Flat Bygrave and Square Compass

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Background

Calculations by slide rule fundamentally involve transferring lengths from one logarithmic scale to another. A Bygrave slide rule consists of two concentric cylindrical scales which are popularly described as being an inner (log-)cotangent scale and an outer (log-)cosine scale. Although the nomenclature is self-consistent log cot(θ) and log cos(θ) are both decreasing functions of θ and the latter is always negative. It is therefore more natural to describe the scales as being logtangent and log-secant scales respectively. Here the standard nomenclature for the scales will be retained but the formulas evaluated by the Bygrave will be written in terms of tan(θ) and sec(θ) which simplifies the interpretation of the operations being performed.

In Gary Lapook's original flat Bygrave https://web.archive.org/web/20220911062426/https://sites.google.com/site/fredienoonan/other-flight-navigation-information/modern-bygrave-slide-rule transferring lengths between scales was achieved by having the cosine scale printed on transparent material that could be overlaid on the cotangent scale. In order to ensure that any required angle can be accessed without running off the edge of the sheet all values on the scale appear twice. That is to say, an angle marker appearing near the middle of some row in the scale will be repeated on the left hand side of the scale in the next row up.

In 2014 https://navlist.net/Flat-Bygrave-alternative-configuration-Stuart-mar-2014-g27166 I pointed out that this repetition could be avoided if two 0° markers were provided on the cosine scale. One of these sits at the left hand end of the bottom row and the other is at the right hand end of the next, unprinted, row down. This means that the scale lengths can be doubled for the same area of paper compared to the original design. The procedures for using them is analogous to those of a standard slide rule in which, depending on where the numbers fall, either the left or right hand end of the sliding scale is aligned with a number on the fixed scale.

Postscript code for producing Bygrave scales with full explanations in various configurations can be found here https://navlist.net/Postscript-code-for-making-Bygrave-Scales-Stuartjan-2015-g29918. Additional information concerning configuring Ghostscript can be found here https://navlist.net/flat-Bygrave-Stuart-feb-2017-g38421.

Before his passing in 2015 Hanno Ix experimented with this design and used a beam compass to transfer lengths from one scale to the other https://navlist.net/Flat-Bygrave-alternativeconfiguration-Stuart-jul-2014-g28182. These scales were designed to be printed on A3 paper. When using a beam compass it is necessary to keep track of the number of horizontal rows the compass pointers span and preserve this when transferring lengths between scales. In order to assist in this he had me number the rows in both scales. He also had me label the two cosine scale pointers "A" and "B" to aid in explaining the steps required to perform calculations https://navlist.net/Borrowed-Bygrave-Stuart-jan-2016-g34208. Although Hanno seemed to be satisfied with this approach I always thought it was inconvenient and that there must be an easier way keep track of this information without having to perform any mental subtractions.

I have built a prototype "square compass" that satisfies this requirements. It consists of pointers that slide on 2 arms at right angles to each other. One of the arms is laid parallel to the horizontal scales when setting the pointers and the orientation is maintained when moving to the other scale. The compass arms however do not need to precisely aligned with the scales. They only need to be placed accurately enough so that there is no doubt about how many rows are spanned in the vertical direction. It is the distance between the tips of the primary importance here. The pointers should however always be placed on the same part of the scale - either the thicker base line or thinner index line that runs parallel.

The formulas that the original cylindrical Bygrave evaluates are derived from Napier's rules for spherical right triangles. Because of its design only those rules that contain combinations of tangent and cosine functions can be used. A flat Bygrave and compass are free from this constraint and in principle can be used to evaluate Napier's rules that contain only sines or cosines. For example the azimuth, Z, at which an object at declination, δ, rises for an observer at latitude, L, is given by

$\sec Z = \dfrac{\sec \delta'}{\sec L}$ or $\operatorname{logsec} Z = \operatorname{logsec} \delta' - \operatorname{logsec} L$, where $\delta' = 90° - \delta$, and can be computed using the cosine scale alone.

Example

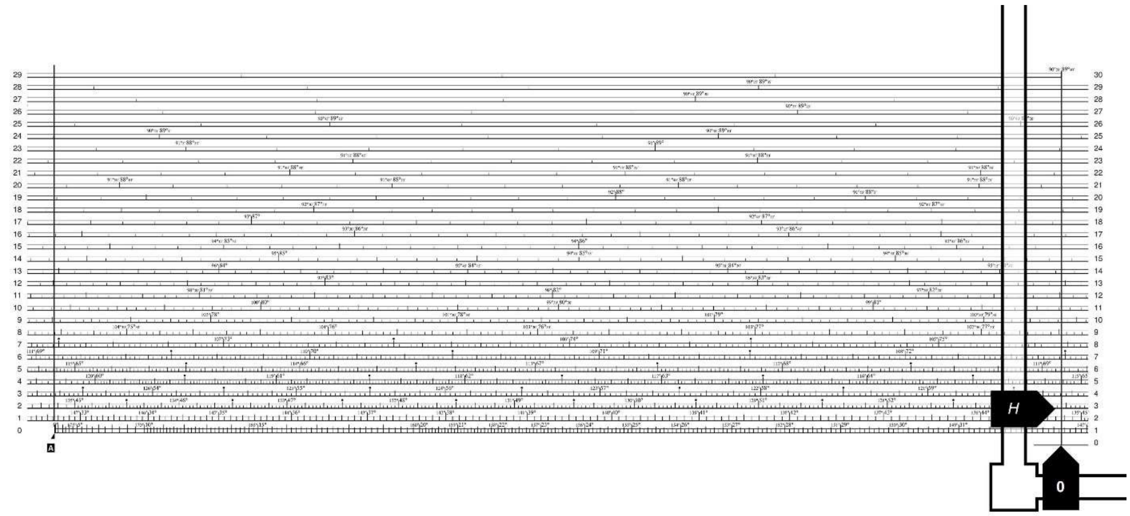
To demonstrate the use of the square compass in conjunction with the flat Bygrave the calculation given as an example in the A.M.L. Position Line Slide instructions (attached) will be replicated.

