HUGHES 'TABLES
for
SEA and AIR NAVIGATION

Leslie J. COMRIE

This collection of tables was developed by Leslie J. Comrie following ideas suggested by PVH Weems during his visit to England in 1936, during the development of the English edition his "Air Navigation" book. These suggestions were also heard by AJ Hughes, supported by several sailors officers, sailors and airmen, convinced his company (Henry Hughes and Son Ltd. [1]) The merits of the publication of this collection of tables.

Leslie J. Comrie [2] (1896-1950) is a mathematician, a specialist in numerical analysis, and astronomer. This is one of the pioneers of mechanical computation by punch cards that apply to calculations astronomical and the development of numerical tables. Besides an intense educational activity and research, LJ Comrie in particular joined HM Nautical Almanac Office in Greenwich in 1926 and was the "Superintendent" of 1930-1936.

The collection was published for the first time in 1938. reprints were published in 1943, 1944, 1946 and 1950.

This collection of tables is not an innovation in the field of navigation tables but the tables being joined and explained by a top scientist, specialist of digital computing, coupled with a teacher, he differs from the other in common use at the time by its wealth of comments and instructions for use.

The volume contains 240 pages; it is relatively lightweight (600g) and compact (250 to 160 mm). The preface and job instructions are developed on 56 pages; 130 pages are devoted to specific tables to calculate the elements of the right height; the pages remaining are assigned to different tables: increasing latitudes, no table, logarithms 4 decimal, conversion tables, heights corrections tables.

Formulation used:

The triangle position of the sun, PZM, is constructed from the highest pole; Latitude, denoted φ, is therefore a positive amount less than 90 °. The declination of the sun, denoted D, is positive if it is the same name as the latitude and negative in the opposite case.

This position of the triangle is divided into two spherical triangles by lowering from zenith Z, ZX the ball perpendicular to the sun meridian. These triangles are Pzx and MZX.

We note the arc ZX R K and the latitude of the point X. The extent of the arc MX is noted that expresses KD absolute value of the algebraic difference between K and D

1Who will become the company Kelvin & Hughes Ltd.
2These biographical elements are extracted from the website: www.columbia.edu/cu/computinghistory.

Based on the configuration shown in Fig namely a point X, ZX foot height, located between the high pole P and the star M, is established, using the rule of the pentagon Napier, formulas following:

Triangle Pzx:

R = sin cos φ. SINP, cost K = cot φ.cos P, Z cot $_1$ P = sin φ.tan

Triangle KZX:

Sinh R. cos = cos (KD) cost Z_2 = Sin cot R. (KD)

These last two formulas are written:

csc H = Comp. dry (KD), Tanz $_2$ = SCCA. tan (KD)

So we recognize here a decomposition and auxiliary variables (R & K) identical to those Ogura and chosen by Weems and a formulation identical to that used by Dreisonstok.

Tab and sign rules:

The convenient use of these tables based on the definition of an auxiliary point. This has the following common characteristics: latitude is the whole number of the nearest degree of estimated latitude and longitude is such that the angle at the auxiliary pole the whole number of the degree closer to the angle estimated Pole.

To calculate the height and azimuth, LJ Comrie uses three tables:

1) Table I (see Annex I):

The horizontal argument is the latitude and the vertical argument is the angle at the pole 3. Both arguments are expressed with an interval of 1 °. The expressed latitude covers the interval [0 °, 89 °] and the angle expressed pole covers the range [0 °, 180 °] with a double scale. Note that a table page I is related to a degree of latitude, and with this arrangement, all the computing elements to a light point appears on the same page.

The table, for each pair of latitude value and angle to the pole:

- the auxiliary arc K in degrees, minutes and tenths of a minute,
- the quantity A = 10 5.log SECR,
- the amount 4D = 10 3. logcscR,
- Z angle 1.

When the angle at the pole is greater than 90 °, the table, in one re column, K '= 180 ° - K. If so case, it should continue the calculation with K = 180 ° - K '.

