

## THE DURATION AND INTENSITY OF TWILIGHT.

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## INTRODUCTION.

Officials of the Weather Bureau frequently receive requests for information relative to the intensity of the light at some specified time, usually during the hours of twilight. Twilight illumination is, of course, markedly influenced by the state of the sky as regards clouds, haze, smoke, etc. Aside from this it is dependent on the angular depression of the sun below the horizon and the position and phase of the moon.

## THE DURATION OF TWILIGHT.

One of the earliest accessible treatises on twilight is by Johann Heinrich Lambert,<sup>1</sup> who states that ancient astronomers had found for the beginning of complete darkness at night and its ending in the morning a depression of the sun below the horizon of from  $18^{\circ}$  to  $19^{\circ}$ .<sup>2</sup> According to Kaemtz<sup>3</sup> they also gave as a rule that at this time stars of the sixth magnitude should be visible near the zenith.

Kaemtz<sup>4</sup> also describes the anti-twilight, which includes what many observers have called the anticrepuscular or anti-twilight arch, below which the sky is of an ashy or a deep blue color, depending on its clearness, and above which it takes on a reddish tinge. Under favorable atmospheric conditions, and specially in arid regions at a considerable elevation above sea level, this arch may be observed to rise in the east soon after sunset. Lambert,<sup>5</sup> at Augsburg, Germany, on November 19, 1759, made measurements from which he computed that it passed his zenith when the sun was  $5^{\circ} 50'$  lower than at its apparent setting; or, allowing  $33'$  for atmospheric refraction, when the depression of the sun below the horizon was  $6^{\circ} 23'$ . Bravais,<sup>6</sup> from observations made on the summit of the Faulhorn, Switzerland, at an elevation of 2,685 meters above sealevel, found it to pass his zenith when the sun was  $6^{\circ} 9'$  below the horizon, and to reach the horizon when the depression of the sun was  $17^{\circ} 30'$ .

On the assumption that the anti-twilight arch represents the limit of direct illumination of the atmosphere by the sun, Lambert<sup>7</sup> shows that a depression of the sun below the horizon of  $18^{\circ} 30'$  at the end of twilight gives  $1/89$ th part of the earth's radius, or about 70 kilometers, as the height to which the atmosphere is capable of reflecting sunlight. This seemed to him to be too high, and he therefore supposed twilight to be divided into two periods, a primary twilight, and a secondary twilight. The primary twilight he attributed to light reflected from portions of the atmosphere directly illuminated by the sun. Secondary twilight he attributed to reflection of light from portions of the atmosphere illuminated by the primary twilight.

Assuming that the end of this secondary twilight corresponds to the beginning of complete darkness, and,

therefore, to a depression of the sun below the horizon of  $18^{\circ} 30'$ , Lambert<sup>8</sup> computed the height to which the atmosphere is capable of reflecting sunlight to be  $1/372$ d part of the earth's radius, or about 17 kilometers. From his own measurements of the height of the anti-twilight arch with the sun  $8^{\circ} 3'$  below the horizon, Lambert<sup>9</sup> computed the height to which the atmosphere reflects sunlight to be  $1/220$ th part of the earth's radius, or about 29 kilometers, which is much less than the height now generally accepted.<sup>10</sup> His computation also placed the anti-twilight arch at about the center of the segment of the sky which he supposed to be illuminated by secondary reflection, and he therefore concluded that the light in one half this segment is so feeble as to be obscured by the light of the fixed stars.

This conclusion of Lambert's found favor with Biot,<sup>11</sup> but was rejected by Grunert,<sup>12</sup> who shows that Lambert's observations give increasingly higher values for the height of the upper limit of the reflecting atmosphere with increasing depression of the sun below the horizon.

Kaemtz<sup>13</sup> also states that "This segment [the deep blue of the anti-twilight] is due to the shadow of the earth projected on the sky," and a note by Martins<sup>14</sup> defines the *second twilight* to be the feeble white light that under favorable conditions is sometimes observed from high mountains after the anti-twilight arch has set, and which has been observed to continue until the sun was  $26^{\circ}$  below the horizon. It is also stated that this second twilight has not been observed at low-level stations.

Laska<sup>15</sup> and also Exner<sup>16</sup> ignore the secondary twilight of Lambert. Following Bezold,<sup>17</sup> however, Exner distinguishes between a "first" and a "second" twilight, the "first" twilight terminating in the evening with the disappearance of the first purple light, which is often a prominent feature of twilight phenomena. According to Bezold's observations, the anti-twilight arch can not be observed to pass the zenith, but can sometimes be seen to reappear in the western sky about  $30^{\circ}$  past the zenith. Its final disappearance marks the end of astronomical twilight.

*Tables of the duration of astronomical twilight.*

In early numbers of the Berliner Astronomisches Jahrbuch<sup>18</sup> will be found tables giving the duration of astronomical twilight for each day in the year. From the explanation of the use of the Ephemeriden in the Jahrbuch for 1776, pages 20-21, it appears that sunrise or sunset is considered to be that instant when the center of the sun coincides with the true horizon, disregarding the effect of atmospheric refraction; and that astronomical twilight begins in the morning and ends in the evening when the true position of the sun's center is  $18^{\circ}$  below the horizon. The interval between the time of sunrise or sunset thus computed and the time the sun is  $18^{\circ}$  below the horizon is the duration of astronomical twilight given.

<sup>1</sup> Lambert, op. cit., p. 102.<sup>2</sup> Op. cit., p. 106.<sup>3</sup> See Heim, Albert, Luft-Farben. Hofer & Co. Zurich, 1912. p. 68.<sup>4</sup> Biot, J. B. Traité élémentaire d'Astronomie physique. 3d ed. Paris, 1841. Vol. 1, p. 309-323.<sup>5</sup> Grunert, Johann August. Beiträge zur meteorologischen Optik. Leipzig, 1848. Erster Theil, Erstes Heft, p. 194-264.<sup>6</sup> Op. cit., p. 408.<sup>7</sup> Kaemtz, op. cit., p. 409; see also p. 499, note "G."<sup>8</sup> Laska, Prof. Dr. W. Lehrbuch der Astronomie und der mathematischen Geographie. II. Auflage. I. Teil: Sphärische Astronomie. Bremerhaven und Leipzig, 1906, p. 74-78. (Kleyers Encyclopädie der gesamten mathematischen, technischen und exakten Naturwissenschaften.)<sup>9</sup> Pernster, J. M., & Exner, Felix M. Meteorologische Optik. Wien und Leipzig, 1910. p. 799-799.<sup>10</sup> See Abbe's translation of Bezold's description of twilight phenomena, with Exner's discussion, p. 630-633, of this number of the REVIEW.<sup>11</sup> See the "Ephemeriden" in the Jahrbücher for 1776-1829.<sup>1</sup> Lambert's Photometrie (1760). Zweites Heft: Theil III, IV, and V, pp. 96-112. (Ostwald's Klassiker der exakten Wissenschaften. Nr. 32. Leipzig, 1892.)<sup>2</sup> For a bibliography summarizing these determinations see Hocquart, J. C. Vademecum de l'astronomie. Bruxelles, 1882. p. 313-316.<sup>3</sup> Meteorology, by L. F. Kaemtz. Notes by Ch. Martins. Translated by C. V. Watker. London, 1843, p. 410.<sup>4</sup> Op. cit., p. 408.<sup>5</sup> Op. cit., p. 104.<sup>6</sup> Bravais, M. A. Observations sur les phénomènes crépusculaires. Annuaire Méteorologique de la France pour 1850, 2<sup>me</sup> Année. p. 185-218. (See Note additionnelle, p. 215.)<sup>7</sup> Op. cit., p. 102.

