ·	106 45	· ·	134 21	· 21
40	323 41.2	21.2	104 23.3	272 37		220 32		109 15		136 45	19
50	326 11.2	21.1	106 53.8	275 07		223 03		111 46		139 10	17
10 00	328 41.2	N15 20.9	109 24.2	277 37	S 4 55	225 33	S20 41	114 16	S 4 35	141 35	S 8 16
10	331 11.2	20.8	111 54.6	280 08		228 03		116 47		143 59	14
20	333 41.2	20.7	114 25.0	282 38		230 34		119 17		146 24	12
30	336 11.3	· 20.6	116 55.4	285 08	· ·	233 04	· ·	121 47	· ·	148 49	· 10
40	338 41.3	20.5	119 25.8	287 38		235 35		124 18		151 13	09
50	341 11.3	20.3	121 56.2	290 08		238 05		126 48		153 38	07
11 00	343 41.3	N15 20.2	124 26.6	292 38	S 4 56	240 35	S20 40	129 19	S 4 35	156 03	S 8 05
10	346 11.3	20.1	126 57.0	295 09		243 06		131 49		158 27	03
20	348 41.3	20.0	129 27.4	297 39		245 36		134 20		160 52	01
30	351 11.4	· 19.8	131 57.9	300 09	· ·	248 07	· ·	136 50	· ·	163 16	8 00
40	353 41.4	19.7	134 28.3	302 39		250 37		139 20		165 41	7 58
50	356 11.4	19.6	136 58.7	305 09		253 07		141 51		168 06	56
Rate	15 00.1	50 00.7		15 01.0	50 00.6	15 02.4	0 00.0	15 02.6	50 00.1	14 27.6	N0 10.5

Moonrise / Diff.

Lat. N °	Moonrise h m	Diff. m
72	20 18	-04
70	20 09	-01
68	20 02	+02
66	19 55	04
64	19 50	06
62	19 45	08
60	19 41	10
58	19 37	11
56	19 34	12
54	19 31	13
52	19 29	14
50	19 26	15
45	19 21	17
40	19 16	18
35	19 13	20
30	19 09	21
20	19 03	23
10	18 58	24
0	18 53	26
10	18 48	28
20	18 43	30
30	18 37	32
35	18 34	33
40	18 30	34
45	18 25	36
50	18 20	38
52	18 17	39
54	18 15	40
56	18 11	41
58	18 08	42
60	18 04	+44
S		

Moon's P. in A.

A	C	A	C
°	'	°	'
0	60	53	35
3	60	54	34
11	59	55	33
15	58	56	32
18	57	58	31
21	56	59	30
23	55	60	29
26	54	61	28
28	53	62	27
30	52	63	26
32	51	64	25
33	50	65	24
35	49	66	23
37	48	67	22
38	47	68	21
40	46	69	20
41	45	70	19
43	44	71	18
44	43	72	17
45	42	73	16
47	41	74	15
48	40	75	14
49	39	76	13
51	38	77	12
52	37	78	11
53	36	79	10
54	35	80	

Sun SD 15.8
Moon SD 16'
Age 15d

Figure 13.6. Error Caused by Atmospheric Refraction.

Figure 13.7. Corrections for Atmospheric Refraction.

REFRACTION
To be subtracted from sextant altitude (referred to as observed altitude in A.P. 3270)

R_0	Height above sea level in units of 1 000 ft.												R_0	$R = R_0 \times f$			
	0	5	10	15	20	25	30	35	40	45	50	55		0·9	1·0	1·1	1·2
	Sextant Altitude													R			
0	90	90	90	90	90	90	90	90	90	90	90	90	0	0	0	0	0
1	63	59	55	51	46	41	36	31	26	20	17	13	1	1	1	1	1
2	33	29	26	22	19	16	14	11	9	7	6	4	2	2	2	2	2
3	21	19	16	14	12	10	8	7	5	4	2 40	1 40	3	3	3	3	4
4	16	14	12	10	8	7	6	5	3 10	2 20	1 30	0 40	4	4	4	4	5
5	12	11	9	8	7	5	4 00	3 10	2 10	1 30	0 39	+0 05	5	5	5	5	6
6	10	9	7	5 50	4 50	3 50	3 10	2 20	1 30	0 49	+0 11	−0 19	6	5	6	7	7
7	8 10	6 50	5 50	4 50	4 00	3 00	2 20	1 50	1 10	0 24	−0 11	−0 38	7	6	7	8	8
8	6 50	5 50	5 00	4 00	3 10	2 30	1 50	1 20	0 38	+0 04	−0 28	−0 54	8	7	8	9	10
9	6 00	5 10	4 10	3 20	2 40	2 00	1 30	1 00	0 19	−0 13	−0 42	−1 08	9	8	9	10	11
10	5 20	4 30	3 40	2 50	2 10	1 40	1 10	0 35	+0 03	−0 27	−0 53	−1 18	0	9	10	11	12
12	4 30	3 40	2 50	2 20	1 40	1 10	0 37	+0 11	−0 16	−0 43	−1 08	−1 31	2	11	12	13	14
14	3 30	2 50	2 10	1 40	1 10	0 34	+0 09	−0 14	−0 37	−1 00	−1 23	−1 44	14	13	14	15	17
16	2 50	2 10	1 40	1 10	0 37	+0 10	−0 13	−0 34	−0 53	−1 14	−1 35	−1 56	16	14	16	18	19
18	2 20	1 40	1 20	0 43	+0 15	−0 08	−0 31	−0 52	−1 08	−1 27	−1 46	−2 05	18	16	18	20	22
20	1 50	1 20	0 49	+0 23	−0 02	−0 26	−0 46	−1 06	−1 22	−1 39	−1 57	−2 14	0	18	20	22	24
25	1 12	0 44	+0 19	−0 06	−0 28	−0 48	−1 09	−1 27	−1 42	−1 58	−2 14	−2 30	5	22	25	28	0