The items supplied are equivalent to those given in Table I of Dreisonstock.

2) Height and Table II (see Annex II):

It is a single entry table giving, with an interval of 0.5 argumentation ', quantities:

- B = 10 5.log dry (argument)
- C = 10 5.log csc (argument).

This table allows the calculation of:

- 10 5. logcsc H = A + 10 5. logsec (KD),
- then the height H by reverse playback.

Note that so far, the only sign of rules to be considered are those relating to (KD) = | K - D | which is a positive amount resulting from an algebraic difference where K is positive (always the name of the latitude) and D is positive if it is the name of the latitude, negative if it is name opposite.

This Table II is identical to that used in the structure HO 211 Ageton.

3) Azimut and Table III (see Annex III):

This table is reserved for the calculation of Z2. In general, the azimuth Z is here counted from the high pole, from 0 ° to 180 °. The result of the algebraic sum of the angles Z_1 and Z_2. The sign rules the following :

- Z_1 is positive if P <90 °, negative if P> 90 °,
- Z_2 is the sign of the difference algebraic K - D
- in these conditions, we always algebraically: $Z = Z_1 + Z_2$.

[3] H noted in the table.
[4] Not to be confused here with the declination Comrie note.

Table III is a single entry table whose argument is expressed from 0 ° to 90 ° with an interval 6 minutes of arguments (or a tenth of a degree) giving $E = 10^3$. logtan (argument) if the argument is greater than 45 ° and $E = 10^4 + 10^3$. logtan (argument) if the argument is less than 45 ° [5] allowing calculation of:

- 10^3. logtan $Z_2 = 10^3$.logcsc + 10^3. logtan (KD) + [10^4éhé] = D + E
- then Z_2 reverse playback.

Table III is particular to the collection of Comrie and allows the calculation of the azimuth to an accuracy sufficient (1/10 °).

example:

	Valued	coordinate schedules		Observation	
φ_e	24 ° 51 'N	AHP	189 ° 31.7 '	H_v	49 ° 58.5 '
G_e	146 ° 29 'W	D	45 ° 04.8 'N		Deneb

$AHAG_e$ = 43 ° 02.7 '; P_e = 43 ° 02.7 'W

Auxiliary point: φ_{at} = 25 ° N, P_{at} = 43 ° W is G_{at} = 146 ° 31.7 'W.

P_{at} =	43 ° W	D =	45 ° 04.8 'N						
φ_{at} =	25 ° N	K =	32 ° 31.3 'N	A =	10453	D =	209	Z_1 =	68.5 ° N
		KD =	12 ° 33.5 'N	B =	1052	E =	9348	Z_2 =	19.8 ° S
				A + B =	11505	D + S =	9557	Z =	N 48.7 ° W
		H_v =	49 ° 58.5 '						
		H_c =	50 ° 06.5 '					**Z =**	**311.3 °**
		int =	**- 8.0 '**						

Extracts useful tables annexed I, II and III.

The exact calculation, based on the auxiliary point H_c gives = 50 ° 06.7 'and the same value for the azimuth.

Some notable items:

We found, in the course of reading the preface and job instructions, many definitions, explanations and remarks LJ Comrie; these specify the critically Scientific, on issues that had hitherto been mainly handled by sailors. The latter are essentially tied to design a tool that combines safety, speed and accuracy computing without bothering too much sometimes, in their presentation of text, including theoretical details the understanding can, moreover, to use a mathematical extensive culture; Comrie comes here to give his lesson in mathematical and scientific rigor.

We note first that the book opens with a list of symbols and abbreviations used in Anglo-Saxon countries and a glossary of key terms of navigation and astronomy Boating. The author indicates that this glossary can not replace a course book but is the

5Because of the construction of Table III, the addition of 10,000 is systematic in all differences of logarithms leading to a result negative; see sample calculation from the reckoning in this card.

handy reference to use the collection tables: it is indeed a course summary (or aide memory) that recalls, as necessary, the calculation formulas and the associated sign rules; rounding rules are also mentioned therein.

