From the *Nautical Almanac*

If p_1, Z_1 are the intercept and azimuth of the first observation, p_2, Z_2, of the second observation and so on, form the summations

$$A = \cos^2 Z_1 + \cos^2 Z_2 + \ldots = \sum_i \cos^2 Z_i$$

$$B = \cos Z_1 \sin Z_1 + \cos Z_1 \sin Z_1 + \ldots = \sum_i \cos Z_i \sin Z_i$$

$$C = \sin^2 Z_1 + \sin^2 Z_2 + \ldots = \sum_i \sin^2 Z_i$$

$$D = p_1 \cos Z_1 + p_2 \cos Z_2 + \ldots = \sum_i p_i \cos Z_i$$

$$E = p_1 \sin Z_1 + p_2 \sin Z_2 + \ldots = \sum_i p_i \sin Z_i$$

where the number of terms in each summation is equal to the number of observations.

With $G = AC - B^2$, an improved estimate of the position at the time of the fix (L_I, B_I) is given by

$$L_I = L_F + \left(AE - BD\right) / \left(G \cos B_F\right), \qquad B_I = B_F + \left(CD - BE\right) / G$$

In the above (L_F, B_F) is the longitude and latitude of the assumed position (AP) of the fix.

For 3 sights this procedure yields the symmedian point of the triangle or cocked hat formed by the 3 lines of position (LOP).

This result is derived follows: It can be shown that a point that is displaced by an amount (X, Y) from the AP lies at a perpendicular distance of $\left| X \sin Z + Y \cos Z - p \right|$ to an LOP with intercept p and azimuth Z. The results in the almanac are now easily derived by least squares regression which involves finding X and Y that minimize the quantity

$$\Delta^2 = \sum_i \left(X \sin Z_i + Y \cos Z_i - p_i \right)^2$$

Least squares regression effectively assumes that observational errors in the measurement of altitudes forms a normal distribution and finds the point on the Earth's surface where the probability density for the set of sights is maximum. In matrix form X and Y are given by

$$\begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} \sum_i \sin^2 Z_i & \sum_i \sin Z_i \cos Z_i \\ \sum_i \sin Z_i \cos Z_i & \sum_i \cos^2 Z_i \end{pmatrix}^{-1} \cdot \begin{pmatrix} \sum_i p_i \sin Z_i \\ \sum_i p_i \cos Z_i \end{pmatrix}$$

$$= \frac{1}{AC - B^2} \begin{pmatrix} A & -B \\ -B & C \end{pmatrix} \cdot \begin{pmatrix} E \\ D \end{pmatrix}$$

Applying Mercator projection scaling in longitude

$$L_I = L_F + X / \cos B_F , \qquad B_I = B_F + Y$$

The method described in the almanac can be extended to the case where an unknown constant error or offset, D, is present in all of the measured altitudes and hence in the intercepts. Such a constant offset might arise as a result of index error, or misestimate of the height of the eye. The least squares regression then becomes

$$\begin{pmatrix} X \\ Y \\ D \end{pmatrix} = \begin{pmatrix} \sum_i \sin^2 Z_i & \sum_i \sin Z_i \cos Z_i & \sum_i \sin Z_i \\ \sum_i \sin Z_i \cos Z_i & \sum_i \cos^2 Z_i & \sum_i \cos Z_i \\ \sum_i \sin Z_i & \sum_i \cos Z_i & N \end{pmatrix}^{-1} \cdot \begin{pmatrix} \sum_i p_i \sin Z_i \\ \sum_i p_i \cos Z_i \\ \sum_i p_i \end{pmatrix}$$

where N is the number of sights taken. The 3×3 matrix is singular when there are less than 3 observations. D is the quantity that should be subtracted from each of the intercepts, p_i, in order to compensate for the constant error and produce the maximum peak probability density. This peak occurs at the point (X, Y) relative to the AP. When the average of the intercepts in zero this produces the same result as the method given in the almanac.