Latitude, L:	36° 51.6′ N.
Hour Angle, H:	53° 15.3′ E.
Declination, d:	10° 27′ S.
Co-latitude, c:	53° 8.4′

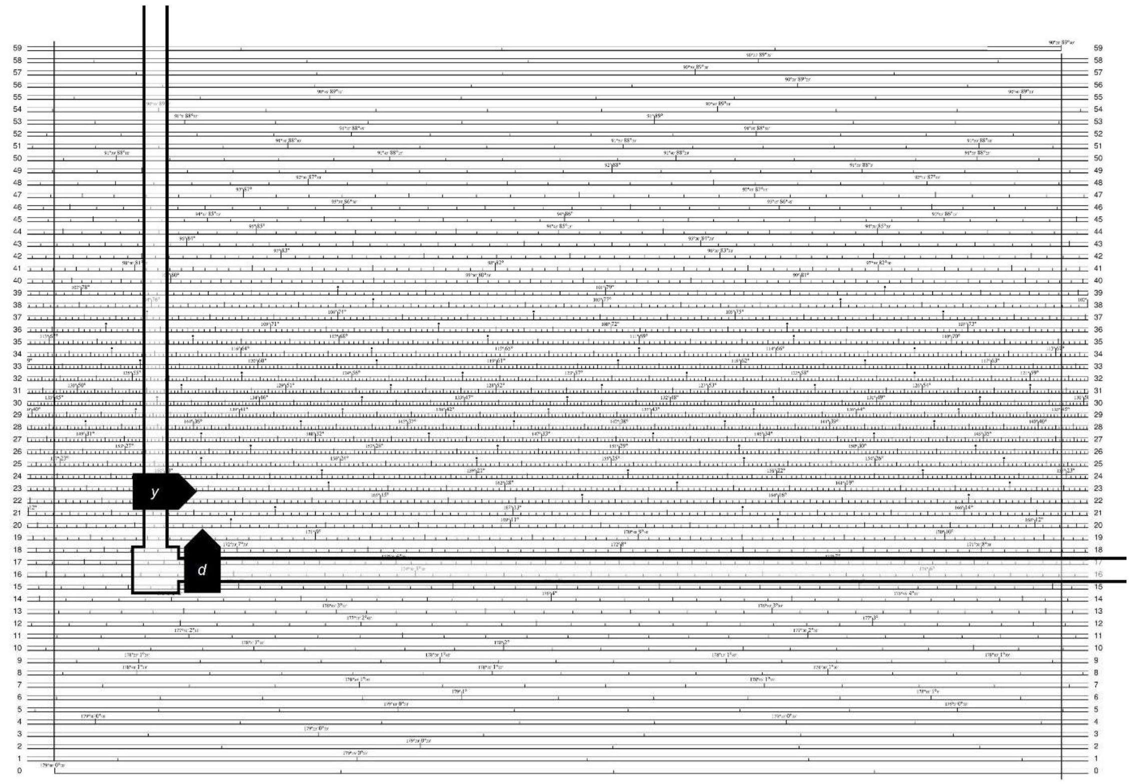
Step 1

Compute y; $\tan y = \tan d \sec H$

Place the horizontal arm pointer tip on zero position **B** of cosine scale and the vertical arm pointer on the local hour angle, H, on the cosine scale. Zero position **B** is chosen in favour of zero position **A** as it remains within the scale boundaries when the setting is transferred to the cotangent scale.



Place the horizontal arm pointer on the declination, d, on the cotangent scale and read $y = 17° \ 8.0'$ from the vertical arm pointer. There are two supplementary labels on the scale markings. If the local hour angle, $H < 90°$ in magnitude then $y < 90°$ and its value should be read from the right hand label. If $H > 90°$ then $y > 90°$ and its value should be read from the supplementary angle on the left.



