S-Diagrams for Solving Problems in Astronomical Navigation

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1. Introduction. The purpose of the *S*-diagrams is to give a rapid and convenient solution of the chief problems in astronomical navigation; they are especially simple for determination of altitude and azimuth. The solution is obtained without logarithms and by referring to no more than two pages of the diagrams. The diagrams are less expensive and more accurate than the *Astronomisches Rechengerät* (ARG),[1] and are, without any doubt, less difficult to use than the *Astronomische Rechenatlas*. The method, which was recently mentioned in a paper by H. C. Freiesleben,[2] may, therefore, be discussed here in some detail.

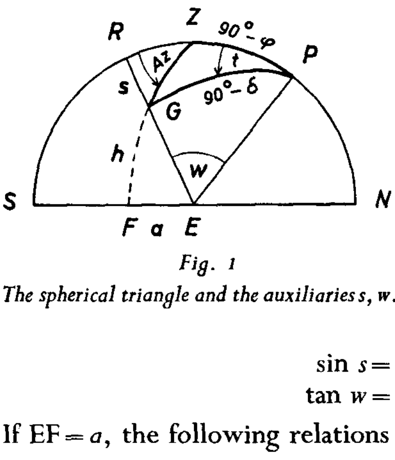

Fig. 1

The spherical triangle and the auxiliaries s, w.

2. Method of solution. The method of solution has already been published by K. Schütte;[3] it will thus suffice to summarize it. Fig. 1 represents the navigational triangle PZG and indicates the construction and notation used. There are two relations for the auxiliary quantities *s*, *w*, where *s* is the perpendicular GR from the star G to the meridian and *w* is the angle PER.

$$\left.\begin{aligned}\sin s &= \sin t \cos \delta\\ \tan w &= \cos t \cot \delta\end{aligned}\right\} \qquad (1)$$

If $EF = a$, the following relations hold for h and a:

$$\left.\begin{aligned}\sin h &= \sin (\varphi + w) \cos s\\ \tan a &= \cos (\varphi + w) \cot s\end{aligned}\right\} \qquad (2)$$

where the azimuth, Az, is given by

$$Az = 90° - a$$

The formulae (1) and (2) are of the same kind, so it must be possible to find the values *s*, *w* as well as *h*, *a* from a single table or diagram, using the appropriate arguments.

3. The performance and use of the diagrams. At the end of the war, the *S*-diagrams were designed, at the author's suggestion, at the Deutsche Seewarte, Hamburg. They consist of eighty-one sheets of curves constructed according to equations (1) and (2).

Each of these diagrams, originally 60 × 60 cm. in size, contains a double network. The first is a fine grid consisting of equally-spaced horizontal and vertical lines at a distance of 10′ with a division on the left, right and lower margins from $2'$ to $2'$ and with a division from 10^{s} to 10^{s} on the upper one; thus there is also a scale of time available. The second consists of the curves s, w = const. and h, a = const.; these curves are in general drawn for each $10'$.

From the original diagrams reduced reproductions have been made, the total size of which is only 20 x 20 cm. The pages are arranged in a

