Diagrammatic Solutions for Astronomical Navigation

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THIS paper discusses the possibilities of using diagrammatic solutions for accurate astronomical navigation and gives a detailed description of two methods of doing so that have been put into effect in Germany. These methods, the ARG1 and the *Astronomischer Rechenatlas*, have previously been described in German publications and referred to in numerous reports and memoranda written in this country and the United States. So far as is known no authoritative description of these two aids to navigation has yet been published in English, and it is appropriate that this, the first, should have been written by Dr. Freiesleben, who played a prominent part in their development.—*Ed.*

1. INTRODUCTION. The development of radio aids to navigation has not made astronomical navigation redundant, but it has given more force to earlier demands for the simplification of methods of reduction. No longer is it acceptable to have a large variety of alternative methods, and it is probable that attempts will be made to select a single method that is both simple and universally applicable.

It seems probable that altitude-azimuth tables such as H.O. 214 will gain wide acceptance for general use; it should, however, be realized that solutions based on nomographical principles are also capable of fulfilling the requirements, and in this paper some of the notable contributions in this field that have been made in the last decade will be described.

2. THE USE OF AUXILIARY TERMS. Numerous attempts to make use of nomograms based on functional ladders for navigational purposes have been made. It has, however, been pointed out by Ramsayer[1] that it is impossible to represent mathematically the true relationship between latitude ϕ, declination δ, hour angle t, and altitude H, as given for example in the formula $\sin H = \sin\phi \sin\delta + \cos\phi \cos\delta \cos t$, by a table of baselines with four straight parallel ladders. This representation requires the use of auxiliary terms, such as are used for example in the Bygrave slide rule. Auxiliary terms have the disadvantage that they entail the use of sign rules, as is the case with the numerous tables based on splitting the astronomical triangle into two right-angled triangles. This is probably the principal reason why diagrams of the type referred to have not gained general acceptance.

3. REPRESENTATION BY PROJECTION. There remains a series of diagrammatic solutions based on a representation of a spherical coordinate system by projections. On these, of course, curves and not straight lines represent the coordinates. In order to solve a spherical triangle it is then

necessary to transfer one system of polar coordinates into another, congruent to it and rotated through a given angle. Thus the altitude and azimuth of a heavenly body represented on the horizontal coordinate system are equivalent to the declination and hour angle represented on the system of the celestial equator, the two systems being congruent; one system can be transformed to the other by rotating the distance from the zenith to the pole, that is, the co-latitude.

Numerous attempts to use these principles to achieve a plane representation of spherical coordinate systems have been made, but in practice most of them have resulted in diagrams whose precision was only adequate for approximate calculations.

It is, however, possible to reduce such a diagram to individual sheets and thus obtain a large enough scale and a degree of accuracy that is sufficient for surface navigational purposes. This principle is used in the well-known star altitude curves first suggested by Weems[2] and used to great advantage in the Astrograph.

In the representation of these equi-altitudes the type of projection used for the maps is of little importance. Although the method is simple, it has the great disadvantage that it is not applicable to the Sun, Moon, and planets, and also that it restricts the observer to certain pairs of stars. Therefore, apart from special occasions, such equi-altitude diagrams are of less value than two other true projections of the spherical coordinate system to the plane which will be described.

These two solutions have been realized, with great accuracy, in the astronomical computing instrument known as the ARG (Astronomisches Rechengerät) made by Zeiss, which is based on the stereographic projection, and in an application of the 'flat map' diagram first suggested by Favé and Rollet de l'Isle[3], and, later, in an atlas compiled by Maurer[4], and finally produced by the German Testing Institute for Aviation (Versuchsanstalt für Luftfahrt) in 1943. Another method of using the same diagrammatic solution, differing from the method mentioned both mathematically and practically, but in some ways similar to it, has been proposed by Schütte.[5]