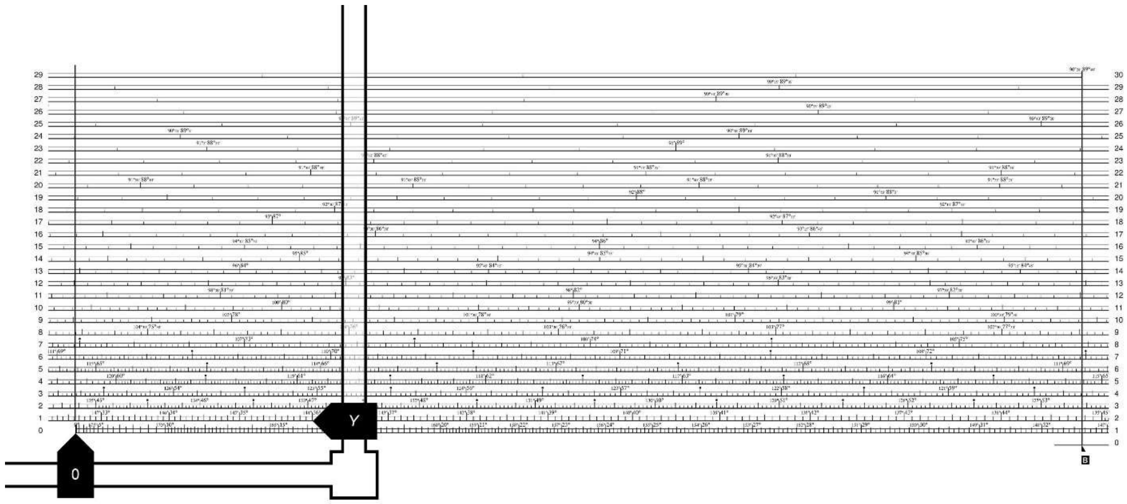
Step 2

Form Y from y and co-latitude, c, adding y to c if the declination, d, and latitude, L, are of the same name and subtracting if they are of opposite names; $Y = | y - c | = 34°0.4'$.

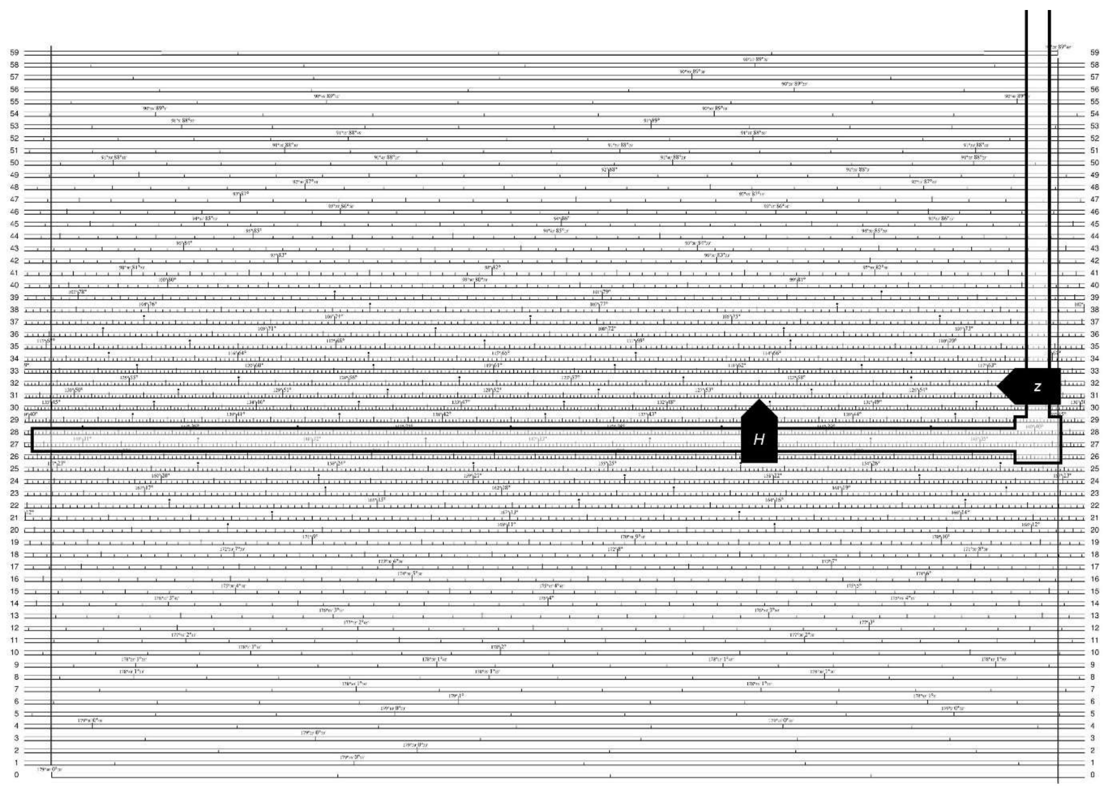
Compute z; $\tan z = \tan H \sec Y$

This is an intermediate quantity that is the length of the dashed side of the spherical triangle shown on page 2 of the Bygrave instructions.

Place the horizontal arm pointer tip on zero position **A** of cosine scale and the vertical arm pointer on Y on the cosine scale. Zero position **A** is chosen in favour of zero position **B** as it remains within the scale boundaries when the setting is transferred to the cotangent scale.



