

Formulas [edit]

See also: [Skew lines § Formulas](#)

A [necessary condition](#) for two lines to intersect is that they are in the same plane—that is, are not skew lines. Satisfaction of this condition is equivalent to the [tetrahedron](#) with vertices at two of the points on one line and two of the points on the other line being [degenerate](#) in the sense of having zero [volume](#). For the algebraic form of this condition, see [Skew lines § Testing for skewness](#).

Given two points on each line [edit]

First we consider the intersection of two lines L_1 and L_2 in two-dimensional space, with line L_1 being defined by two distinct points (x_1, y_1) and (x_2, y_2) , and line L_2 being defined by two distinct points (x_3, y_3) and (x_4, y_4) .^[1]

The intersection P of line L_1 and L_2 can be defined using [determinants](#).

$$P_x = \frac{\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \\ x_4 & y_4 & 1 \end{vmatrix}}{\begin{vmatrix} x_1 & 1 \\ x_2 & 1 \\ x_3 & 1 \\ x_4 & 1 \end{vmatrix}} \quad P_y = \frac{\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \\ x_4 & y_4 & 1 \end{vmatrix}}{\begin{vmatrix} y_1 & 1 \\ y_2 & 1 \\ y_3 & 1 \\ y_4 & 1 \end{vmatrix}}$$

The determinants can be written out as:

$$P_x = \frac{(x_1 y_2 - y_1 x_2)(x_3 - x_4) - (x_1 - x_2)(x_3 y_4 - y_3 x_4)}{(x_1 - x_2)(y_3 - y_4) - (y_1 - y_2)(x_3 - x_4)}$$
$$P_y = \frac{(x_1 y_2 - y_1 x_2)(y_3 - y_4) - (y_1 - y_2)(x_3 y_4 - y_3 x_4)}{(x_1 - x_2)(y_3 - y_4) - (y_1 - y_2)(x_3 - x_4)}$$

When the two lines are parallel or coincident, the denominator is zero. If the lines are almost parallel, then a computer solution might encounter numeric problems implementing the solution described above: the recognition of this condition might require an approximate test in a practical application. An alternate approach might be to rotate the line segments so that one of them is horizontal, whence the solution of the rotated parametric form of the second line is easily obtained. Careful discussion of the special cases is required (parallel or coincident lines, overlapping or non-overlapping intervals).