4. THE ARG. The main constituent of the ARG of Messrs. Zeiss (Fig. 1) is a stereographic diagram of a hemisphere projected on to a plane parallel to the plane of the meridian from east and/or west. The diagram was drawn with the radius of the hemisphere measuring one metre. It was then reduced photographically, and finally copied on to a glass plate. The vertical circles of the diagram between $+60°$ and $-60°$ are shown at intervals of 10 minutes of arc; thence, towards the pole, up to 80°, at intervals of 30 minutes of arc; and then at gradually increasing intervals. The parallels of declination and/or altitude are drawn at intervals of 10 minutes. As a tenth of the intervals can be estimated without difficulty, the instrument can be set to exactly 1 minute of arc. The instrument is set, looking through the central microscope ($\times 28$ magnification), by means of a cross-shaped micrometer. On the outer edge of the diagram

is a scale marked for every 10 minutes of arc; this can be set with great accuracy with the aid of a second microscope. The star's place (declination and time angle) is set on the instrument, looking through the central microscope, while the pointer on the rim, seen through the outer microscope, is at 90°. The time angle τ, as used in Germany is counted from 0 – 360° and/or 0 – 24^h from the lower meridian through east, south and west. It therefore differs from the hour angle by 180°, or 12^h. This definition, which was introduced when astronomical time began to be reckoned from midnight to midnight, has numerous advantages and is closely related to local time.

Two sets of numbers, from 0–180 and 180–360, have to be considered when setting the time angle. This is taken into account in the numbering which has the incidental advantage that circles marked with two sets of numbers are instantly recognizable as vertical circles, whereas those marked from + 90° to 0 and 0 to – 90° represent parallels of declination or altitude (see Fig. 2). The accurate setting of the graticule through the central microscope to minutes is, as has been mentioned, done by means of a micrometer adjustment.

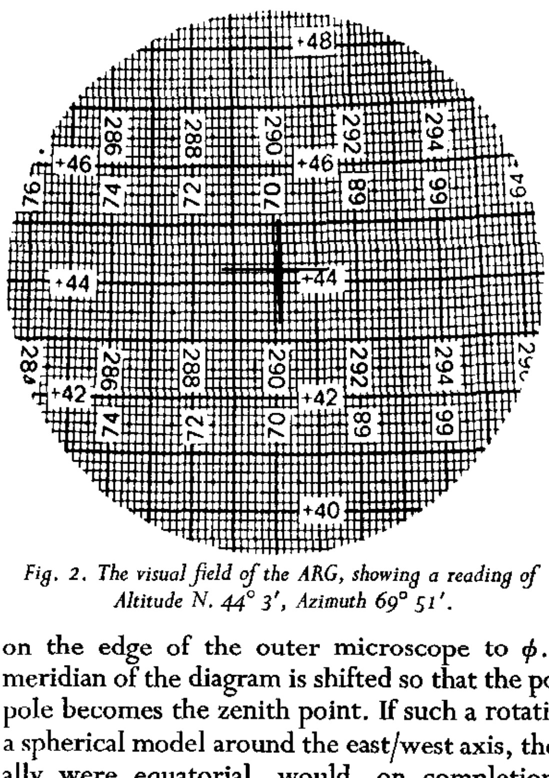

Fig. 2. The visual field of the ARG, showing a reading of Altitude N. 44° 3′, Azimuth 69° 51′.

The diagram is then rotated through $90° - \phi$, by shifting the scale on the edge of the outer microscope to ϕ. During this process, the meridian of the diagram is shifted so that the point previously marking the pole becomes the zenith point. If such a rotation could be carried out on a spherical model around the east/west axis, the coordinates which originally were equatorial, would, on completion of the rotation, be coordinates of the horizontal system. The same is true for the stereographic projection drawn from the direction of the east/west axis.

The lines marking hour circles and parallels of declination prior to the rotation now become azimuth circles and parallels of altitude. The position of the graticule seen through the microscope will now show the altitude and azimuth of the star whose declination and time angle was set. The azimuth is reckoned from 0 to 360° and its value lies in the same hemisphere as the time angle, with which it has the initial meridian of the reckoning in common.

The instrument must be carefully centred. If this has been done, the accuracy to be obtained depends on the errors of estimation during the setting and reading of the instrument, and the errors probably existing in the graticule. The latter are small, owing to the care taken by the manufacturer Dr. G. Förstner-Heidenheim. Experience has shown that the setting errors are also small, and investigations of the accuracy [6] of the instrument have shown a maximum error of 3′, the mean error being between 1′ and 1′·5. It is impossible to repair or adjust the instrument on board but this should not be necessary under normal conditions as there are hardly any mechanical parts which can be put out of order.