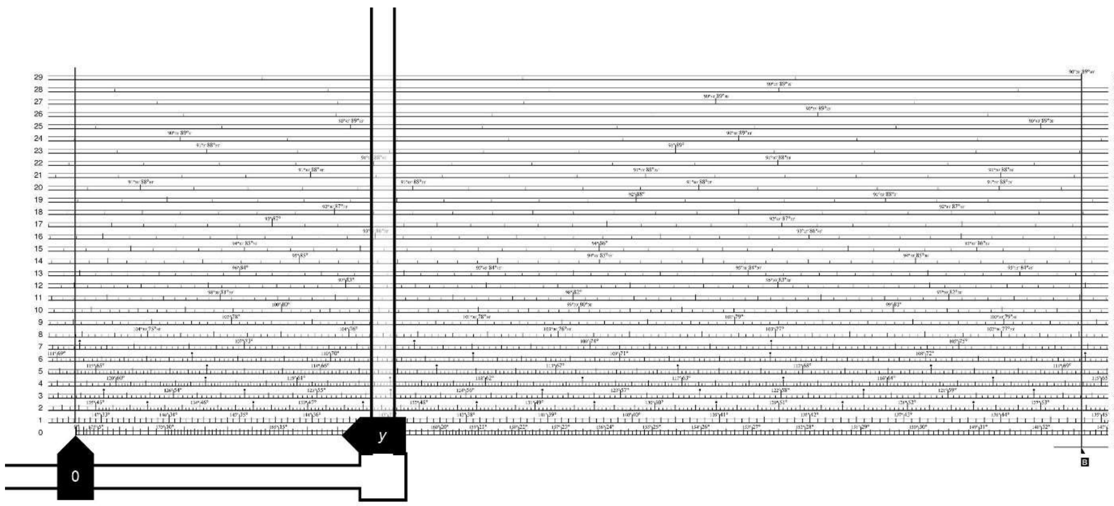
Place the horizontal arm pointer on the local hour angle, *H*, on the cotangent scale and read $z = 58° \; 52.2'$ from the vertical arm pointer.



Step 3

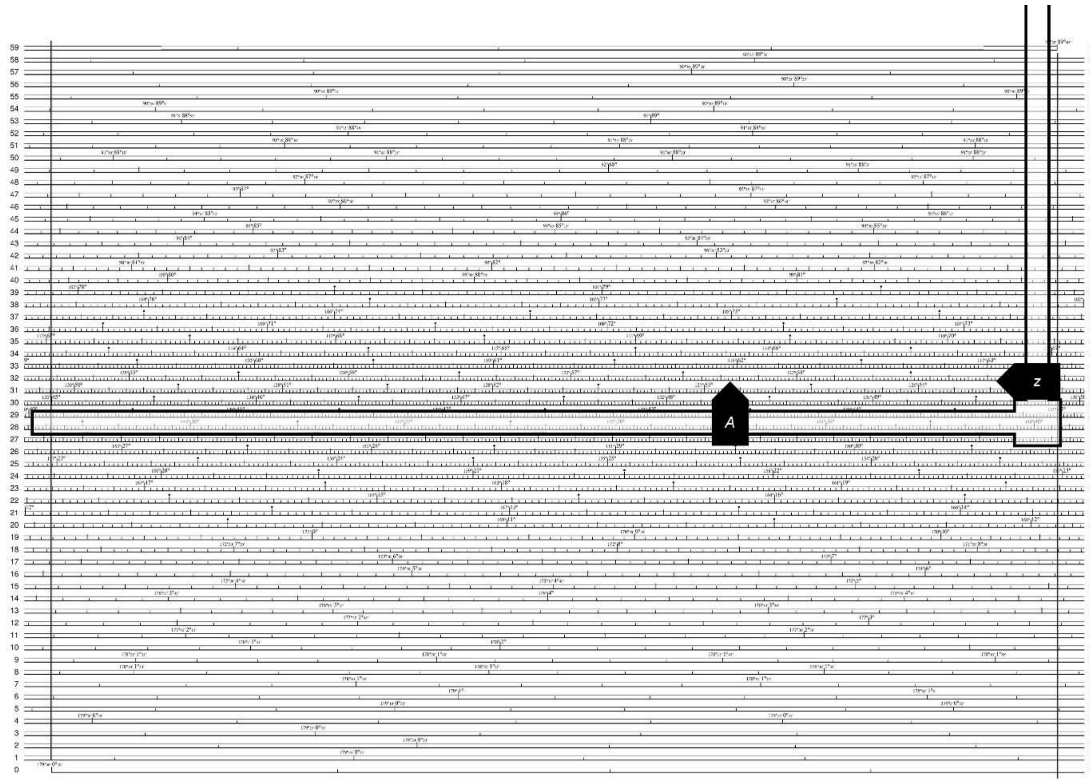
Compute azimuth angle A; $\tan A = \tan z / \sec y$

Place the horizontal arm pointer tip on zero position **A** of cosine scale and the vertical arm pointer on y on the cosine scale. Zero position **A** is chosen in favour of zero position **B** as it remains within the scale boundaries when the setting is transferred to the cotangent scale.



Place the vertical arm pointer on z on the cotangent scale and read the azimuth angle, $A = 57° 42.4'$, from the horizontal pointer. Once again if $Y < 90°$ in magnitude then $A < 90°$ and its value should be read from the right hand label. If $Y > 90°$ then $A > 90°$ and its value should be read from the supplementary angle on the left. The azimuth, Zn, is obtained from A by reading it from the opposite pole as the name of the latitude, south in this case, and to the east or west according to whether the local hour angle, H, is east or west.

