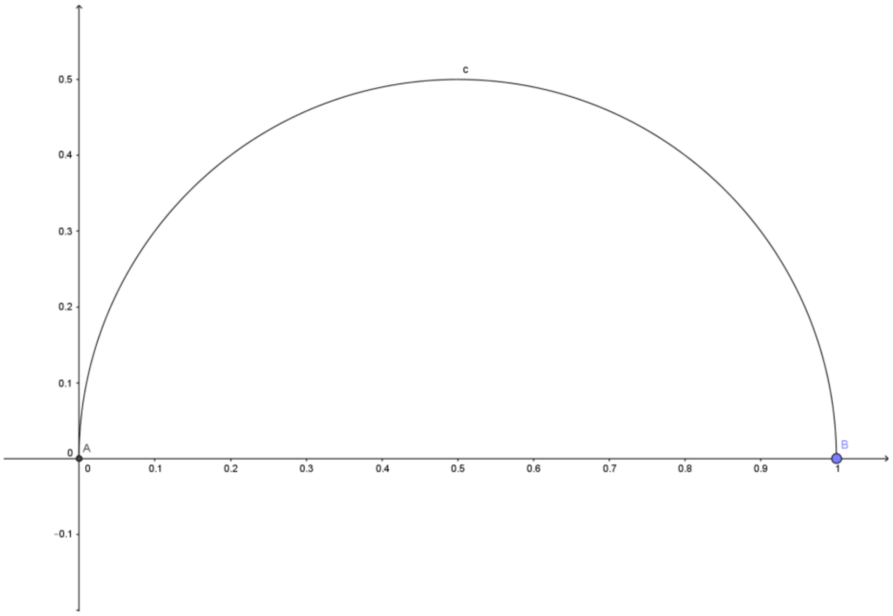
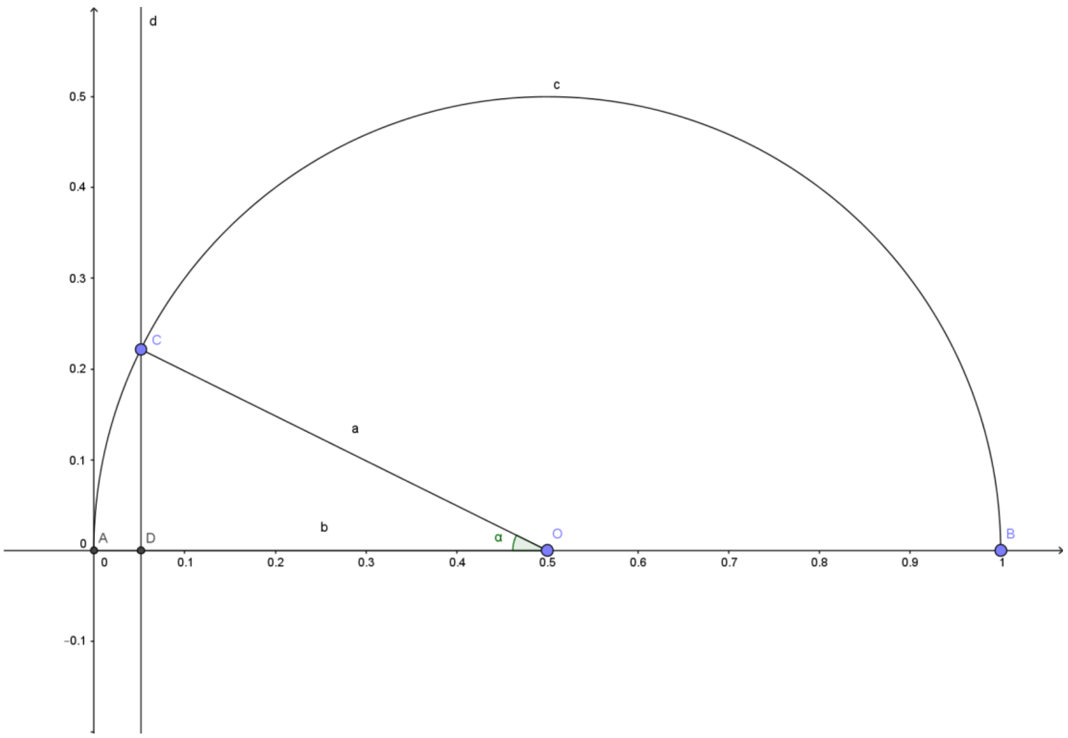
# Determining Hc by graphical use of the all-haversine formula

Triggered by the worksheet for graphical sight reduction on Erik de Man's nautical pages [ http://www.siranah.de/html/sail008h.htm ] I set out to find if it would be possible to see if it could be done for the all haversine formula also.

I came up with a circle (or semi-circle for compactness) between 0 and 1.