Besides this concern for terminological rigor, LJ Comrie details several themes concern:

- precision concepts
- control techniques of a table,
- the use of an auxiliary point and inconvenience to its distance from the view point
- using the table from a reckoning,
- the calculation of a great circle.

1) Precision Concepts:

In his preface, LJ Comrie says the air navigator works in 1 'near and so can neglect without hesitation interpolation in Table II while the marine works, he usually 0.1 'close without that it is actually justified. Comrie proposes the adoption of a new standard level precision could be, for sailors, 0.5 '. The declination and K from the table I could be taken, as now, to 1/10 of a minute but KD could be to the nearest half-minute round nearest and all tweens II deleted table. Comrie says the height and not calculated would differ by more than 0.4 "of the rigorously calculated height with all its decimal places.

In the instruction manual, following several examples, Comrie says, as he notes performed in the examples the various interpolations but neglecting the lead:

- a maximum error of up to 0.2 'if we neglect the only interpolation of KD,
- a maximum error of up to 0.4 'if, in addition, extracted H_c Table II without interpolating half-minute[6].

2) Control Techniques table:

Comrie then makes several observations on the construction of tables indicating that Originally, it was proposed to resume in table I the elements K and A provided by Weems and of D and Z_1 provided by Dreisonstok. After analysis and comparison of multiple tables use (Aquino, Dreisonstok, Gingrich, Ogura, Pinto, Smart and Shearme and Weems[7]), Has Comrie found that, with the exception of Aquino tables, none of which was rigorously reliable[8]. Following the use of verification procedures, Comrie has taken all the calculations and presents tables in which "*... error is less than one unit of the last decimal in K and A, and less than half a unit in D and Z_1 ...*"

Audit procedures in question are extremely simple, such as on Table I, where entries are φ and P and which are calculated from the formula R = sin cos φ.sin P, the

[6]Values closer to those presented in Annex II Ogura.
[7]All authors cited by Comrie.
[8]Comrie uses the word "reliable".

page 6

rated A and D values equal, the factor 10^5 (or 10^3) Close, the logarithms of the transversal and cosecant of R. We must indeed find strictly identical values by swapping assigned variables simultaneously complementary values. For example, for φ = 20 ° and P = 30 °, we have A = 5416 and D = 328. We find these values for φ = 60 ° and P = 70 °.

The same kind of process extends to controls of K and Z_1:

$$\text{cost } K = \cos \varphi \text{ P cot, cot } Z_1 = \text{Tan P. sin}\varphi$$

It is clear that replacing the values φ P and the first equation their additions respectively in the second equation (and in the same order operators), we arrive at the same results. For example :

φ	P	K	AT	D	Z_1
32 °	34 °	37 ° 00.4 '	5532	324	70.3 °
56 ° = 90 ° - 34 °	58 ° = 90 ° - 32 °	70 ° 19.9 '	5532	324	37.0 °

A difference in the results to detect an error and resume calculation with table logarithms, the number of decimal places is expressed enough (at least 7, see below).

Table II was compared with the results of tables of logarithms to decimal 7 and 8 and the table presented is *"okay ... to the last decimal.* "He then compares his table II that of Ageton (HO 211) and there highlights inaccuracies[9] that allow Comrie to deduct qu'Ageton conducted its calculations with a table of logarithms to 6 decimal places. Indeed, they do not allow determine with certainty whether a "5" 6 e decimal shall be rounded in 5 e decimal place above or below.

Example: the two logarithms of the left column are rounded to 6 decimal identically this which could lead to write, to 5 decimals, 0.78645.

7 decimal places	6D	5 decimals
0.7864445	0.786445	0.78644 (rounded below)

0.7864454 0.786445 0.78645 (rounded on top)

Knowing the value to 7 decimal allows for rounded correct to 5 decimal places.

