THE DIP OF THE HORIZON*

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The dip of the horizon, geometrically determined, is, in minutes of arc

$$D = 1.927\sqrt{h}$$

where "h" is the height of the eye in meters. Taking into account normal refraction to the sea horizon, this formula becomes

$$D = 1.78\sqrt{h}$$

The latter formula is used in computing most published dip tables. If the temperature of the air, T_h, and of the water at the surface of the sea, T_w, are very different, then the true dip may differ from that computed by the foregoing second formula. In general,[1] the dip is given by an equation in the form

$$D = a\sqrt{h} + b(T_w - T_h) \qquad (1)$$

For Centigrade temperatures, Kohlschütter[2] gives the values of the coefficients in the formula (1) as $a = 1.82$ and $b = 0.37$. These coefficients have not been generally accepted because many series of observations, made for the purpose of their verification, have yielded discordant results.

Calculating the theoretical path of a ray of light through an atmosphere divided into a number of concentric layers of refractive index "n," we find that the value of the dip is given by the expression

$$D = \int_{r_o}^{r} \frac{r_o n_o\, dr}{r\sqrt{r^2n^2 - r_o^2 n_o^2}} + \int_{n_o}^{n} \frac{r_o n_o\, dn}{n\sqrt{r^2n^2 - r_o^2 n_o^2}}$$

where "r" is the radius from the center of the Earth, and the subscript "o" indicates the lowest layer. Putting $r - r_o = h$ and $n = 1 + c/1 + \alpha T_h$, where $c = 0.000293$ and $\alpha = 0.003665$, there emerges, after some transformations

$$D = 5.04\sqrt{0.1123h + T_o + T_h}$$

The effects on the value of D yielded by equation (2), caused by variations in pressure and humidity are negligible. Under stationary atmospheric and ocean temperature conditions, the temperature T_o may be replaced by T_w. Making this substitution and expressing equation (2) in the form of a series gives us the equation

$$D = 1.69\sqrt{h} - 7.5\frac{\Delta T}{\sqrt{h}}, \qquad (3)$$

$$\text{where } \Delta T = T_h - T_o$$

Equation (3) is in the form of equation (1), from which it differs in that the coefficient "b" is smaller, and the second or correction term is a function of "h" as well as of the temperature difference.

We have made many actual measurements of the dip, at various places, but especially on the shore of Heligoland. The Heligoland observations were made at three different heights of eye, with the following results:

h	D
4 meters	$1.74\sqrt{h} - 0.47\Delta T$
8 meters	$1.74\sqrt{h} - 0.23\Delta T$
50 meters	$1.77\sqrt{h} - 0.04\Delta T$

$$\Delta T = T_h - T_w$$

These results were generalized into the empirical formula

$$D = 1.8\sqrt{h} - \frac{1.1\Delta T}{\sqrt{h}};\ \Delta T = T_h - T_w \qquad (4)$$

Reconciliation of equation (4) with the theoretical equation (3) is possible only for stationary conditions and then only by arbitrary alteration of the numerical coefficients. Under stationary meteorological conditions, T_o is the same temperature as T_w, in accordance with

* This article is abstracted from a detailed report[3] made jointly by the author and Dr. G. Prüfer, based on observations inaugurated by the late Rear Admiral Dr. F. Conrad.

modern theories of the atmosphere and of turbulent motion.

A graph of the exact formula (2) shows that it can be adapted, for certain heights of eye and temperature differences, to the form of equation (1), but both "*a*" and "*b*" no longer remain constant. See Figure 1. "*b*" must decrease when "*h*" increases, as was shown by the Heligoland observations. The coefficient "*a*," on the other hand, will vary with changes in ΔT.

References

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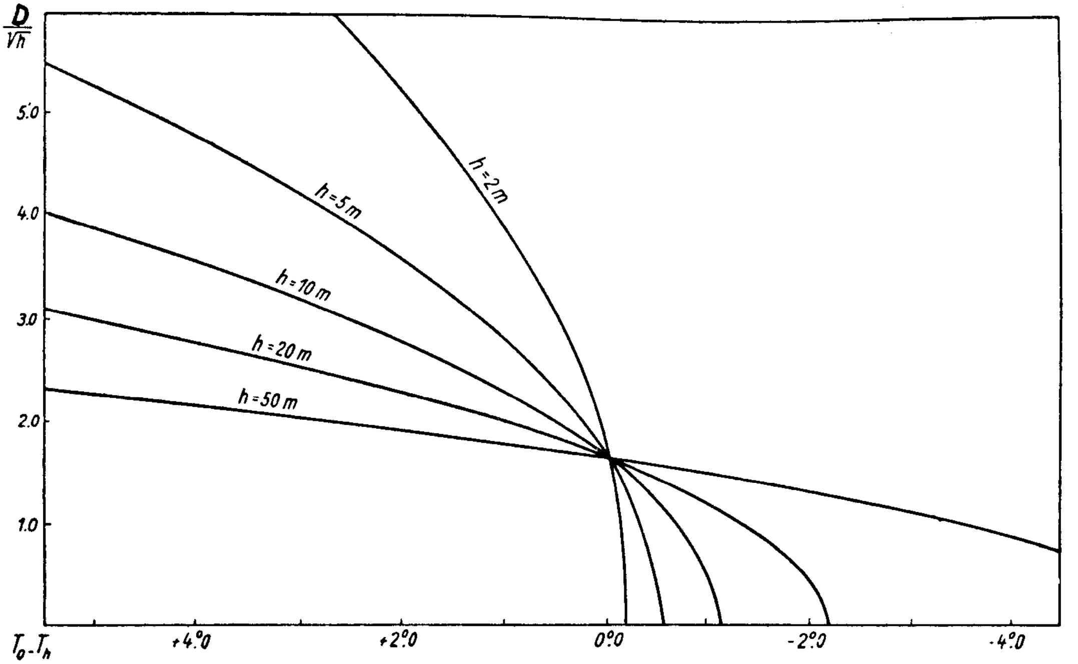

Figure 1.