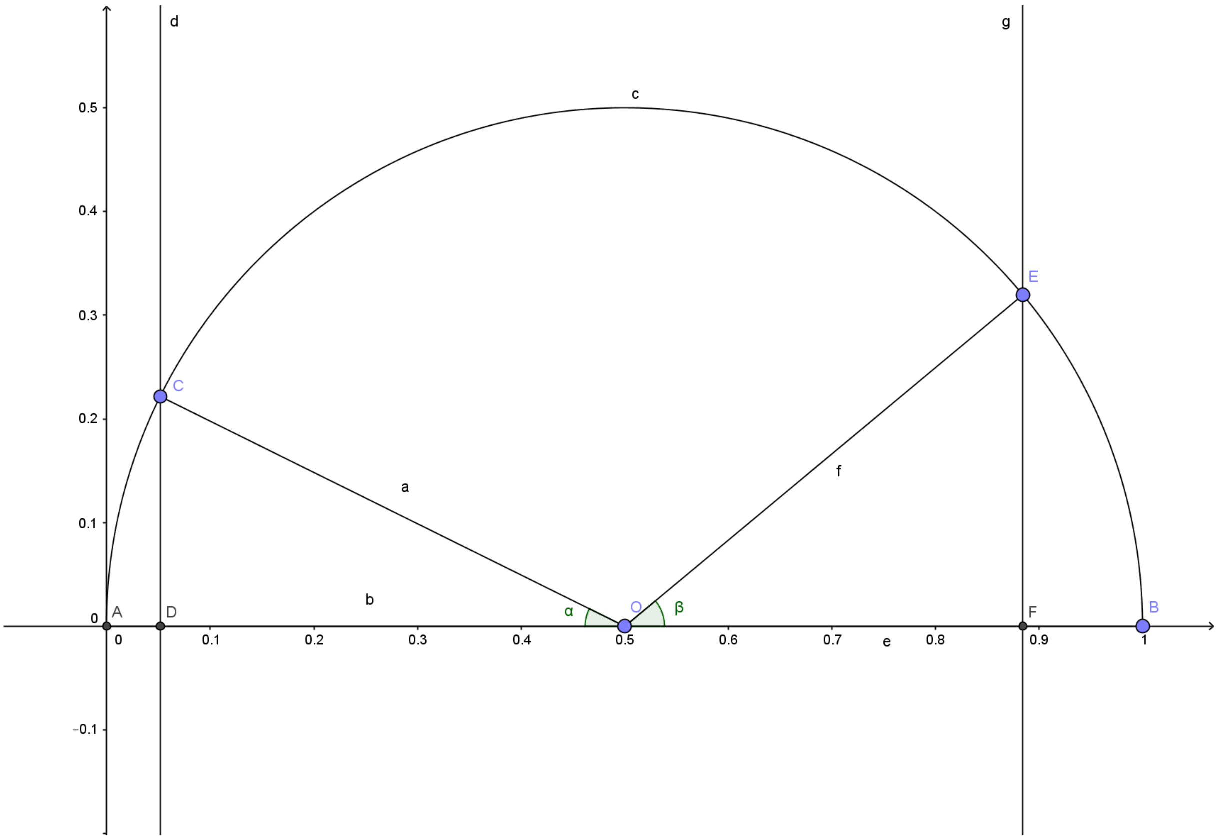

When a line is drawn from the center of the (semi-)circle at angle of α as the included angle of AOC, the distance AD, with D being the projection of C on the x-axis, is the haversine of α.



Can this be used to model the formula

Hav (|B - Dec|) + (1 – hav (|B - Dec|) – hav (|B + Dec|)) * hav (LHA) ?