In the *Annuaire Astronomique de l'Observatoire Royal de Belgique*<sup>19</sup> is published a table giving the duration of astronomical twilight for each fifth degree of latitude at the time of the equinoxes and of the summer and winter solstices that is in accord with the above.

In the *Comptes Rendus* for 1860, 50:81, M. F. Petit presents, for latitudes 48° and 49° N., some preliminary computations of the duration of twilight, or the interval between the time the center of the sun appears to be on the horizon, allowing 33' 30" for atmospheric refraction, and the time the true position of the sun's center is 18° below the horizon. In the *Comptes Rendus* for 1860, 51:486-489, he published a table giving the duration of twilight for each degree of solar declination from -24° to +24° and for each degree of longitude from 0° to 70°.

This table appears to be the basis for most of the tables giving the duration of astronomical twilight since published. Thus, in the "*Annuaire Astronomique et Météorologique pour 1893 par Camille Flammarion*" is a table giving the duration of astronomical twilight for each fifth degree of latitude and for 15-day intervals, beginning with January 1. No authority is given, but it appears to be in accord with Petit's table, and might, indeed, have been obtained by interpolation in it for the solar declination on the days for which data is given.

Likewise, Láska<sup>20</sup> published a table giving the duration of astronomical twilight that is attributed to the *Annuaire* for 1905. His table states in the heading that the data refer to the first day of the respective months; but they seem to have been copied directly from the *Annuaire*, taking for the first day of the respective months the data for the date given in the table that falls nearest the first, although in some cases this may have been as early as the 26th of the preceding month. In a few cases only does it appear that an attempt has been made to obtain correct data by interpolation. This table of Láska's has been copied by Exner.<sup>21</sup>

From the above brief review of the literature it is apparent that the uniform practice in the computation of tables of the duration of astronomical twilight has been to regard the time of its beginning in the morning, or its ending in the evening, as that instant when the true position of the center of the sun is 18° below the horizon; although the earlier observers, as has already been shown, gave a slightly greater depression of the sun at the beginning or end of complete darkness, and later observers have generally found a somewhat less depression, namely, from 16° to 18°. The tables of duration of twilight, however, show variations due to differences with respect to the position of the sun at its rising and setting. The older German writers, as already shown, considered the sun to rise or set when its center coincided with the true horizon, disregarding atmospheric refraction; modern French and Belgians consider sunrise or sunset to be that instant when the center of the sun appears to be on the true horizon; while modern English and German writers consider it to be the instant when the upper limb of the sun appears to be on the horizon. These two latter, therefore, apply a correction for atmospheric refraction to the computed position of the sun at sunrise or sunset.

Table 1 of the present paper gives the length of the period between the time when the upper limb of the sun

appears to coincide with the true horizon and the time the true position of its center is 18° below the horizon. Allowing 16' for the sun's semidiameter, and 34' for atmospheric refraction, the sun is then only 17° 10' lower than at the time of sunrise or sunset.

The computations may be made from the equation

$$h = \frac{\sin a - \sin \phi \sin \delta}{\cos \phi \cos \delta}$$

where *h* is the sun's hour angle from the meridian, *a* is the sun's altitude, considered minus below the horizon,  $\delta$  is the solar declination, and  $\phi$  is the latitude of the place of observation.

TABLE 1.—Duration of astronomical twilight. (Interval between sunrise or sunset and the time when the true position of the sun's center is 18° below the horizon.)