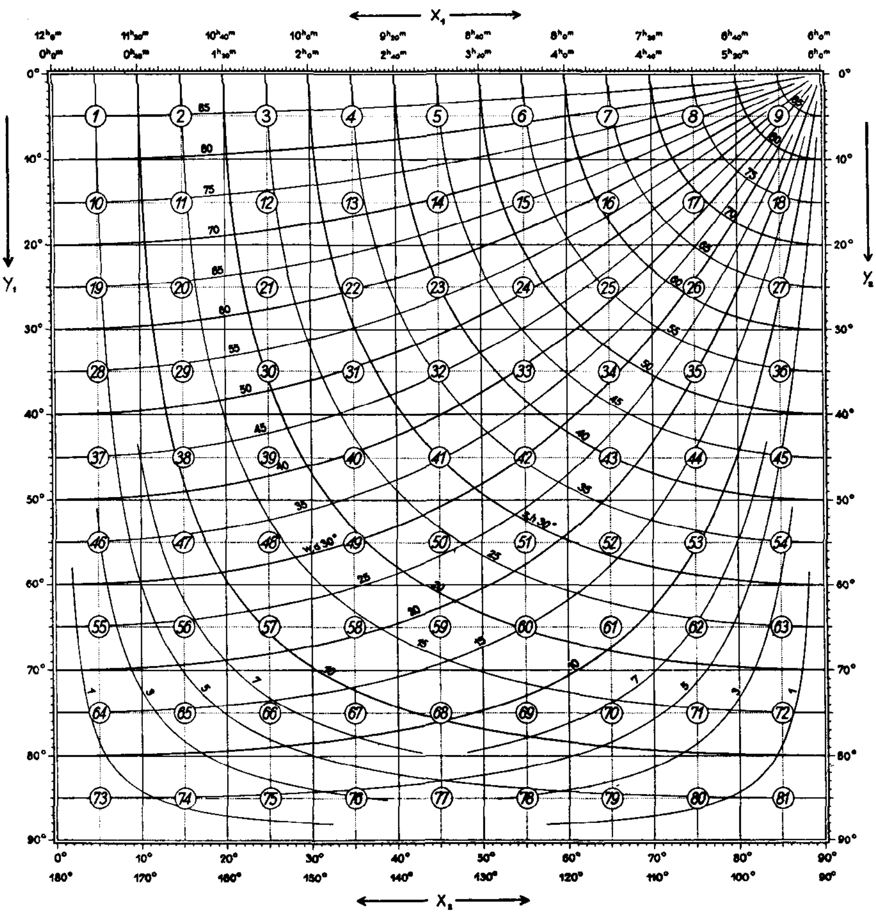

Fig. 2. Plan and mapping of the S-diagrams.

handy volume, in which nine horizontal pages follow one another and form a section.[4] An additional page provides a key and facilitates the rapid finding of the two pages which in each case are necessary for the solution of a problem (see Fig. 2).

For practical use the following should be borne in mind:

(1) The s-curves are similar to circles (the centre of these lies on page 9 on the upper right edge) and nearly equidistant; their inclination is usually from upper left to lower right.

(2) The w-curves are convergent and coincide at the above-mentioned point on the upper edge of page 9.

(3) The inscription of all curves increases from below.

(4) In the spherical triangle the s-curves are always sides, the w-curves always angles.

(5) w and a are always to be given a sign according to a simple rule given on each page of the diagrams (see also (4)).

(6) Instead of the sides in the main triangle the complements are always used. (This is not so for the auxiliary s which does not occur in the main triangle.)

In the following examples the horizontal axes are always marked by x, and the vertical ones by y. Thus in the first part x_1, y_1 is used, in the second x_2, y_2.

4. DETERMINATION OF ALTITUDE AND AZIMUTH. For the most important problem of astronomical navigation, the determination of altitude and azimuth, there are given two sides and the included angle, viz. the geographical latitude φ, the hour angle t, and the declination δ of the body. The solution can be found as follows:

After consulting the index page, the page is selected which contains the given declination on the vertical scale and the given hour angle on the horizontal scale. By the rectangular coordinates:

$$y_1 = \delta \quad x_1 = t$$

a point is fixed, the position of which between both neighbouring bands of curves can be estimated. Thus are found the values of s and w. w must always be supplied with a sign according to the simple rules which hold also for all other problems:

$$x < 6^h, \text{ sign of } w \text{ (or } a\text{) like } y$$
$$x > 6^h, \text{ sign of } w \text{ (or } a\text{) opposite to } y.$$

Now $\varphi + w$ is formed and the diagram opened at the page containing the arguments:

$$y_2 = s \quad x_2 = \varphi + w$$

By the rectangular coordinates x_2, y_2 a point on this second page of diagrams is fixed, the position of which can be estimated between the neighbouring curves. According to equations (2) the values so read will be h and a. The sign of a is fixed by means of the same rule as for w.