13.10.2. The displacement of the liquid and the bubble in the chamber may be divided into two vectors and each vector may be considered separately. These vectors may be thought of as a lateral vector (along the wings) and a longitudinal vector (along the nose-tail axis of the aircraft). Any change in GS can

cause a longitudinal displacement. This change can be brought about by a change in the airspeed or the wind encountered, or the change in GS brought about by a change in heading due to other factors (gyro precession, rhumb line error, etc.). A lateral displacement results from a number of causes, most of which will occur in spite of any efforts to hold them in check. These causes are Coriolis, rhumb line, and wander errors.

13.11. Coriolis Force. Any free-moving body traveling at a constant speed above the earth is subject to an apparent force that deflects its path to the right in the Northern Hemisphere and to the left in the Southern Hemisphere. This apparent force and the resulting acceleration were first discovered shortly before the middle of the 19th century by Gaspard Gustave de Coriolis (1792-1843) and given quantitative formulation by William Ferrel (1817-1891). The acceleration is known as Coriolis acceleration (or force) or simply Coriolis and is expressed in Ferrel's law.

13.11.1. You must realize that the bubble sextant indicates the true vertical only when the instrument is at rest or moving at a constant speed in a straight line as perceived in space. If the earth were motionless, this straight path in space would also be a straight path over the surface of the earth; conversely, a straight path over the motionless earth would also be a straight path in space.

13.11.2. When the aircraft is flying a path curved in space to the left, the fluid in the bubble chamber is deflected to the right and the bubble is deflected to the left of the aircraft's path over the earth. When the aircraft is flying a curved path in space to the right, the reverse is true.

13.11.3. In Figure 13.8, the aircraft is represented as flying on a curved path to the left. Note that in the inset representing the bubble chamber, the heavy black bubble is indicated in its approximate position representing the true vertical.

Figure 13.8. Error Caused by Coriolis Force.
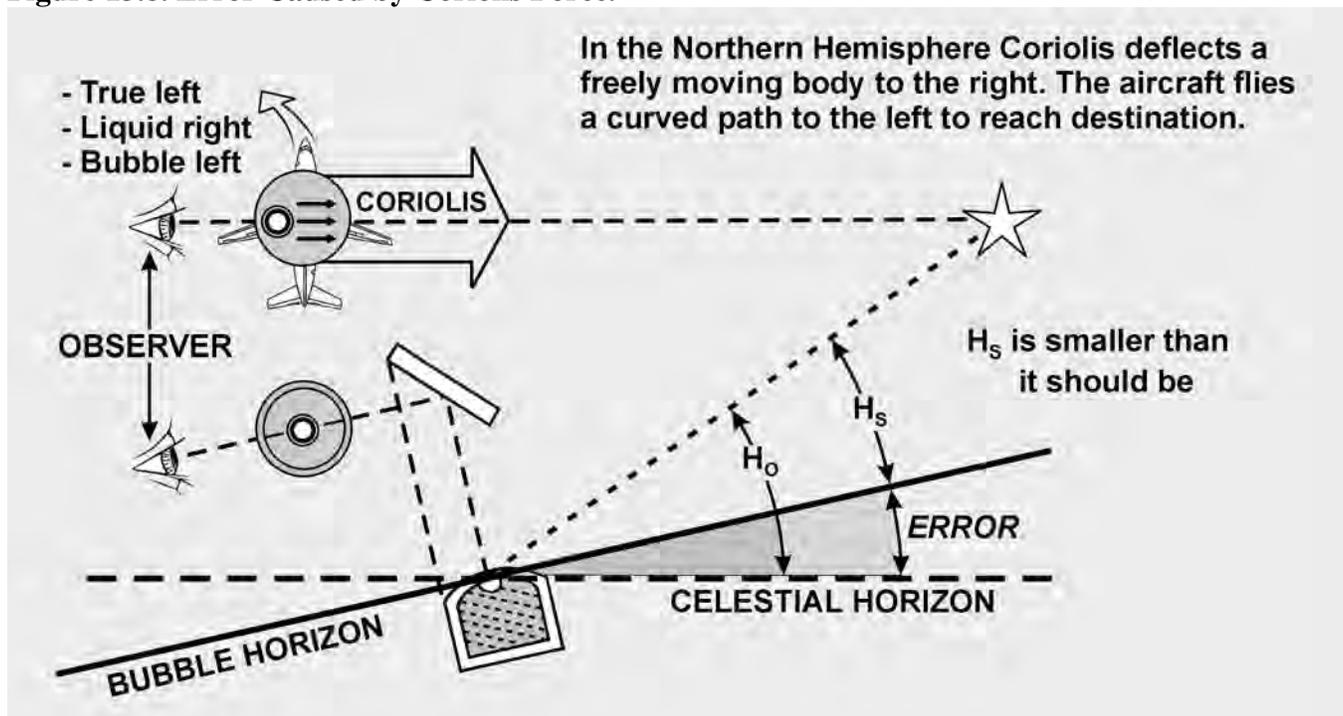
13.11.4. The observer always seeks to center the bubble and, on this beam shot facing to the right side of the aircraft to observe the body, tip the sextant up. This would tilt the bubble horizon from its true position, producing a smaller sextant reading than the true value. The smaller the height observed (Ho), the greater the radius of the circle of equal altitude—the LOP will fall farther from the subpoint than the true LOP. Obviously, if the erroneous LOP falls farther from the subpoint, it will fall to the left of the true LOP and the correction to the right is valid. Corrections for Coriolis error are shown on the inside back cover of the *Air Almanac* as well as in all volumes of Pub. No. 249.

