Sight Reduction using the Polar Key on a Calculator

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In celestial navigation, one crucial task is to reduce the data in the nautical almanac to a form that can be directly compared to sextant measurements. Specifically, the observer begins with an assumed position at (*lat, lon*) at a known time, and extracts the Greenwich Hour Angle (*GHA*) and declination (*dec*) of a body e.g. star, from pre-computed tables. A Local Hour Angle (*lha*) is found using *lha= GHA+lon*. This data is then converted to the body's altitude (*Hc*) above the horizon, and its azimuth (*Zn*) relative to due north.

Herein, we describe a short formula[1] pertinent to a calculator that performs this calculation. It is not well-known, but perhaps it should be.

$$\langle r, \theta \rangle = \text{pol}(\tan dec \cos lat - \sin lat \cos lha, -\sin lha)$$
$$Hc = \cos^{-1}(r \cos dec)$$
$$Zn = \theta \bmod 360°$$

This formula computes both the altitude *Hc* and azimuth *Zn*, using the pol key, which performs a conversion from (*x, y*) to (*r, θ*) coordinates. The CASIO calculator shown in the figure has a particularly convenient version of this rectangular to polar
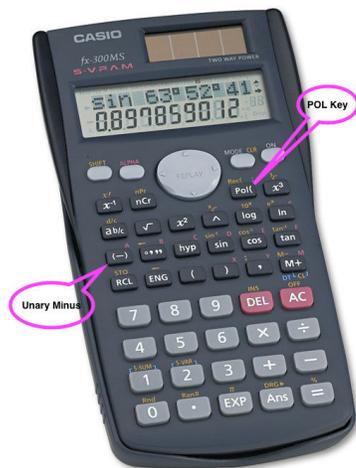
conversion. In one operation, the calculator computes both an *r* value i.e. length which is automatically stored in variable E, and an angle value which is stored in variable F. Thus one formula simultaneously provides two values. However, we note that not all calculators are this user-friendly. For example, the TI-30x requires choosing from a menu to calculate one or the other of *r* and θ, and multiple executions of the formula detract from the simplicity of this approach.

With the CASIO, after executing the pol function, we extract the altitude *Hc* by multiplying the displayed result i.e. the *r* value, by cos *dec*, and then taking the inverse cosine. Alternatively, execute SHIFT cos(*alpha* E cos *dec*), which references the *r* value stored in variable E. There is <u>one caveat</u>: the computed altitude is always

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[1] I became aware of this formula from a post by Robin Stuart. More than ten years ago, Lars Bergman also mentioned it. See
http://fer3.com/arc/m2.aspx/POL-arctan2-Bergman-nov-2005-w26304
http://fer3.com/arc/m2.aspx/Which-calculator-use-for-arctantan269-Stuart-feb-2017-g38110.

positive. In most instances, this is the case, but if the body is below the horizon (but visible due to refraction effects), we should negate the computed altitude. The danger in error arises if we are uncertain if the object is just above or just below the horizon, and one more operation must be performed to resolve the ambiguity.

The azimuth, as measured clockwise from due north is read directly from variable F. If the value is negative (equivalent to measuring counter-clockwise from due north), add three hundred sixty degrees.

Because the variables declination (*dec*), local hour angle (*lha*) and assumed latitude (*lat*) are used multiple times, it is convenient to store them in variables.   A reasonable procedure is to organize the information alphabetically, and store the values in variables A,B, and C:

- *dec* SHIFT STO A
- *lat* SHIFT STO B
- *lha* SHIFT STO C

Now execute the line

pol(tanA cosB - sinB cosC , - sinC)=

Notice that we use the variable A once, B twice in consecutive order, and ditto for C. A mnemonic I use (is there a better one?) is

 less , no

Taco less sock, no sink

where Ta is tanA, co is cosB (the B reminds me of an O?), less means minus sign, and sock represents sinBcosC. Finally, no sink represents the second entry, minus sinC.

