Fix From a Set of Sights Taken over a short period of time that spans Meridian Transit

Typically we use our DR position to calculate the time of meridian transit for taking a noon sight to confirm our Latitude. Below is a numerical method for calculating the sextant altitude & time of meridian transit that does not depend on an accurate DR Position. This method allows us to determine both Latitude & Longitude from a set of sextant altitudes taken over a short period of time that spans meridian transit.

■ Let $\mathbf{T_i}$ represent the observation times in decimal hours and $\mathbf{H_i}$ represent the associated sextant altitudes in decimal degrees, **for i =1 to n** where **n** is the number of sights. Minimum number of sights is 3, Maximum number of sights is 12.

■ For best results approximately half (6) of the sights should be taken during a six minute period of time before meridian transit and approximately half (6) of the sights should be taken during a six minute period of time following meridian transit.

■ Use the sextant altitudes ($\mathbf{H_i}$)and their associated times of observation($\mathbf{T_i}$) to calculate the coefficients $(\mathbf{a_0, a_1, a_2})$ of a second order polynomial where $\mathbf{H = a_o + a_1 T + a_2 T^2}$

First calculate the following for **i = 1 to n** where **n** is the number of sights:

ΣT_i

ΣT^2_i

ΣT^3_i

ΣT^4_i

ΣH_i

$\Sigma T_i H_i$

$\Sigma T^2_i H_i$

Then use the following matrix operations* to determine the coefficients:

$$\begin{vmatrix} a_0 \\ a_1 \\ a_2 \end{vmatrix} = \begin{vmatrix} n & \Sigma T_i & \Sigma T^2_i \\ \Sigma T_i & \Sigma T^2_i & \Sigma T^3_i \\ \Sigma T^2_i & \Sigma T^3_i & \Sigma T^4_i \end{vmatrix}^{-1} \times \begin{vmatrix} \Sigma H_i \\ \Sigma T_i H_i \\ \Sigma T^2_i H_i \end{vmatrix}$$

*Most hand held scientific calculators such as TI83 & TI84 provide these matrix operations as built in functions.

Then calculate $\mathbf{T_{MT}}$ (time of meridian transit) and $\mathbf{hs_{MT}}$ (sextant altitude of the body at meridian transit) Where $\mathbf{T_{MT} = -a_1/(2a_2)}$ and $\mathbf{hs_{MT} = a_0 + a_1 T_{MT} + a_2 T^2_{MT}}$

For all the above calculations convert time to decimal hours

Example 12:10:09   T = 12 +10/60 +09/3600 = 12.16917

Convert the date and calculated zone time of meridian transit (T_{MT}) to Greenwich date & GMT then lookup the **GHA** & **Dec** of the body in the ***Nautical Almanac.***

■ Calculating Longitude of the observer at Meridian Transit.

Using the Greenwich Date & GMT of Meridian Transit obtain the **GHA** value of the body from the ***Nautical Almanac*** corresponding to the Greenwich Date & GMT. The value of **GHA** is then used to determine the observer's longitude based on the following:

Observer's West Longitude = **GHA**
or
Observer's East Longitude = 360° -**GHA**

Note that each second of error in the calculated zone time of Meridian Transit (T_{MT}) will result in an error of ±0.25 arc minutes in the calculated value of the observer's longitude. If your sights are of "*acceptable*" accuracy, the calculated time of Meridian Transit should be within ±4 seconds of the actual time of Meridian Transit, which would produce a calculated value for the observer's longitude accurate to within ±1.0 arc minute.

■ Calculating Latitude of the observer at Meridian Transit.

Convert sextant altitude at meridian transit (hs_{MT}) to observed altitude at meridian transit (Ho_{MT}) by applying index correction, dip, atmospheric refraction, parallax in altitude and semi-diameter corrections. Then determine Zenith Distance (**Z**), also known as co-altitude. **Z** is the angular distance from the observer's zenith to the body (the arc of a vertical circle between the observer's zenith and the body).

$$Z = 90° - Ho_{MT}$$

Z's name (**N** or **S**) is the direction (**N** or **S**) from the Body to the observer's Zenith. The Latitude of Observer can then be determined by the following equation:

$$L = Z \pm Dec^*$$

***** If **Z** & **Dec** have opposite names subtract **Dec**

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Meridian Transit Sight Data Plot

Sight #	Zone Time of Sight hr.	min.	sec.	Sextant Altitude (hs) deg.	min.
1	13	2	28	61	12.7
2	13	3	10	61	13.1
3	13	4	9	61	13.7
4	13	5	42	61	14.4
5	13	7	11	61	14.9
6	13	8	35	61	15.2
7	13	10	8	61	15.3
8	13	11	42	61	15.2
9	13	13	16	61	14.9
10	13	14	44	61	14.4
11	13	16	12	61	13.7
12	13	17	52	61	12.6