3) Auxiliary point:

It has been demonstrated [10]that the auxiliary point may be located at relatively large distance from the point estimated (about 42 miles to the maximum in the case of the use of these tables) which can lead to finding a distant observed point the determinative points. This distance can be causing a gap, more or less acceptable, right between (supposed place the vessel on the card) pitch curve (actual location) in the vicinity of the observed position. To state a limit rule validity, LJ Comrie is based on the work [11]WM Smart appeared in May 1919 in the journal of the Royal Astronomical Society and, considering the difficulties of accurate observation at sea, concludes

[9]Inaccuracies without real consequences in practice as Comrie said.
[10]The expression of the distance between the view point and auxiliary was established in several forms, including one devoted to tables Souillagouët.
[11]Work closer (but the method is different) from those of the limits of use of a straight high; detail dedicated. Also the work of Admiral Perrin.

that the use of an auxiliary point is not a source of significant error as the height observed does not exceed 75 °.

If the height is greater than 75 °, and whether the differences in latitude and longitude between the view points Auxiliary exceed 20 'Comrie recommends the calculations from the reckoning by a particular formulation.

Note: The rule set can remember if the observed point is relatively close to the point valued ; if this is not the case, it should check that the substitution conditions of the right to the curve in the vicinity of the observed point are verified.

4) Calculated from the estimated point

LJ Comrie briefly mentions the calculation formula cosine-Haversine common practice in Great Britain and the method Ageton [12]1931 (with the disadvantage that we know him, and Comrie recalls) then presents a formulation based on the decomposition of the position of the triangle according to After spherical height Z (see figure on page 2), directly usable with tables.

The formulation is established using the rule of the pentagon Napier then using exclusively secant functions, cosecant and tangent whose logarithms are tabulated in Tables II and III. This formulation is as follows:

$$\csc R = \sec\varphi \,.\csc P;\ \csc K = \frac{\csc\varphi}{\text{SECR}};\ \text{CSCh}_c = \text{SecR}.\sec(KD)$$

$$\text{Tanz}_1 = \frac{\csc\varphi}{\text{TANP}};\ \text{Tanz}_2\ \text{Csc} = \text{R}\tan(K - D)$$

All elements R, K, H_c, Z_1 and Z_2 then calculated by logarithms. The above example below, based on the application data processed on page 4, shows that the calculation is relatively laborious.

Valued		coordinate schedules		Observation	
φ_e	24 ° 51 'N	AHP	189 ° 31.7 '	H_v	49 ° 58.5 '
G_e	146 ° 29 'W	D	45 ° 04.8 'N		Deneb
$AHAG_e$ = 43 ° 02.7 '; P_e = 43 ° 02.7 'W					

φ_e =	24 ° 51 'N	logcscφ_e =	37650	logsecφ_e =	4220	logcscφ_e =	10377
P_e =	43 ° 02.7 '			+ logcscP_e =	16588	-logtanP_e =	9970
		-logsecR =	10505	logcscR =	20808	logtanZ_1 =	407
K =	32 ° 21.5 '	logcscK =	27145				
D =	45 ° 04.8 '	logsecR =	10505			Z_1 =	N68,6 °
KD =	12 ° 43.3 '	+ logsec (KD) =	1080				
		logcscHc =	11585	H_c =	49 ° 59.0 '	logcscR =	208
				H_v =	49 ° 58.5 '	+ logtan (KD) =	9353
				int =	- 0.5 '	logtanZ_2 =	9561
				Z = Z_1 + Z_2 =	N48,6 ° W	Z_2 =	S20,0 °

We note that it took to add 10,000 "logcscφ_e"For a positive difference.

[12] See Publication 211 HO.

page 8

The exact calculation, based on the estimated spot gives He = 49 ° 58.9 'and Z = 311.4 ° (N48,6 ° W).