Date.	NORTH LATITUDE.														
	0°	10°	20°	25°	30°	32°	34°	36°	38°	40°	42°	44°	46°	48°	50°
Jan. 1.....	1 14	1 15	1 18	1 21	1 26	1 28	1 29	1 31	1 34	1 37	1 41	1 45	1 49	1 53	1 57
11.....	1 14	1 14	1 18	1 21	1 25	1 27	1 29	1 31	1 33	1 36	1 39	1 43	1 47	1 51	1 55
21.....	1 13	1 13	1 17	1 20	1 23	1 25	1 28	1 30	1 32	1 34	1 38	1 41	1 45	1 49	1 54
Feb. 1.....	1 13	1 12	1 15	1 18	1 22	1 24	1 26	1 28	1 30	1 33	1 36	1 39	1 43	1 47	1 52
11.....	1 11	1 12	1 14	1 17	1 21	1 23	1 25	1 27	1 29	1 32	1 34	1 37	1 41	1 45	1 49
21.....	1 10	1 11	1 13	1 16	1 20	1 22	1 24	1 26	1 28	1 31	1 33	1 36	1 40	1 44	1 48
Mar. 1.....	1 10	1 11	1 13	1 16	1 20	1 21	1 23	1 25	1 28	1 30	1 33	1 36	1 39	1 43	1 48
11.....	1 09	1 10	1 13	1 16	1 19	1 21	1 23	1 25	1 28	1 30	1 33	1 36	1 39	1 43	1 48
21.....	1 09	1 10	1 13	1 16	1 20	1 22	1 24	1 26	1 29	1 31	1 34	1 37	1 41	1 45	1 50
Apr. 1.....	1 09	1 11	1 14	1 17	1 21	1 23	1 25	1 27	1 30	1 33	1 36	1 40	1 44	1 49	1 54
11.....	1 10	1 11	1 15	1 18	1 22	1 24	1 27	1 30	1 33	1 36	1 39	1 43	1 47	1 52	1 57
21.....	1 11	1 12	1 16	1 20	1 24	1 27	1 29	1 32	1 36	1 39	1 43	1 48	1 52	1 57	1 62
May 1.....	1 12	1 13	1 18	1 22	1 27	1 30	1 33	1 36	1 39	1 43	1 48	1 52	1 57	1 62	1 67
11.....	1 13	1 14	1 19	1 24	1 30	1 33	1 36	1 40	1 43	1 48	1 52	1 57	1 62	1 67	1 72
21.....	1 13	1 15	1 21	1 26	1 32	1 36	1 39	1 43	1 48	1 52	1 57	1 62	1 67	1 72	1 77
June 1.....	1 14	1 16	1 23	1 28	1 35	1 38	1 41	1 46	1 52	1 59	1 67	1 74	1 81	1 88	1 95
11.....	1 15	1 17	1 24	1 29	1 36	1 40	1 44	1 49	1 56	1 62	1 70	1 78	1 85	1 92	1 99
21.....	1 15	1 18	1 24	1 29	1 37	1 41	1 45	1 50	1 58	1 62	1 70	1 78	1 85	1 92	1 99
July 1.....	1 15	1 17	1 24	1 29	1 36	1 40	1 44	1 49	1 56	1 62	1 70	1 78	1 85	1 92	1 99
11.....	1 14	1 16	1 23	1 28	1 35	1 38	1 41	1 46	1 52	1 59	1 67	1 74	1 81	1 88	1 95
21.....	1 13	1 15	1 21	1 26	1 32	1 36	1 39	1 43	1 48	1 52	1 60	1 68	1 75	1 82	1 89
Aug. 1.....	1 13	1 14	1 19	1 24	1 30	1 33	1 36	1 40	1 44	1 48	1 54	1 62	1 70	1 78	1 85
11.....	1 12	1 13	1 18	1 22	1 27	1 30	1 33	1 36	1 39	1 43	1 48	1 52	1 57	1 62	1 67
21.....	1 11	1 12	1 16	1 20	1 24	1 27	1 30	1 33	1 36	1 39	1 43	1 48	1 52	1 57	1 62
Sept. 1.....	1 10	1 11	1 14	1 18	1 22	1 24	1 27	1 30	1 33	1 36	1 39	1 43	1 48	1 52	1 57
11.....	1 09	1 11	1 13	1 17	1 21	1 23	1 25	1 27	1 30	1 33	1 36	1 39	1 44	1 49	1 54
21.....	1 09	1 10	1 13	1 16	1 20	1 22	1 24	1 26	1 29	1 31	1 34	1 37	1 41	1 45	1 50
Oct. 1.....	1 09	1 10	1 13	1 16	1 19	1 21	1 23	1 25	1 28	1 30	1 33	1 36	1 39	1 43	1 48
11.....	1 10	1 11	1 13	1 16	1 19	1 21	1 23	1 25	1 28	1 30	1 33	1 36	1 39	1 43	1 48
21.....	1 10	1 11	1 13	1 16	1 20	1 22	1 24	1 26	1 29	1 31	1 33	1 36	1 40	1 44	1 48
Nov. 1.....	1 11	1 12	1 14	1 17	1 21	1 23	1 25	1 27	1 30	1 32	1 34	1 38	1 41	1 46	1 49
11.....	1 12	1 12	1 16	1 18	1 22	1 24	1 26	1 28	1 31	1 33	1 36	1 40	1 43	1 47	1 52
21.....	1 13	1 13	1 17	1 20	1 24	1 26	1 28	1 30	1 32	1 35	1 38	1 42	1 46	1 49	1 55
Dec. 1.....	1 14	1 14	1 18	1 21	1 25	1 27	1 29	1 31	1 33	1 36	1 40	1 44	1 47	1 52	1 57
11.....	1 14	1 15	1 18	1 22	1 26	1 28	1 30	1 32	1 34	1 37	1 41	1 45	1 49	1 53	1 59
21.....	1 15	1 16	1 19	1 22	1 26	1 28	1 30	1 32	1 35	1 38	1 41	1 45	1 49	1 54	1 59

The solar declinations employed are those given in the *American Ephemeris and Nautical Almanac*, 1899, pp. 377-384, *Solar Ephemeris for Washington*, which Prof. Marvin has found to be very close average values.<sup>23</sup>

The computations have been simplified by the use of Ball's *Altitude Tables*,<sup>24</sup> from which the value of *h* has been determined for true altitudes of the sun of -50' and -18°. The difference between these two values is the duration of astronomical twilight. It is, therefore, from 1 to 2 minutes less than that given in Petit's table and from 3 to 7 minutes less than that given in the

<sup>19</sup> See, for example, the *Annuaire* for 1907, p. 193.

<sup>20</sup> *Op. cit.*, p. 77.

<sup>21</sup> *Pernter & Exner. op. cit.*, p. 743.

<sup>22</sup> See Láska, *op. cit.*, p. 74; Grünert, *op. cit.*, p. 221; *Pernter & Exner. op. cit.*, p. 743, 766, 767; *Schmitt, J. F. Julius. Ueber die Dämmerung. Astronomische Nachrichten* No. 1495-1496, Altona, 1865, 36, col. 97-116.

<sup>23</sup> *Marvin, C. F. Sunshine Tables. Edition of 1905, p. 3. (W. B. No. 320.)*

<sup>24</sup> *Ball, Frederick. Altitude Tables for lat. 31° to 60°. London, 1907; [same] for lat. 0° to 30°, London, 1910.*

Berliner Astronomisches Jahrbuch and in the *Annuaire Astronomique de Belgique* above quoted, the magnitude of the differences increasing with the latitude.

*Tables of the duration of civil twilight.*

*Definitions of civil twilight.*—There is a lack of definiteness and of uniformity in the definitions of *civil twilight*. Thus the Berliner Astronomisches Jahrbuch for 1776, in the explanation of the use of the Ephemeriden, page 22, gives what it calls Lambert's definition of the beginning of civil twilight in the morning or its ending in the evening, namely, when the center of the sun is  $6^{\circ} 23\frac{1}{2}'$  below the horizon,<sup>25</sup> or  $5^{\circ} 50'$  lower than when its center appears to coincide with the true horizon, at which time the [anti-] twilight arch passes through the zenith. Also, Bravais<sup>26</sup> states that "The results obtained show that the passage of the [anti-twilight] curve at the zenith, the commencement or end of the civil twilight of Lambert, occurs when the sun reaches a zenith distance of  $96^{\circ}$ ." Abbe, sr.,<sup>27</sup> gives a similar definition, except that he does not give the position of the sun. Kaemtz<sup>28</sup> distinguishes astronomical from ordinary twilight, the latter terminating "when darkness compels us to suspend labour that is going on in the open air." Láska's definition is similar, except that he adds that the sun is then about  $6\frac{1}{2}^{\circ}$  below the horizon. Bezold<sup>29</sup> identifies the end of civil twilight with the disappearance of the first purple light, when the sun is about  $6^{\circ}$  below the horizon. This is also Gruner's definition.<sup>30</sup> Flammarion<sup>31</sup> states that civil twilight ends and the day closes at the moment when the sun is  $6^{\circ}$  below the horizon, and the planets and stars of the first magnitude begin to appear. In the *Annuaire Astronomique de l'Observatoire Royal de Belgique*<sup>32</sup> it is stated that civil twilight ends when the sun is about  $6^{\circ}$  below the horizon, at the time the [anti]crepuscular arch passes the zenith. Heim<sup>33</sup> states that with a depression of the sun of  $6\frac{1}{2}^{\circ}$  civil twilight ends; that is to say, one can no longer read and write without artificial light. According to Vincent<sup>34</sup> civil twilight begins in the morning at the instant when we are first able to read ordinary print with the back turned toward the east; the sun is then  $6^{\circ}$  below the horizon. It ends in the evening when we cease to be able to read with the back turned toward the sun's setting. A note in the *Journal of the Royal Astronomical Society of Canada*, May, June, 1916, p. 265, also gives as the end of civil twilight the moment when the [anti]crepuscular curve passes the zenith, and planets and stars of the first magnitude begin to appear, the depression of the sun below the horizon being about  $6^{\circ}$ .