13.11.5. Coriolis acceleration is directly proportional to the straight-line velocity, directly proportional to the angular velocity of the earth, directly proportional to the sine of the latitude, and at right angles to the direction of flight.

13.12. Rhumb Line Error. The straight Coriolis table (Figure 13.9) found in the *Air Almanac* or Pub. No. 249 has a limited application. As long as a constant TH is flown, the path of the aircraft will be a rhumb line. Because a rhumb line on the earth's surface is a curve, it is also a curved line in space. If the aircraft is headed in a general easterly direction in the Northern Hemisphere, the apparent curve is to the left and becomes an addition to the Coriolis error. By the same token, if headed in a westerly direction in the Northern Hemisphere, the apparent curve is to the right, or opposite that of Coriolis force as shown in Figure 13.10.

13.12.1. There are notable exceptions to this. When flying north or south, the aircraft is flying a great circle and there is no rhumb line error. Also, when steering by a free running, compensated gyro, the track approximates a great circle and eliminates rhumb line error.

Figure 13.9. Coriolis Correction.

CORIOLIS (Z) CORRECTION																				
To be applied by moving the position line a distance Z to starboard (right) of the track in northern latitudes and to port (left) in southern latitudes.																				
G/S KNOTS	Latitude										G/S KNOTS	Latitude								
	0° 10°		20° 30°		40° 50°		60° 70°		80° 90°			0° 10°		20° 30°		40° 50°		60° 70°		80° 90°
	′	′	′	′	′	′	′	′	′	′		′	′	′	′	′	′	′	′	′	′
150	0	1	1	2	3	3	3	4	4	4	550	0	3	5	7	9	11	12	14	14	14
200	0	1	2	3	3	4	5	5	5	5	600	0	3	5	8	10	12	14	15	16	16
250	0	1	2	3	4	5	6	6	6	7	650	0	3	6	9	11	13	15	16	17	17
300	0	1	3	4	5	6	7	7	8	8	700	0	3	6	9	12	14	16	17	18	18
350	0	2	3	5	6	7	8	9	9	9	750	0	3	7	10	13	15	17	18	19	20
400	0	2	4	5	7	8	9	10	10	10	800	0	4	7	10	13	16	18	20	21	21
450	0	2	4	6	8	9	10	11	12	12	850	0	4	8	11	14	17	19	21	22	22
500	0	2	4	7	8	10	11	12	13	13	900	0	4	8	12	15	18	20	22	23	24