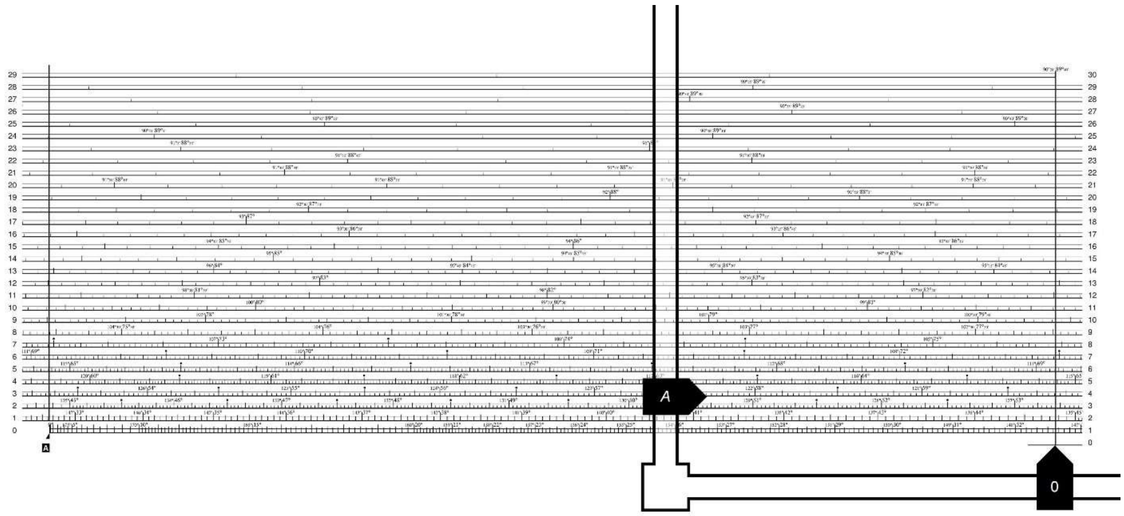
Hence the azimuth is S 57° 42.4′ E or Zn =122° 17.6′.



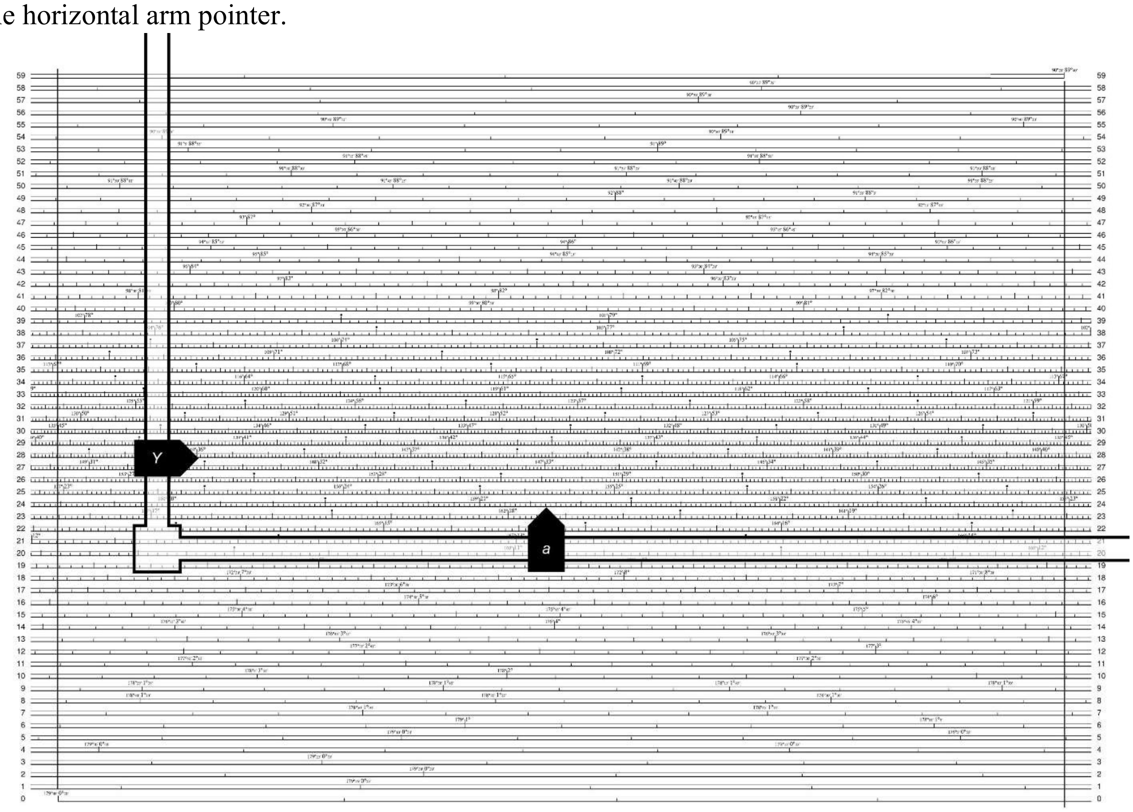
Step 4

Compute altitude a; $\tan a = \tan Y / \sec A$

Place the horizontal arm pointer tip on zero position **B** of cosine scale and the vertical arm pointer on the azimuth angle, A, on the cosine scale. Zero position **B** is chosen in favour of zero position **A** as it remains within the scale boundaries when the setting is transferred to the cotangent scale.