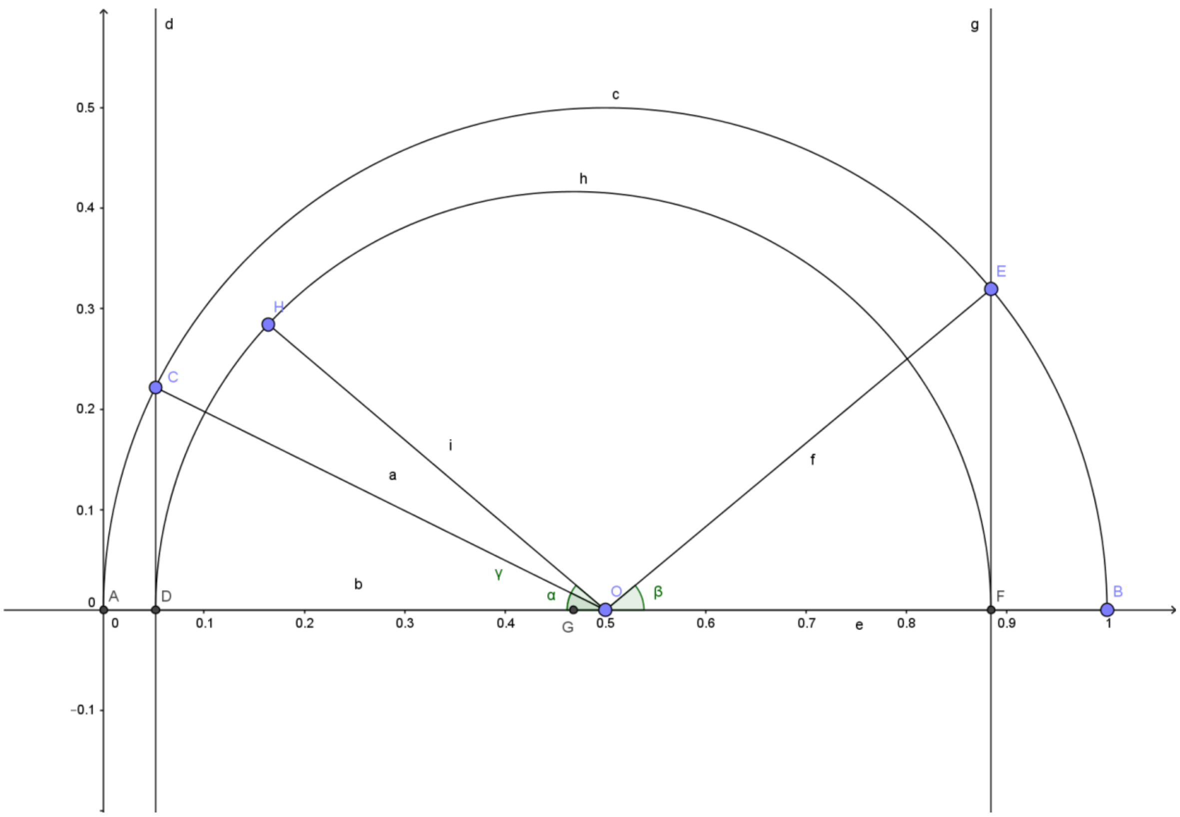
I think so. As the total distance between A and B is 1 (by definition) and we take α = |B – Dec|, it is then also possible to define β as |B + Dec| and put it in the same figure.




In this figure AD = hav (|B – Dec|) , FB = hav (|B + Dec|) and thus

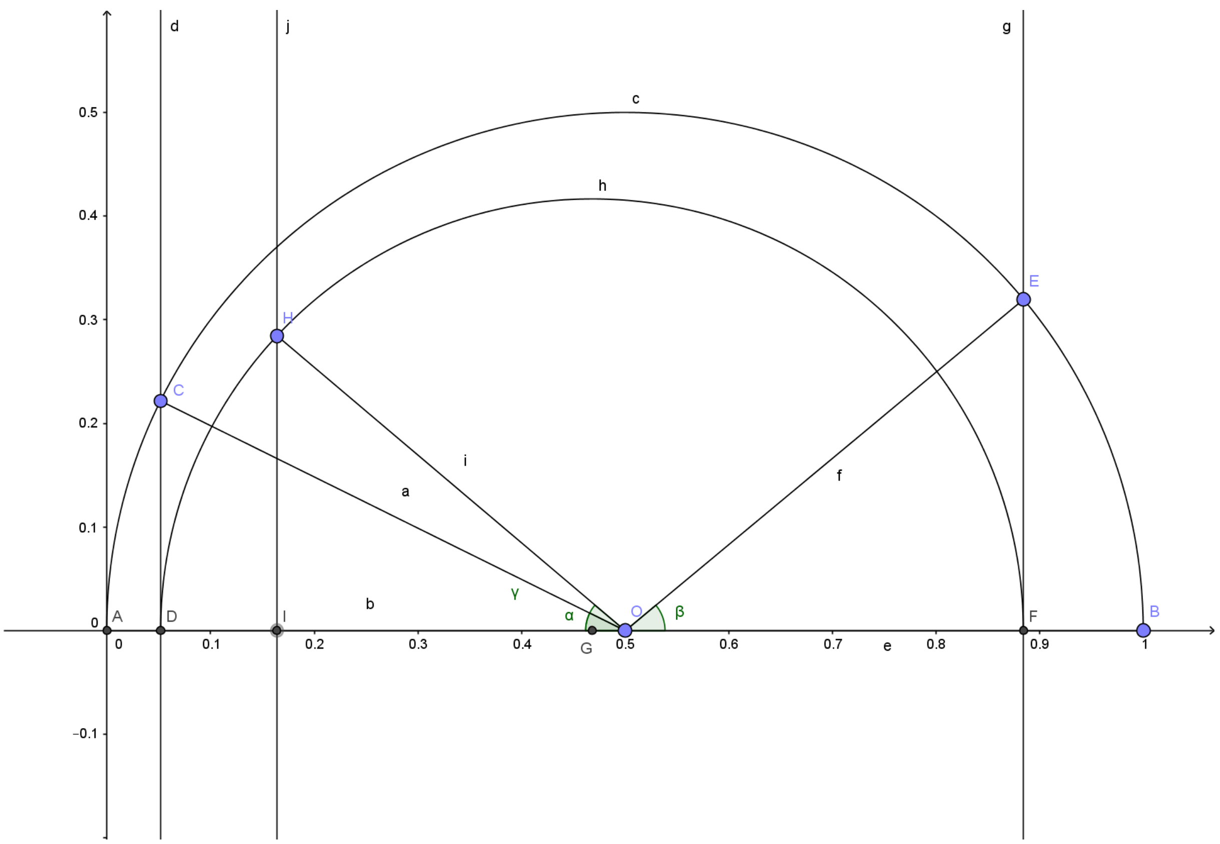
DF = (1 – hav (|B – Dec|) – hav (|B + Dec|)