It thus appears that five distinct definitions are given for the ending of civil twilight in the evening or its beginning in the morning, as follows:

(1) The moment when the anticrepuscular, or anti-twilight arch passes the zenith.

(2) The moment when the sun is  $6^{\circ}$  to  $6\frac{1}{2}^{\circ}$  below the horizon. This latter, however, appears to be dependent upon (1).

(3) The moment when stars and planets of the first magnitude are just visible.

(4) The moment in the evening when darkness compels us to suspend labor in the open air, or the moment in the morning when the light is sufficient for its resumption.

(5) With the disappearance of the first purple light in the evening, or with its reappearance in the morning. This, in general, appears to coincide with (2) and observations likewise appear to make it coincide with (3) and (4).

Likewise, there are discrepancies to be found in tables giving the duration of civil twilight. Thus, in the Berliner Astronomisches Jahrbücher for 1776-1783, the tables which give the duration of astronomical twilight (already referred to) also give the duration of civil twilight; or the interval between the time the center of the sun coincides with the true horizon, disregarding atmospheric refraction, and the time its center is  $6^{\circ} 23\frac{1}{2}'$  below the horizon. In the *Annuaire Astronomique de l'Observatoire Royal de Belgique*<sup>35</sup> is a table giving the duration of civil twilight for every fifth degree of latitude, at the times of the summer and winter solstices and the equinoxes; and the time given is that required for the center of the sun to pass from the horizon to a point  $6^{\circ}$  below, *true positions being understood*. This table was copied by Exner.<sup>36</sup> In the *Annuaire Astronomique et Météorologique*, par Camille Flammarion,<sup>37</sup> is a table that gives the time interval between the instant when the center of the sun appears to be on the true horizon, allowing for atmospheric refraction, and the instant when its center is  $6^{\circ}$  below the horizon; and this table has been copied by Láska.<sup>38</sup>

Finally, in the Ephemerides of the *Annuaire Astronomique de l'Observatoire Royal de Belgique* is given the time of beginning of civil twilight at Uccle, the time of sunrise, the time of sunset, and the time of ending of civil twilight, for each day in the year. The duration of civil twilight as determined from these data agree quite closely with Flammarion's table, although on some days the duration of morning and evening twilight differs by two minutes. The time given for sunrise or sunset is the instant when the center of the sun will appear to be on the true horizon, assuming  $34.5'$  for atmospheric refraction.

With the exception of the table in the Berliner Astronomisches Jahrbuch, it is seen that all the above are in accord in considering that civil twilight ends in the evening and begins in the morning when the true position of the sun's center is  $6^{\circ}$  below the horizon; and the differences found in them, as in the case of tables of the duration of astronomical twilight, are to be attributed to differences in the conception of the position of the sun at sunrise or sunset.

The older definition of the beginning or ending of civil twilight, the moment when the anticrepuscular, or anti-twilight arch passes the zenith, depends on a phenomenon that can only be observed under the most favorable circumstances.<sup>39</sup> It therefore appears that we should abandon this definition for the newer one of Bezold, namely,

<sup>25</sup> See, for example, the *Annuaire* for 1907, p. 191.

<sup>26</sup> *Pernter & Exner*, op. cit., p. 745.

<sup>27</sup> See, for example, the *Annuaire* for 1893, p. 39.

<sup>28</sup> Láska, op. cit., p. 77.

<sup>29</sup> One of the few authentic observations of the transit of the "upper boundary of the earth's shadow" was made by Heim from a steamer in the Indian Ocean. See Heim, *Luft-Farben*, p. 75.

<sup>25</sup> Lambert's value for this depression is  $6^{\circ} 23'$ . See Lambert, op. cit., p. 108. The writer is unable to confirm Lambert's use of the terms *civil twilight* and *twilight arch* in this connection, although both terms are repeatedly attributed to him.

<sup>26</sup> Bravais, M. A. Observations crépusculaires faites en Suisse, à une élévation de 2,690 mètres au-dessus de la mer. *Comptes Rendus*, 1884, 18: 723.

<sup>27</sup> Abbe, Prof. Cleveland. Notes from the reports of State sections. MONTHLY WEATHER REVIEW, March 1898, 24: 114-15.

<sup>28</sup> Kaemtz, op. cit., p. 409.

<sup>29</sup> Bezold, Wilhelm von. Beobachtungen über die Dämmerung. Gesammelte Abhandlungen aus den Gebieten der Meteorologie und des Erdmagnetismus. Braunschweig, 1906, p. 29. See also Abbe's translation, this REVIEW, p. 620-623.

<sup>30</sup> Gruner, P. Nouvelles remarques concernant les lueurs crépusculaires du ciel. *Arch. d. sci. phys. et nat.*, Geneva, 1916, 4me. pér., t. 42, p. 39.

<sup>31</sup> Flammarion, op. cit., 1893, p. 29.

<sup>32</sup> See, for example, the *Annuaire* for 1907, p. 193.

<sup>33</sup> Heim, op. cit., p. 63.

<sup>34</sup> Vincent, J. *Traité de météorologie*. Bruxelles, 1914 p. 54.

the moment when the first purple light disappears from the sky in the evening or reappears in the morning; since under favorable atmospheric conditions this is a prominent feature of twilight phenomena.

We will therefore define the end of civil twilight in the evening and its beginning in the morning as *the instant when the true position of the sun's center is 6° below the horizon*. At this time stars and planets of the first magnitude are just visible. In the evening the first purple light has just disappeared, and darkness compels the suspension of work in the open air unless artificial illumination is provided. In the morning the first purple light is beginning to appear, and the illumination is sufficient for the resumption of outdoor occupations.

TABLE 2.—Duration of civil twilight. (Interval between sunrise or sunset and the time when the true position of the sun's center is 6° below the horizon.)

[Minutes.]