Then we have

$$h = \text{altitude of the body.}$$
$$\text{Az.} = 90° - a = \text{azimuth of the body.}$$

5. AN EXAMPLE OF THE DETERMINATION OF ALTITUDE AND AZIMUTH, WITH REFERENCE TO SIMILAR PROBLEMS (Case I).

Given:

$$\delta = y_1 = +42° \ 04'$$
$$t = x_1 = 4^h \ 10^m \ 00^s$$
$$\varphi = +38° \ 10'$$

The first entry is made on diagram 43; Fig. 3a shows an enlarged part of the page. By the rectangular coordinates x_1, y_1 we find point I and at the same time the corresponding values of the curves:

$$s = 41^\circ\ 11' \quad w = +27^\circ\ 06'$$

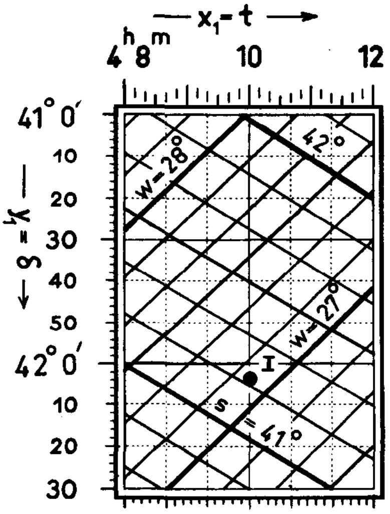

Fig. 3a

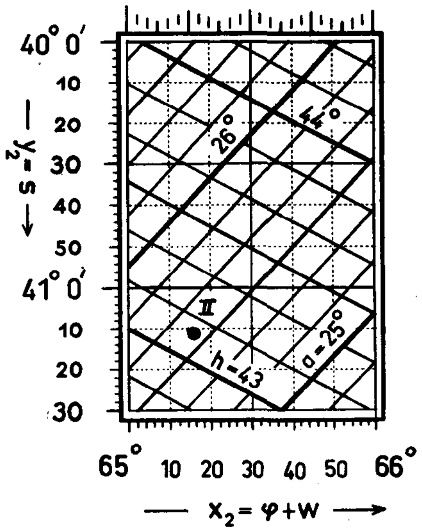

Fig. 3b

Example for determination of altitude and azimuth (enlarged parts of S-diagrams, page 43).

The second entry can, by the way, be made on the same diagram; the corresponding part is shown in Fig. 3b. By the coordinates:

$$y_2 = s = 41^\circ\ 11'$$
$$x_2 = \varphi + w = +65^\circ\ 16'$$

we find point II and, with it the corresponding values of the curves:

$$h = +43^\circ\ 07'.5$$
$$a = +25^\circ\ 34'$$

The azimuth is

$$\text{Az.} = 64^\circ\ 26'$$

Of course, there are other problems of the same type in which two sides and the included angle are given, such as great-circle sailing and radio direction finding, which can be solved in the same manner.

When solving Case I we derive those elements that lie opposite to the side and the angle used to determine the auxiliaries s, w. Accordingly we must chose the appropriate page to begin the solution; whenever we wish to derive altitude and azimuth we must begin with the side $90^\circ - \delta$, i.e. declination. If, however, we start with the side $90^\circ - \varphi$, i.e. latitude, we still derive the altitude, but instead of the azimuth we get the parallactic angle. In problems of this kind it is essential to begin with the side opposite to the desired angle.

6. The solution of other problems. By means of the S-diagrams four of the six general problems involving solution of spherical triangles can be solved; they fail only in the two cases in which the given parts are 3 sides or 3 angles. So the following four cases may be solved:

I. Given two sides and the included angle.
II. Given two angles and a side opposite one of them.
III. Given two sides and an angle opposite one of them.
IV. Given two angles and the included side.

Case IV can only be solved by using the polar triangle in which two sides and the included angle are given, so that this problem is reduced to Case I.

When dealing with the different problems it simplifies solutions if, as a matter of principle, the determination of the auxiliaries s, w is always carried out in exactly the same way. For all problems we can use the following schedule:

First entry: determination of the auxiliaries s, w *for all problems.*

entry $\begin{cases} y_1 \text{ (vertical)} = \text{complement of a given side} \\ x_1 \text{ (horizontal)} = \text{included or adjacent angle.} \end{cases}$

reading: curves s, w.