Hav (LHA) is then constructed by drawing a new semi-circle between D and F. From the center G of this semi-circle a line GH is drawn so that the included angle γ in DGH equals LHA.




With I being the projection of H on the x-axis and DF being (1 – hav (|B – Dec|) – hav (|B + Dec|), DI becomes (1 – hav (|B – Dec|) – hav (|B + Dec|)) * hav (LHA) and

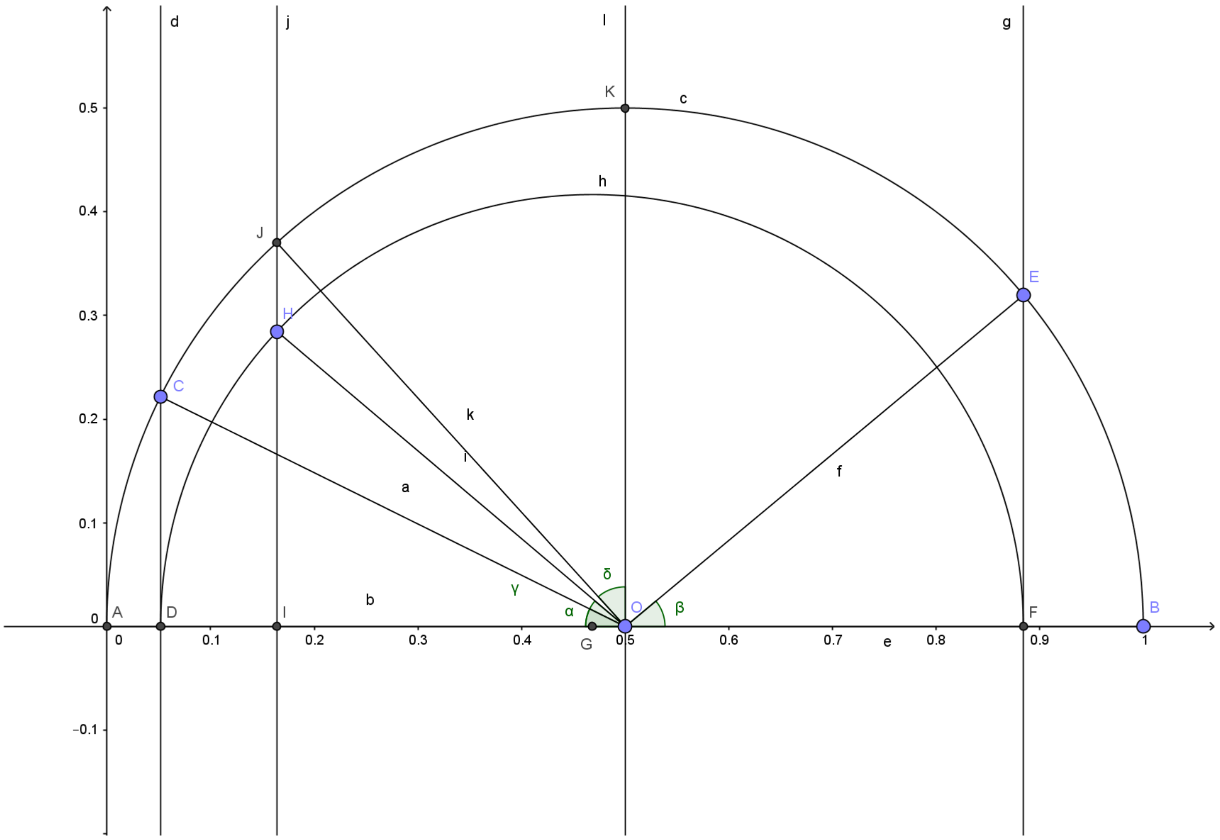
AI = hav (|B – Dec|) + (1 – hav (|B – Dec|) – hav (|B + Dec|)) * hav (LHA)



