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Astro-Navigation Methods and Software for the PC

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T and the speed V (in knots) of the observer are constant then *Long* and *Lat* at the time of observation are calculated from

$$Long = L_F + t\,(V/60)\sin T/\cos B_F$$
$$Lat = B_F + t\,(V/60)\cos T$$

where L_F and B_F are the estimated longitude and latitude at the time of fix and t is the time interval in hours from the time of fix to the time of observation, t is positive if the time of observation is after the time of fix and negative if it was before.

The position line of an observation is plotted on a chart using the intercept p where

$$p = H_O - H_C$$

and azimuth Z with origin at the calculated position (*Long*, *Lat*) at the time of observation, where H_C and Z are calculated using the method in section 7.2. Starting from this calculated position a line is drawn on the chart along the direction of the azimuth to the body. Convert p to nautical miles by multiplying by 60. The position line is drawn at right angles to this azimuth line, distance p from the position (*Long*, *Lat*), towards the body if p is positive and distance p away from the body if p is negative. Provided there are no gross errors the navigator should be somewhere on or near the position line at the time of observation. Two or more position lines are required to determine a fix.

The equation of a position line with origin at the estimated position at the time of fix (L_F, B_F) is given by

$$x\sin Z + y\cos Z = p$$

where the x-axis is parallel to lines of constant latitude, positive to the east, and the y-axis is along the meridian , positive to the north. If p is expressed in nautical miles (nm) then x and y will be in nautical miles also.

The position line may be drawn as described above or by the method described in section 7.6.

7.4 Position from intercept and azimuth by calculation Alternatively the position of the fix may be calculated directly from two or more sextant observations, by using the method of least squares as follows:

If p_1, Z_1 are the intercept and azimuth of the first observation, p_2, Z_2 of the second observation and so on, form the summations

$$A = \cos^2 Z_1 + \cos^2 Z_2 + \cdots$$
$$B = \cos Z_1 \sin Z_1 + \cos Z_2 \sin Z2 + \cdots$$
$$C = \sin^2 Z_1 + \sin^2 Z_2 + \cdots$$
$$D = p_1 \cos Z_1 + p_2 \cos Z_2 + \cdots$$
$$E = p_1 \sin Z_1 + p_2 \sin Z_2 + \cdots$$
$$F = p_1^2 + p_2^2 + \cdots$$

where the number of terms in each summation is equal to the number of observations. As a check verify that $A+C = n$ where n is the total number of observations included in the solution.

Calculate

$$G = AC - B^2$$

Then an improved estimate of the position at the time of fix (L_I, B_I) is given by

$$L_I = L_F + dL, \quad \text{and} \quad B_I = B_F + dB$$

where $dL = (AE - BD)/(G \cos B_F)$ and $dB = (CD - BE)/G$

Additional observations may be included in the solution by simply adding the extra terms to the summations A, B, C, D, E and F and calculating dL and dB again. Similarly, observations may be rejected by subtracting the terms that were originally added to A, B, C, D, E and F and calculating dL and dB again.

Calculate the distance d between the initial estimated position (L_F, B_F) at the time of fix and the improved estimated position (L_I, B_I) in nautical miles from

$$d = 60\sqrt{(dL^2 \cos^2 B_F + dB^2)}$$

If d exceeds about 20 nautical miles set $L_F = L_I$, $B_F = B_I$ and repeat the calculation until d, the distance between the position at the previous estimate and the improved estimate, is less than about 20 nautical miles.

It is possible but not advisable to start the iterations with a position that is in a different hemisphere. Provided L_I is kept in the range $-180°$ to $+180°$ and B_I in the range $-90°$ to $+90°$ the solution in most cases will begin to converge after a few iterations.

7.5 Estimated position error If three or more position lines are obtained an estimate of the error in position may be calculated. The standard deviation of the estimated position σ in nautical miles is given by

$$\sigma = 60\sqrt{(S/(n-2))}$$

where $S = F - D\,dB - E\,dL \cos B_F$

The standard deviations σ_L, σ_B in longitude and latitude are given by

$$\sigma_L = \sigma\sqrt{(A/G)} \quad \sigma_B = \sigma\sqrt{(C/G)}$$

In general as the number of observations increases the error in the estimated position decreases. Statistical theory shows that the estimated position has a probability P of lying within a confidence ellipse which is specified by the lengths of its axes a and b and the azimuth θ of the a-axis, where

$$\tan 2\theta = 2B/(A - C)$$

$$a = \sigma k/\sqrt{(n/2 + B/\sin 2\theta)}$$

$$b = \sigma k/\sqrt{(n/2 - B/\sin 2\theta)}$$

and $k = \sqrt{(-2 \log_e(1 - P))}$ is a scale factor

Values of the scale factor k for selected values of P are given in the table below.

Probability, P	0·39	0·50	0·75	0·90	0·95
Scale factor, k	1·0	1·2	1·7	2·1	2·4

Normally a confidence level of 95% is chosen, that is $P = 0{\cdot}95$.