Figure 13.10. Coriolis and/or Rhumb Line Errors in the Northern Hemisphere.
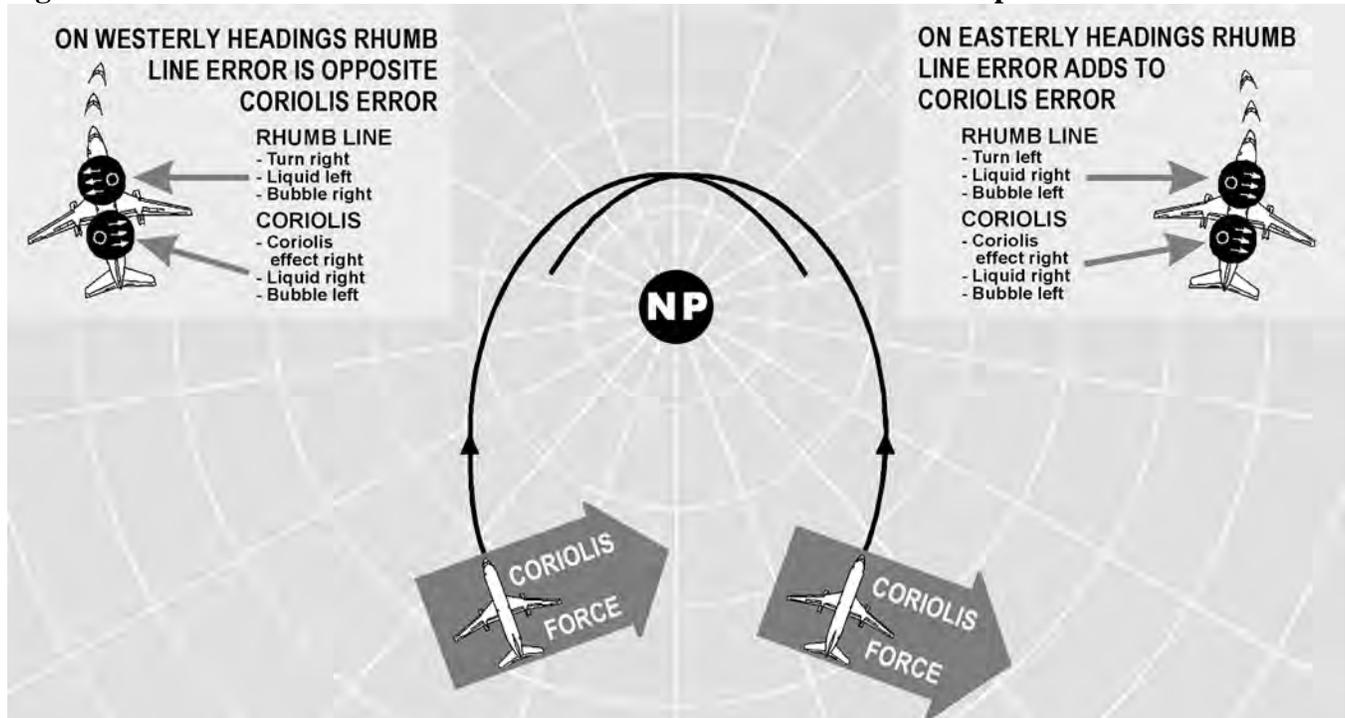
13.12.2. At speeds under 300 knots, the error is negligible. However, at high speeds or high latitudes, rhumb line error is appreciable. For example, at 60° N latitude with a track of 100° and a GS of 650 knots, the Coriolis correction is 15 NM right and the rhumb line correction is 10 NM right. Use the following steps and Figure 13.11 to determine the correction for rhumb line error and Coriolis correction:

13.12.2.1. Enter the nearest latitude on the left side. Interpolate if necessary.

13.12.2.2. Enter the nearest track across the top of the chart. Interpolate if necessary.

13.12.2.3. Choose the closest GS and extract the correction; that is, 50N, track 080°, GS 500 knots = 14.3 Right.

13.13. Groundspeed Acceleration Error. Changes in airspeed or wind velocity cause this error. Prevent changes of airspeed through good crew coordination.

Figure 13.11. Combined Coriolis and Rhumb Line Correction.

GROUND SPEED 300 KNOTS

TR. → LAT. ↓	270 / 270	260 / 280	250 / 290	240 / 300	230 / 310	220 / 320	210 / 330	200 / 340	190 / 350	180 / 0	170 / 10	160 / 20	150 / 30	140 / 40	130 / 50	120 / 60	110 / 70	100 / 80	90 / 90
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	1.1	1.1	1.1	1.2	1.2	1.2	1.2	1.3	1.3	1.4	1.4	1.4	1.5	1.5	1.6	1.6	1.6	1.6	1.6
20	2.2	2.2	2.2	2.3	2.3	2.4	2.4	2.5	2.6	2.7	2.8	2.8	2.9	3.0	3.1	3.1	3.1	3.2	3.2
30	3.2	3.2	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.1	4.2	4.3	4.4	4.5	4.6	4.6	4.7	4.7
40	4.0	4.0	4.0	4.1	4.2	4.4	4.5	4.7	4.9	5.1	5.2	5.4	5.6	5.8	5.9	6.0	6.1	6.1	6.2
50	4.5	4.5	4.5	4.7	4.8	5.0	5.2	5.5	5.7	6.0	6.3	6.6	6.8	7.0	7.2	7.4	7.5	7.6	7.6
60	4.6	4.6	4.7	4.9	5.1	5.4	5.7	6.0	6.4	6.8	7.2	7.6	7.9	8.2	8.5	8.8	8.9	9.0	9.0
70	3.8	3.8	4.0	4.3	4.6	5.1	5.6	6.1	6.8	7.4	8.0	8.6	9.2	9.7	10.2	10.5	10.8	10.9	11.0
80	0.3	0.4	0.8	1.3	2.0	3.0	4.0	5.2	6.4	7.7	9.0	10.3	11.5	12.5	13.5	14.2	14.7	15.0	15.1
89	**67.2**	**66.1**	**62.6**	**57.4**	**50.1**	**40.2**	**30.0**	**17.9**	**5.3**	7.9	21.0	33.6	45.7	55.9	65.8	73.1	78.3	81.8	82.9

