

## The equations of the observations

### Altitude

The equation of the circumference of position is:

$$\sin H = \sin Dec \sin B + \cos Dec \cos B \cos (GHA + L)$$

$$H = \arcsin (\sin Dec \sin B + \cos Dec \cos B \cos (GHA + L))$$

$$H = H(B, L, Dec, GHA)$$

$$Dec = Dec(t), GHA = GHA(t) \rightarrow H = H(t, B, L)$$

If time is in error:

$$dH = \frac{\partial H}{\partial B} dB + \frac{\partial H}{\partial L} dL + \frac{\partial H}{\partial t} dt$$

$$dH = H_0 - H$$

$$\frac{\partial H}{\partial B} = \frac{1}{\cos H} (\sin Dec \cos B - \cos Dec \sin B \cos (GHA + L))$$

$$\frac{\partial H}{\partial L} = \frac{-1}{\cos H} (\cos Dec \cos B \sin (GHA + L))$$

$$\frac{\partial H}{\partial t} = \frac{H_{i+1} - H_i}{t_{i+1} - t_i} = \frac{H(t_{i+1}, B_i, L_i) - H(t_i, B_i, L_i)}{t_{i+1} - t_i}$$

Time derivatives:

$$\cos H \frac{dH}{dt} = \cos B \frac{dB}{dt} \sin \delta + \sin B \cos \delta \frac{d\delta}{dt} - \sin B \frac{dB}{dt} \cos \delta \cos LHA - \cos B \sin \delta \frac{d\delta}{dt} \cos LHA - \cos B \cos \delta \sin LHA \frac{dLHA}{dt}$$

$$\cos H \frac{dH}{dt} = \cos B \frac{dB}{dt} \sin \delta + \sin B \cos \delta \frac{d\delta}{dt}$$

$$- \sin B \frac{dB}{dt} \cos \delta \cos LHA$$

$$- \cos B \sin \delta \frac{d\delta}{dt} \cos LHA$$

$$- \cos B \cos \delta \sin LHA \frac{dLHA}{dt}$$

$$\frac{dLHA}{dt} = \frac{dGHA}{dt} + \frac{dL}{dt}$$

For a stationary observer:

$$\frac{dB}{dt} = 0$$

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