In the following, upper case letters mean true values, lower case apparent values.

We have, for the triangle star$_1$, star$_2$ and zenith

$\cos D = \sin H_1 \sin H_2 + Q\ (\cos d - \sin h_1 \sin h_2)$, where $Q = \frac{\cos H_1}{\cos h_1} \frac{\cos H_2}{\cos h_2}$

where *H* and *h* are the altitudes and *D* and *d* the distances.

We want an additive term to the apparent distance to get the true distance, i.e.

$$\boxed{D = d + \rho}$$

Now, $\cos D = \cos\,(d + \rho) = \cos d \cos \rho - \sin d \sin \rho = \cos d - \rho \sin d$, as ρ is a small quantity.

Making the approximation that Q = 1, we then get

$$\cos D - \cos d = \sin H_1 \sin H_2 - \sin h_1 \sin h_2$$

$$\rho \sin d = \sin h_1 \sin h_2 - \sin H_1 \sin H_2 = \sin h_1 \sin h_2 - \sin(h_1 - r_1) \sin(h_2 - r_2),$$

where *r* is the refraction. Thus

$$\rho \sin d = \sin h_1 \sin h_2 - (\sin h_1 \cos r_1 - \cos h_1 \sin r_1)\ (\sin h_2 \cos r_2 - \cos h_2 \sin r_2) =$$

$$= \sin h_1 \sin h_2 - (\sin h_1 - r_1 \cos h_1)\,(\sin h_2 - r_2 \cos h_2) =$$

$$= \sin h_1 \sin h_2 - (\sin h_1 \sin h_2 - r_2 \sin h_1 \cos h_2 - r_1 \sin h_2 \cos h_1 + r_1 r_2 \cos h_1 \cos h_2) =$$

$$\approx r_2 \sin h_1 \cos h_2 + r_1 \sin h_2 \cos h_1$$

as r_1 and r_2 are small quantities. We define $r_i = c \cot h_i$ for I = 1, 2 where c is a constant. Then

$$\rho \sin d = c \cot h_2 \sin h_1 \cos h_2 + c \cot h_1 \sin h_2 \cos h_1 = c \left(\frac{\sin h_1}{\sin h_2} \cos^2 h_2 + \frac{\sin h_2}{\sin h_1} \cos^2 h_1 \right) =$$

$$= c \left(\frac{\sin h_1}{\sin h_2} (1 - \sin^2 h_2) + \frac{\sin h_2}{\sin h_1} (1 - \sin^2 h_1) \right) = c\left(\frac{\sin h_1}{\sin h_2} + \frac{\sin h_2}{\sin h_1} - 2 \sin h_1 \sin h_2\right) =$$

$$\approx 2c \left(\frac{1}{2} \left(\frac{\sin h_1}{\sin h_2} + \frac{\sin h_2}{\sin h_1} \right) - \cos d \right) = 2c(x - \cos d)$$

with $x = \frac{1}{2} \left(\frac{\sin h_1}{\sin h_2} + \frac{\sin h_2}{\sin h_1} \right)$

Hence, $\rho = \frac{2c(x - \cos d)}{\sin d}$

Comparing the HMNAO formula for refraction with the simple $c \cot h$ used above, for altitudes equal or larger than 10°, we find that c = 0.97′ minimizes the sum of squared differences. We thus get

$$\boxed{\rho = 1.94'\ (x - \cos d)/\sin d}$$

to be compared with Letcher's
$R' = 1.90'\ (x - \cos d)/\sin d$

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