The instrument very nearly approaches the optimum to be attained in this way. A drawback is, however, the high price caused by the cost of the optical constituents and the careful centring. Another disadvantage is the necessity of artificially illuminating the diagram from below, so as to enable the observer to adjust the instrument carefully and take precise readings under all circumstances and in spite of the high magnification. Messrs. Zeiss planned therefore to manufacture an instrument with a diagram of 20 cm.—the present type being 10 cm. only—and to do without the artificial illumination and use a magnifying glass instead of a microscope for the reading. Such a type would have been a bit more unwieldy but would have been easier to operate. No model of this type was, however, manufactured.

The instrument has some predecessors, notably Kohlschütter's *Meskarte*[7] and the Bastien-Morin Calculateur du Point Astronomique,[8] both of which were based on the same principles. The accuracy attained in either case was, however, insufficient.

5. Conical and cylindrical projection. Theoretically, the ARG method is not restricted to the stereographic projection. As far as plane projections are concerned, the rotation of the coordinate system on the sphere by $90^\circ - \phi$ means a rotation by the same angle in the projection as well. In the case of conical projections, if the rotation on the sphere is 90° the corresponding rotation around the top of the cone is $90^\circ - \phi/n$, n being cosec α where α is half the opening angle of the cone. Assuming the top of the cone to be at an infinite distance, the projection is no longer a conical projection but a cylindrical one in which the rotation around the top of the cone is replaced by a shifting by $90^\circ - \phi$. A diagram based on such a cylindrical projection replaces the rotation which takes place in the ARG by a shifting along a track dependent on ϕ. Generally this method can be applied with every true cylindrical projection, e.g. the Mercator projection. The simplest formulae and a scale which is equal both on the equator and the initial meridian, however, are obtained from a projection based on the principle of the 'flat map', the so-called Cassini-Soldner projection.

In this projection the surface of the Earth is portrayed on a cylinder which touches the globe along a meridian, and the intervals of the points portrayed are made equal to their spherical intervals. For the solution of the spherical astronomical triangle, the meridian of the sky is the tangential meridian, and the formulae for the portrayal are

$$\sin y = \cos \delta \ \sin t; \ \sin x = \sin \delta \ \sec y; \text{ and } Y = y$$

(Fig. 3), where OP is the meridian, OM the Equator, and S any point on the sphere the portrayed point of which has the rectangular coordinates x and Y, Y being assumed equal to the arc y. The formulae given show the relations to the polar coordinates δ and t with regard to the pole of the equator.

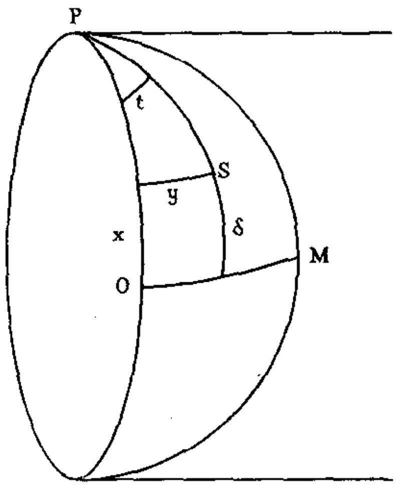

Fig. 3.

As stated previously, the diagram has been considered in different publications. But Maurer's [4] suggestion to give it the form of an atlas which would make it possible for the values required to be ascertained with an accuracy of 1′ was first put into effect in the astronomical calculation atlas of the German Testing Institute for Aviation; in this an attempt was made to make the system foolproof by using the appropriate forms and directions for use. Considering its limited possibilities this attempt can be regarded as having been successful.

6. THE ASTRONOMICAL CALCULATION ATLAS. The *Astronomischer Rechenatlas* contains a single octant of the sky portrayed on 126 pages; this is deemed sufficient, for reasons of symmetry, for all cases possible, but makes it necessary for a number of sign rules to be observed if all the possibilities involved in this restriction are to be taken into account.