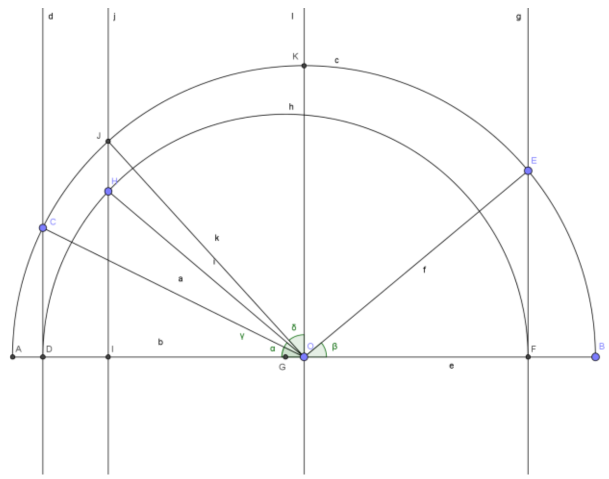
We then complete the determination of the zenith distance ZD, knowing

Hav (ZD) = hav (|B – Dec|) + (1 – hav (|B – Dec|) – hav (|B + Dec|)) * hav (LHA)

We can do this because we can see from the above hav (ZD) can be found by drawing a line from center O to a point J on the semi-circle AB such that the angle AOJ is the zenith distance. Hc being 90° - ZD means angle δ included in JOK is Hc.




As AB equals 1 by definition, it really isn't necessary to draw an axis with values. One could just start by drawing a line between to arbitrary points on an empty piece of paper and proceed from there.



## Determining azimuth by graphical use of the all-haversine formula

Next up was the azimuth angle Zn. From rewriting the spherical law of cosines into an all haversine formula as has been done for Hc (ZD really) I found

Hav (coDec) = hav (|B – Hc|) + (1 – hav (|B – Hc|) – hav (|B + Hc|)) * hav (Zn)

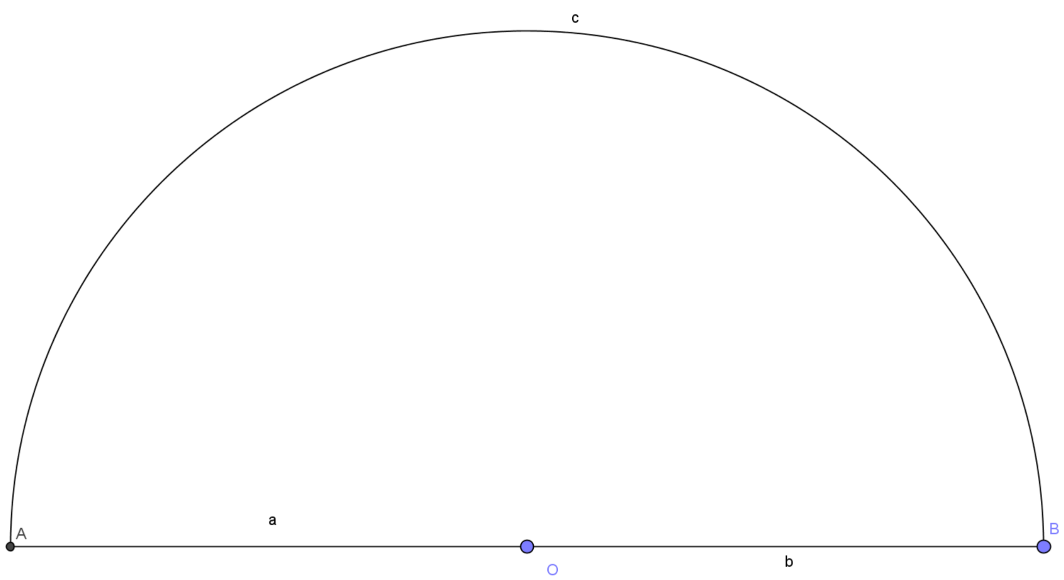
So this can be modelled in the same way as Hc above, δ being Dec, γ being azimuth and α and β being hav (|B – Hc|) and hav (|B + Hc|) respectively.

Normally the azimuth is calculated by rearranging the formula used. In this case, that would mean writing the haversine azimuth formula as