The rule for the sign of w and a is given above.

The second entry is analogous when solving Case I. With all other cases the *second* entry also begins with the entry vertically

$$y_2 = s$$

From now the cases vary; and it will be best to follow the schedule below for the second entry:

Schedule for the second entry for the Cases I–III

Case	I	II	III
Entry	*vertical* $y_2 = s$ *horizontal* $x_2 = k_2 + w$ (k_2 = complement of the second given side)	like I a-curve = complement of the angle opposite to the given side	like I h-curve = complement of the second given side
Reading	h-curve = complement of the side opposite to the given angle	like I	*horizontal* $x_2 = k_3 + w$ (k_3 = complement of the third side)
	a-curve = complement of the angle opposite to the side used for first entry	*horizontal* $x_2 = k_3 + w$ (k_3 = complement of the third side)	like I

If we denote the values from the second reading of the curves as (h) and (a), the rule is:

Elements to be found: side $= 90° - (h)$, angle $= 90° - (a)$. (h) must be taken negatively if the sign of x_2 is opposite to y_1; but this is only possible if the required side is greater than 90°. The missing sixth element is always an angle, which can most easily be found by using Case I.

It must be mentioned that the w-values seem to become inaccurate when the w-curves are convergent, especially on page 9. The inaccuracy, however, can easily be avoided, if we exchange w and δ according to formula (1); all that is necessary is to enter the diagrams with the same $x = t$, and find that w-curve, for which the value is equal to δ. Then the corresponding y-coordinate will give the exact value of w.

Example: for $t = 5^h\ 50^m\ 00^s$ and $y = +2°\ 10'$ we can find on page 9 a value for w of about $+49°$, which here can scarcely be made out more exactly. But with $x = t = 5^h\ 50^m\ 00^s$ we find on page 45 the coincidence with the curve $(w) = \delta = 2°\ 10'$ and, in the right-hand margin $y = 49°\ 06'$; in this case the exact value is $w = +49°\ 06'$.

7. EXAMPLE FOR GREAT-CIRCLE SAILING WITH A DISTANCE GREATER THAN 90°. As an example for great-circle sailing the distance D, Valparaiso–Nagasaki, and course K_2 from Valparaiso are to be determined. We have:

Nagasaki	*Valparaiso*
$\varphi_1 = +32°\ 45'$	$\varphi_2 = -33°\ 02'$
$\lambda_1 = 129°\ 52'$ east	$\lambda_2 = 71°\ 38'$ west

The difference of longitudes, therefore, will be $\Delta\lambda = 158°\ 30' = 10^h\ 34^m$. On page 30 we can solve the first part (see Fig. 4a, which is a part of the diagram).

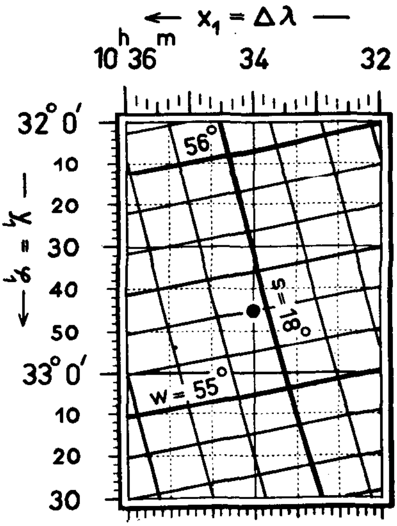

Fig. 4a

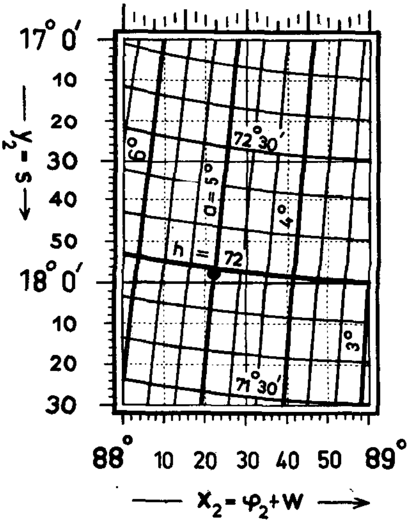

Fig. 4b

Great-circle sailing with a distance greater than 90° (enlarged parts of S-diagrams, pages 30, 18).

