## E. BACKGROUND

**1. Accuracy of Tables.** The tabular values as given in these tables have maximum and probable (50%) errors of ±0.05′ and ±0.025′ in altitude and ±0.05° and ±0.025° in azimuth angle.

The maximum error arising from the use of the Interpolation Table for the first-difference correction is ±0.14′, with a probable error of ±0.03′, when used for the interpolation of altitude for declination.

The maximum error arising from the use of the correction for second differences obtained from the Interpolation Table is ±0.12′ with a probable error of ± 0.03′.

When second differences are completely negligible, the maximum error of an interpolated altitude is ±0.19′ with a probable error of ±0.04′; when the second differences are not negligible and the second- difference correction is included in the interpolation, the maximum error of the calculated altitude will be ±0.31′ with a probable error of ±0.05′.

The largest value of the double-second difference when the value of d is not printed in italics is 3.9′, and if the correction for this value is neglected, an error of up to –0.24′ may be introduced into the computed altitude. But such an error is only possible when the altitude is greater than 60° and when the value of Dec. Inc. is close to 30′. The neglect of the second-difference correction when d is not printed in italics will rarely introduce an error as large as –0.2′.

For altitudes less than 86°, i.e., for zenith distances greater than 4°, interpolation of the tabular altitude for declination, utilizing both first and second differences and the Interpolation Table, may be made to within about 0.2′; linear interpolation for azimuth angle can be made to about 0.2°. Closer to the zenith, not only do second differences exceed the limits of the tables but higher differences are also significant.

When the body is in the zenith, its azimuth is indeterminate, that is when LHA is 0° and when latitude and declination are equal and have the Same Name. In these cases Z is tabulated as 90° or as one-half the preceding value. There are 91 of these cases.

When latitude is 90° and declination is 90°, the altitude is 90° for all hour angles. Here the value of Z tabulated is one-half the preceding value. There are 182 of these cases, two of which are included in the previous set. In the above cases the tabulated azimuth angles are the mathematical limits of the azimuth angle when the limit is approached in a specified direction.

In the special cases when the latitude is 90°, i.e., at the poles, all directions from the North Pole are south and from the South Pole are north; the criterion adopted in these cases has been to tabulate the azimuth as equal to 180° minus LHA, i.e., the directions are tabulated as the angular directions from the lower branch of the Greenwich Meridian. There are 90 × 180 of these cases not included in the previous sets.

**2. Computation formulas.** For latitude (L), declination (d) and local hour angle (LHA), the altitude (Hc) and the azimuth angle (Z) were calculated from the following formulas:

$$\sin Hc = \sin L \sin d + \cos L \cos d \cos LHA$$

$$\tan Z = \frac{\cos d \sin LHA}{\cos L \sin d - \sin L \cos d \cos LHA}$$

All values of altitude within 1°30′ of the zenith were recalculated using a more appropriate formula because determination of these high altitudes from their sines with only nine figures could introduce errors of the order of 0.0005′, which would sometimes affect the rounding off of the altitude to 0.1′. The formula used is equivalent to:

$$\sin^2 \tfrac{1}{2} z = \cos^2 \tfrac{1}{2} LHA \sin^2 \tfrac{1}{2} (L-d) + \sin^2 \tfrac{1}{2} LHA \cos^2 \tfrac{1}{2} (L+d),$$ where z is the zenith distance.