Date.	North latitude.														
	0°	10°	20°	25°	30°	32°	34°	36°	38°	40°	42°	44°	46°	48°	50°
Jan. 1.....	22	22	24	25	27	27	27	28	29	30	32	33	34	36	39
11.....	22	22	24	25	26	27	28	28	29	30	31	32	33	35	38
21.....	22	22	23	24	26	26	27	27	28	29	30	32	33	34	37
Feb. 1.....	22	22	23	24	25	26	27	27	27	28	29	31	32	34	35
11.....	22	22	23	24	25	26	27	27	28	29	30	31	33	34	34
21.....	21	22	23	23	24	25	25	26	27	28	28	29	30	32	33
Mar. 1.....	21	22	23	24	24	25	26	27	28	28	29	30	31	31	33
11.....	21	22	23	24	24	25	26	27	27	28	29	30	31	32	32
21.....	21	21	22	23	24	24	25	26	26	27	27	28	29	31	33
Apr. 1.....	21	21	22	23	24	25	26	27	28	28	29	30	32	33	33
11.....	21	22	23	24	25	26	26	27	28	28	29	31	32	32	34
21.....	22	22	23	24	25	26	27	28	28	29	30	32	34	35	35
May 1.....	22	22	23	24	25	26	27	28	29	30	32	33	35	36	36
11.....	22	23	24	25	26	27	28	29	30	31	33	35	36	39	39
21.....	22	23	24	25	27	28	28	29	30	31	33	35	36	41	41
June 1.....	22	23	24	25	27	28	28	29	30	31	33	34	36	40	43
11.....	22	23	24	26	28	28	29	30	31	32	34	36	38	41	44
21.....	22	23	25	26	28	29	29	30	31	33	34	36	38	42	44
July 1.....	22	23	24	26	28	28	29	30	31	33	34	36	38	41	44
11.....	22	23	24	25	27	28	29	31	32	34	36	37	40	43	43
21.....	22	22	24	25	27	28	28	29	30	31	33	35	36	38	41
Aug. 1.....	22	22	23	24	26	27	28	29	30	31	33	35	36	39	39
11.....	22	22	23	24	25	26	27	28	29	30	32	33	35	36	36
21.....	22	22	23	24	25	26	27	28	28	29	30	32	34	35	35
Sept. 1.....	21	22	23	24	25	26	26	27	28	28	29	31	32	34	34
11.....	21	22	23	24	25	26	27	28	29	30	31	32	33	33	33
21.....	21	21	22	23	24	24	25	26	27	27	28	30	31	31	33
Oct. 1.....	21	21	22	23	24	24	25	26	26	27	27	29	30	31	32
11.....	21	22	23	24	24	25	26	27	28	28	29	30	31	31	33
21.....	21	22	23	24	25	25	26	27	28	28	29	30	32	33	33
Nov. 1.....	22	22	23	24	25	26	27	28	28	29	30	31	32	33	34
11.....	22	22	23	24	25	26	27	28	28	29	30	31	32	33	35
21.....	22	22	23	24	26	26	27	28	28	29	30	32	33	34	37
Dec. 1.....	22	22	24	25	26	27	28	28	29	30	31	33	34	35	38
11.....	22	22	24	25	27	27	28	28	29	30	32	33	34	36	39
21.....	22	23	24	25	27	27	28	28	29	31	32	33	34	37	39

Table 2 gives the length of the period between the time when the upper limb of the sun appears to coincide with the true horizon and the time the true position of its center is 6° below the horizon. Allowing 16' for the sun's semidiameter and 34' for atmospheric refraction the sun is then only 5° 10' lower than at the time of sunrise or sunset. The length of this interval has been determined in the same manner as the values of astronomical twilight in Table 1. The values of civil twilight thus determined are from 1 to 3 minutes less than the values given by Flammarion, and, likewise, than those that appear in the Ephemerides of the *Annuaire Astronomique de Belgique*. They are from 3 minutes to 7 minutes less than those given in the above *Annuaire* for 1907, page 194, which were copied by Exner, and are also less by

the same amount than the values given in the table published by me in the *Transactions of the Illuminating Engineering Society*, 1916 (reprinted in this REVIEW for January, 1916, 44: 13). This latter table gives, as stated in its heading, the time required for the sun to pass from the horizon to a point 6° below, or vice versa.

Applying the duration of astronomical or civil twilight, as given in Tables 1 and 2, to the time of sunrise or sunset as derived from *Weather Bureau Sunshine Tables*<sup>40</sup> we may obtain the time of the ending in the evening or of the beginning in the morning of either astronomical or civil twilight as desired.

THE INTENSITY OF TWILIGHT.

With a standard photometer we may measure the intensity of the illumination at intervals during the twilight period, and this has been done by the writer at Mount Weather, Va.,<sup>41</sup> and later, with the same photometer, by Mr. A. H. Thiessen at Salt Lake City, Utah. The results are summarized in Table 3, and are shown graphically in figure 1. The sun's altitude refers to the true position of its center, and has been determined from *Ball's Altitude Tables*, already referred to.

It is to be understood that the discussion which follows refers to a practically cloudless sky, unless otherwise stated.

TABLE 3.—Photometric measurements of twilight illumination.

Mount Weather, Va.				Salt Lake City, Utah.					
Nov. 4, 1913.		Nov. 5, 1913.		Nov. 6, 1913.		Dec. 15, 1914.		May 17, 1916.	
Sun's altitude.	Illumination.	Sun's altitude.	Illumination.	Sun's altitude.	Illumination.	Sun's altitude.	Illumination.	Sun's altitude.	Illumination.
°	Foot-candles.	°	Foot-candles.	°	Foot-candles.	°	Foot-candles.	°	Foot-candles.
-0.6	37	-1.4	57	±0.0	98	-0.6	41.9h	+0.5	57.8
-1.9	13	-2.9	14	-2.0	27	-1.1	20.0	-0.5	32.8
				-4.2	3	-4.8	1.6h	-1.0	23.6h
				-5.9	0.4	-7.2	0.12h	-1.7	17.7
						-6.2	0.07	-2.2	15.2h
									9.4