Entry:	*Reading*:
$y_1 = \varphi_1 = +32^\circ\ 45'$	$s = 17^\circ\ 57'$
$x_1 = \Delta\lambda = 10^h\ 34^m$	$w = -55^\circ\ 20'$

The second entry is made on page 18 (see Fig. 4b, which is a part of the diagram).

Entry:	*Reading*:
$y_2 = s = 17^\circ\ 57'$	$(h) = -71^\circ\ 59'$
$x_2 = \varphi_2 + w = -88^\circ\ 22'$	$(a) = +5^\circ\ 00'$

Note: the sign of x_2 must be opposite to y_1; so (h) must be negative and the distance is greater than 90°. Thus, for the distance D and the course K_2 from Valparaiso we find:

$$D = 90^\circ - (h) = 161^\circ\ 59' = 9719 \text{ miles}$$
$$K_2 = 85^\circ\ 00'$$

8. Example for Case III. A ship is sailing on course $K_1 = 22^\circ\ 06'$ from the point $A_1(\varphi_1 = +16^\circ\ 24'.6)$. What will be the distance D and the course K_2 when it reaches latitude $\varphi_2 = +50^\circ\ 00'$ sailing on a great-circle course?

In this example there are given two sides and an angle opposite one of them and so Case III must be solved.

The first entry is possible on page 12 (see Fig. 5a, which is a part of the diagram):

Entry:	*Reading*:
$y_1 = \varphi_1 = +16^\circ\ 24'.6$	$s = 21^\circ\ 09'$
$x_1 = K_1 = 22^\circ\ 06'$	$w = +72^\circ\ 21'.5$

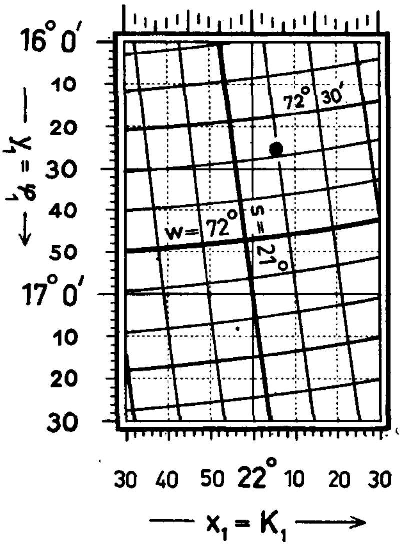

Fig. 5a

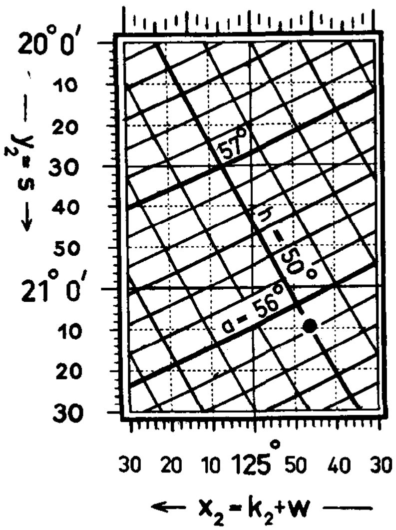

Fig. 5b

Example for solving Case III (enlarged parts of S-diagrams, pages 12, 24).

For the second part of the solution we open page 24 (see Fig. 5b, which is a part of the diagram) and find:

Entry:	*Reading*:
$y_2 = s = 21° \ 9'$	$x_2 = k_2 + w = 124° \ 47'$
curve $(h) = \varphi_2 = +50° \ 0'$	$(a) = -55° \ 51'$

It is necessary to take $x_2 > 6^h$ in order to get a positive distance D. Then (a) will also be negative and we find:

$$D = x_2 - w = 52° \ 25'.3 = 3145 \text{ miles}$$
$$K_2 = 90° - (a) = 145° \ 51'$$

REFERENCES

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