The shape of the confidence ellipse depends only upon n and the distribution of the observations in azimuth; whilst the size of the ellipse, apart from the scale factor, depends upon the errors of observation. The method assumes that the observations have equal weight. The ideal situation is to produce a circular distribution of errors, with $A = B$ and $C = 0$, so that the errors are the same in all directions. This situation is approached when the bodies are equally spaced in azimuth. In addition if the bodies are observed at similar altitudes, this has the effect of minimising the systematic errors of the final calculated position.

7.6 Plotting position lines and the confidence ellipse with a personal computer Personal computers usually have facilities for plotting either on a TV monitor screen or on some other plotting device. They normally use the principle that a cursor or pen is moved from a point A with rectangle coordinates x_A, y_A, to a point B with rectangular coordinates x_B, y_B, and a straight line is drawn joining A and B in the process. A curve, such as an ellipse, is drawn by approximating it to a succession of short straight lines; the curve looks smooth to the eye provided that the lines are short enough.

The origin for plotting is chosen initially to be the estimated position of the fix with the x-axis along the line of latitude, positive to the east, and the y-axis along the meridian, positive to the north.

Suppose the position lines and confidence ellipse are drawn inside a square of length 20 nautical miles centred on the estimated position at the time of fix with sides parallel to the x and y axes. For the position line to enter this square p must be less than about 14 nm.

To draw the position line for intercept p (in nautical miles) and azimuth Z first find the two points where this line crosses the sides of the square. Set $x = \pm 10$. Then $y = (p \pm 10 \sin Z)/\cos Z$. The line crosses the left or right side of the square if $-10 \leq y \leq 10$. Similarly set $y = \pm 10$. Then $x = (p \pm 10 \cos Z)/\sin Z$. The line crosses the top or bottom of the square if $-10 \leq x \leq 10$.

The confidence ellipse is centred at the latest calculated position of the fix at $(60 dL \cos B_F,\ 60 dB)$, and is defined by the values of a, b and θ. The curve is drawn by connecting up the rectangular coordinates (x, y) of a series of points that are distributed closely along the ellipse. The values of x and y are calculated from

$$x = a \cos\alpha \sin\theta - b \sin\alpha \cos\theta + 60\, dL \cos B_F$$

$$y = a \cos\alpha \cos\theta + b \sin\alpha \sin\theta + 60\, dB$$

where α takes values from 0° to 360° in steps of, say 15°. By trial and error find the optimum step length for α below which there is no noticeable improvement in the smoothness of the ellipse which the computer draws.

At the next iteration move the origin to the latest calculated position of the fix.

Example On 2001 February 9 at $12^h 00^m 00^s$ UT the dead reckoning (DR) position of a ship was W 15° 30′, N 32° 45′. Before sunrise, at $06^h 58^m 52^s$ the sextant altitude of *Vega* (No 49) was 49°5850. At $07^h 01^m 45^s$ the sextant altitude of *Spica* (No 33) was 38°7067, and at $07^h 03^m 52^s$ the sextant altitude of the lower limb of the Moon was 20°4300. After sunrise, at $09^h 53^m 45^s$ the sextant altitude of the lower limb of the Sun was observed to be 22°6733. The ship maintained a constant track and speed of 315° and 12 knots, respectively. Calculate the position of the ship at $12^h 00^m 00^s$ from these data, assuming that the height of eye above the horizon is 6 m and the sextant index error is zero, using the formulae and methods described in section 7.

The data have been chosen to illustrate the method, especially the part involving the calculation and plotting of the confidence ellipse. In real situations the errors are normally much smaller than those in this example.

Table D: Calculation of the Sight Reduction

Body	Vega	*Spica*	*Moon*	Sun
Time of observation	$06^h 58^m 52^s$	$07^h 01^m 45^s$	$07^h 03^m 52^s$	$09^h 53^m 45^s$
t	−5·0189	−4·9708	−4·9356	−2·1042
Long (section 7.3)	−14·6561	−14·6642	−14·6701	−15·1462
Lat	32·0402	32·0470	32·0520	32·4524
GHA (section 2, 3, 5)	324·9181	43·5659	87·3397	324·8852
DEC	+38·7813	−11·1657	+13·4597	−14·5874
HP or *S*			1·0186	0·2705
LHA (section 7.2)	310·2620	28·9018	72·6696	309·7390
Z	66·0954	217·4137	272·6852	126·1928
H_C	49·4070	38·7001	21·6579	22·7632
H_S	49·5850	38·7067	20·4300	22·6733
D_h	0·0718	0·0718	0·0718	0·0718
I	0·0000	0·0000	0·0000	0·0000
H	49·5132	38·6349	20·3582	22·6016
R	0·0138	0·0203	0·0437	0·0389
PA			0·9549	0·0022
Oblateness of Earth			−0·0008	
S			0·2775	0·2705
H_O	49·4994	38·6146	21·5462	22·8353
p (section 7.3)	+0·0924 (+5·5 nm)	−0·0855 (−5·1 nm)	−0·1117 (−6·7 nm)	+0·0722 (+4·3 nm)
Iteration 1 (section 7.4)	$A = +1·1460$ $B = +0·3297$ $C = +2·8540$ $D = +0·0575$ $E = +0·3062$ $F = +0·0335$ $G = +3·1619$ $n = 4$ $dL = +0°.1248$ $dB = +0°.0200$ $d = 6·413$ nm			
Errors (section 7.5)	$\sigma = 0·648$ nm $\theta = 349°$ $P = 0·95$ $k = 2·448$ $a = 1·523$ nm $b = 0·929$ nm			
Improved estimate	$L_I = -15°.3752$ = W 15° 22′.5 $B_I = +32°.7700$ = N 32° 46′.2			