GROUND SPEED 650 KNOTS

TR. → LAT. ↓	270 / 270	260 / 280	250 / 290	240 / 300	230 / 310	220 / 320	210 / 330	200 / 340	190 / 350	180 / 0	170 / 10	160 / 20	150 / 30	140 / 40	130 / 50	120 / 60	110 / 70	100 / 80	90 / 90
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	1.9	1.9	1.9	2.0	2.1	2.3	2.4	2.6	2.8	3.0	3.1	3.3	3.5	3.7	3.8	3.9	4.0	4.0	4.0
20	3.6	3.6	3.7	3.9	4.1	4.4	4.7	5.1	5.4	5.8	6.2	6.6	7.0	7.3	7.6	7.7	7.9	8.0	8.1
30	5.0	5.0	5.2	5.4	5.8	6.3	6.7	7.3	7.9	8.5	9.1	9.7	10.3	10.8	11.2	11.6	11.8	12.0	12.1
40	5.8	5.9	6.1	6.5	7.0	7.7	8.4	9.2	10.1	11.0	11.8	12.7	13.5	14.2	14.9	15.5	15.8	16.0	16.1
50	5.7	5.8	6.1	6.6	7.4	8.3	9.3	10.5	11.8	13.0	14.3	15.6	16.7	17.7	18.6	19.4	19.9	20.2	20.3
60	4.2	4.3	4.8	5.6	6.6	8.0	9.4	11.1	12.9	14.7	16.5	18.3	20.0	21.5	22.8	23.9	24.6	25.1	25.2
70	**0.9**	**0.6**	0.2	1.3	3.0	5.2	7.5	10.2	13.1	16.0	19.0	21.8	24.5	26.8	29.0	30.7	31.8	32.6	32.9
80	**18.0**	**17.5**	**15.9**	**13.6**	**10.1**	**5.6**	**0.8**	4.8	10.7	16.8	22.8	28.7	34.3	39.1	43.6	47.1	49.4	51.0	51.5
89	**334.8**	**329.2**	**313.0**	**288.9**	**254.5**	**208.2**	**160.1**	**103.6**	**44.6**	17.0	78.7	137.6	194.1	242.2	288.5	322.9	347.0	363.2	368.8

Figures in **BOLD FACE** type are plotted in a direction opposite to that of coriolis force.

*Coriolis corrections alone are the figures in the 0° or 180° column.

13.13.1. Changes in wind velocity with resultant changes in GS are more difficult to control. The change in GS will cause the liquid to be displaced, with the subsequent shifting of the bubble creating a false horizon. Notice in Figure 13.12 how the horizon is automatically displaced by keeping the bubble in the center while these changes are taking place. A very simple rule applies to acceleration and deceleration forces. If the aircraft accelerates while a celestial observation is in progress, the resultant LOP will fall ahead of the actual position. Accelerate—Ahead. The more the LOP approaches a speed line, the greater the acceleration error will become. Refer to Figure 13.13:

**Figure 13.12. Acceleration/Deceleration Errors.**
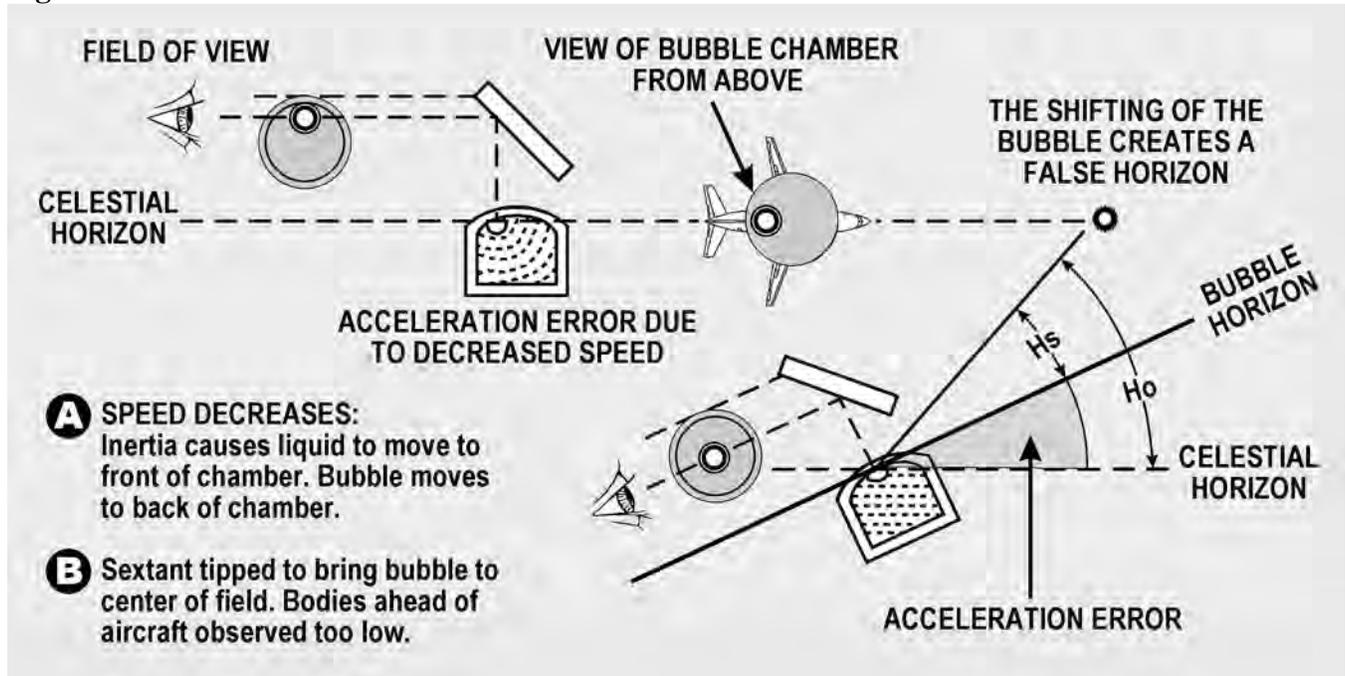
Figure 13.13. Groundspeed Acceleration Error.

