2. From the versine of the zenith distance subtract the versine of the sum or difference.

3. Add together the log-secants of the latitude and declination, and the log difference of the versines - the result is the log versine of the Hour Angle, which take from Table II.'

Example: Given lat. 35° 55' N, declination 21° 40' S, true altitude Sun 22° 39' E. Find the Apparent Time.

lat	35° 55' N	sec			....................0.0915
dec	21 40 S	sec			....................0.0318
sum	57 35	n.vers	4639		
Z.D.	67 21	n.vers	6149		
		l.diff	1510		..........3.1790
H.A. = 2h.27m.42s ⟵		l.vers	⟵		3.3023

Apparent Time = 21h.32m.18s

24. Guyou's Pocket Tables

H. B. Goodwin, in a paper[35], published in 1895, wrote:

> 'In the development of modern navigation we owe so much to the originality and mathematical skill of our neighbours across the Channel, that the reception from that quarter of a new system of nautical astronomy, unlike anything to be found in existing textbooks, will hardly excite surprise...'

The new system to which Goodwin referred was proposed in a small pamphlet[36] published in 1884, by Lieut. Emile Guyou, Professor at the Naval Academy and Instructor on the Training Ship 'Borda' at Brest. In the same year, 1884, a small set of tables of 52 small pages (14cm x 9cm) made their appearance. The characteristic feature of these - Guyou's Pocket Tables - is that all the common problems of nautical astronomy may be solved by one single-entry table of logarithms. Sines and cosines, tangents and cotangents, secants and cosecants, and haversines and versines, are all made redundant by a table whose function is in general limited to Mercator Sailing; namely the Meridional Parts Table.

The meridional parts (mp) for any latitude ϕ is given by the formula:

$$mp(\phi) = r\int_0^{\phi} \sec\phi \,.\, d\phi$$

The integral here may be expressed as $\log_e \tan(45^\circ + \phi/2)$, so that:

$$mp(\phi) = r \log_e \tan(45^\circ + \phi/2)$$

where r is the value, in minutes of arc, equivalent to the radius of the Earth. This is, for a spherical Earth, equal to $(180^\circ \times 60)/\pi$, or 3438 very nearly.

According to Gougenheim[38], the French Naval Officer Hilleret had foreseen the use of the Table of Meridional Parts for nautical astronomical purposes, but it is to Guyou that is due the merit of having been the first to indicate a method of computation of a ship's position based on the exclusive use of the Table of Meridional Parts. The idea was not received favourably by practical navigators, and even a further paper by Guyou[39] failed to bring about an appreciation of the procedure he recommended. And Goodwin's attempt to bring Guyou's method to the notice of British navigators, in the paper referred above, seems to have made little or no impact.

Guyou's proposed method applied to the 'time sight' and it was based on the half-angle formula, using the tangent form, for finding an angle in a PZX-triangle given the three sides.

According to Goodwin, whose paper was aimed at generalising Guyou's method for nautical astronomical problems, the Table of Meridional Parts may be regarded as a table of natural logarithms each term of which is multiplied by a constant; and it is in this capacity that it lends itself to nautical astronomy.

To extend the use of Guyou's method for problems other than the one for which it was originally intended, Goodwin pointed out that it is necessary to deal with angles over 90° and with negative angles. Accordingly, he defined mp($\emptyset$) ($\emptyset$ being any angle positive or negative) as: 'the natural logarithm of tan ($45^\circ + \emptyset/2$) multiplied by the value in minutes of the radius of a circle'. He then enunciated and proved certain propositions connected with Meridional Parts. These included:

$$\text{mp}\,(180^\circ - \emptyset) = \text{mp}\,(\emptyset) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$\text{mp}\,(-\emptyset) = -\text{mp}\,(\emptyset) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

$$\log \tan \emptyset = \text{mp}\,(2\emptyset - 90^\circ)/r \quad \ldots\ldots\ldots\ldots(3)$$

If: $\tan x = \cos \emptyset,$

Then: $\text{mp}\,(2x) = 2\,\text{mp}\,(90^\circ - \emptyset) \quad \ldots\ldots\ldots\ldots(4)$

If: $\tan x = \tan a \tan b,$

Then: $\text{mp}\,(90^\circ - 2x) = \text{mp}\,(90^\circ - 2a) + \text{mp}\,(90^\circ - 2b)\ldots(5)$

From these relationships, Goodwin demonstrated how the various problems connected with PZX-triangles could be solved by the Meridional Parts Table.