Adopting the DR position as the estimated position at the time of fix then $L_F = -15°.5000$ and $B_F = +32°.7500$. Intermediate values obtained in the calculation are shown in Table D.

Since the distance d between the DR position and the improved estimate is 6·4 nm, another iteration is not necessary, and thus the position at the time of fix is W 15° 22′.5 N 32° 46′.2. If further iterations were required the quantities H_O, *GHA*, *DEC*, *HP*, *S*, and t in Table D do not change.

The figure shows the four position lines and the confidence ellipse, centred in a square of dimension 20 nm. The coordinates of the points where the position lines cross the square which were used for plotting are

Vega	(+10·0, −8·9) : (+1·6, +10·0)	*Spica*	(+10·0, −1·2) : (−4·6, +10·0)
Moon	(+6·2, −10·0) : (+7·2, +10·0)	Sun	(+10·0, +6·3) : (−2·0, −10·0)

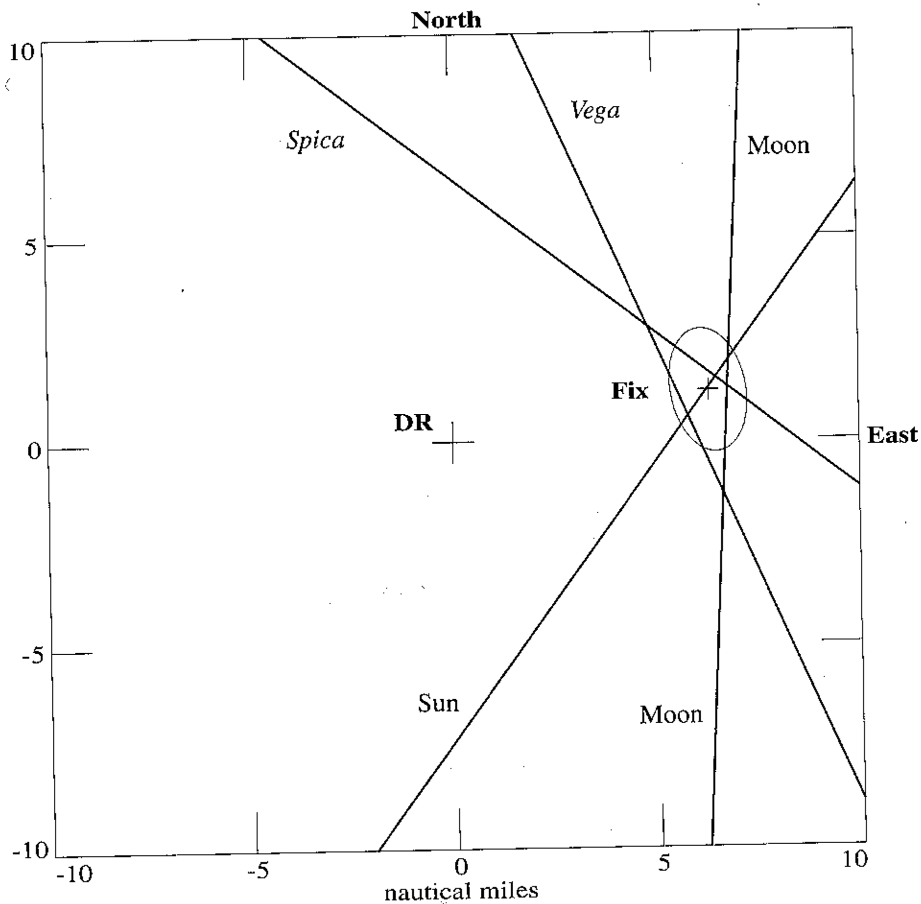

Figure: The four position lines, centred at the DR position, illustrate the numerical example. The confidence ellipse is drawn at the 95% probability level.

8. TIMES OF RISING AND SETTING AND TRANSIT

8.1 Times of rising and setting The times of rising and setting of a celestial body at longitude *Long*, latitude *Lat* may be calculated from its *GHA* and *DEC* by using the following iterative process.

Method

Step 0 Start with an initial estimate T_0 for the UT of the phenomenon.

For the Sun a suitable estimate is $T_0 = 6^h - Long^h$ for sunrise and $T_0 = 18^h - Long^h$ for sunset, where $Long^h$ is the longitude (in hours) measured east of the Greenwich meridian.