ZN-TR		Groundspeed Acceleration Error/Knot
- / +	+ / -	
000/180 005/175	180/360 185/355	1.50
010/170 015/165	190/350 195/345	1.48
020/160 025/155	200/340 205/335	1.40
030/150 035/145	210/330 215/325	1.30
040/140 045/135	220/320 225/315	1.15
050/130 055/125	230/310 235/305	0.97
060/120 065/115	240/300 245/295	0.75
070/110 075/105	250/290 255/285	0.51
080/100 085/095	260/280 265/275	0.26
090/090	270/270	0.00

13.13.1.1. Enter with Zn-Track.

13.13.1.2. Extract acceleration error and apply sign.

13.13.2. Example: Track = 080°, Zn = 060°, Beginning GS – 500 knots, ending GS – 515 knots.

   060° – 080° = 340° = -1.40
   515 – 500 = 15 knots
   -1.40 x 15 = -21 correction to the Ho.

13.14. Wander Error. A change in track can be produced by changes in the wind, heading changes caused by the autopilot or changing magnetic variation, or by heading changes caused by pilot manual steering errors. As with the Coriolis force and rhumb line errors, correction tables have been developed for wander error. Values extracted from the wander correction table, shown in Figure 13.14, are to be applied to the Ho. Use the following information as entering arguments for the determination of the correction taken from the table: (1) the heading at the beginning of the observation was 079°, (2) the heading at the end of the observation was 081°, (3) the observation was taken over a 2-minute period, (4) the GS was 450 knots, and (5) the Zn of the body was 130°.