Salt Lake City, Utah.							
June 6, 1916.		June 7, 1916.		June 10, 1916.		June 29, 1916.	
Sun's altitude.	Illumination.	Sun's altitude.	Illumination.	Sun's altitude.	Illumination.	Sun's altitude.	Illumination.
°	Foot-candles.	°	Foot-candles.	°	Foot-candles.	°	Foot-candles.
+0.1	95.8	+0.5	57.8	+0.5	35.6	+1.6	97.7
-1.2	34.5	-0.8	58.3h	+0.2	40.4	+1.1	76.8
-1.8	37.1h	-1.1	33.2	-0.2	45.6	+0.8	66.8
-2.6	16.6	-1.6	55.6h	-1.0	25.7	-0.2	51.0
-3.0	18.3h	-1.8	24.5	-1.4	18.3h	-0.7	29.7
-3.8	7.8(a)	-1.9	30.8h	-2.1	15.4	-1.0	28.8
-4.4	4.2h	-2.0	8.0	-2.2	18.2h	-1.3	30.3h
-6.6	0.6h	-3.0	11.8h	-3.0	6.4	-1.8	19.5
-7.4	0.02h	-4.1	2.3	-3.2	7.4h	-2.6	16.5h
		-4.4	4.5h	-3.6	2.8	-2.9	8.3
		-4.7	1.0	-3.8	2.7h (b)	-3.1	13.6h
		-4.9	2.6h	-5.2	0.55	-3.4	5.6
		-5.6	0.3h	-5.3	1.00h (c)	-3.5	8.2
		-5.9	0.04h	-6.2	0.20	-3.7	4.2
		-6.5	0.20	-6.4	0.32h	-4.2	4.4h
		-6.7	0.46h	-7.3	0.082	-4.6	2.0
		-7.3	0.078	-7.4	0.118h	-4.6	2.6h
		-7.4	0.186h	-8.3	0.015	-4.8	1.4
		-8.3	0.037	-8.6	0.034h(d)	-6.9	0.14h
		-8.6	0.069h	-9.0	0.008	-8.0	0.047 (e)
		-9.5	0.015	-9.3	0.016h	-8.6	0.063h
		-9.6	0.027h			-10.8	0.004
		-10.0	0.008			-11.4	0.007h
		-10.3	0.023h				

<sup>40</sup> *Marvin, C. F. Sunshine Tables*. Edition of 1905. Washington, 1905. (W. B. No. 320.)  
<sup>41</sup> *Kimball, Herbert H.* Photometric determinations of daylight illumination on a horizontal surface at Mount Weather, Va. MONTHLY WEATHER REVIEW, December, 1914, 42:650-653.

Remarks on Table 3.

At Mount Weather the sun appeared to set when the true position of its center was about 0.7° below the horizon. At Salt Lake City, on December 15, 1914, it appeared to set when the true position of its center was 2.8° above the horizon; and in May and June, 1916, when the true position of its center was about 0.3° below the horizon.

At Mount Weather on November 4, 1913, dense haze prevailed, and at sunset the sun disappeared in a bank of haze; while on November 5 and 6 the sky was clear and the twilight colors were brilliant—yellow with purple light, followed by red.

At Salt Lake City, on December 15, 1914, the sun set clear; on May 17, 1916, the sun was obscured at sunset by Ci.St. clouds, which covered about half the sky; on June 6, 7, and 10 the sky was clear; on June 29 there were a few Ci.St. clouds in the west.

*h* indicates illumination intensities measured with the photometer tube pointed toward the western horizon; all others were measured with tube pointed toward the zenith.

June 6—(a) Moon at 45° altitude.

June 10—(b) Venus appeared; (c), first star in NE.; (d), North Star discernible.

June 29—(e) Dipper plainly discernible.

From figure 1 and the notes on Table 3 it is apparent that the twilight is more intense on clear days than on hazy days, and that a cirrus cloud sheet diminishes the light intensity only slightly. Elevated mountains on the horizon near the point where the sun sets diminish the twilight intensity.

The illumination measurements with the photometer pointed toward the western horizon (*h* in the table) do not become markedly higher than measurements with the photometer pointed toward the zenith until the sun is about 2° below the horizon, or shortly before the first purple light begins to appear. The measurements show no increase in illumination during the prevalence of this light. In this respect they are in accord with Gruner's<sup>42</sup> measurements, and disprove many eye observations of an apparent increase in illumination at this time. This apparent increase must be attributed to light contrasts. That part of the sky covered by the first purple light has increased in brightness since sunset, as compared with other parts, and, in consequence, at this time the outlines of buildings and of mountains facing this light stand out with unusual clearness.

From figure 1 it is seen that when the upper limb of the sun appears to coincide with the true horizon (depression of the sun's center 50') the zenith illumination is about 33 foot-candles. At the end of civil twilight the illumination is about 0.4 foot-candle.

To the unscientific reader light intensities expressed in units of illumination, as above, have little significance. It will therefore be useful to express these intensities in another way.

From two papers by Russell<sup>43</sup> we are able to compare the intensities of sunlight, twilight, moonlight, and starlight with considerable accuracy. Thus, Russell<sup>44</sup> gives the stellar magnitude of the zenith sun as  $-26.72 \pm 0.04$  and the stellar magnitude of the zenith moon<sup>45</sup> as  $-12.55 \pm 0.07$ . The difference in these stellar magnitudes is 14.17. A difference of one stellar magnitude represents light intensities in the ratio of 1:2.512. Therefore, the sun exceeds the moon in brightness 465,000 times. From Weather Bureau photometric measurements made at Mount Weather, Va., Russell<sup>46</sup> obtains for the zenith sun a light intensity of 9,600 foot-candles, or 9.96 stellar magnitudes brighter than a foot-candle, and

gives for the foot-candle a stellar magnitude of  $-16.76$ . This is 4.21 stellar magnitudes, or 48.3 times, brighter than the full moon in the zenith.

TABLE 4.—Photometric measurements of moonlight illumination, Salt Lake City, Utah.

Photometer tube pointed toward the zenith.						Photometer tube pointed toward moon.			
June 13, 1916.		June 14, 1916.		June 15-16, 1916.		June 14, 1916.		June 15-16, 1916.	
105th meridian time.	Illumination.	105th meridian time.	Illumination.	105th meridian time.	Illumination.	105th meridian time.	Illumination.	105th meridian time.	Illumination.
<i>H. m.</i>	<i>Foot-candles.</i>	<i>H. m.</i>	<i>Foot-candles.</i>	<i>H. m.</i>	<i>Foot-candles.</i>	<i>H. m.</i>	<i>Foot-candles.</i>	<i>H. m.</i>	<i>Foot-candles.</i>
9:02 p	0.00451	9:10 p	0.00303 <sub>2</sub>	10:28 p	0.00558	11:40 p	0.0127	10:34 p	0.0129
9:13 p	0.00458	9:25 p	0.00400	10:50 p	0.00434	11:53 p	0.0117	10:47 p	0.0127 <sub>e</sub>
9:32 p	0.00370	9:43 p	0.00443	11:01 p	0.00546	.....	.....	10:39 p	0.0131 <sub>f</sub>
9:34 p	0.00382	9:58 p	0.00426	11:19 p	0.00691	.....	.....	Midn't	0.0143
9:36 p	0.00496	10:14 p	0.00409	11:35 p	0.00555	.....	.....	12:42 a	0.0171
9:41 p	0.00419	10:28 p	0.00424	12:05 a	0.00574	.....	.....	12:45 a	0.0185 <sub>g</sub>
10:01 p	0.00468	10:45 p	0.00405	12:24 a	0.00569	.....	.....	12:52 a	0.0174
10:03 p	0.00468	11:00 p	0.00426	12:38 a	0.00574	.....	.....	.....	.....
10:15 p	0.00415	11:15 p	0.00514	12:55 a	0.00707	.....	.....	.....	.....
10:17 p	0.00420	11:33 p	0.00491	12:58 a	0.00707	.....	.....	.....	.....
10:29 p	0.00464	11:47 p	0.00541	.....	.....	.....	.....	.....	.....
10:31 p	0.00582	Midn't.	0.00528 <sub>f</sub>	.....	.....	.....	.....	.....	.....
10:35 p	0.00453	.....	.....	.....	.....	.....	.....	.....	.....

