Steady Observer position from *H culm* and *UT culm*

Iterative Solution

with h_o : geocentric height, D : Body Declination, T : Body Local Hour Angle and ϕ : Observer's Latitude

1 - Height Formula

(1) $\sin h = \sin \phi \sin D + \cos \phi \cos D \cos T$

2 - Differential and Derivative of Formula (1) ***with $d\varphi = 0$***

$\cos h \, dh = \sin \phi \cos D \, dD - \cos \phi \sin D \, dD \cos T - \cos \phi \cos D \sin T$

$dD/dUT \,(\sin \phi \cos D - \cos \phi \sin D \cos T) = dT/dUT \cos \phi \cos D \sin T$

(2) $\sin T = (dD/dUT \,/\, dT/dUT) (tg\, \phi - tg\, D \cos T)$

3 - Iteration from the following starting elements :

$T_{oo} = 0°$, $\phi_o = D +/- (90° - h)$ *+ if Body North, - if Body South*

3.1 - Iteration on T

Everything else being known, compute *$\sin T_{o1} = (dD/dUT \,/\, dT/dUT) (tg\, \varphi_o - tg\, D \cos T_{oo})$*, and iterate as necessary :
$\sin T_{o2} = (dD/dUT \,/\, dT/dUT) (tg\, \varphi_o - tg\, D \cos T_{o1})$
$\sin T_{o3} = (dD/dUT \,/\, dT/dUT) (tg\, \varphi_o - tg\, D \cos T_{o2})$ until no significant change in T, hence get *T_{1o}*

3.2. - Update on ϕ

Then from (1) compute :

$\sin h_1 = \sin \varphi \sin D + \cos \varphi \cos D \cos T_{1o}$, from which we compute *h_1*

$\phi_1 = \phi_o +/- (h_1 - h_o)$ *+ if Body North, - if Body South*

3.3 - Back to 3.1

Then restart iteration with :

T_{1o} and ϕ_1 , to get T_{2o} and ϕ_2 , then as necessary with :

T_{2o} and ϕ_2 , to get T_{3o} and ϕ_3 , then again until no significant change in T and ϕ .

The Observer's Coordinates at Time of Culmination UT culm are then:

Latitude = ϕ , and Longitude = Body LHA - T

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Numerical Example

From https://navlist.net/Weird-moonrise-direction-Ritchie-nov-2023-g55024 :

23Nov2023, TT-UT=+69.2s, HoE=15’, Temp=50° F, QFE=29.83”, UT culm=06:19:44 , Moon UL=31°09.2’

0 - Moon data

From an Ephemeris - here own Software accurate to +/- 4” - get for the Moon:
GHA=149°16.54’, Dec=+1°39.17’ , Par.=59.20’, dD/dUT = 0.279°/h, dRA/dUT=0.520°/h, dT/dUT=14.621°/h

From the ***Éphémérides Nautiques (2D) Tables***, let’s perform a *preliminary estimate* of h_0 :
Dip = -3.7’ , LL Correction (-Refraction, +Parallax +SD) = +65.2’ , UL D Correction = -32.2’
Total Correction = +33.0’ , hence h_0 *provisional value* = 31°38.5’ from which we get ϕ_0 = 60°00.67’

1.1 - First iteration on T

1.1 - Provisional computation carried out with h_0 *provisional value* = 31°38.5’

With T_{00} = 0° , ϕ_0 = 60°00.67’ , from **(2)** get T_{01} = 1.8759° , T_{02} = 1.8759°, hence *provisional* T_{10}= 1.8759°

1.1.2 - Refinement of h_0 value.

From provisional T_{10} and ϕ_0 , get Observer’s Approximate Longitude at GHA-T_{10}= 147°23.99’
From Observer’s approximate coordinates (ϕ_0 = 60°00.67’, Longitude = 147°23.99’) get a better refined 3D correction onto Moon UL height, and get : **h_0 = 31°38.18’**, to be kept from now on.

1.1 bis - From refined h_0, let’s start 1.1 all over again.

This time, from refined h_0 and again assuming T_{00} = 0° , then ϕ_0 = 60°00.99’.

With T_{00} = 0° , ϕ_0 = 60°00.99’ , from **(2)** get T_{01} = 1.8763° , T_{02} = 1.8763°, hence T_{10}= 1.8763°

1.2 - First Update on ϕ

With T_{10}= 1.8763°, from **(1)** get : h_1 = 31°37.10’ . With $h_1 - h_0$ = - 1.08’, we need to decrease ϕ in order to bring h closer to h_0

Hence $\phi_1 = \phi_0 + (h_1 - h_0)$ = 59°59.91’

2.1 - Second iteration on T

With T_{10}= 1.8763°, ϕ_1 = 59°59.91’, from **(2)** get T_{11} = 1.8749° , T_{12} = 1.8749°, hence T_{20}= 1.8749°

1.2 - Second Update on ϕ

With T_{20}= 1.8749°, from **(1)** get : h_2 = 31°38.18’ . **With $h_1 - h_0$ = - 0.00’ , our iteration is complete.**

Position at UT culm 06:19:44 : N 59°59.9$_1$’ / W 147°24.0$_4$’ *(vs. Author’s at N60°00.0’/W147°24.0’)*

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