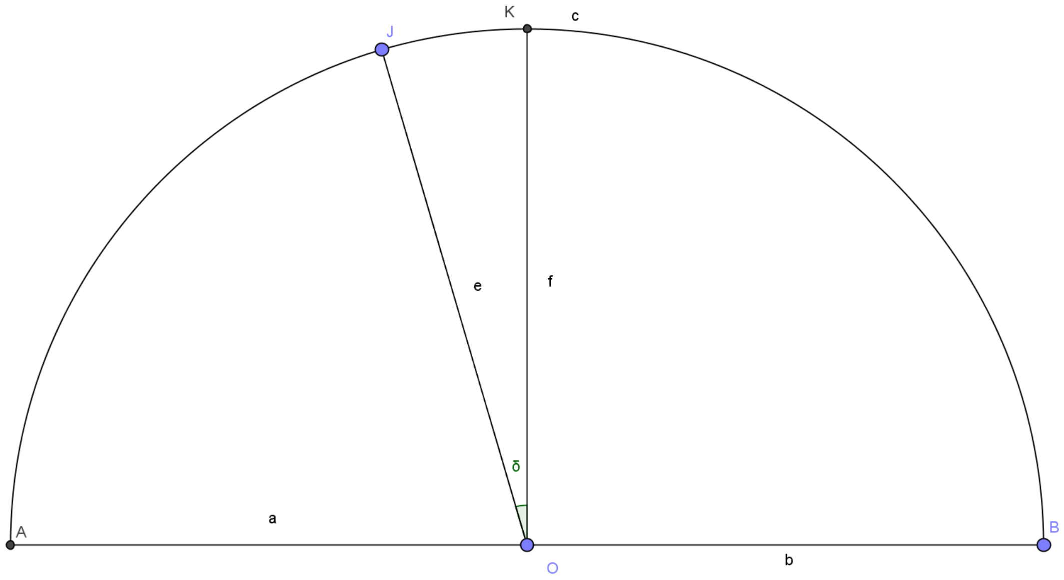
Hav (Zn) = (hav (coDec) - hav (|B – Hc|)) / (1 – hav (|B – Hc|) – hav (|B + Hc|))

But in case of using the diagram, that isn't necessary. The same is achieved by "working from two sides towards the middle".

The start is the same, points A, B and O, line AB, (semi-)circle AB



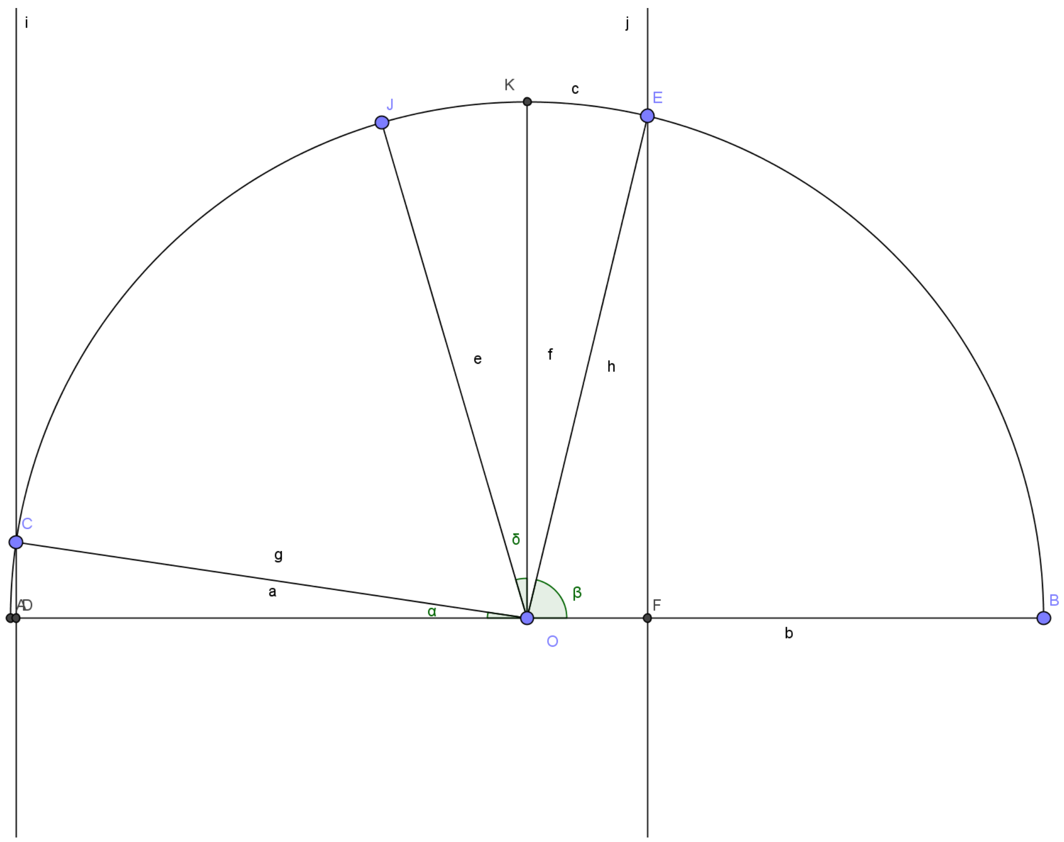

Next come a vertical line through O (representing East-West), points J and K and lines connecting J and K to O, in such a way that the angle δ included JOK equals declination Dec. To the left of the vertical can be taken as North of the equator, to the right is South. (you may take A to be the North pole, B the South pole ☺ )