Figure 13.14. Wander Correction Tables.
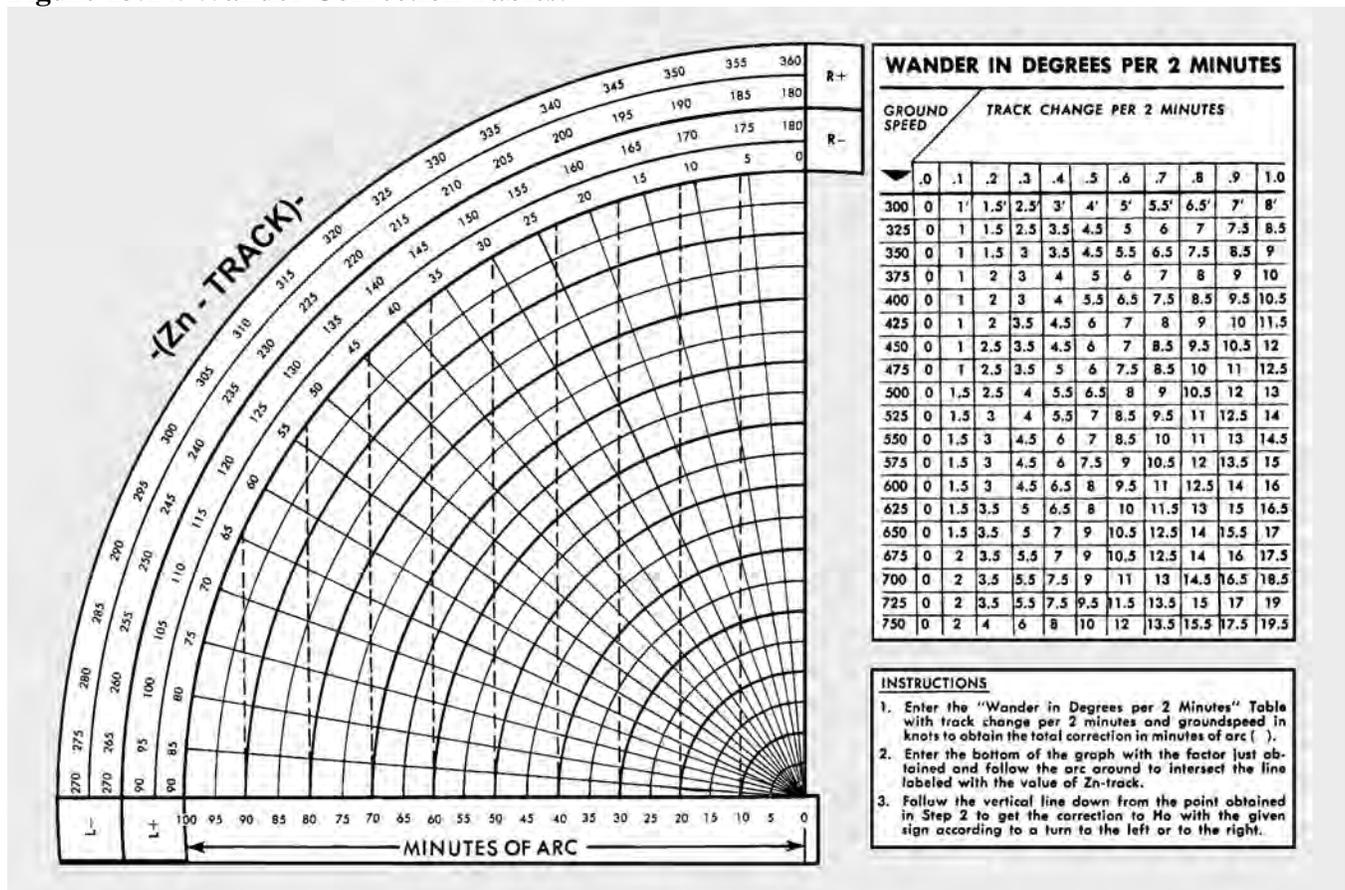
13.14.1. Following the instructions shown at the bottom of the table, enter the numerical portion of the table with the values of GS and the change of track per 2 minutes. In this case, the GS is 450 knots and the change in track per 2 minutes is 2°. Since the heading at the end of the observation is greater than the heading at the beginning, the change is 2° to the right. Notice that you must know whether the change is to the right or to the left to determine the sign of the correction. The factor obtained from the table is 12 x 2 = 24.

13.14.2. Next, enter the graph portion of the table with the value of the factor (24) and the value of the azimuth of the body minus the value of track. The graph is so constructed that it must be entered with Zn - Tr. Zn - Tr = 130° – 080°; so use 050°. Following the rules in steps two and three in the table; the correction is 19'. Since the change in track is to the right, the correction is subtracted from the Ho. This is determined by referring to the signs shown at the ends of the arc in the table. Figure 13.15 shows the effect of this correction.

13.14.3. If the track and groundspeed are the same at the beginning and the end of a shooting period, there will be no wander error.

Figure 13.15. Wander Correction Applied to Ho.
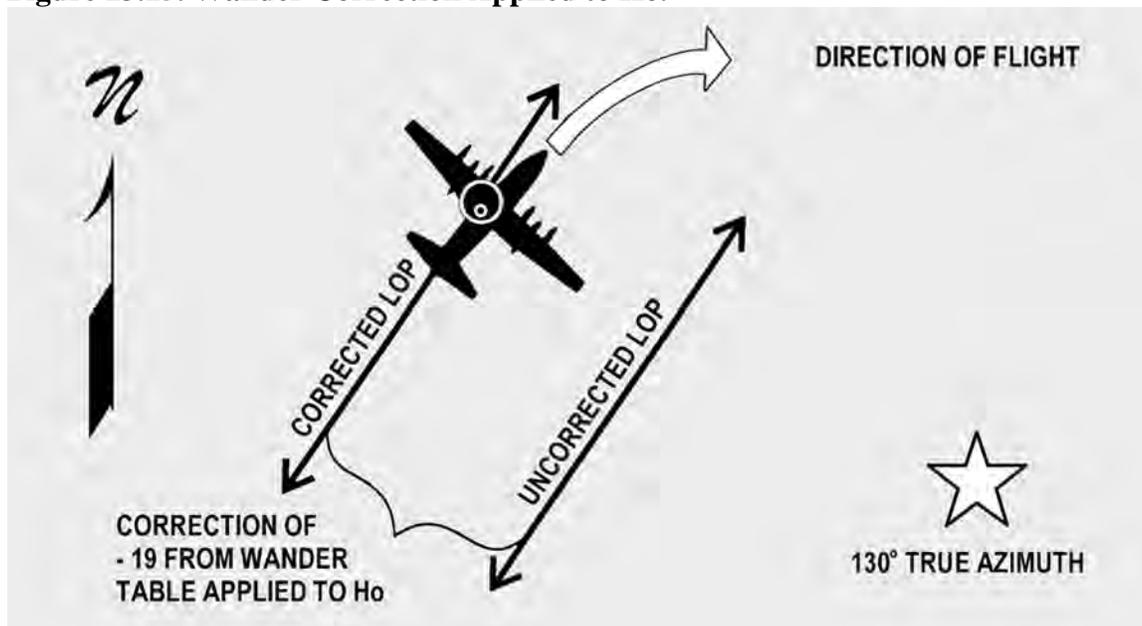
13.15. Instrument Error. Index error is usually the largest mechanical error in the sextant. This error is caused by improper alignment of the index prism with the altitude counter. No matter how carefully a sextant is handled, it is likely to have some index error. If the error is small, the sextant need not be readjusted; rather, each Hs can be corrected by the amount of the error. This means that the index error of the sextant must be known to obtain an accurate celestial LOP. Another mechanical error found in sextants is backlash. This is caused by excessive play in the gear train connecting the index prism to the altitude counter.