Gougenheim criticised Goodwin's attempt to popularise Guyou's method in the following terms:

> '...When the angles do not appear in the formula by their tangents it is necessary to have recourse to auxiliary variables the use of which makes the calculation singularly more involved, besides which the advantage inherent in the use of one single table is lost. It may well be thought then that, by this excess of zeal, Goodwin's explanation ran counter to the end sought for and did not contribute to make evident the simplicity characterising Guyou's solution...'

Guyou's Table contains values, for each angle of latitude, $\emptyset$, of 'Lambda $\emptyset$' ($\lambda\emptyset$), and 'Co-Lambda $\emptyset$ (co λ ($\emptyset$)).

($\lambda\emptyset$) is equivalent to the meridional parts for latitude $\emptyset$, and (co $\lambda(\emptyset)$) is equivalent to the meridional parts for latitude ($90^\circ - \emptyset$).

$$\lambda(\emptyset) = (10{,}800/\pi)\,\log_e \tan\,(45^\circ + \emptyset/2)$$

$$\text{co-}\lambda(\emptyset) = (10{,}800/\pi)\,\log_e \cot\,(\emptyset/2)$$

Guyou used the following property of the term:

$$\log_e \tan(45^\circ + c/2) = \log_e \tan(45^\circ + a/2) - \log_e \tan(45^\circ + b/2)$$

and: $$\tan(45^\circ + c/2) = \frac{\tan(45^\circ + a/2)}{\tan(45^\circ + b/2)}$$

From which: $$\cot c/2 = \frac{1 - \tan a/2 \tan b/2}{\tan a/2 - \tan b/2}$$

That is: $$\cot c/2 = \frac{\cos(a + b)/2}{\sin(a+b)/2} \quad \ldots\ldots\ldots\ldots\ldots (6)$$

Now, in any spherical triangle ABC:

$$\tan A/2 = \sqrt{\frac{\sin(s - b)\sin(s - c)}{\sin s \sin(s - a)}}$$

Where: $$s = \tfrac{1}{2}(a + b + c)$$

This formula applied to the PZX-triangle for angles P and Z can be expressed as follows:

$$\cot^2 P/2 = \frac{\cos(1/2 + (d - z)/2)\cos((d + z)/2 + 1/2)}{\sin(1/2 - (d - z)/2)\cos((d + z)/2 + 1/2)} \quad \ldots\ldots (7)$$

$$\cot^2 Z/2 = \frac{\cos(1/2 + (d - z)/2 \sin((d + z)/2 - 1/2}{\sin(1/2 - (d - z)/2 \cos((d + z)/2 + 1/2} \quad \ldots\ldots (8)$$

in which 1, d and z denote, respectively, latitude, declination and zenith distance.

If: $$\cot x/2 = \frac{\cos(1/2 + (d - z)/2)}{\sin(1/2 - (d - z)/2)}$$

and: $$\cos y/2 = \frac{\cos((d + z)/2 + 1/2)}{\sin((d + z)/2 - 1/2)}$$

relations which, according to the property of the term discussed above, are equivalent to:

$$\lambda(x) = \lambda(1) - \lambda(d - z)$$
$$\lambda(y) = \lambda(d + z) - \lambda(1)$$

Formulae (7) and (8) appear, respectively, as:

$$\cos^2 P/2 = \cot x/2 \cot y/2 \quad \ldots\ldots\ldots\ldots\ldots (9)$$
$$\cot^2 Z/2 = \frac{\cot x/2}{\cot y/2} \quad \ldots\ldots\ldots\ldots\ldots (10)$$

or, by taking co-λ's:

$$2\ \text{co-}\lambda\,(P) = \text{co-}\lambda\,(x) + \text{co-}\lambda\,(y) \qquad \text{..........(11)}$$

$$2\ \text{co-}\lambda\,(Z) = \text{co-}\lambda\,(x) - \text{co-}\lambda\,(y) \qquad \text{..........(12)}$$

Finally, if altitude a replaces zenith distance z, we have:

$$(x) = \lambda(1) + \text{co-}\lambda\,(d + a) \qquad \text{..........(13)}$$

$$(y) = \text{co-}\lambda\,(d - a) - \lambda(1) \qquad \text{..........(14)}$$

and formulae (7) and (8) reduce to:

$$2\ \text{co-}\lambda\,(P) = \text{co-}\lambda\,(x) + \text{co-}\lambda\,(y) \qquad \text{..........(15)}$$

$$2\ \text{co-}\lambda\,(Z) = \text{co-}\lambda\,(x) - \text{co-}\lambda\,(y) \qquad \text{..........(16)}$$

Two cases are distinguished: the first in which latitude and declination have the same name; and the second, in which latitude and declination have different names. Moreover it is necessary to name the azimuth from the declination.