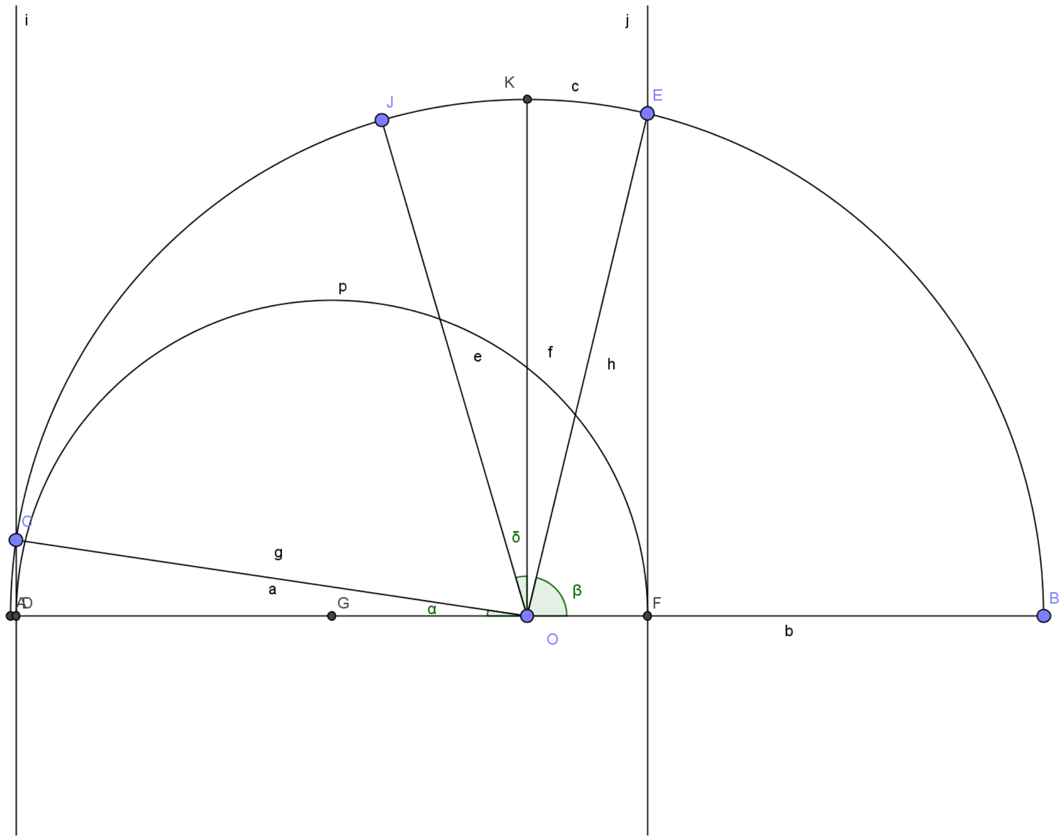

Then construct hav (|B – Hc|) and hav (|B + Hc|).

hav (|B – Hc|) : point C, vertical line through C, point D (α = |latitude B – Hc|)

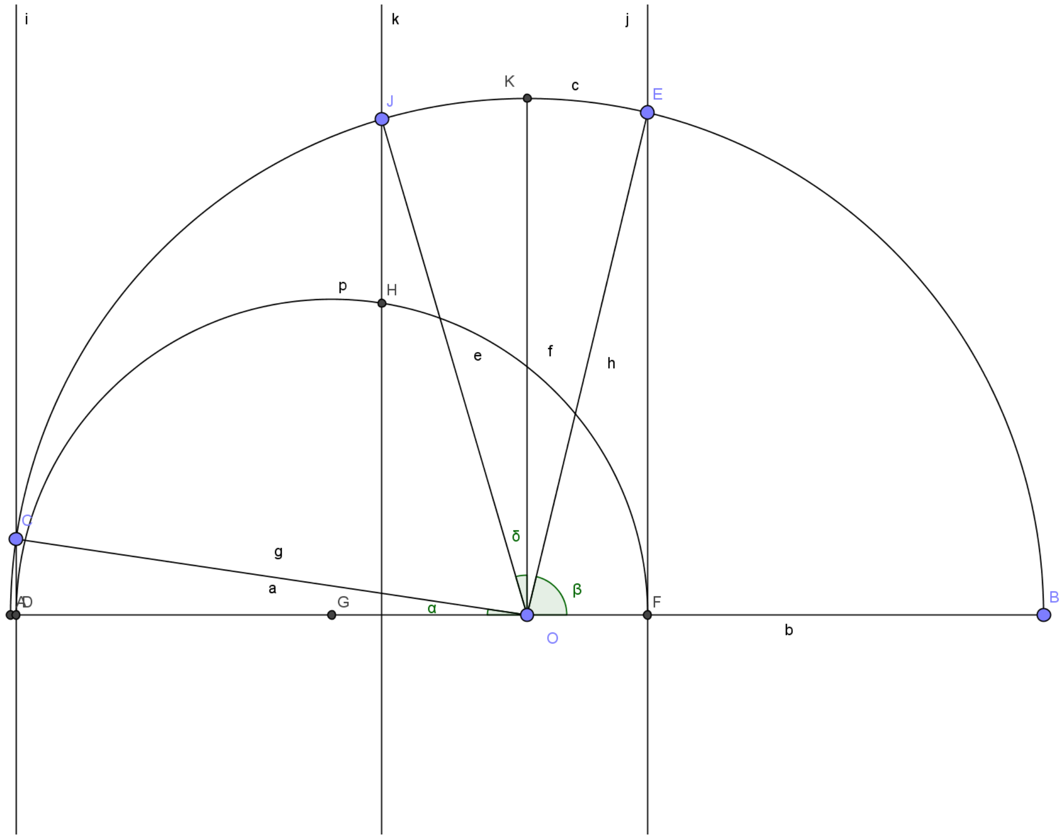
hav (|B + Hc|) : point E, vertical line through E, point F (β = |latitude B + Hc|)