13.15.1. Usually, index and backlash errors are nearly constant through the altitude range of the sextant. Therefore, if the error at one altitude setting is determined, the correction can be applied to any Hs or Hc. The correction is of equal value to the error but the opposite sign.

13.15.2. The sextant should be checked on the ground before every celestial flight. Preflighting the sextant can determine the sextant error of an individual instrument. The sextant error can also be determined in-flight and a correction can be applied to the precomp to compensate for the error. To determine the error and correction in-flight, one must have a celestial LOP, a Zn and the actual (or best-known) position of the aircraft at the same time. Refer to Figure 13.16.

Figure 13.16. Determining Sextant Error Correction.

Zn = 220
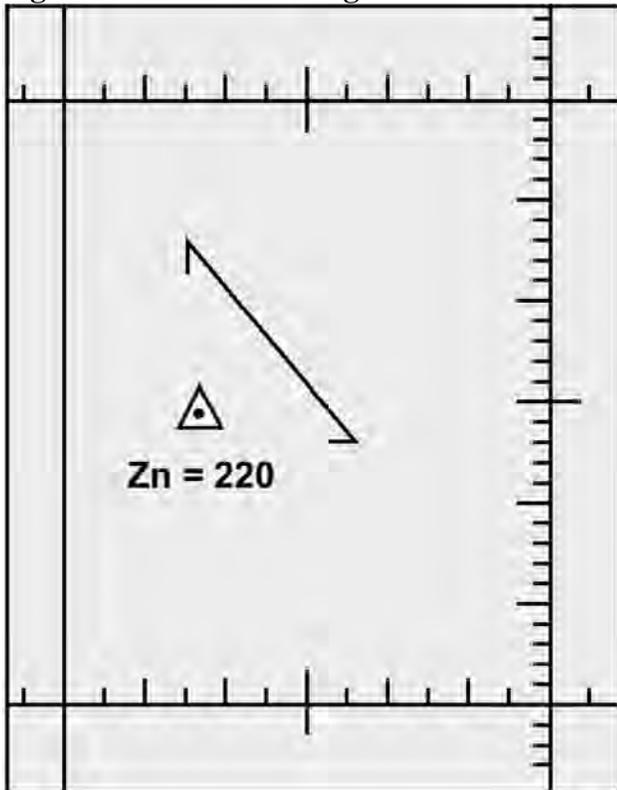
13.15.3. The fix symbol represents the best-known position at the time of the celestial LOP. To determine the actual value of the correction, measure the shortest distance between the position and the LOP. This tells you how many minutes of arc (NM) the Ho must be adjusted on subsequent shots to get an accurate LOP (in this case, the value is 10'). To determine whether this value must be added or subtracted, note whether the LOP needs to be adjusted toward the Zn or away from the Zn. Remember the rule HOMOTO? It applies here, too. If the LOP needs to be moved toward the Zn in order to be made more accurate, the Ho needs to be made larger, thus the correction is added to the Ho to make the Ho value increase. If the LOP needs to be moved away from the Zn, the correction will be subtracted from the Ho to make the Ho less. In Figure 13.16, the LOP needs to be moved 10 miles toward the Zn in order to be accurate; thus, the sextant error correction is +10 to the Ho and can be used on subsequent shots obtained from the same sextant.

13.15.4. An important thing to remember is that the sextant error correction assumes conditions will be consistent. As a technique, it is wise to obtain several LOPs with a sextant, noting the sextant errors on each, before establishing a value to be carried on the precomp. Once using that correction, make sure you use the same sextant.

13.16. Summary. The first half of this chapter described the parts and operation of the sextant, and the second half explained sextant errors. Remember to apply parallax, semidiameter, and refraction errors on every applicable shot. Corrections for acceleration errors can be applied only if you know the track and groundspeed before and after each shot, so be aware of your speed and direction when shooting. Time permitting, always try to evaluate the accuracy of your sextant on the ground.