Place the vertical arm pointer on *Y* on the cotangent scale and read the altitude $a = 21° \; 13.1'$ from the horizontal arm pointer.

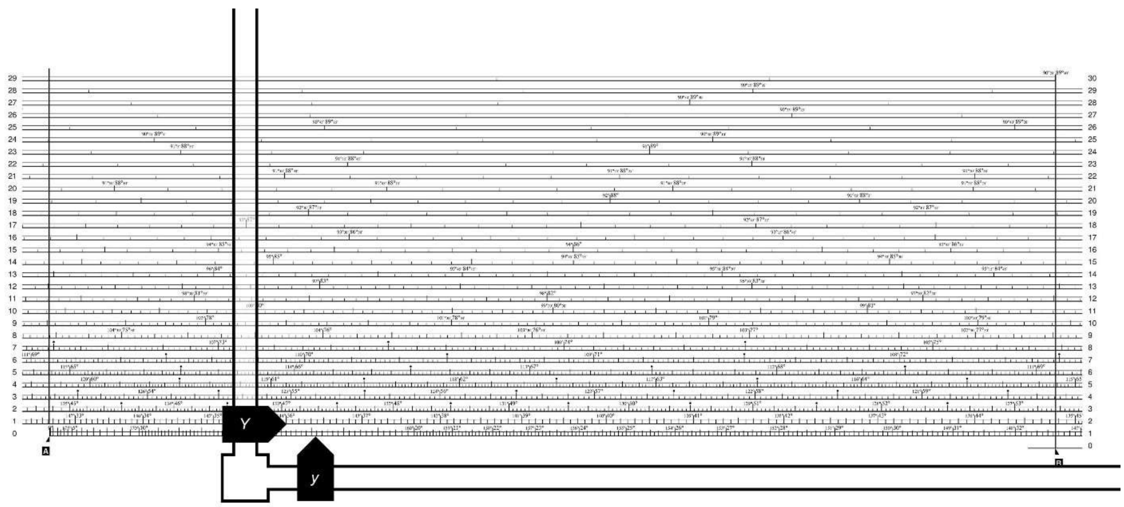


A Short Cut

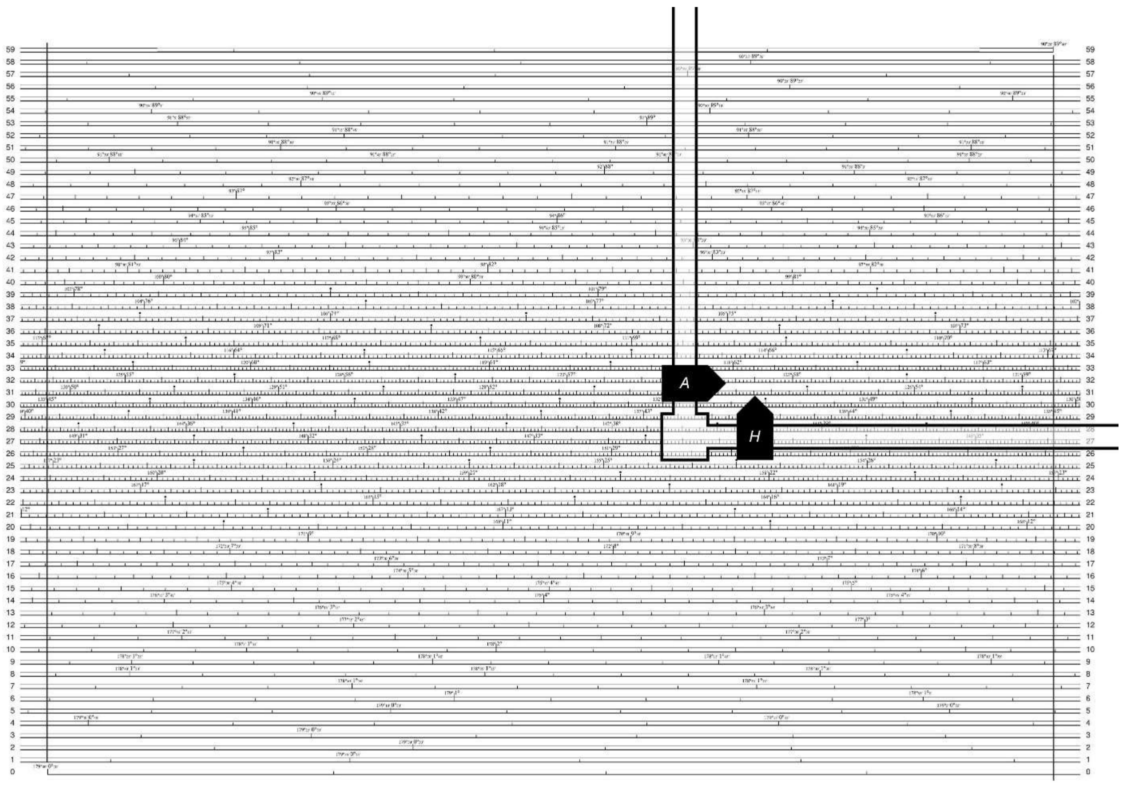
In many, but not all, instances it is possible to combine Step 2 and 3 into a single operation without computing the intermediate quantity, z, and without running off the end of the cotangent scale.