The general form of the solution for P and Z, as given by Gougenheim, is as follows:

$$1 \longrightarrow \lambda(1)$$

$$d$$

$$a$$

$$(d + a)\begin{cases} <90^\circ \longrightarrow \text{co-}\lambda\,(d + a) \\ >90^\circ \longrightarrow \lambda(d + a - 90^\circ) \end{cases}$$

$$(d - a) \longrightarrow \underline{\text{co-}\lambda\,(d - a)}$$

$$\lambda(x)$$

$$\lambda(y)$$

$$\lambda(x) \longrightarrow \text{co-}\lambda\,(x)$$

$$\lambda(y) \longrightarrow \text{co-}\lambda\,(y)$$

$$\left.\begin{array}{ll} \text{sum} & \tfrac{1}{2}\text{sum} = \text{co-}\lambda\,(u) \\ \text{Diff}\begin{cases} +\text{ve} \\ -\text{ve} \end{cases} & \begin{array}{l} \tfrac{1}{2}\text{diff} = \text{co-}\lambda\,(v) \\ \tfrac{1}{2}\text{diff} = (v - 90^\circ) \end{array} \end{array}\right\} \quad \begin{array}{l} d < a,\ P = u,\ Z = v \\ d > a,\ P = v,\ Z = u \end{array}$$

The following example illustrates Guyou's method:

<u>Example</u>: Given $l = 17^\circ\ 19'$ N, $d = 22^\circ\ 52'$ N, $a = 10^\circ\ 15'$ W. Find P and Z using meridional parts to the nearest unit.

$l = 17^{\circ}\ 19'$ N

$d = 22\ \ 52$ N

$a = 10\ \ 15$

$d + a = 33\ \ 07$

$d - a = 12\ \ 37$

$\lambda(l) = 1055$

co- λ $(d + a)$ = 4170

co- λ $(d - a)$ = 7571

$\lambda(x) = 5225$

$\lambda(y) = 6516$

co- λ (x) = 1528

co- λ (y) = 1041

sum = 2569

Diff = 487

co- λ(u) = 1285

co- λ(v) = 244

Z = N 69° 04' W

P = 85° 56'

The solution, using a meridional parts table, is facilitated if the table is designed to give values of λ and co-λ abreast of each other against angle as argument. Friocourt's[40] Meridional Parts Table, a fragment of which is illustrated in figure 8.5, is designed in this way.

LATITUDES CROISSANTES						
'	15°		16°			'
	λ	co-λ	λ	co-λ		
0	910.5	6970.3	.	.		
1	911.5	6966.5	.	.		
2	912.5	6962.6				
3	913.6	6958.8				
.	.	.	.			
.	.	.	.			
58	971.7	6749.4	1034.3	6537.9		
59	972.7	6745.7	1035.3	6534.4		
	74°		73°			

Figure 8.5

Goodwin, after outlining the several advantages of Guyou's method, summed up by writing:

> '...In fact the Nautical Astronomy worked out by Meridional Parts may be said to present the mariner with a new system of computation, the rules for which he may carry in his waistcoat pocket, and the necessary tables (nine out of 553 pages which form the Inman Collection) in the crown of his hat...'

25. Altitude Azimuth Tables (H.O.200)

The United States Hydrographic Office published, in 1913, H.O.200 under the title:

> 'Altitude, Azimuth and Line of Position, Marcq St. Hilaire, cosine-haversine formula and also Aquino's Altitude and Azimuth Tables'

This work comprises 320 pages of diverse tables for solving PZX-triangles using the cosine-haversine method and also Aquino's Method (See Chapter 9). A fourth edition of this work appeared in 1918.

It was the publication of the combined haversine table in this work about which Percy L. H. Davis bitterly complained of the infringement of what he claimed to be his copyright.

26. Armistead Rust's Tables

Captain Armistead Rust of the United States Navy, published his Practical Tables[41] in 1918. The formula used by Rust for the basis of his tables is, in effect, an adaptation of Borda's rule. In a spherical triangle PZX we have:

$$\cos P = \frac{\cos ZX - \cos PZ \cos PX}{\sin PZ \sin PX}$$

$$= \frac{\sin a \pm \sin l \sin d}{\cos l \cos d}$$

i.e. $$1 - \cos P = 1 - \frac{\sin a \pm \sin l \sin d}{\cos l \cos d}$$

i.e. $$\text{vers } P = \frac{\cos l \cos d \pm \sin l \sin d - \sin a}{\cos l \cos d}$$

$$= \frac{\cos (l \pm d) - \sin a}{\cos l \cos d}$$

i.e. $$\text{hav } P = \sec l \sec d \tfrac{1}{2}(\cos (l \pm d) - \sin a)$$