Remarks on Table 4.

Clear skies prevailed on all three nights.

Moon full, June 15, 1916, at 2:42 p. m.

Moon on meridian, June 13, 10:44 p. m.; June 14, 11:48 p. m.; June 16, 12:54 a. m. Moon's declination, from  $-25^\circ$  to  $-26^\circ$ .

NOTES.

June 14:

- a. 18-point type read with ease; 8-point type read with difficulty.
- b. 12-point type read with ease; 6-point type read with difficulty.
- c. 10-point type read with ease; 5½-point type read with difficulty.
- d. 9-point type read with ease.

June 15-16:

- e. 14-point type read with ease; 10-point type read with difficulty.
- f. 14-point type read with ease; 10-point type read with difficulty.
- g. 12-point type read with ease; 10-point type read with difficulty.

Century expanded type on white paper was used, and the sizes used are illustrated below.

- 18-point Century expanded type
- 14-point Century expanded type
- 12-point Century expanded type
- 10-point Century expanded type
- \*10-point Roman type
- 8-point Century expanded type
- 6-point Century expanded type
- \*5-point Roman type

\*Approximately 9-pt. and 5½-pt. "Century," respectively.

This result is in good accord with recent measurements made by Thiessen, which are summarized in Table 4. For the total moonlight illumination we must add about 10 per cent for diffuse sky radiation, so that the illumination from the full moon in the zenith as compared to a foot-candle is about 1:43.5. With the moon 66.5° from the zenith its illumination is about 70 per cent of its zenith illumination, and the proportion to a foot-candle is

<sup>42</sup> Gruner, P. Quelques remarques concernant les lueurs crépusculaires du ciel. Arch. sci. phys. et nat., Geneva, 1914, 37(42):226-248.

<sup>43</sup> Russell, Henry Norris. Stellar magnitudes of the sun, moon, and planets. Astrophysical journal, March 1916, 43:103-126.

On the albedo of the planets and their satellites. Ibid., Apr. 1916, 43:173-196.

<sup>44</sup> Op. cit., p. 105.

<sup>45</sup> Op. cit., p. 125.

<sup>46</sup> Op. cit., p. 126-129.

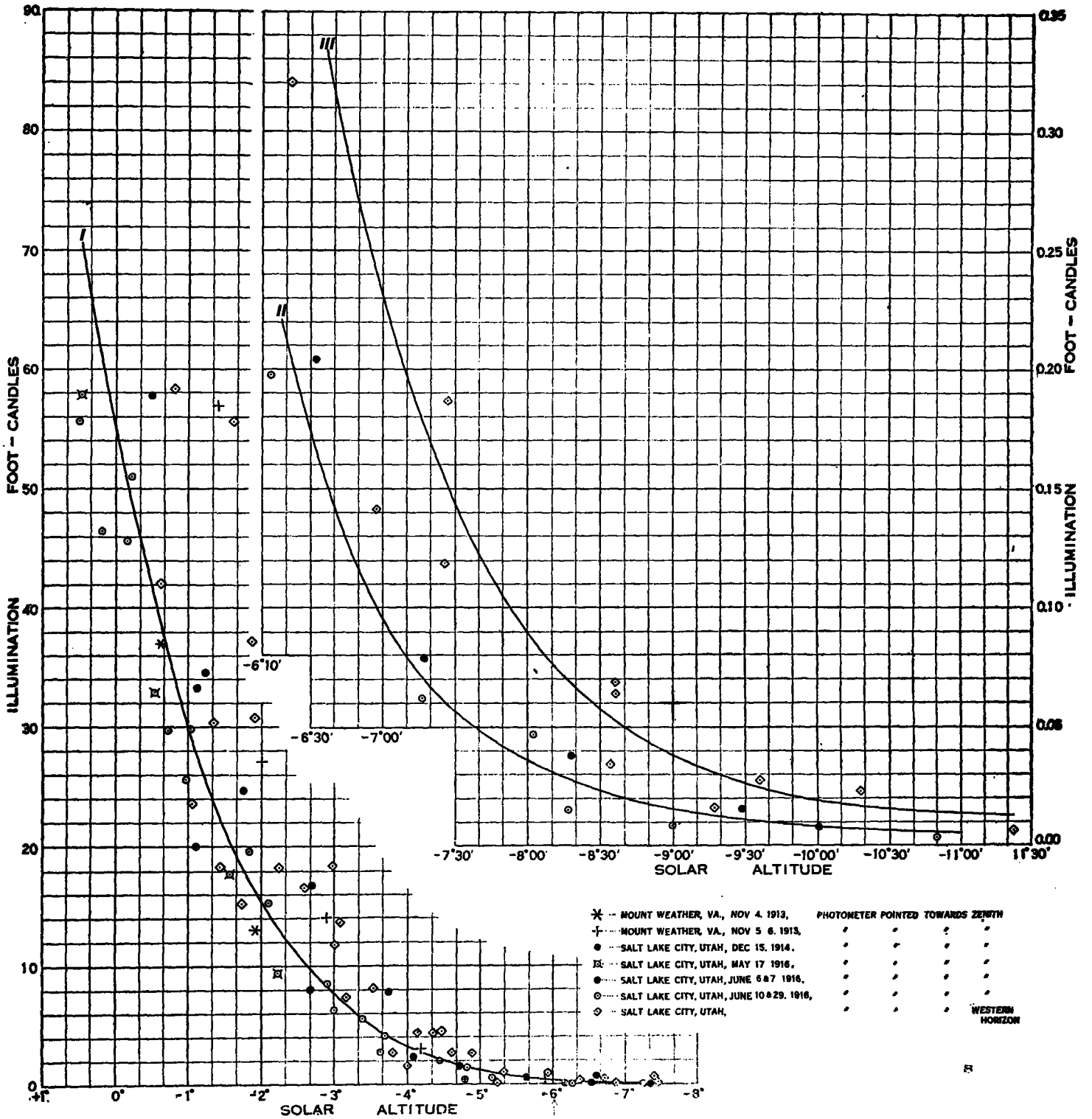


Fig. 1. Photometric measurements of twilight illumination.