Compute azimuth angle A; $\tan A = \tan H \sec Y / \sec y$

Place the horizontal arm pointer tip on y and the vertical arm pointer on Y on the cosine scale.



Place the horizontal arm pointer on the local hour angle, H, on the cotangent scale and read the azimuth angle, A = 57° 42.4′, from the vertical arm pointer. Follow the instructions under Step 3 on how to interpret it and obtain Zn.


A
H

A.M.L.

POSITION LINE SLIDE RULE

BYGRAVE SLIDE RULE

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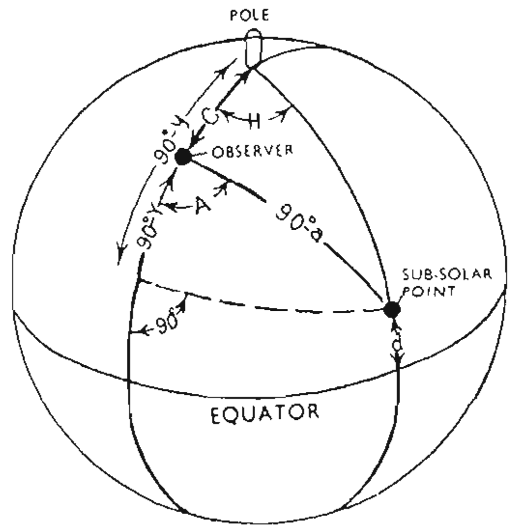
A.M.L. POSITION LINE SLIDE RULE

BYGRAVE SLIDE RULE



H = Hour Angle.
A = Azimuth.
C = Co-Latitude.
d = Declination.
a = Altitude.

Dotted Line—Perpendicular from Sub-Solar point to Observer's Meridian.

This slide rule has been designed to calculate the altitude of a celestial object as it would be seen from a given point on the earth's surface at a given time.

THEORY OF THE METHOD.

The three points—the Pole, the observer's position, and the sub-solar (or sub-stellar) point—determine a spherical triangle, sufficient elements of which are known to enable an unique solution to be obtained; this triangle is usually solved by direct logarithmic calculation or by the use of special tables based on logarithmic functions.

In order to solve the triangle by a slide rule it was necessary to re-arrange the formulæ involved in the solution so that each step involved not more than four logarithms or three numbers.

The method adopted is illustrated in Fig. 1, which shows the theoretical diagram and the formulæ employed. From the sub-solar point a perpendicular is drawn to the observer's meridian forming two right-angled spherical triangles. These two triangles have the perpendicular and the right angle at the meridian in common; the angles opposite the common side are the Hour Angle and the Azimuth respectively; the sides opposite the right angle are the complements of the declination and the altitude; the parts of the meridian forming the third sides are the complements of the auxiliary angles *y* and Y *respectively*, the difference of these being the co-latitude.

By applying Napier's Rules of Circular Parts to the triangle containing the Hour Angle we get

$$\tan y = \frac{\tan d}{\cos H}$$

enabling the second auxiliary angle Y to be obtained by the use of the co-latitude.

The tangent of the perpendicular drawn, the common side of both triangles, can now be determined in terms of the elements of both triangles and these values equated to each other, giving

$$\tan A = \frac{\tan H \cos y}{\cos Y}$$

From the triangle containing the azimuth angle A the relation $\tan a = \cos A \tan Y$ is derived.

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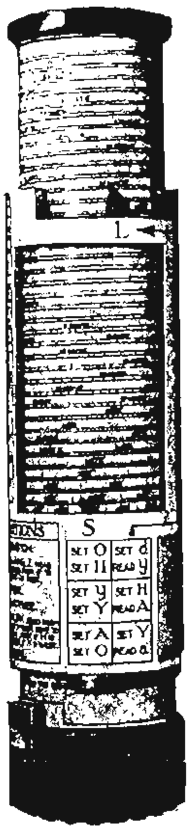

On examination of the original spherical triangle it will be at once seen that the altitude could have been found equally well if the perpendicular had been drawn from the observer's position to the meridian through the sub-solar point, in which case the corresponding angle to the azimuth in the above case would be meaningless as regards the Position Line Problem. This enables a check to be applied to any problem worked on the slide rule by interchanging the latitude and declination in which case the final altitudes should agree, although the second false azimuth of the check must be disregarded. As the true azimuth was used to determine the altitude, this has been checked by checking the altitude.

DESCRIPTION OF THE SLIDE RULE.

