## Northern and Southern Limits of an Occultation

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Under the orthographic projection, that forms the basis of Besselian Elements approach to analysing occultations, parallels of latitude become ellipses. If the occulted star is at declination, *δ*, then the parallel for latitude, *L*, is an ellipse centred at the point ( 0, *y*0 ) ≡ ( 0, sin*L* cos*δ* ) with semi-major and minor axes *a* = cos *L*, *b* = cos *L* sin *δ*.

Around the time of the occultation the upper (northern) and lower (southern) limbs of the Moon trace out something very close to straight line paths, *y* = *m* *x* + *c*. If *q*0, *p*′and *q*′ are the usual Besselian Elements then  and  where the upper and lower signs correspond to the upper and lower limbs respectively and *k* = 0.2725 is the ratio of the Moon’s radius to that of the Earth.

Assume that the star’s declination, *δ*, is positive (N). The Earth’s north pole is visible to an observer located on the star but they can see no further south than latitude −90°+*δ*. Determining the northern limit of occultation requires determining the largest, or most northerly, value of *L* that is intersected by the line representing the track of the Moon’s upper limb. This can be done by solving for the point (*x*, *y*) where that line intersects the ellipse representing the given parallel of latitude. Substituting *y* = *m* *x* + *c* into the equation  yields separate quadratic equations for *x* and *y*. The limiting latitude is attained when the discriminant of those quadratic equations vanishes. This requirement yields the equation



which has solutions



with the upper sign giving the northern limit. The limit will be 90° if the argument of the square root is negative.

The southern limit is represented by a point where the track of the straight line path of the Moon’s lower limb intersects the Earth’s limb. That point can be found by solving the simultaneous equations, *y* = *m* *x* + c and *x*2 + *y*2 = 1. Points on the Earth’s limb satisfy can be shown to satisfy the condition  and hence the southern limit is given by

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When the argument of the square root is negative the occultation is constrained only by the southern visibility of the star and the southern limit is then L = −90°+*δ*.

Analogous expression can be derived when *δ* < 0.

## Conditions for an Occultation

For an occultation to be visible somewhere on the surface of the Earth the orthographic projection of the centre of the Moon must approach the centre of the Earth, ( 0, 0 ) within a distance 1 + *k* for their discs to overlap. It can be shown that the distance of closest approach is . Therefore an occultation will only be observed if .

### Limit Calculation

**Northern declination (*δ* ≥ 0)**

Northern Limit (*LN*)



*LN* = 90° if 

Southern Limit (*LS*)



*LS* = −90° + *δ* if 

**Southern declination (*δ* < 0)**

Northern Limit (*LN*)



*LN*  = 90° + *δ* if 

Southern Limit (*LS*)



*LS* = −90° if 