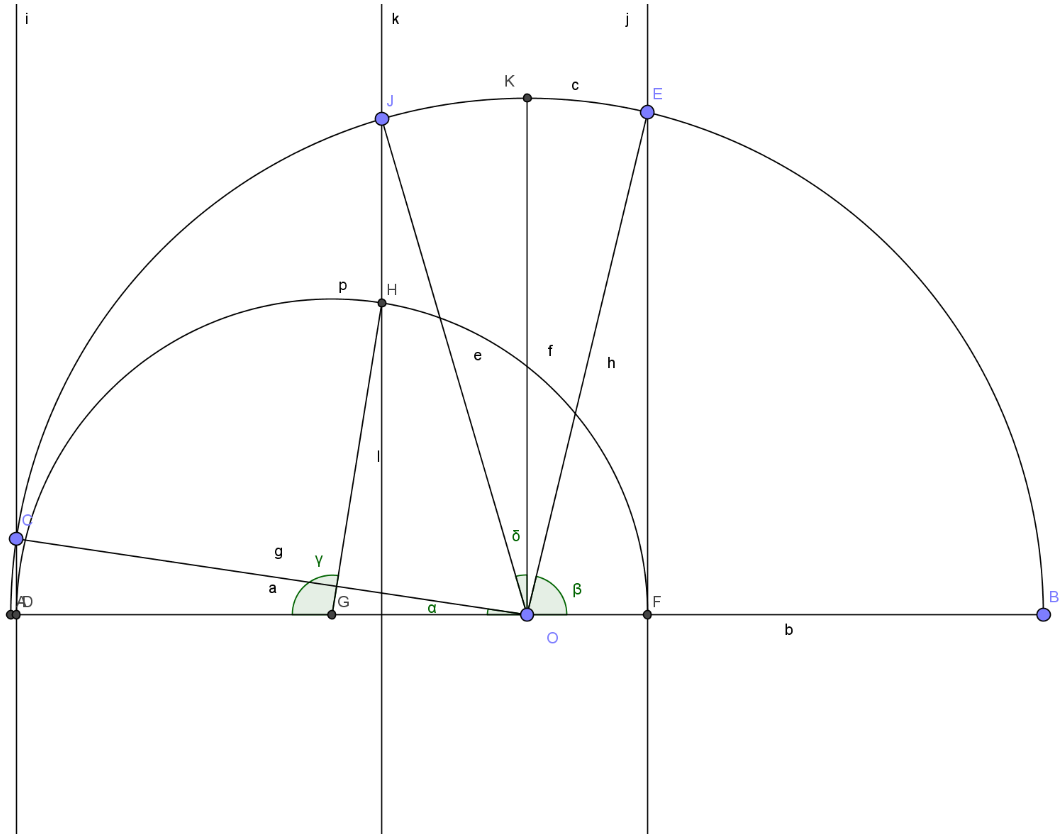
Between D and F draw semi-circle DF having center point G




Then draw a vertical line through J and point H on the intersection of this line and the inner semi-circle.




Finally draw a line GH. The angle DGH (marked γ) is the azimuth Zn. (the angle in between North and the bearing of the sun - or any other heavenly body)



## Bearing (azimuth) at sunrise and sunset

From the last figure it can easily be seen that:

At spring and fall equinox at sunrise/sunset (Hc = 0) the center of the inner semi-circle and the center of the outer semi-circle coincide. So, wherever you are on Earth, the azimuth at sunrise/sunset will be due East/West.

It can also be seen that between spring and fall equinox the inner semi-circle will be shifted to the left, so the azimuth at sunrise/sunset will always be North of East-West, wherever you are on Earth. Likewise, between fall and spring equinox the inner semi-circle is shifted to the right so the azimuth at sunrise/sunset will always be South of East-West, wherever you are on Earth.