5) Calculation of a great circle:

The problem here is presented very theoretically. Having defined the great circle passing a starting point A and end point B and clarified the symbols used, LJ Comrie fixed objective calculation:

- angle drive starting at A [13],
- regularly spaced points of contact on the great circle arc,
- of the route to follow to get from one point of the arc of great circle to the next.

The formulation presented includes only the intersecting functions, cosecant and tangent to use the tables II and III only. This formulation is built on the decomposition of triangle PAB (P being a terrestrial pole) into two right triangles leading from the heights A or B.

The points of the great circle in question to calculate the coordinates are determined by their distances each other (eg every 300 miles - or 5 - from the starting point).

LJ Comrie then book a long mathematical analysis of successive differences coordinates of these points in order to check the calculations. This analysis, applied to the angles starting at road [14] of each point, to determine the average road to follow between two points calculated great circle.

As we have said, the problem is analyzed from a theoretical angle; the elements that are essential to the marine circle distance and the vertex position, including its latitude due to weather, dark. Note here that the conventional calculation in Using the formulas in the spherical triangle PAB is much more expeditious.

Other tables:

Several other tables included in the collection of LJ Comrie. Without making a study in detail, seemed interesting to give some indication of some of them.

1) Table of increasing latitudes:

This table shows the increasing latitudes, at intervals of one minute, for latitudes between 0 ° 80 °. Values are calculated for 1880. Clarke ellipsoid The formula is next :

$$\Lambda = 7915.7045.\ \mathrm{logtan}\,(45° + \frac{\varphi}{2}) - 23.4285.\ \sin\varphi + 0.0133 \sin(3\varphi)$$

Expression in which Λ is expressed in minutes.

13 See note below.
14 Road angle from a point is the angle formed at this point as the tangent to the great circle and the meridian.

2) heights correction tables:

The heights correction tables are calculated according to the latest theories of the time and make subject to detailed explanations. Six tables are included in the collection including LJ Comrie distinguished:

- tables calculated tenth of a minute for the use of marine observing with "Marine sextant" and the hydrographic and geodetic using a theodolite,
- tables calculated to the minute with the use of fliers using a bubble sextant.

LJ Comrie warns against hasty certain uses relating to the calculation of an overall correction depending on the elevation of the eye and observed height:

"The Practice of Giving altitude correction tables in double entry form, in qui one argument is, the altitude and the --other eight of the eye HAS grown up During the last century. While it is feasible to compute a table Correctly in this form, Does not this APPEAR to-have-been done, as compilers-have lost sight of the fact That year Observed altitude must be corrected for dip *before* the altitude is used as an argument for refraction. The effect of this oversight is negligible at small heights of eye and moderate or wide elevations goal is not negligible When the eye of eight Approaches 100 feet (or more in aircraft) and the altitude is small ... "

As a result, tables lead to the calculation of corrections in several stages:

- calculation of the apparent dip of horizon, depending on the elevation of the eye,

- calculation of the correction relative to the average refraction, parallax and the half diameter,
- Correction of temperature and pressure (if any).

Tables correction heights of the Nautical Almanac today are very design close to those in the collection of Comrie.

Final comment:

This is a reference book and tabulation accurate and precisely controlled, allows easy method execution of Ogura to calculate the height; the result is precise without interpolate. The tables presented also allow the implementation of all water calculations common, sometimes at the price of a certain heaviness due to the use of only intersecting features, and cosecant tangent.

Particular attention was paid to typography using suitable characters and respecting sufficient space for the eye to unambiguously isolate the digital group research.

In a tribute to LJ Comrie delivered after his death in 1950, DH Sadler, then superintendent the Nautical Almanac Office, said that the "Hughes Tables for Sea and Air Navigation" are *"... probably the finest book of navigational tables of Their kind.* " [15]

[15] See Ch. Cotter "A History of Nautical Astronomy" on page 334.

ANNEX I

EXTRACT TABLE I

ANNEX II

EXTRACTS TABLE II

ANNEX III

EXTRACT TABLE III