A general map is provided to show which sectional sheet should be used (Fig. 4). This is used by entering it with the values of declination δ and the hour angle t. With the aid of these two quantities, the place in the sheet where the relevant curves intersect is found. All curves occurring for δ are given for all areas, and for t for most areas, at intervals of 10 minutes. Where δ is over 60° the number of curves provided for values of t approaching the pole diminishes.

A covering sheet made of transparent plastic is laid over the appropriate place in the atlas so that it is fixed precisely to a fraction of 1 mm. in position. So that this can be done accurately the covering sheet bears two marks, a circular one and one shaped like a double dash which enables this object to be attained very accurately (Fig. 5). The point desired is marked on the covering sheet. The sheet is then shifted by $90° - \phi$; this shifting must naturally also be carried out to an accuracy of fractions of 1 mm. if the desired accuracy of 1′ is to be obtained.

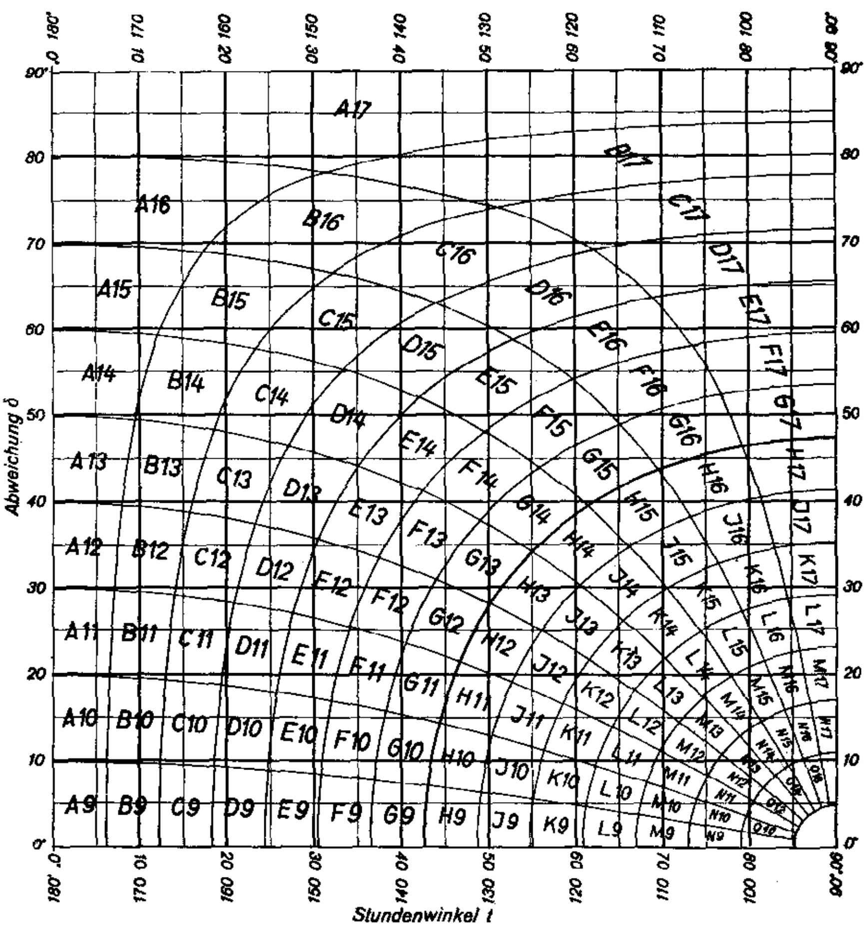

Fig. 4. The index map in the Astronomischer Rechenatlas.

A whole number obtained from the use of a covering sheet is added to the value of $90° - \phi$ (to the nearest degree). This number is an auxiliary term which would be found by carrying out an arithmetical reduction of a triangle and is used for setting the covering sheet correctly on another page of the atlas from which *H* and Az. are obtained. This second sheet is moved against the first by $90° - \phi$, and the exact place on the atlas where the covering sheet is to be placed is determined from the maximally three-digit whole number mentioned. These figures are given in large print at the sides of the sheets and, as a succession of 40 figures are printed on one sheet, they serve as an index for each individual page. Some of these figures are in red and, by their position, indicate alternative ways of putting the covering sheet on; in short, they are used for dealing with all possible cases in an appropriate manner so that in these figures and their correct application, the sign rules of the problem are contained. If the covering sheet has been laid over the proper place in the correct manner, for which the circular mark and the dash should be used, the previously marked point indicates a point on the second sheet whose coordinates in the printed curves represent the altitude and the azimuth; this can be estimated to an accuracy of 1′. The curves previously denoting δ have now become the curves of altitude, and those previously denoting t are the azimuths.