Two scales are printed on two cylinders sliding with reference to each other. The inner cylinder, on which all results are read, is graduated with log tangents and the spiral scale is about twenty-four feet long. It is divided into minutes of arc throughout its length, and the smallest degree division, occurring at the middle of the scale, is over an inch in length. The outer scale is graduated with log cosines. Two pointers are provided, one for each scale, and are attached to a sliding ring, a stop being provided to register the cosine pointer on the zero of its scale. The pointer, which has to be used for each setting, is clearly marked, and the full instructions for dealing with all possible cases are printed at the bottom of the outer cylinder. After a little practice, the calculation can be performed in about two minutes, and the result should be accurate to one minute of arc, with careful use.

A third cylinder sliding inside the others is used for carrying notes, secured by rubber bands, and can be partly withdrawn from below.

On the back of the slide rule are given scales of dip, refraction, moon's parallax, and for converting time into arc, so that the whole of the reduction of a sight can be done without any reference book whatever, other than the abridged Nautical Almanac.

DIRECTIONS FOR USE.

The data required are :—

The Dead Reckoning or Assumed Position of the observer.

The exact time of the observation.

The Right Ascension and Declination of the body observed.

From these the Local Hour Angle is calculated by the usual methods. Turning to the Slide Rule, and holding it by the lower corrugated fibre ring at the bottom in the left hand, the value of y is found by the following operations :—

Set pointer S to zero.

Set pointer L to the declination by sliding and turning the inner cylinder secured to the top end cap of the slide rule.

Set pointer S to the Hour Angle H by turning and sliding the outer cylinder.

Read off the value of y by the pointer L. As two figures (supplementary) are shown to each mark on the scales it is necessary to remember that y is greater or less than 90° according to whether H was greater or less than 90°.

Next form Y from y and c, the co-latitude, adding y to c if the declination and latitude are of the same name and subtracting if they are of opposite names.

The Azimuth A is now found from the Slide Rule by setting the pointer S to y, then the pointer L to the hour angle H, the pointer S to the value of Y, and then reading off the azimuth from the pointer L. Here again we have the rule that A is greater or less than 90° according to whether Y is greater or less than 90°. The Azimuth is noted for future use and the altitude found from the Slide Rule by a third series of settings.

Set Pointer S to the Azimuth A, pointer L to the value of Y, return pointer S to zero, and read off the altitude by pointer L.

The calculation is now complete, having found both Azimuth and Altitude.

As described in the Theory of the Method, the calculation can be checked by re-working with the declination and latitude interchanged, in which case the new azimuth is to be rejected but the altitude will be correct.

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THE OBSERVED ALTITUDE.

Before the observed altitude can be compared with the calculated altitude, it must be corrected for Dip of the Horizon (height of eye) Refraction, semi-diameter, and Parallax, and this is done either by the usual nautical tables or by the Auxiliary tables found on the slide rule; while the latter are sufficiently accurate for aircraft observations, for marine work the nautical tables are to be preferred.

Here it is essential to note that if the observations were made with a bubble or artificial horizon, then the corrections for Height of Eye (Dip) and semi-diameter must not be applied.

The Azimuth and the difference of the calculated and corrected observed altitudes are now used to draw the position line.

CALCULATION OF GREAT CIRCLE COURSES.

Reference to the figure shows at once that the Slide Rule can be used to calculate Great Circle Courses by the following method :—

The latitude and longitude of the point of departure are used in place of the Dead Reckoning Position.

The latitude of the point of arrival is used instead of the declination.

The difference of the longitude S is used for the Hour Angle.

The Azimuth so found is the Initial Great Circle Course, and the zenith distance or 90° — altitude found is the length of the Great Circle Course which, when reduced to minutes of arc, is the length in Geographical Miles.

Accuracy of the Results obtained with the Slide Rule.

A number of examples have been worked with the Slide Rule, and it is found that the accuracy of the altitude obtained is one minute of arc.

For the convenience of the user, a concise summary of the directions for use and the correction tables for the observed altitude are printed on the slide rule cursor, available for instant reference when required.

EXAMPLES.

I.

Latitude	36° 51′.6 N.
Hour Angle	53° 15′.3 E.
Declination	10° 27′ S.
C = 53° 8′.4	
y = 17° 8′	
Y = 36° 0′	
Azimuth = S.57° 42′ E.	
Altitude = 21° 13′	

By five figure logarithms the values are S.57° 42′.4 E. and 21° 13′.1.

II. What is the Great Circle Course and Distance between Lat. 36° 51′.6 N., Long. 70° W., and Lat. 10° 27′ S., Long. 16° 44′.7 W.

The first position is the observer's position.

The second Latitude is used as the declination.

The difference of Longitude is used as the Hour Angle.

This gives :—

Latitude	36° 51′.6 N.
Hour Angle	53° 15′.3 E.
Declination	10° 27′ S.

which is exactly the same as the first example, and therefore gives Course S.57° 42′ E. and distance 5.400—1,273 miles=4,127 miles.

III. Check on Example I.

New Latitude	10° 27′ S.
Hour Angle	53° 15′.3 E.
New Declination	36° 51′.6 N.
c = 79° 33′	
y = 51° 25′	
Y = 28° 8′	
False Azimuth	43° 26′
Altitude	21° 13′ showing the original working is correct.

Height of Slide Rule, 9 inches; Diameter of Slide Rule, 2½ inches; Weight in metal case, under 2 lbs.

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