Curve I, photometer pointed toward the zenith.  
 Curve II (insert), continuation of Curve I on more open scale with observations of June 7, 10, and 29, 1916.  
 Curve III, photometer pointed toward western horizon.  
 ↑ End of civil twilight. True position of sun, 6° 0' below the sensible horizon.

about 1:62, which is very close to Thiessen's value with the photometer pointed toward the full moon, the zenith distance of the latter being about 66°. According to Russell<sup>47</sup> the illumination of the moon in its first or last quarter is about one-tenth of the full-moon illumination.

From papers by Fabry<sup>48</sup> and Yntema<sup>49</sup> it appears that the total starlight of a hemisphere is somewhat in excess of 1,000 stars of the first magnitude, or about 1/250th of the brightness of the full moon.

From the above data Table 5 has been constructed.

TABLE 5.—Relative illumination intensities.

Source of illumination.	Intensity.	Ratio to zenithal full moon.
Zenithal sun.....	9,600.0	465,000.0
Sky at sunset.....	33.00	1,650.0
Sky at end of civil twilight.....	0.40	20.0
Zenithal full moon.....	0.02	1.0
Quarter moon.....	0.002	0.1
Starlight.....	0.00008	0.004

From Table 5 and figure 1 it appears that the twilight illumination exceeds the illumination from the zenithal full moon until the sun's center is about 8° 40' below the horizon. As this is an illumination intensity of some interest, the time after sunset or before sunrise when the center of the sun will be 8° 40' below the horizon is given in Table 6 for certain latitudes at the time of the equinoxes and the solstices.

TABLE 6.—Time after sunset, or before sunrise, during which the twilight intensity exceeds zenithal full-moonlight.

Latitude.	Winter solstice.	Equinoxes.	Summer solstice.	Latitude.	Winter solstice.	Equinoxes.	Summer solstice.
°	H. m.	H. m.	H. m.	°	H. m.	H. m.	H. m.
0	0 35	0 31	0 35	38	0 34	0 40	0 49
10	0 35	0 32	0 35	40	0 46	0 41	0 51
20	0 36	0 33	0 37	42	0 48	0 42	0 54
25	0 38	0 35	0 39	44	0 50	0 44	0 57
30	0 40	0 36	0 42	46	0 52	0 45	1 00
32	0 41	0 37	0 43	48	0 54	0 47	1 05
34	0 42	0 38	0 45	50	0 57	0 49	1 11
36	0 43	0 39	0 47				

SUMMARY.

1. A review of the literature indicates that from an early date *astronomical* twilight has been considered to end in the evening and begin in the morning when the true position of the sun's center is 18° below the horizon. At this time stars of the sixth magnitude are visible near the zenith, and generally there is no trace on the horizon of the twilight glow.

2. It also appears that *civil* twilight ends in the evening and begins in the morning when the true position of the sun's center is 6° below the horizon. At this time stars and planets of the first magnitude are just visible. In the evening the first purple light has just disappeared, and darkness compels the suspension of out-door work unless artificial lighting is provided. In the morning the first purple light is beginning to be visible, and the illumination is sufficient for the resumption of out-door occupations.

3. Some confusion has arisen in the computation of tables of the duration of both astronomical and civil twilight, due to the fact that in some instances the time of sunrise or sunset has been considered to be that instant when the *center* of the sun is on the true horizon; in others, when its center *appears* to be on the true horizon; and in still others when the *upper limb* of the sun appears to coincide with the true horizon. In the United States this latter is regarded as defining the time of sunrise and sunset.

4. In the tables here presented the duration of astronomical twilight is the interval between sunrise or sunset, according to this latter definition, and the instant the true position of the sun's center is 18° below the horizon. Likewise, the duration of civil twilight is the interval from sunrise or sunset to the instant the true position of the sun's center is 6° below the horizon.

5. At the instant of sunrise or sunset the illumination is about 1,650 times as intense as that from the zenithal full moon; at the end of civil twilight it is about 20 times as intense; with the sun 8° 40' below the horizon it about equals zenithal full moon illumination; while at the end of astronomical twilight, in the absence of the moon, it is only about 0.004 as intense.

The above refer to average clear sky conditions. The twilight will be more intense in a dry climate than in a moist one, will be greatly reduced by smoke or haze, and may be almost completely obliterated by a dense cloud layer. On the other hand, the intensity may be increased by the presence of ice crystals in the atmosphere, especially if they are at a considerable elevation above the place of observation.

I wish to acknowledge my indebtedness to the editor, Dr. Cleveland Abbe, jr., for valuable assistance in reading many of the foreign books and papers consulted in the preparation of this paper, and to Prof. C. F. Talman for his criticism of the manuscript, and for bringing to my attention certain publications that had been overlooked.

VON BEZOLD'S DESCRIPTION OF TWILIGHT.<sup>1</sup>

The oldest and best description of the phenomenon of twilight is that by von Bezold.<sup>2</sup> It is a summary of many observations of all phases of the phenomenon into a single picture, although not all phases usually appear with equal distinctness on any one day, and is based on the evening twilight of Central Europe.<sup>3</sup> It is here given literally, since nearly all systematic and theoretical concepts and investigations of twilight are based upon it.

As soon as the sun begins to approach the horizon on a cloudless<sup>4</sup> evening, the lowest part of the sky all about begins to assume a color in greater contrast to the higher parts of the sky than it has when the sun is higher in the heavens.

In the west—which may be defined once for all as that portion of the sky lying on the sun's side of the

<sup>1</sup> Translated from *Pernter & Euner, Meteorologische Optik, IV. Abschnitt.* Wien, etc., 1910. Pp. 745-751, 754-755. By C. Abbe, jr.

<sup>2</sup> Poggendorff's *Annalen*, 1864, 123:240-276. Republished in full on pp. 1-30 of von Bezold's "Gesammelte Abhandlungen aus den Gebieten der Meteorologie und des Erdmagnetismus." Braunschweig, 1906. viii, 448 p. 4<sup>o</sup>.

<sup>3</sup> In von Bezold's "Gesammelte Abhandlungen", p. 2, he states this description to be based largely on 24 observations by himself at Munich from Oct. 20, 1863, to Apr. 15, 1864.

<sup>4</sup> "Cloudless" (wolkentfrei) as here used indicates only the absence of greater cloud masses, particularly in the western and eastern skies. Detached small clouds, particularly if located in the north and south, of course can not influence the general course of the phenomenon.—"Gesammelte Abhandlungen," p. 2, footnote 1.

<sup>47</sup> Op. cit., p. 117.  
<sup>48</sup> Fabry, Charles. The intrinsic brightness of the starlit sky. *Astrophysical journal*, 1910, 31: 399.  
<sup>49</sup> Yntema, Lambertus. On the brightness of the sky and the total amount of starlight. Groningen, 1909. 4<sup>o</sup> p. 37.