True zenith distance of lunar star	$=\{49° 43' 28''-(-17''\cdot23)\}=49° 43' 45''\cdot23$
Corresponding altitude of „	$=40° 16' 14''\cdot77$
*Refraction, Riddle, Table III.	$= 1' 9''$
†Apparent altitude of α Pegasi is	$H'=40° 17' 24''$

20. From the apparent lunar distance d' on the earth's surface, to find the geocentric lunar distance d—that is, "to clear the distance."

		°	′	″
Observed lunar distance between the moon's farther limb and the lunar star, α Pegasi	=	103	43	45
Index error	=		0	25 –
Observed lunar distance free from instrumental error between moon's farther limb and the lunar star	$d''=$	103	43	20
Moon's augmented semi-diameter (p. 15)	$s'=$	0	16	56·0
Apparent lunar distance between moon's centre and the lunar star is d'	=	103	26	24
Let h = geocentric altitude of moon's centre	=	36	26	1
„ H = „ „ lunar star's centre	=	40	16	15
„ d = geocentric lunar distance between the centre of the moon and the lunar star.				
„ h' = apparent altitude of moon's centre	=	35	37	28
„ H′ = „ „ lunar star's centre	=	40	17	24
„ d' = apparent lunar distance between the centre of the moon and the lunar star	=	103	26	24

$$h'+H'+d'=179\ 21\ 16$$

$$\frac{h'+H'+d'}{2}=89\ 40\ 38$$

$$(H'+h'-d')=(75°\ 54'\ 52''-103°\ 26'\ 24'')=27\ 31\ 32\ n$$

$$\frac{h'+H'-d'}{2}=13\ 45\ 46\ n$$

* By Chauvenet, vol. ii. Table I., the refraction is 1′ 9″.
From Table VI. Cape's *Logarithms*, refraction may easily be found.

Refraction	Barometer 29″·9	Thermometer 57°·0
Alt. 40° = 1′ 9″·3	0″·1 × 2″·32 = 0·232 –	7° × 0″·139 = 0″·973 –
17′·3 × ·04″ = 0 0·692	0·973 –	
1 8·608	1·205 –	
0 1·205 –		
1 7·40		

The method is easy; the table gives the signs to be used.

† It will be observed that, in the case of the lunar star, step No. 14 has been omitted.
This is because the star's parallax is insensible. If the sun had been observed, instead of a lunar star, the process followed in step No. 14 would have been followed.
The sun's horizontal parallax P is found in *Nautical Almanac*, p. I., or under the head "Obliquity of the Ecliptic," see Index to *N. A.*
Then sun's parallax in altitude = P . cos apparent altitude.

*From Riddle, pp. 233, 289:—

$$\sin^2\frac{d}{2}=\cos\left(\frac{h+H}{2}+\theta\right).\cos\left(\frac{h+H}{2}-\theta\right)$$

in which—
$$\sin^2\theta=\frac{\cos h.\cos H.\cos\frac{1}{2}(h'+H'+d').\cos\frac{1}{2}(h'+H'-d')}{\cos h'.\cos H'}$$

Log $\cos h$	=	9·905552
,, $\cos H$	=	9·882523
,, $\cos\frac{1}{2}(h'+H'+d')$	=	7·750778
,, $\cos\frac{1}{2}(h'+H'-d')$	=	9·987348†
,, $\sec h'$	=	0·117600
,, $\sec H'$	=	0·089986
		2)37·733789

Log $\sin\theta$ $=18·866894=\log\sin 4°\ 13'\ 15''$

$\therefore\ \theta = 4°\ 13'\ 15''$

$h = 36°\ 26'\ 1''$

$H = 40°\ 16'\ 15''$

$h+H = 76°\ 42'\ 16''$

$\frac{h+H}{2} = 38°\ 21'\ 8''$

$\theta = 4°\ 13'\ 15''$

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* There are various forms of this formula—

Godfrey, p. 256, gives:—

$$\sin\frac{d}{2}=\cos\tfrac{1}{2}(h+H).\cos\theta$$

in which—
$$\sin^2\theta=\frac{\cos h.\cos H.\cos\frac{1}{2}(h'+H'+d').\cos\frac{1}{2}(h'+H-d)}{\cos h'.\cos H'.\cos^2\frac{1}{2}(h+H)}$$