There is no doubt that the particular qualities of the calculation atlas guarantee sufficient accuracy and ease of handling. The complicated system of sign rules, however, can only be managed by using a form (Fig. 6), the careful application of which is indispensable. But even so the rules are by no means simple. The main difficulties arise owing to the fact that no complete hemisphere is available, as in the case of the ARG, but only one-eighth of a sphere; though this is all that is absolutely necessary for all possible cases.

It may be seen from Fig. 5 that, for a given year of publication the δ of certain bright stars can be printed in advance, but these remain of course valid only for a few years.

7. Conclusion. It is the writer's view that the best way of reducing astronomical observations for navigation by diagrammatic methods is to use diagrams of equal altitudes, the ARG, and the calculation atlas. This does not, however, imply that other methods, such as tables and geometrical or numerical calculating machines could not be used with greater advantage.

As regards the difference in accuracy of different parts of the diagrams, it should be noted that, both in the ARG and in the calculation atlas, the accuracy seems to be reduced in the same places, i.e. near the pole in the case of hour angles, near the zenith with azimuths. This is, however, in the nature of things. In order to determine accurately the azimuth of the Pole Star, a very rough knowledge of the hour angle is sufficient. Azimuths in the vicinity of the zenith will not be observed, because of the inaccuracy of the observations. It would therefore seem unjustified to raise objections to the two methods on this account; their accuracy appears sufficient for all cases that occur in practice.

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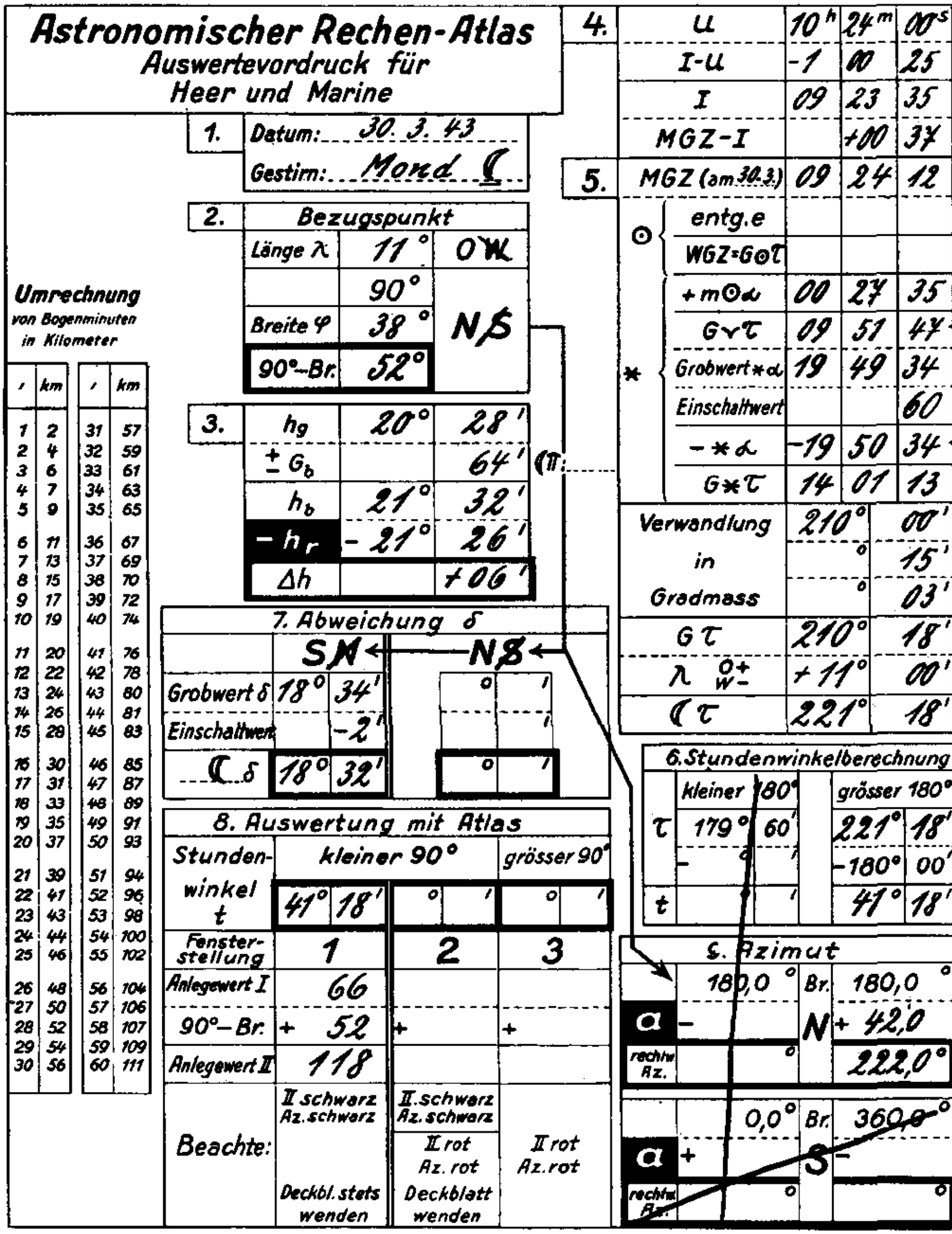