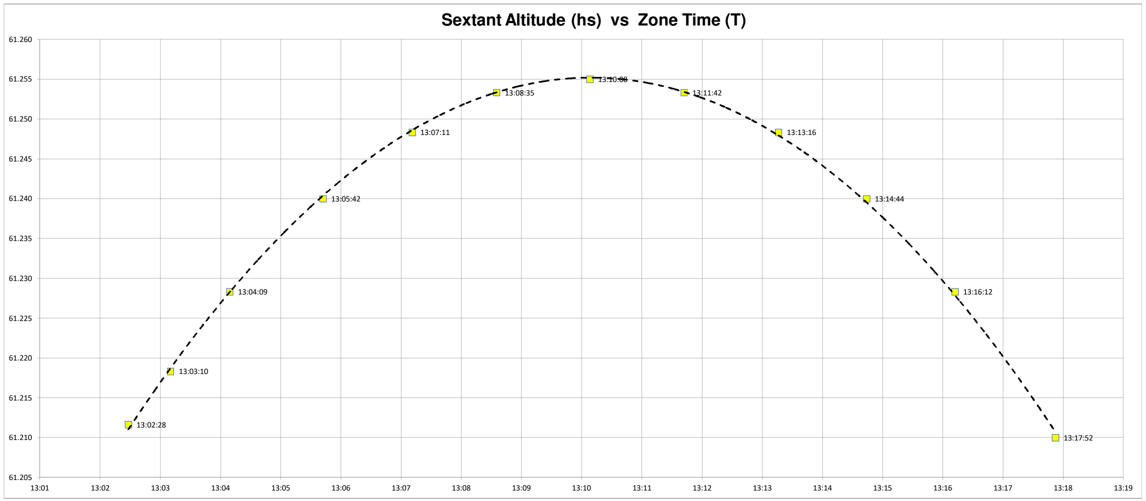
DST **Yes** HE **6.0** ft. IC **-1.2** min.

Click here to clear user input data cells

Click here to update sight data plot

Body SUN **Limb** LL **DR L** 48 deg. 8.50 min. N

Date @ DR Position 18-May-03 **DR Lo** 123 deg. 26.00 min. W



- - - - - Trend Line   Sight Data Used to Compute Trend Line

Notes:
- Before using this worksheet, change the **Formula Calculations Options** to **"Manual"** by clicking the above **"Click here to clear user data cells".**
- **Sight #1** must contain a valid **Time** & **Sextant Altitude**.
- **Time** must be increasing with **Sight #**
- After entering the new sight data, press the **"F9" key** or click on the above **"Click here to update sight data plot"** to refresh the **Sight Data Plot**.
- To remove a bad sight from the list, click on the yellow square that contains the **Sight #** to be removed.
- The **Sight Data Plot** will automatically update after a bad sight is removed.

Before leaving this worksheet click here to change the **Formula Calculations Options back to "Automatic"**

Enter Date into Yellow Cells

Zone Time of MT 13:10:09

Click to view calculated Latitude & Longitude via Meridian Transit Sight

Dip Short Distance ____ Yards   Natural Horizon

Air Temperature 10 ° C

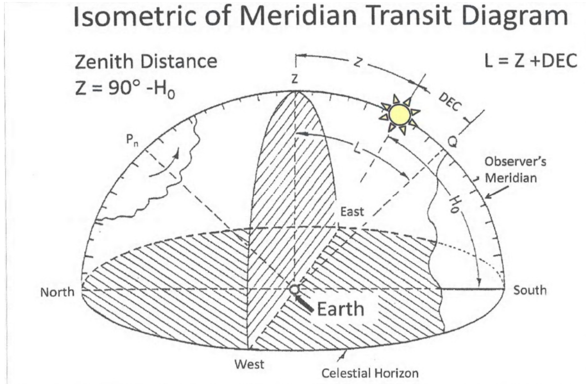
Atmospheric Pressure 1010 mb

Greenwich Date & GMT of MT
18-May-03 20:10:09

$hs = a_0 + a_1T + a_2T^2$

a_0	-405.4725876
a_1	70.88154125
a_2	-2.691179454

Sextant Altitude at MT	61	deg.	15.31	min.	
GHA of Body at MT	123	deg.	26.20	min.	
Dec of Body at MT	19	deg.	35.80	min.	N



At Meridian Transit the LHA of the Body is equal to 0° 0.00′

GHA = Observer's West Longitude
or
GHA = 360° - Observer's East Longitude

Enter Data into Yellow Cells
To Clear User Data Cells Click On This Box

90°	89	deg.	60.00	min.		
- Ho	61	deg.	27.14	min.		
Z	28	deg.	32.86	min.	N	
+ Dec	19	deg.	35.80	min.	N	
Calculated L	48	deg.	8.66	min.	N	
Calculated Lo	123	deg.	26.20	min.	W	
a	0.16 n. mi. North of DR Latitude					
DR L	48	deg.	8.50	min.	N	
DR Lo	123	deg.	26.00	min.	W	

Date & Zone Time of Meridian Transit

18-May-03	13:10:09

Body SUN **Limb** LL

Apply corrections to Sextant Altitude to determine Ho

Height of eye 6 ft.

	(+)		(-)	
hs	61	deg.	15.31	min.
IC	0.00	min.	1.20	min.
Dip			2.38	min.
(±) Totals	0.00	min.	3.58	min.
Corr	-3.58	min		
ha	61	deg.	11.74	min
Moon's HP				min
Main	15.40	min.		min.
Add'l M, PI		min.		min.
If Body is UL of Moon	-30.0'		0.00	min.
Add'l Ref.	0.00	min.	0.00	min.
(±) Totals	15.40	min.	0.00	min.
Corr	15.40	min		
Ho	61	deg.	27.14	min.

The values for Height of Eye, IC, Dip Short Distance, Atmospheric Pressure, Air Temperature, Date, DR Position, Body, hs, GHA, Dec & Time of MT are from the **Meridian Transit Sight Data Plot** section of this worksheet. See Columns A --> T, Rows 1 --> 35.

Click to view Meridian Transit Sight Data Plot

Horizon Natural

Dip Short Distance Yards

Atmospheric Pressure 1010 mb

Air Temperature 10 ° C

Nautical Almanac Page A4
Add'l Ref. Correction
(R_0 -R) = 0.00 min.

R_o 0.55 R 0.55 λ 1.000

Distance to Visible Horizon 5805 **Yards**

For sight taken on a natural horizon: Dip = 2.38 **min**

Dip 2.38 **min.**

Dip Short = **min**

U.S. Naval Observatory

Celestial Navigation Data
Provides computed altitudes and azimuths and other data for an assumed position and time.