The minus sign in front of sinC is the "unary" operator and not the "subtract" operator, and most calculators have a separate key for it, often labeled (-). The TI-30x will give an error if the "subtract" key is used in place of the "unary" operator. The CASIO is more forgiving.

To find *Hc* we multiply cosA (I remember all variables are used twice, but A was used only once in the formula, so we use A here!) times the displayed value (or var E), and then an inverse cosine. I remember it is a cosine by thinking that each term in the original formula ends in a cos term, but it is "missing" from the -sinC… so use it in this next calculation.

Here is a worked example.

Assumed position is S 40° E 160° at 17:00:00 UT. (Treat north and east as positive values, and south and west as negative values.)

At this time, the Nautical Almanac tells us:

Body	Declination	SHA	GHA	LHA
Jupiter	N 17° 20.6'		195° 03.4'	355° 03.4'
Aries			331° 12.3'	
Rigil Kentaurus	S 60° 47.8'	140° 15.7'	111° 28.0'	271° 28.0'

where we computed the Greenwich Hour Angle of Rigil Kentaurus from

$GHA= SHA$+Aries= 471° 28.0'= 111° 28.0',

and the local hour angle from

$lha= lon+GHA$.

In our example, the assumed longitude lon is +160°, and we get the values noted in the table.

Example 1: Jupiter

Assume we are using a CASIO calculator.

Enter the *dec*, *lat*, and *lha* (note they are sorted alphabetically) for Jupiter:

1) 17 ⟦° ' "⟧ 20.6 ⟦° ' "⟧ SHIFT STO A   The ⟦° ' "⟧ key is used to enter degrees/minutes
2) -(40) SHIFT STO B   Note that south is treated as negative. It is a habit that I use parentheses after the <u>unary minus sign</u>. Usually we enter degrees and minutes.
3) 355 ⟦° ' "⟧ 3.4 ⟦° ' "⟧ SHIFT STO C
4) pol**(** tan ALPHA A cos ALPHA B - sin ALPHA B cos ALPHA C **,** -sin ALPHA C**)=**

Note that we reference the variables by first pressing the ALPHA key, and then the letter. Remember to enter a comma between the first and second entries, and to use a unary minus operator on the second entry.

After pressing the = key, the calculator displays:

0.883839348 which is the *r* value.

Now execute:

5) SHIFT cos **(** ANS x cos ALPHA A **) =**   displays 32.47176309
6) press ⟦° ' "⟧ to convert to degrees, minutes   displays 32 28 18.35 = _Hc 32° 28.3'_
7) ALPHA F   displays 5.59497389 = _Zn 5.6°_

Example 2: Rigil Kentaurus

For the second body, star Rigil Kentaurus, we enter its declination and *lha*; the assumed latitude remains the same:

1) -(60 °'" 47.8 °'")SHIFT STO A
2) 271 °'" 28 °'" SHIFT STO C

Next, scroll up through the formulas using the "replay button" until the pol formula shows in the display, and press = to execute it. The calculator will recompute *r* and θ using the new declination (in A) and *lha* (in C) values.

3) on the display and in variable E: 1.683079642; and in var F: 143.5618779
4) SHIFT cos (ANS x cos A)= 34.79569045 °'" = 34° 47' 44.49' = *Hc* 34° 47.7'

Examining var F (use ALPHA F), we find *Zn*= 143.6°. Note: to perform step 4), it is convenient to scroll up to find the inverse cosine statement, rather than keying it in again. This is possible because we use variable names rather than numbers (which change with each reduction.)

In summary, the conveniences of this method are:

* Only one complicated formula to remember and key in.
* The altitude *Hc* and azimuth *Zn* are computed at the same time.
* There are no complicated "sign rules" to remember to find the azimuth.

One caveat: ambiguity of the sign of *Hc* for a body near the horizon. The sign can resolved by evaluating the sign of the expression:

tanA sinB + cosB cosC

A negative value means *Hc* is also negative.

Alternatively, use the well-known formula

$$\sin Hc = \sin A \sin B + \cos A \cos B \cos C$$

where A=*dec*, B=assumed latitude, and C= *lha*.