In his Series of Observed Lunar Distances, p. III. Dr. Pearson gives:—

$$\sin\frac{d}{2}=\sin(A+B).\sin(A-B)$$

in which—
$$\cos^2 B=\cos^2\tfrac{1}{2}(h+H)$$
$$\cos^2 A=\frac{\cos h.\cos H.\cos\frac{1}{2}(h'+H'+d').\cos\frac{1}{2}(h'+H'-d')}{\cos h'.\cos H'}$$

In his *Astronomy*, vol. i. p. 389, Chauvenet gives:—

$$\sin\frac{d_,}{2}=\cos\tfrac{1}{2}(h_,+H_,)\cos\theta$$

in which—
$$\sin^2\theta=\frac{\cos h_,.\cos H_,.\cos\frac{1}{2}(h'+H'+d').\cos\frac{1}{2}(h'+H-d')}{\cos h'.\cos H'.\cos^2\frac{1}{2}(h_,+H_,)}$$

This gives the lunar distance referred to the normal centre, the point O, where the vertical of the observer intersects earth's polar axis.

To reduce this lunar distance $d_,$ into a geocentric lunar distance d Chauvenet gives:—

$$(d-d_,)=A.\pi.\sin\phi\left(\frac{\sin\Delta}{\sin\delta_,}-\frac{\sin\delta}{\tan d_,}\right)$$

In Chauvenet's formulæ,

$h_,$ = moon's altitude referred to the point O
$H_,$ = sun's ,, ,, ,, ,,
$d_,$ = lunar distance ,, ,, ,,
$\delta_,$ = moon's declination ,, ,, ,,
δ = ,, geocentric declination
Δ = sun's geocentric declination

† Note that cos (−) 13° 45′ 46″ = +cos 13° 45′ 46″

$\left(\frac{h+H}{2}+\theta\right) = 42° \ 34' \ 23''$

$\left(\frac{h+H}{2}-\theta\right) = 34° \ 7' \ 53''$

$$\sin^2\frac{d}{2} = \cos 42° \ 34' \ 23'' . \cos 34° \ 7' \ 53''$$

$$\therefore \frac{d}{2} = 51° \ 19' \ 45''$$

$$\text{And } d = 102° \ 39' \ 30''$$

21. To find the Greenwich mean time T_0 corresponding to the measured geocentric lunar distance d.

The measured distance d falls between :—

		prop. log.	
(d)	100° 59′ 41″	0·2184=Q	at 3^h, 31st Dec. 1884, *N. A.* Lunar Distances.
(d')	102° 48′ 33″	0·2188	at 6^h, „ „ „
		0·0004=ΔQ	

Taking the mode laid down in *N. A.* (see explanations to pp. XIII.–XVIII.), we have :—
Nautical Almanac, 31st Dec. 1884, p. XVII., Lunar Distances :—

	° ′ ″	prop. log.	h. m. s.
(d)=	100 59 41 at 3^h	0·2184	
d =	102 39 30		
Δ' =	1 39 49	0·2560	
	=5989″		
Approximate interval is t		=0·0376	=2 45 2
Greenwich mean time of lunar distance (d) immediately preceding the measured lunar distance d is (T)			=3 0 0
Approx. Greenwich mean time of observation is $\{(T)+t\}$			=5 45 2

Nautical Almanac, 31st Dec. 1884, p. XVII., Lunar Distances :—

		prop. log.	
Lunar distance $(d_{\prime})$	at noon	0·2182	a
„ (d)	at 3^h	0·2184	b
„ $(d_{\prime})$	at 6^h	0·2188	c

$$\text{Mean difference of prop. logs} = \frac{(b-a)+(c-b)}{2} = 3$$

With approx. interval $t=2^h \ 45^m \ 2^s$, and with mean diff. of prop. logs=3; the *N. A.*, in its Table of Second Differences (see Index to *N. A.*), gives (p. 476) $\Delta t = 0·5$ secs.

Since the prop. logs in the lunar distances are increasing, this correction Δt is to be subtracted.