Astronomischer Rechen-Atlas
Auswertevordruck für
Heer und Marine

1. Datum: 30. 3. 43
Gestirn: Mond ☾

2.	Bezugspunkt	
Länge λ	11°	O W
	90°	
Breite φ	38°	N S
90°–Br.	52°	

3.		
h_g	20°	28′
$\pm G_b$		64′
h_b	21°	32′
$-h_r$	−21°	26′
Δh		+06′

Umrechnung von Bogenminuten in Kilometer

′	km	′	km
1	2	31	57
2	4	32	59
3	6	33	61
4	7	34	63
5	9	35	65
6	11	36	67
7	13	37	69
8	15	38	70
9	17	39	72
10	19	40	74
11	20	41	76
12	22	42	78
13	24	43	80
14	26	44	81
15	28	45	83
16	30	46	85
17	31	47	87
18	33	48	89
19	35	49	91
20	37	50	93
21	39	51	94
22	41	52	96
23	43	53	98
24	44	54	100
25	46	55	102
26	48	56	104
27	50	57	106
28	52	58	107
29	54	59	109
30	56	60	111

4.			
U	10^h	24^m	00^s
I-U	-1	00	25
I	09	23	35
MGZ-I		+00	37
5. MGZ (am 30.3.)	09	24	12
⊙ entg. e			
⊙ WGZ=G⊙τ			
+ m⊙α	00	27	35
G♈τ	09	51	47
✱ Grobwert ✱α	19	49	34
Einschaltwert			60
− ✱α	−19	50	34
G✱τ	14	01	13

Verwandlung in Gradmass		
	210°	00′
	°	15′
	°	03′
Gτ	210°	18′
λ O+ W−	+11°	00′
☾τ	221°	18′

7. Abweichung δ	S N		N S	
Grobwert δ	18°	34′	°	′
Einschaltwert		−2′		′
☾δ	18°	32′	°	′

6. Stundenwinkelberechnung	kleiner 180°		größer 180°	
τ	179°	60′	221°	18′
	− °	′	−180°	00′
t	°	′	41°	18′

8. Auswertung mit Atlas	kleiner 90°		größer 90°
Stundenwinkel t	41° 18′	° ′	° ′
Fensterstellung	1	2	3
Anlegewert I	66		
90°–Br.	+ 52	+	+
Anlegewert II	118		
Beachte:	II schwarz Az. schwarz Deckbl. stets wenden	II. schwarz Az. schwarz II rot Az. rot Deckblatt wenden	II rot Az. rot

9. Azimut				
	180,0°	Br.	180,0°
α	−	N	+ 42,0
rechtw. Az.	°		222,0°
	0,0°	Br.	~~360,0°~~
α	+	S	−
rechtw. Az.	°		°

Fig. 6. Form for use with the Astronomischer Rechenatlas.